EBERHARD KARLS UNIVERSITÄT TÜBINGEN

Neutron Stars:

Oscillations, Instabilities and Gravitational Waves

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Theoretical Astrophysics

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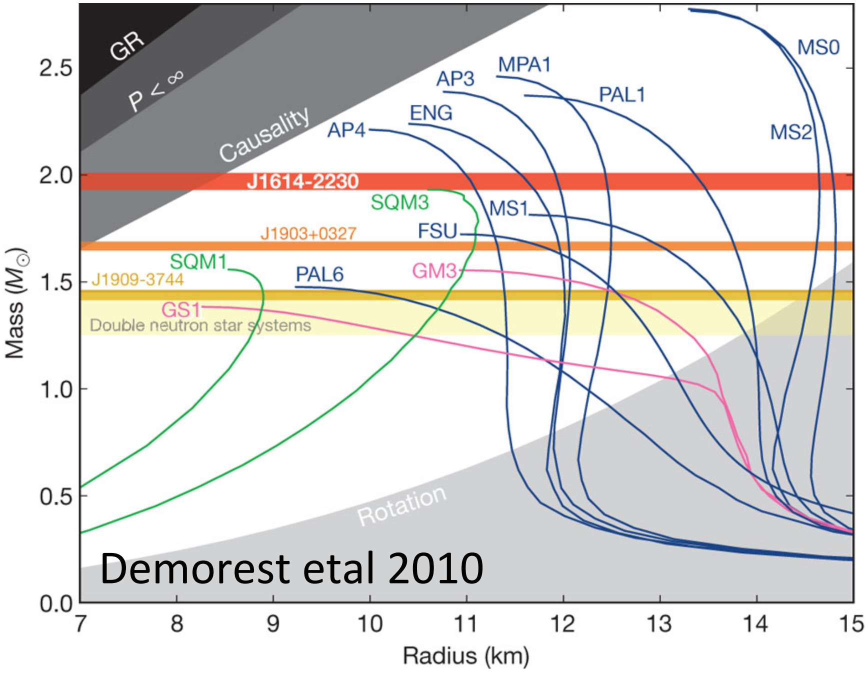
Neutron Stars: Mass vs Radius

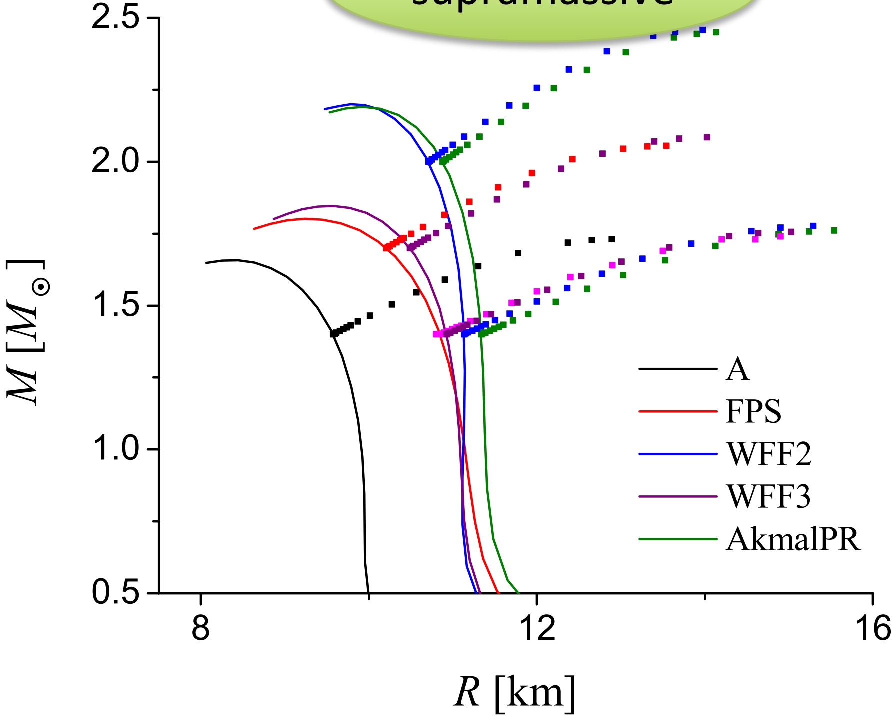
Static Models



Demorest etal 2010

Rotating Models

supramassive



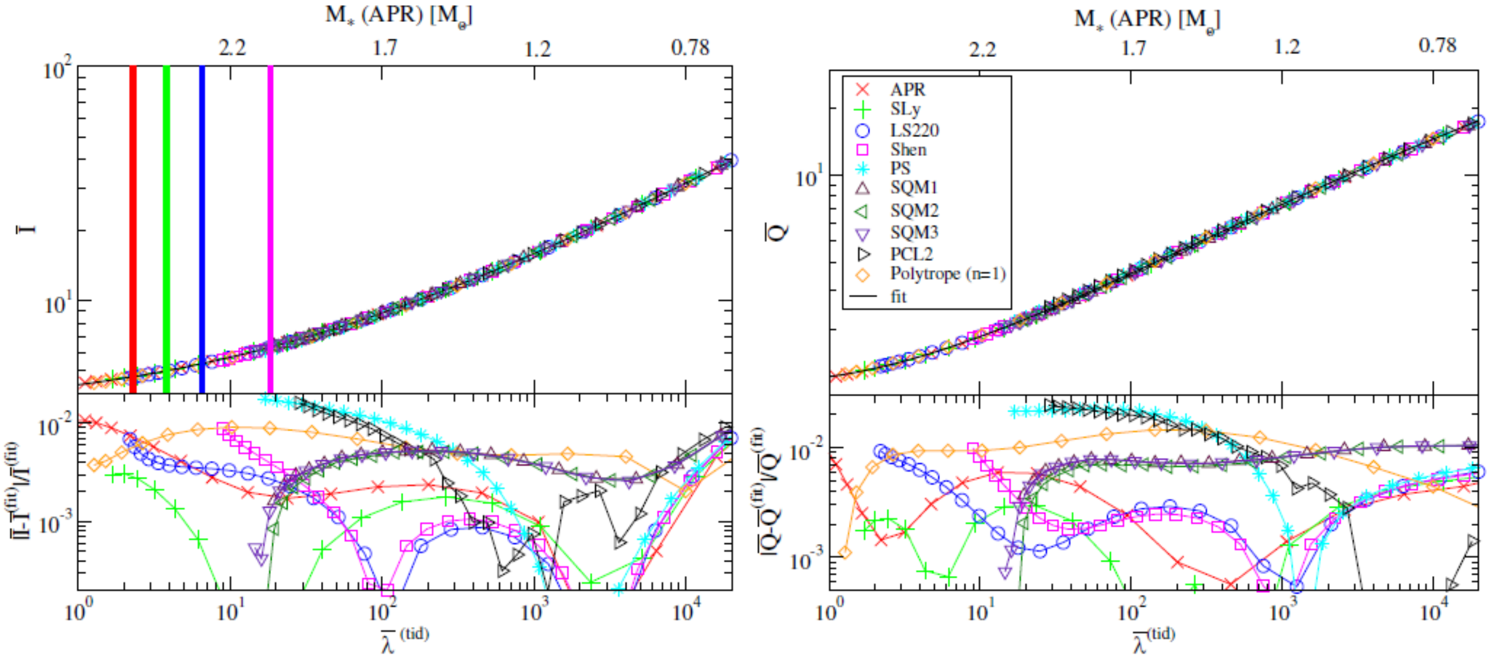
Neutron Stars & “universal relations”

Need for relations between the “observables” and the “fundamentals” of NS physics

Average Density	$\bar{\rho} \sim M / R^3$	
Compactness	$z \sim M / R$	$\eta = \sqrt{M^3 / I}$
Moment of Inertia	$I \sim MR^2$	$I \sim J / \Omega$
Quadrupole Moment	$Q \sim R^5 \Omega^2$	
Tidal Love Numbers	$\lambda \sim I^2 Q$	

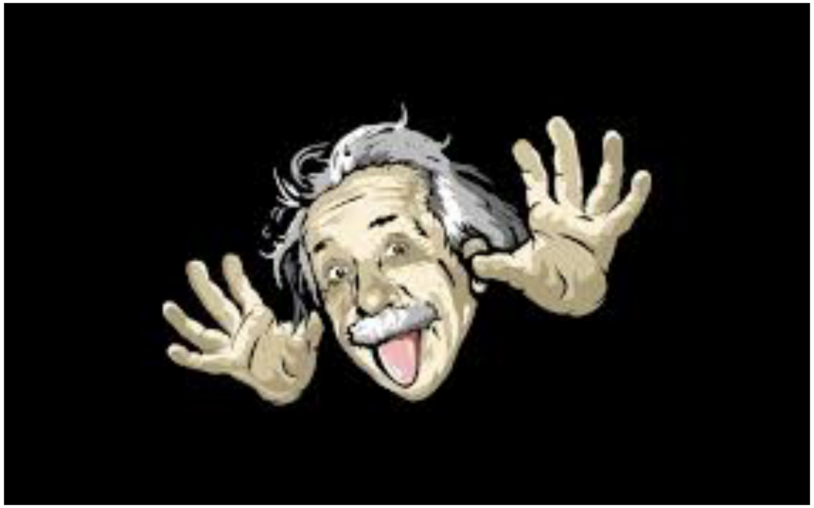
I-Love-Q relations

EOS independent relations were derived by Yagi & Yunes(2013) for non-magnetized stars in the slow-rotation and small tidal deformation approximations.



… the relations proved to be valid (*with appropriate normalizations*) even for *fast rotating* and *magnetized* stars

NEUTRON STARS &
ALTERNATIVE THEORIES OF GRAVITY



STT of gravity - Motivation

- The Scalar Tensor Theory (STT) is one of the most natural generalizations of the Einstein's Theory of Gravity (ETG)
- Their essence is in one or several scalar fields that are mediators of the gravitational interaction in addition to the spacetime metric of classical ETG
- Scalar fields appear in the reduction of the Kaluza-Klein theories to 4 dimensions, in string theory and in higher dimensional gravity but STT can be defined completely independently
- STT can be considered as an ETG with variable gravitational constant
- They fit to the observational data very well
- They are also an essential part of dark energy and dark matter models
- The f(R) theories are mathematically equivalent to the STT

STT of gravity – Action

- **Physical (Jordan) frame action**:

$$S = \frac{1}{16\pi G_*}\int d^4x\sqrt{-\tilde{g}}\left[F(\Phi)\tilde{R} - Z(\Phi)\tilde{g}^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi - 2U(\Phi)\right] + S_m\left[\Psi_m;\tilde{g}_{\mu\nu}\right]$$

- **Einstein frame action** **(much simpler):**

$$S = \frac{1}{16\pi G_*}\int d^4x\sqrt{-g}\left(R - 2g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - 4V(\varphi)\right) + S_m[\Psi_m;\mathcal{A}^2(\varphi)g_{\mu\nu}]$$

  - **Coupling function**

$$\boxed{k(\varphi) = \frac{d\ln\left(A(\varphi)\right)}{d\varphi}}$$

$$A(\varphi) = e^{\frac{1}{2}\beta\varphi^2}$$

$$k(\varphi) = \beta\varphi$$

  - **We set the potential to zero**

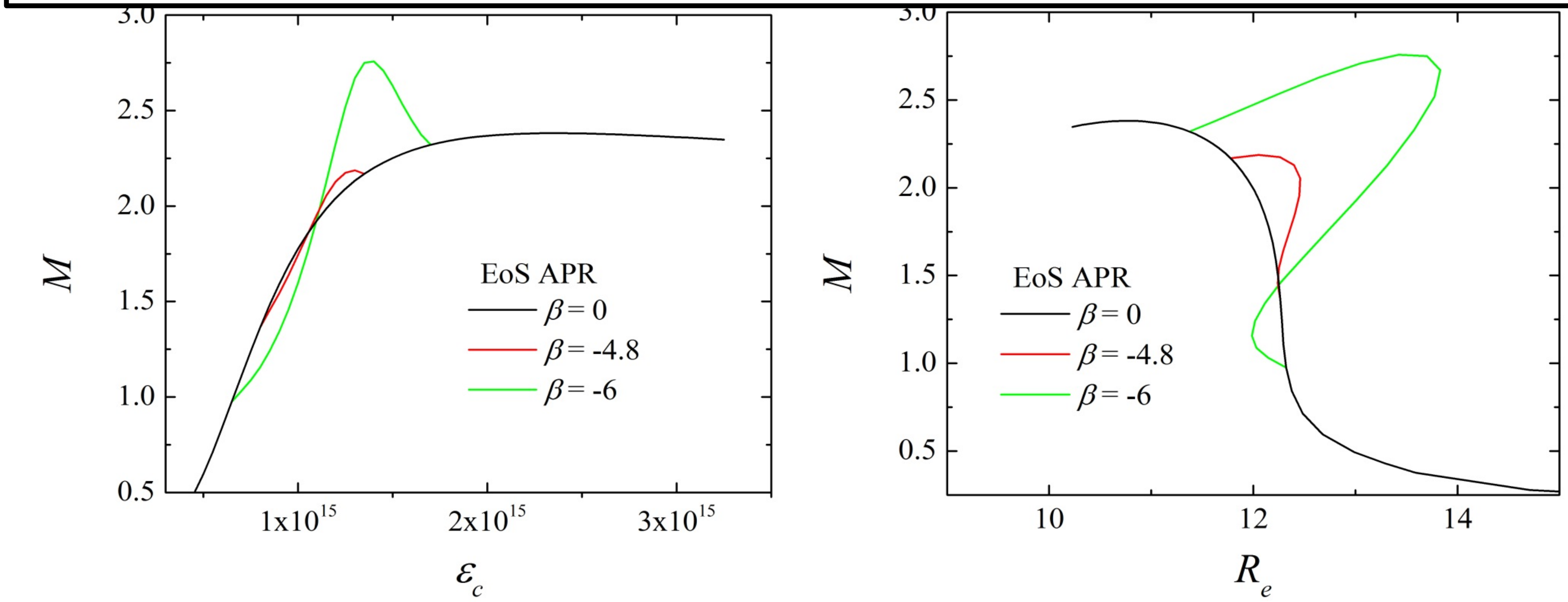
$$\boxed{V(\varphi) = 0}$$

STT of gravity – Neutron Stars

Spontaneous Scalarizarion is possible for β <-4.35

(Damour+Esposito-Farese 1993)

Properties of the static scalarized neutron stars



The solutions with nontrivial scalar field are *energetically more favorable* than their GR counterpart (Harada 1997, Harada 1998, Sotani+KK 2004).

STT of gravity - Observations

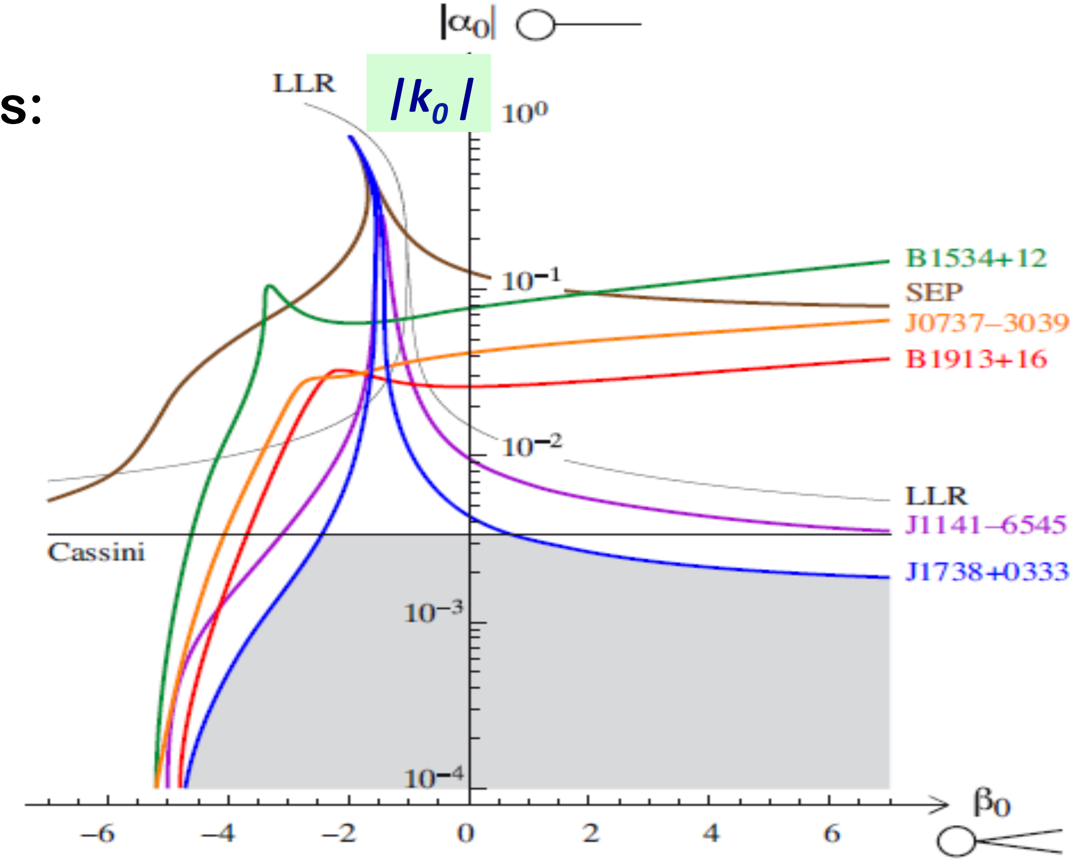
Observational constraints:

$k_0 < 0.004$

$\beta > -4.8\ (-4.5)$

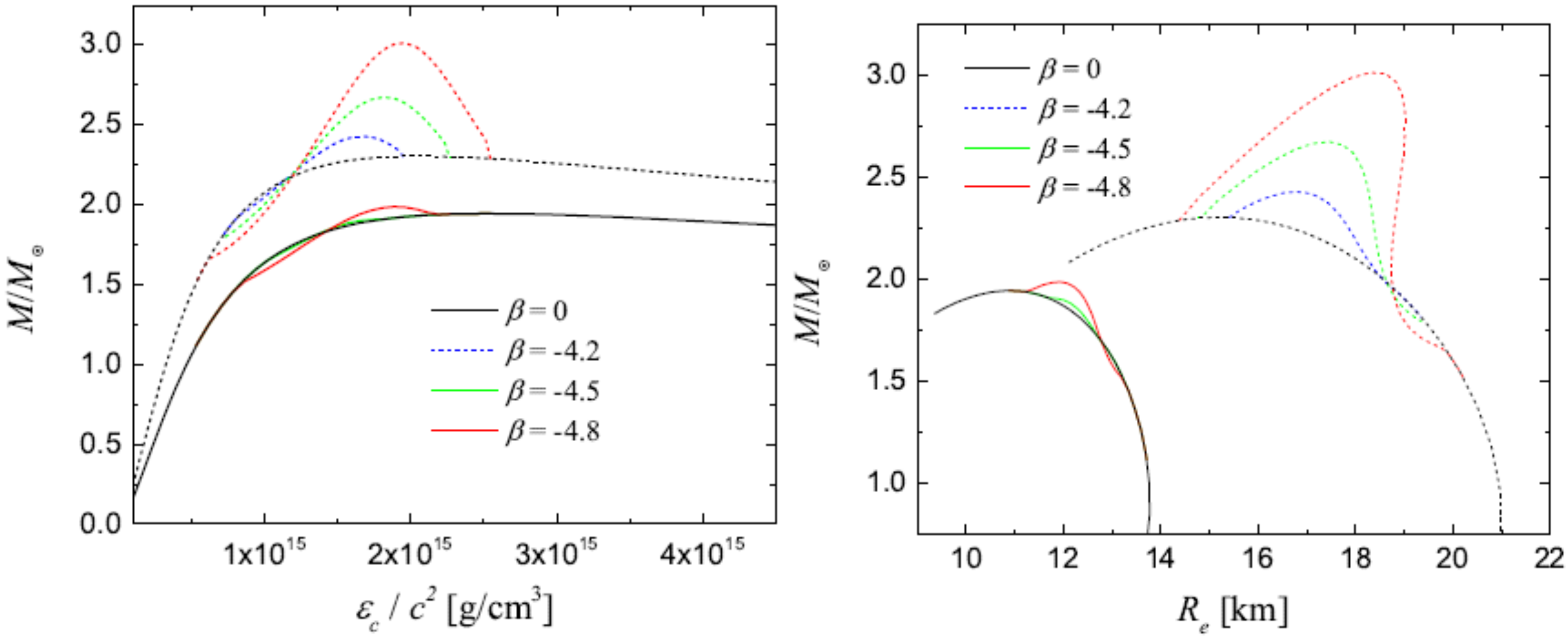
Damour & Esposito-Farese (1996, 98)

Will (2006), Freire etal (2012)



Freire at al (2012)

STT of gravity – Fast Rotating Stars



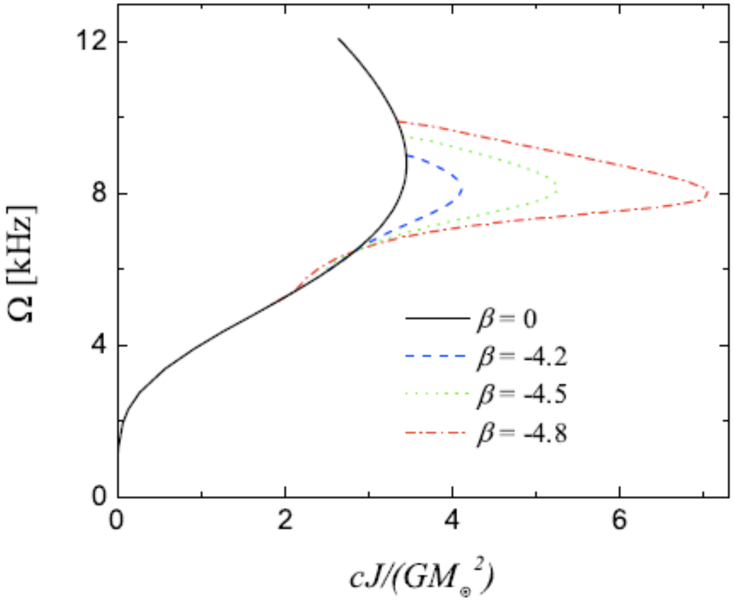
- The effect of scalarization is *much stronger* for fast rotation.
- Scalarized solutions exist for a *much larger range of parameters* than in the static case

Doneva, Yazadjiev, Stergioulas, Kokkotas 2013

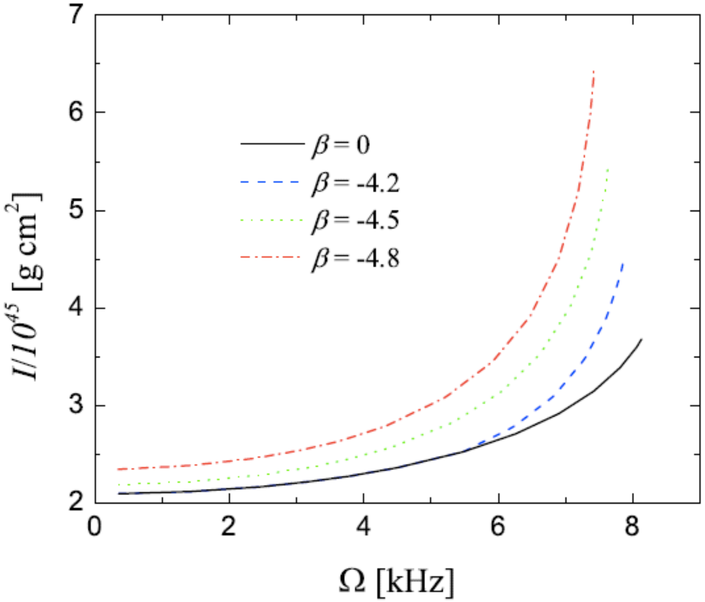
STT of gravity

Angular Momentum & Moment of Inertia

Sequences of models rotating at the Kepler limit



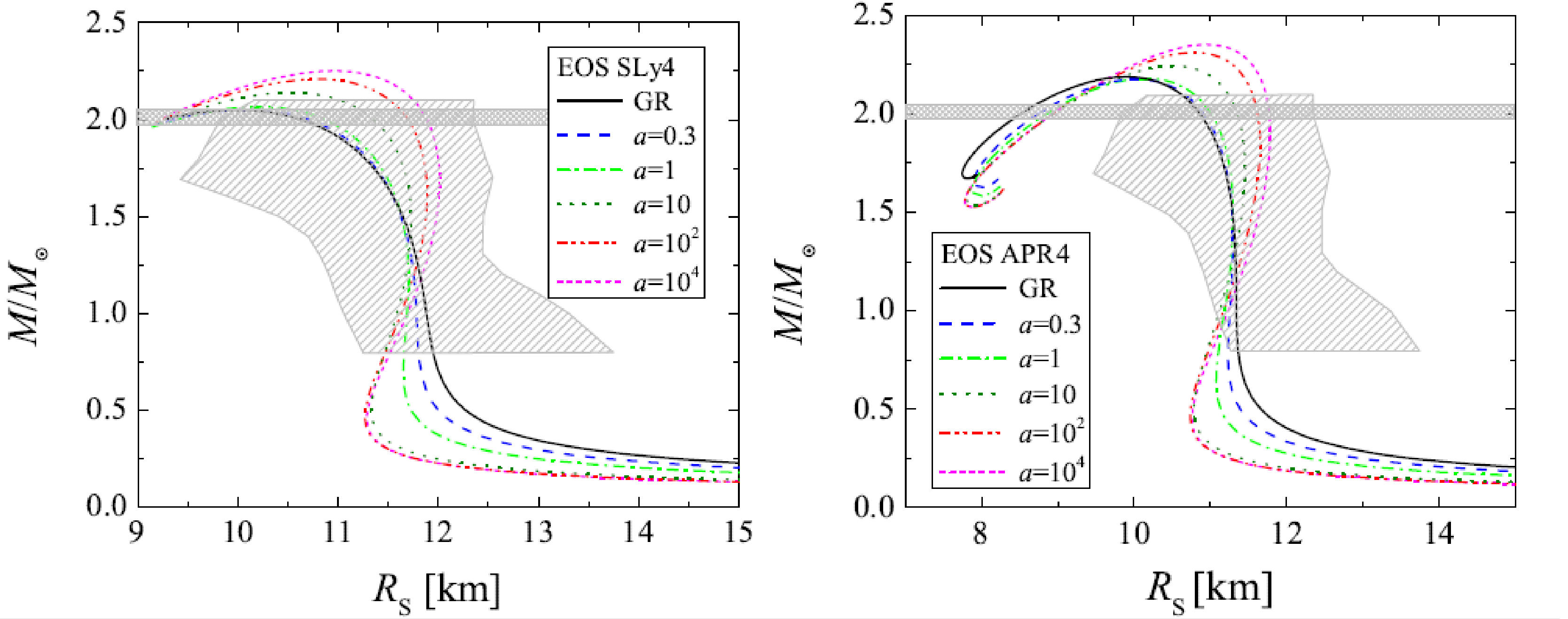
Models with constant central energy density



Not surprizing that both **angular momentum** and **moment of inertial** could *differ twice* for scalarized solutions

NSs in f(R)-gravity: Static Models

$$f(R) = R + aR^2$$



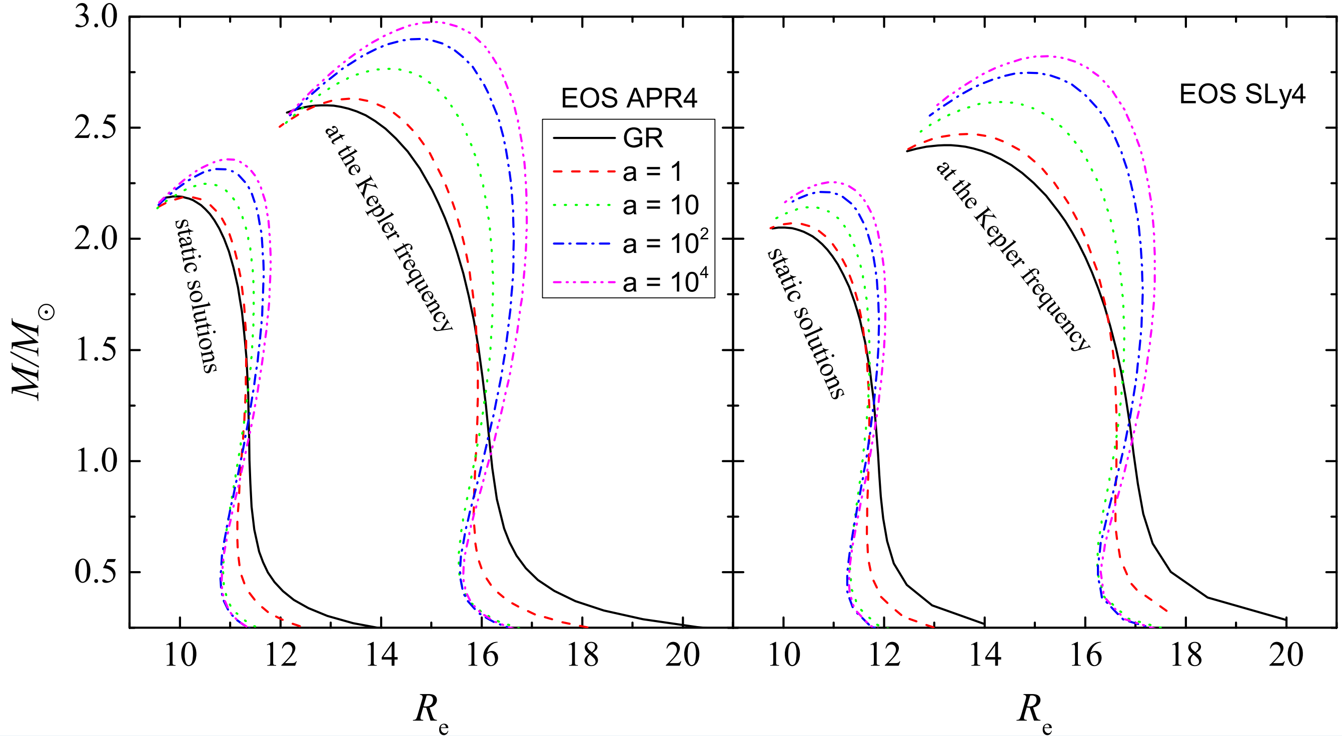
- The differences between the R^2 and GR are comparable with the uncertainties in the nuclear matter equations of state.
- The current observations of the NS masses and radii alone can not put constraints on the value of the parameters a, **unless the EoS is better constrained in the future.**

Yazadjiev, Doneva, Kokkotas, Staykov (2014)

NSs in f(R)-gravity: Fast Rotation

$$f(R) = R + aR^2$$

Mass of radius diagrams for two realistic EOS



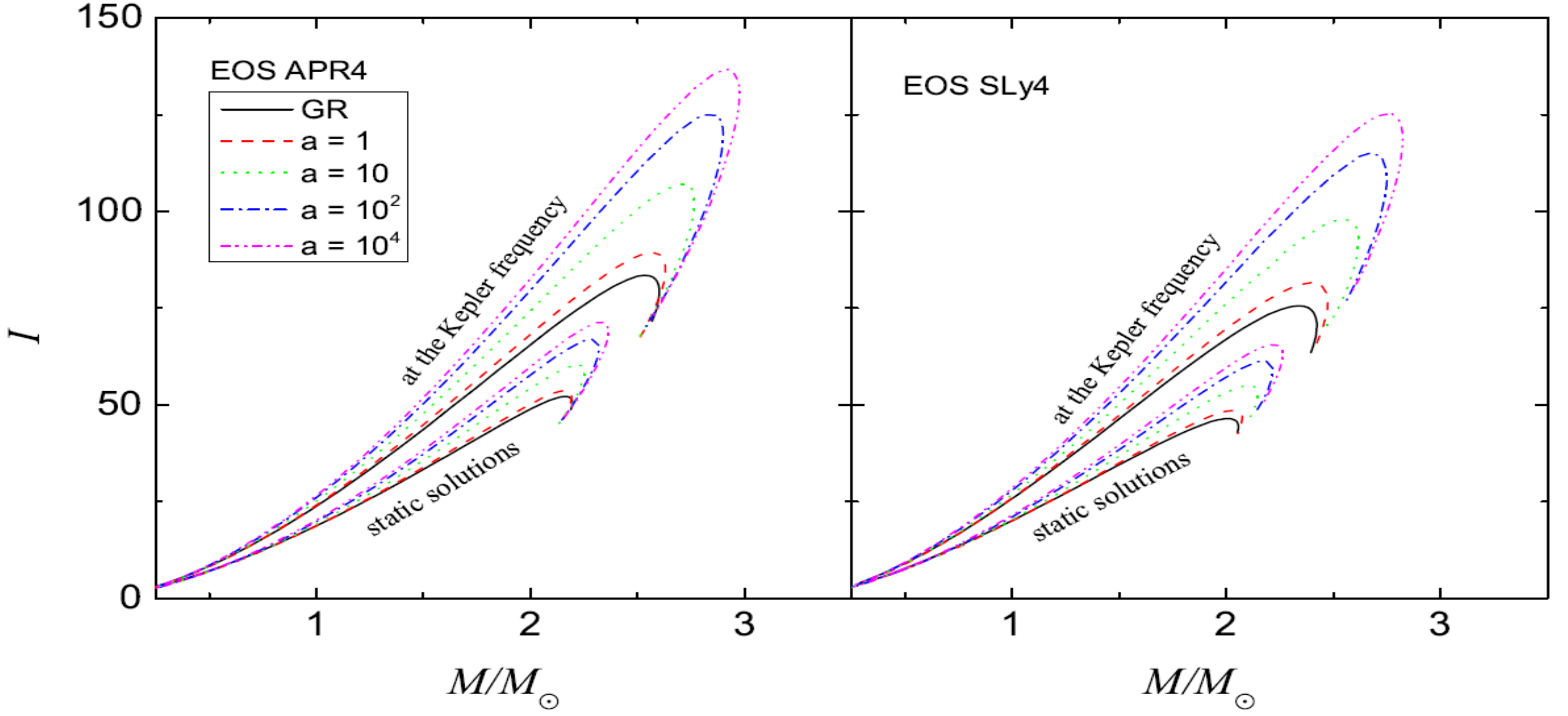
Difficult to set constraints on the *f* (*R*) theories using measurement of the neutron star **M** and **R** alone, until the EOS can be determined with smaller uncertainty.

Yazadjiev, Doneva, Kokkotas, (2015)

NSs in f(R)-gravity: Fast Rotation

$$f(R) = R + aR^2$$

Yazadjiev, Doneva, Kokkotas (2015)



- ✓ The differences in the neutron star moment of inertia on the other hand can be much more dramatic.
- ✓ Large deviations can be potentially measured by the forthcoming observations of the NS moment of inertia [Lattimer-Schutz 2005, Kramer-Wex 2009] that can lead to a direct test of the R^2 gravity.

NSs in f(R)-gravity: I-Q relations / Fast Rotation

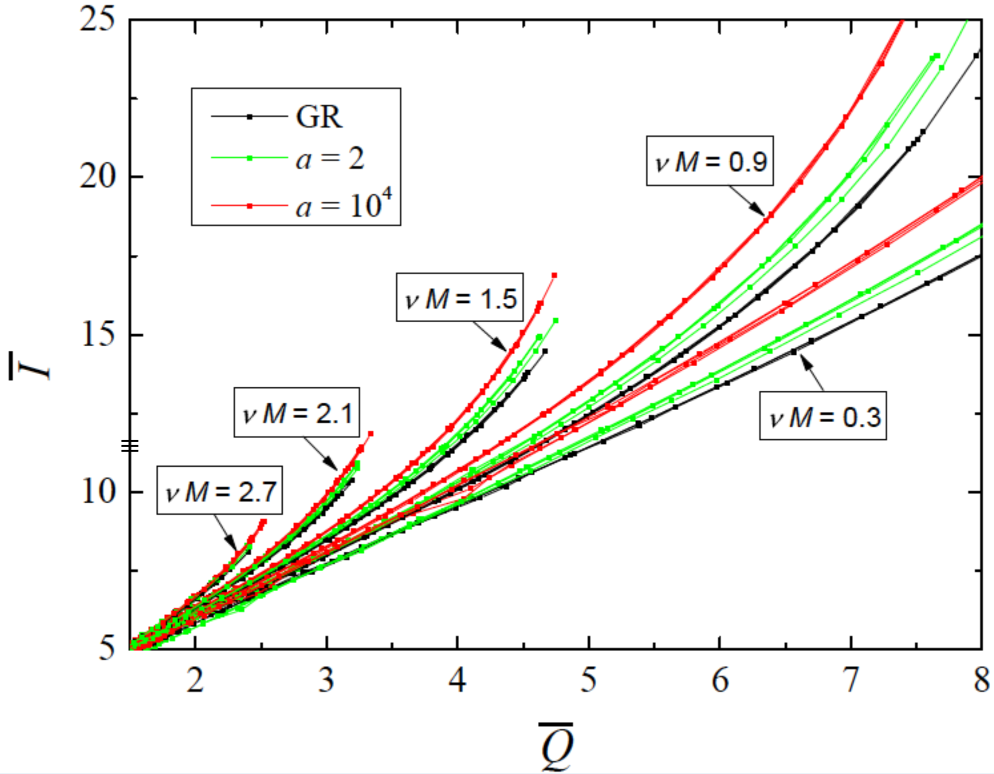
$$f(R) = R + aR^2$$

$$\bar{I} \equiv I / M^3$$

$$\bar{Q} \equiv Q / (M^3 \chi^2)$$

$$\chi \equiv J / M^2$$

ν : rot. frequency (Hz)



Doneva, Yazadjiev, Kokkotas (2015)

- The results show that the I-Q relation remain **nearly EoS independent** for fixed values of the normalized rotational parameter
- The differences with the pure Einstein's theory can be large reaching **above 20%** for **lower masses** and **slow rotation.**

Neutron Star “ringing”

p-modes: main restoring force is the pressure (**f-mode**) *(>1.5 kHz)*

$$\sigma \approx \sqrt{\frac{GM}{R^3}}$$

Inertial modes: (r-modes) main restoring force is the Coriolis force

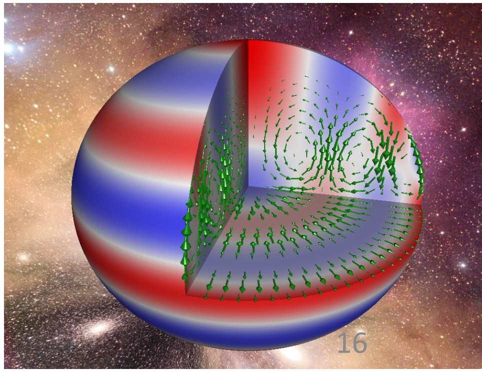
$$\sigma \approx \Omega$$

w-modes: pure space-time modes (only in GR) *(>5kHz)*

$$\sigma \approx \frac{1}{R}\left(\frac{GM}{Rc^2}\right)$$

Torsional modes (t-modes) *(>20 Hz)* shear deformations. Restoring force, the weak Coulomb force of the crystal ions.

$$\sigma \approx \frac{v_S}{R} \sim 16\ \ell\ \text{Hz}$$

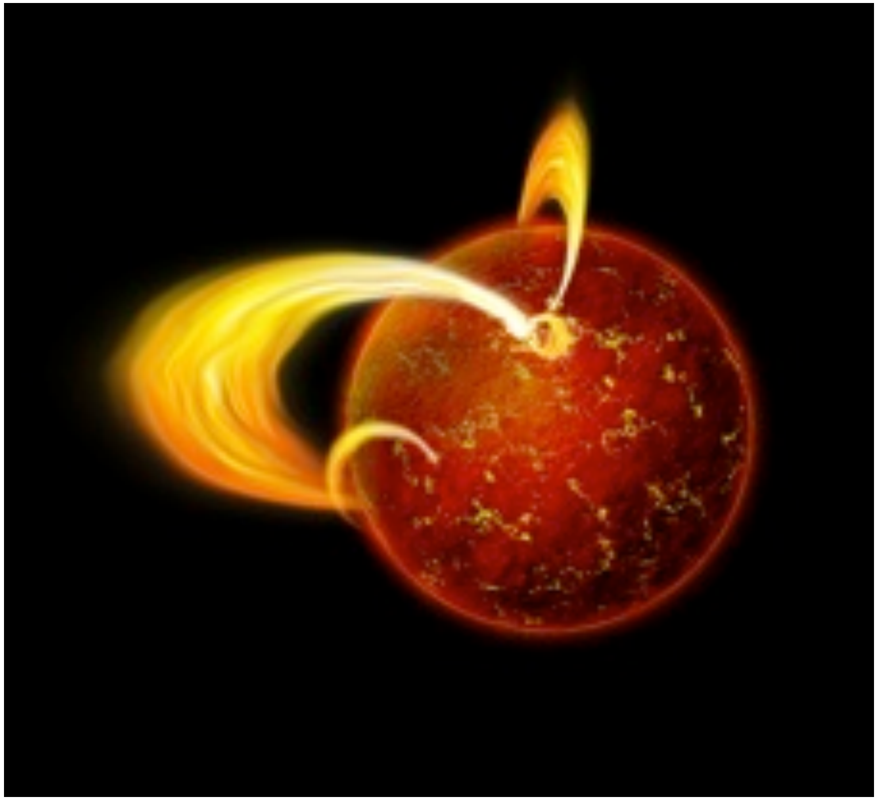


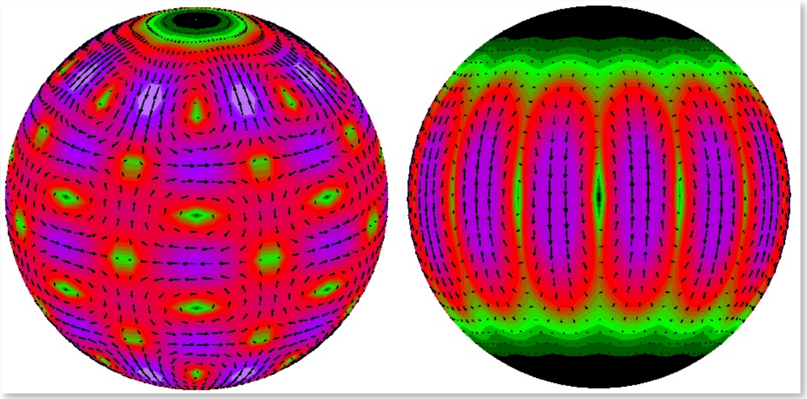
… and many more

shear, g-, Alfven, interface, … modes

MAGNETARS:

A PROMISING CASE FOR ASTEROSEISMOLOGY





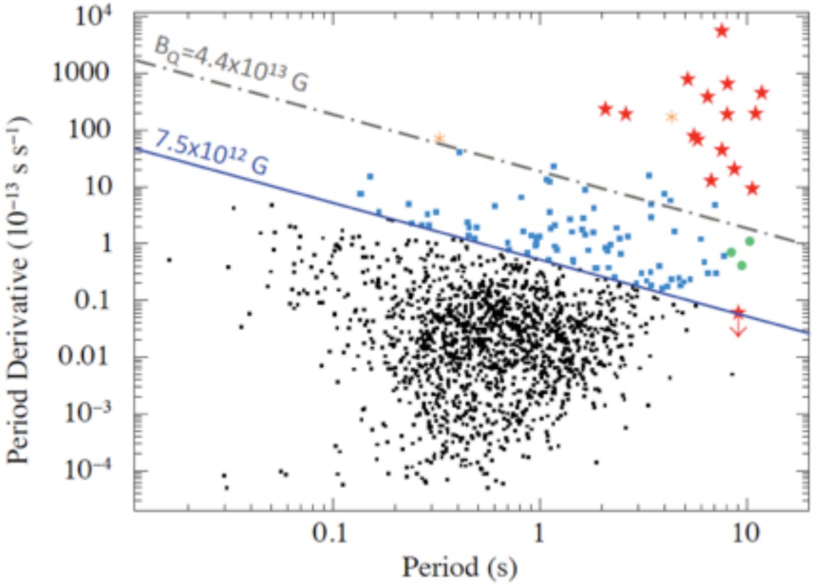
Magnetars

- **Young, slowly spinning (P~10s) systems (20+)**

- Exhibit regular γ-ray flares
  - Believed to be powered by magnetic field
  - Either trigger or are preceded by starquakes
  - Some linked to glitches or **anti-glitches**

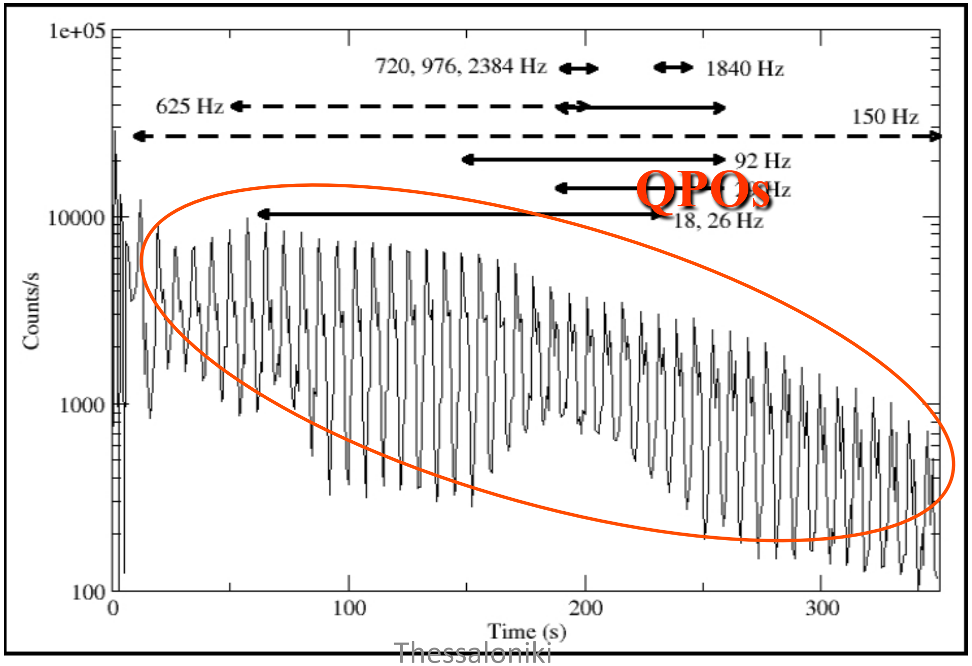
- **Three giant flares observed with peak luminosities ~10^{47} erg/s**
  - March 5, 1979 : SGR 0526-66
  - August 27, 1998 : SGR 1900+14
  - December 27, 2004: SGR 1806-20
  - Recently few medium ones

- **Giant flares**
  - QPOs – 10's -100's of Hz
  - Magnetic field reconstruction



Magnetars:
Quasi-Periodic Oscillations

- ✓ **Giant flares in SGRs**
  - A decaying tail for several hundred seconds follows the flare.
- ✓ **QPOs in decaying tail** (Israel *et al*. 2005; Watts & Strohmayer 2005, 2006)
  - **SGR 1900+14** : 28, 54, 84, and 155 Hz
  - **SGR 1806-20** : 18, 26, 29, 92.5, 150, 626.5, 720, 976, 1837, 2384 Hz
  - **SGR 1806-20** : Additional frequencies 22, 16, 116 Hz, also 720 & 2384 Hz; (Hambaryan, Neuhaeuser, Kokkotas 2011)



Alfven Continuum and/or Discrete oscillations

Only Crust Oscillations
- Sotani, Kokkotaa, Stergioulas 2007,2008
- Samuelsson, Andersson 2007
- Sotani, Colaiuda, Kokkotas 2008
- Steiner, Watts 2009
- …
- Sotani etal 2012-15

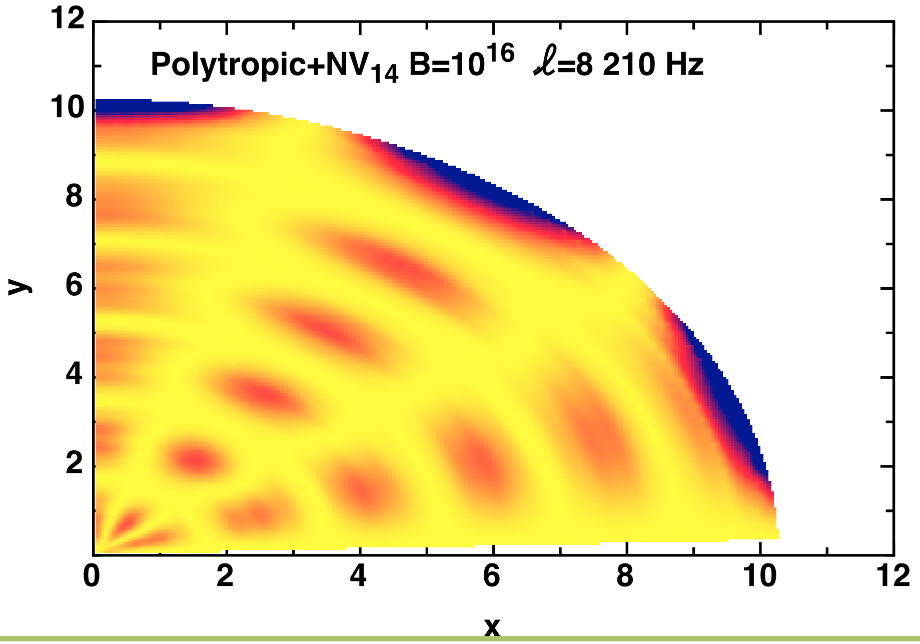
Without Crust
- Levin 2007
- Sotani, Kokkotas, Stergioulas 2008
- Colaiuda, Beyer, Kokkotas 2009
- Cerda-Duran, Stergioulas, Font 2009

Fluid + Crust
- Van Hoven, Levin 2011, 2012
- Cerda-Duran, Stergioulas, Font 2011
- Colaiuda, Kokkotas 2011
- Gabler etal 2012
- Gabler etal 2013

Superfluidity
- Passamonti, Lander 2012
- Sotani etal 2013
- Gabler etal 2013

Mixed axial-polar
- Colaiuda, Kokkotas 2012
- Lee, Yoshida 2015

Non-axisymmetric
- Sotani, Kokkotas 2012



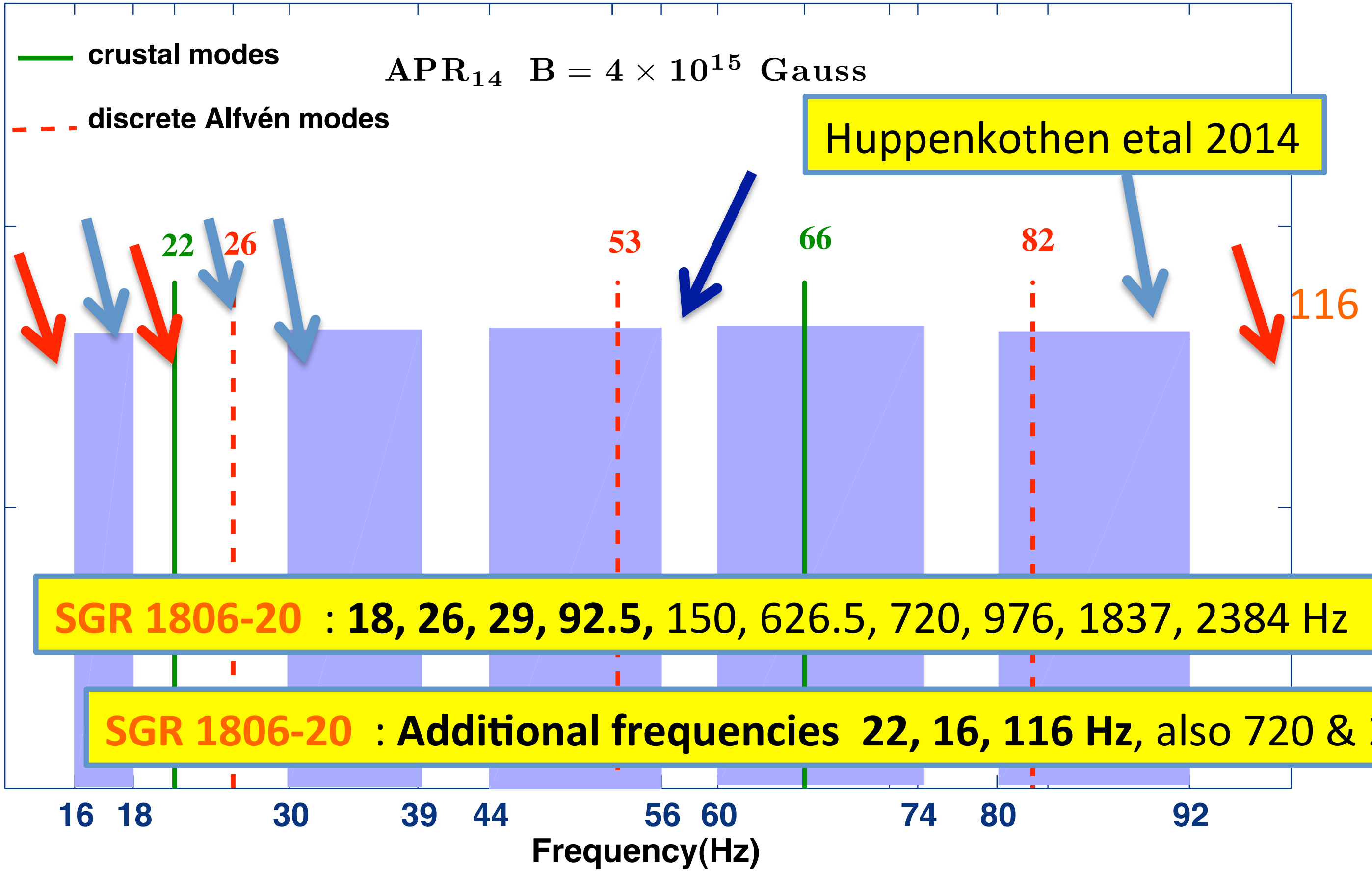
✓ The combination of poloidal + toroidal magnetic fields +crust leads to PURE discrete spectrum

✓ The main results of the magnetar seismology remain unchanged !

(Colaiuda-KK 2012)

SGR 1806-20

Colaiuda, KK (2011-12)



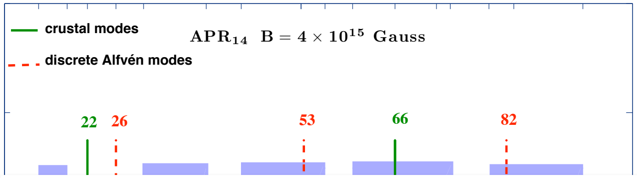
Huppenkothen etal 2014

SGR 1806-20 : **18, 26, 29, 92.5,** 150, 626.5, 720, 976, 1837, 2384 Hz

SGR 1806-20 : **Additional frequencies 22, 16, 116 Hz**, also 720 & 2384 Hz;

SGR 1806-20

Colaiuda, KK (2011-12)



Do we understand how the QPOs are excited?

The answer is **NO! (?)**

Great progress in the last 7-8 years

A new event of the type of SGR 1806-20, might be catalytic for understanding:

- The mechanism that triggers the hyperflares
- The QPOs in the decaying tail
- The EOS, the Mass, Radius, B-field of magnetars

…and predicts new that found after careful analysis of data

Hambaryan, Kokkotas, Neuhauser (2012)

EoS :	APR (NV)
Mass:	$M = 1.4M_{\odot}$
Radius:	11.57 km
B-field :	$2\text{x}10^{15}$ Gauss
Crust :	0.099 R

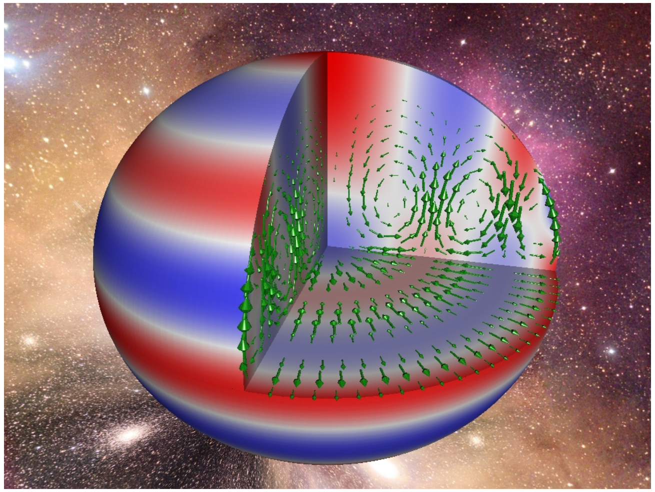
oscillations both poloidal & toroidal B-fields are unstable!

The observed QPOs or some of them can be due to magnetospheric phenomena (breathing of the fireball)

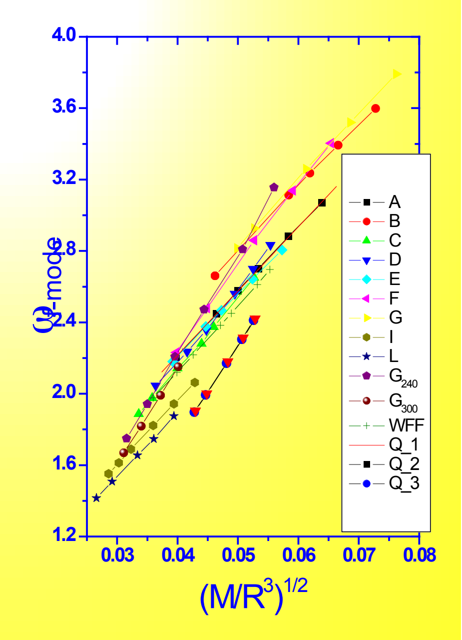
Gravitational Wave Asteroseismology

Oscillation patterns can reveal the **internal structure** of neutron stars :

- ✓ mass,
- ✓ radius,
- ✓ EoS,
- ✓ rotation,
- ✓ B-field,
- ✓ crust,…



Andersson,Kokkotas 1996,1998,2001



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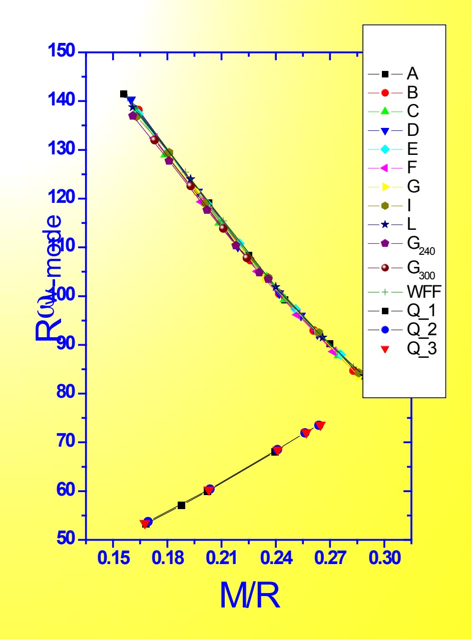
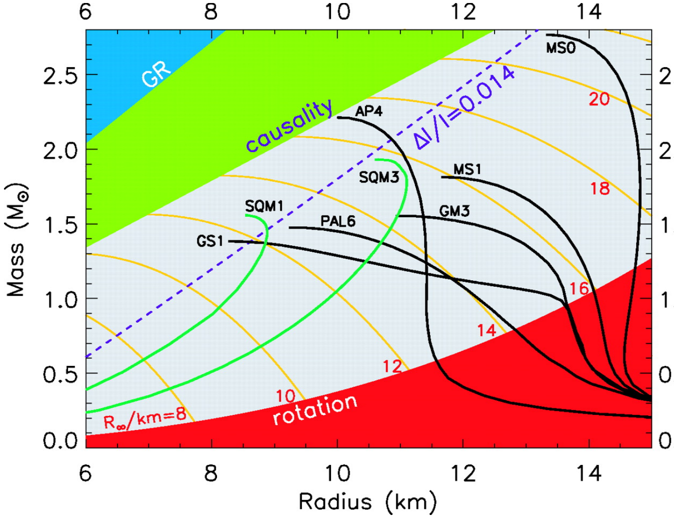




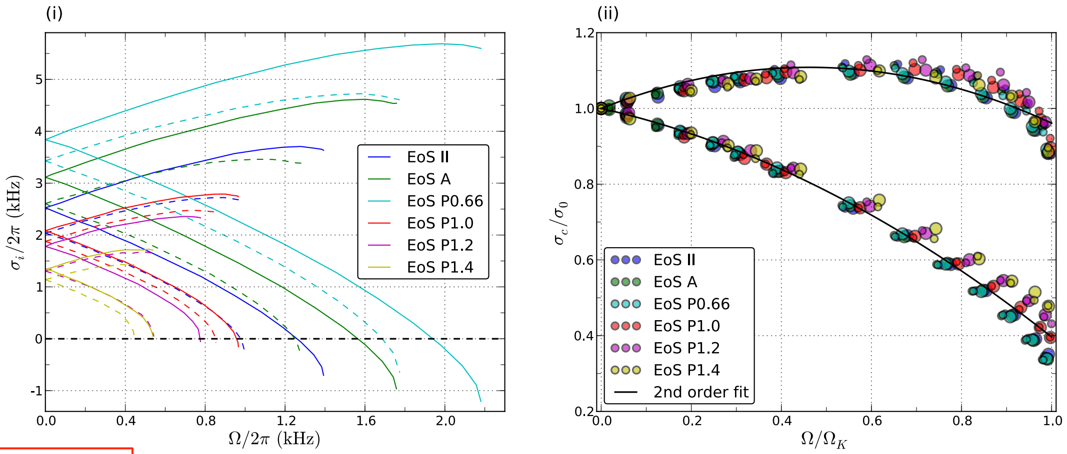
Lattimer+Prakash 2007

f-modes: Asteroseismology

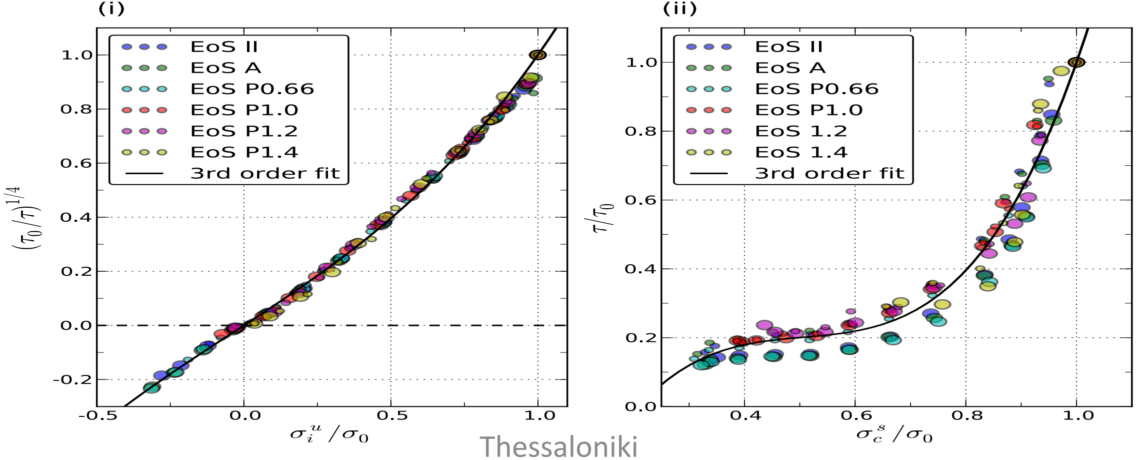
We can produce empirical relation relating the parameters of the *rotating neutron stars* to the observed frequencies.

Gaertig-Kokkotas 2008, 2010, 2011

Frequency



Damping/Growth time



Cowling Approximation

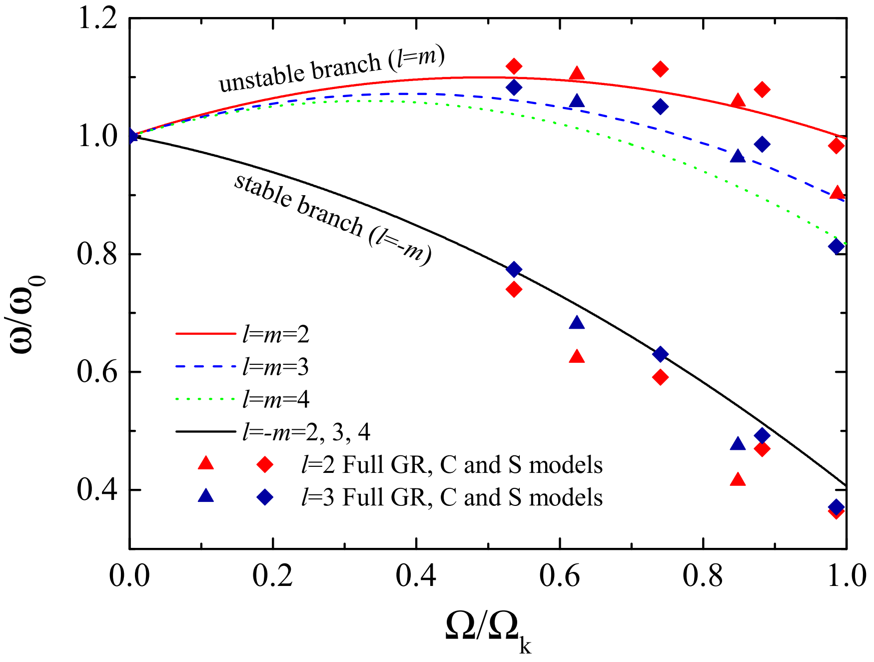
Asteroseismology: Realistic EoS

Doneva, Gaertig, KK, Krüger (2013)

$$\left(\frac{\omega_c}{\omega_0}\right)_{\ell=2,3,4} \approx f\left(\frac{\Omega}{\Omega_K}\right)$$

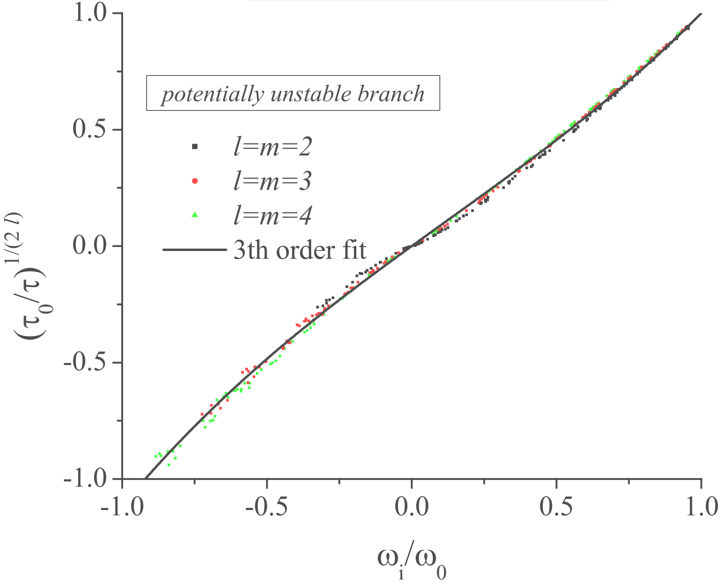
Nearly "universal" fitting formulae for :

- **the frequencies**
- **the damping times**
- **Independent of GR or Cowling**



Oscillation frequencies

$$\left(\frac{\tau_0}{\tau}\right)^{1/2\ell} \approx f\left(\frac{\omega_i}{\omega_0}\right)$$



Damping/Growth Times

Asteroseismology

Stable Branch

$$\frac{\omega_c^s}{\omega_0} = 1 - 0.235\left(\frac{\Omega}{\Omega_K}\right) - 0.358\left(\frac{\Omega}{\Omega_K}\right)^2$$

Unstable Branch

$$\frac{\omega^u_{c\; l=2}}{\omega_0} = 1 + 0.402\left(\frac{\Omega}{\Omega_K}\right) - 0.406\left(\frac{\Omega}{\Omega_K}\right)^2$$

$$\frac{\omega^u_{c\; l=3}}{\omega_0} = 1 + 0.373\left(\frac{\Omega}{\Omega_K}\right) - 0.485\left(\frac{\Omega}{\Omega_K}\right)^2$$

$$\frac{\omega^u_{c\; l=4}}{\omega_0} = 1 + 0.360\left(\frac{\Omega}{\Omega_K}\right) - 0.543\left(\frac{\Omega}{\Omega_K}\right)^2$$

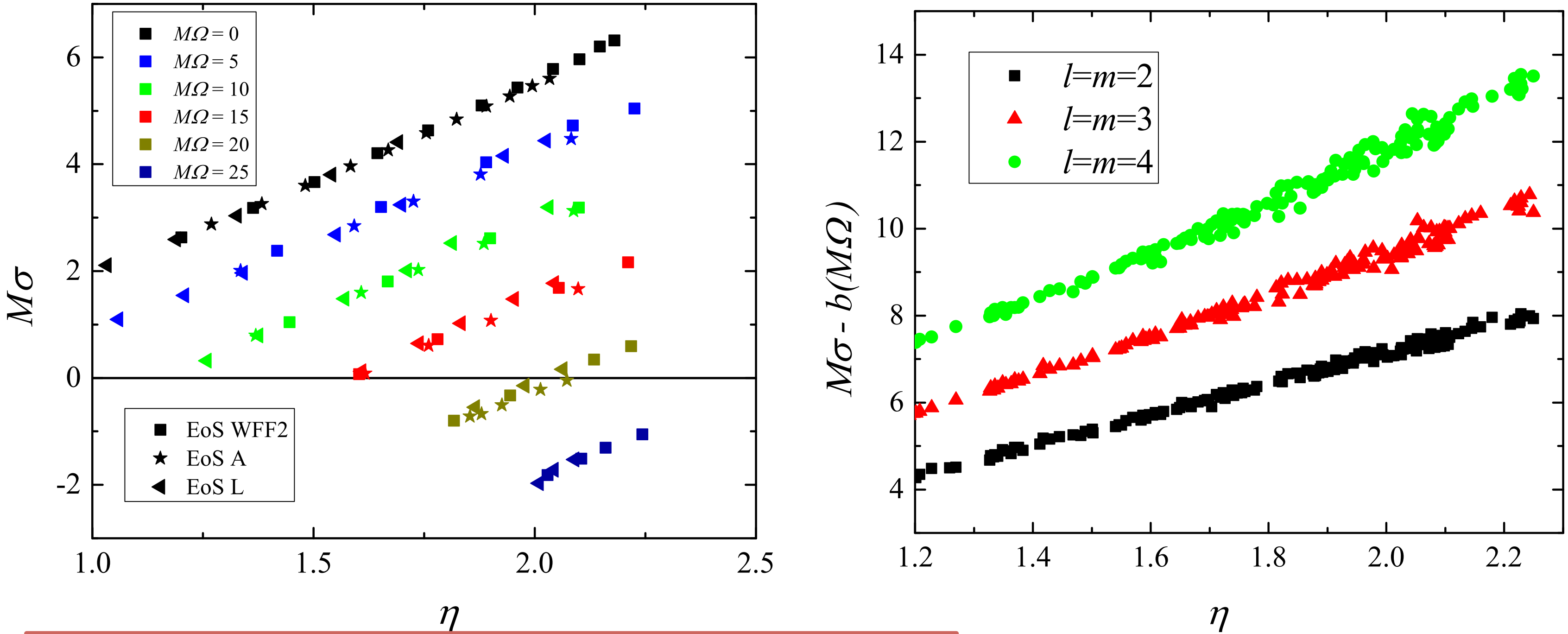
Unstable Branch

$$\frac{\tau_0}{\tau} = \mathrm{sgn}(\omega_i^u)\left(0.900\left(\frac{\omega_i^u}{\omega_0}\right) - 0.057\left(\frac{\omega_i^u}{\omega_0}\right)^2 + 0.157\left(\frac{\omega_i^u}{\omega_0}\right)^3\right)^{2l}$$

Doneva, Gaertig, KK, Krüger (2013)

Asteroseismology: alternative scalings

$$M\sigma_i^{unst} = \left[(0.56-0.94\ell)+(0.08-0.19\ell)M\Omega+1.2(\ell+1)\eta\right]$$

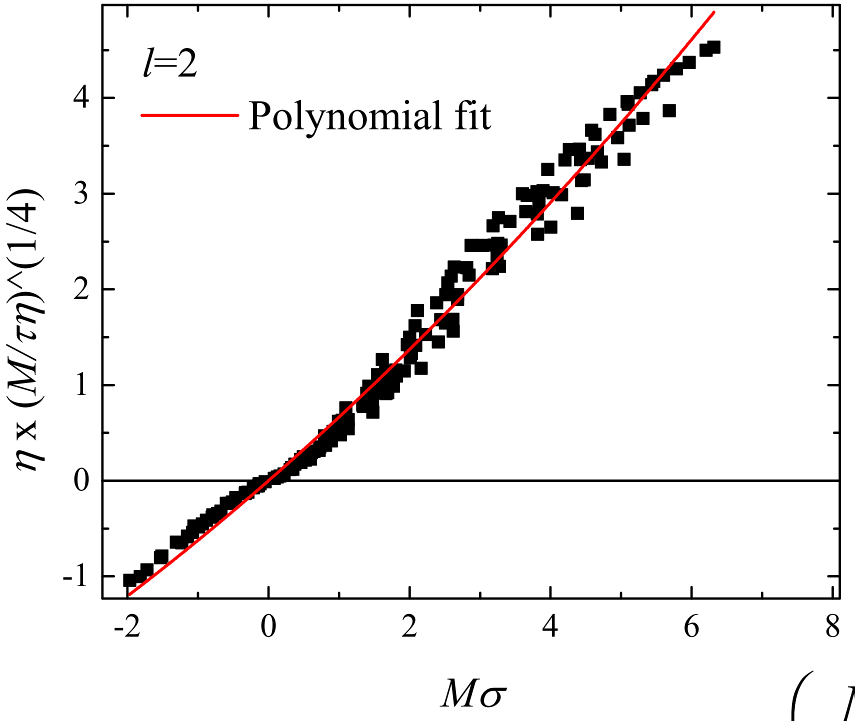


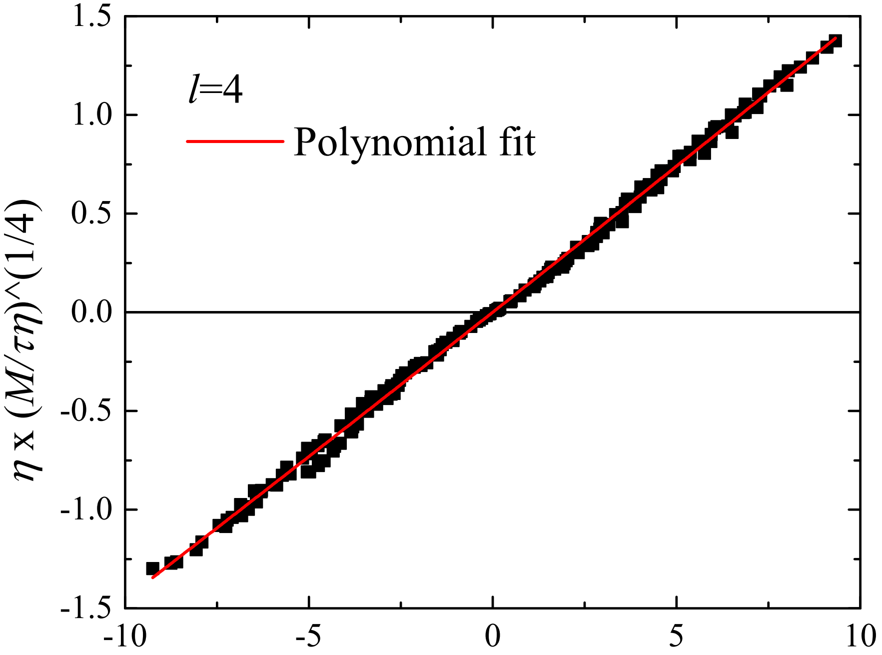
The l = 2 f-mode oscillation frequencies as functions of the parameter η

$$\eta = \sqrt{M^3/I}$$

Doneva-KK 2015

Asteroseismology: alternative scalings





The normalized damping time

$$\eta\left(\frac{M}{\tau\eta^{2}}\right)^{(1/2\ell)}$$

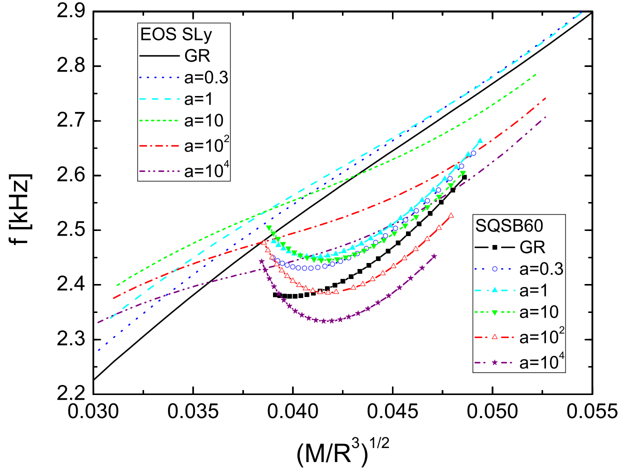
$$\eta=\sqrt{M^{3}/I}$$

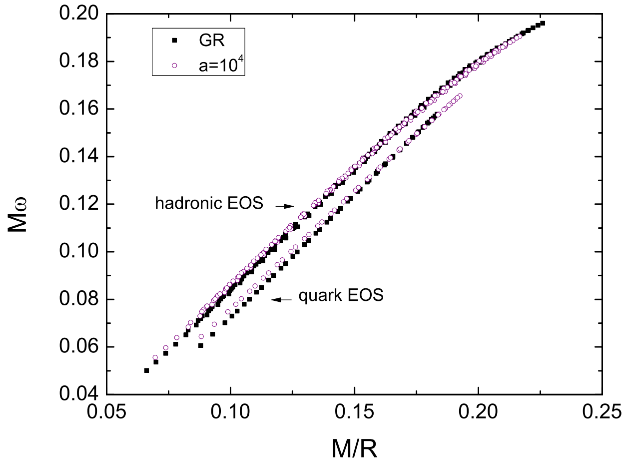
as a function of the normalized oscillation frequency ***Mσ*** for l = m = 2 & l = m = 4 f-modes.

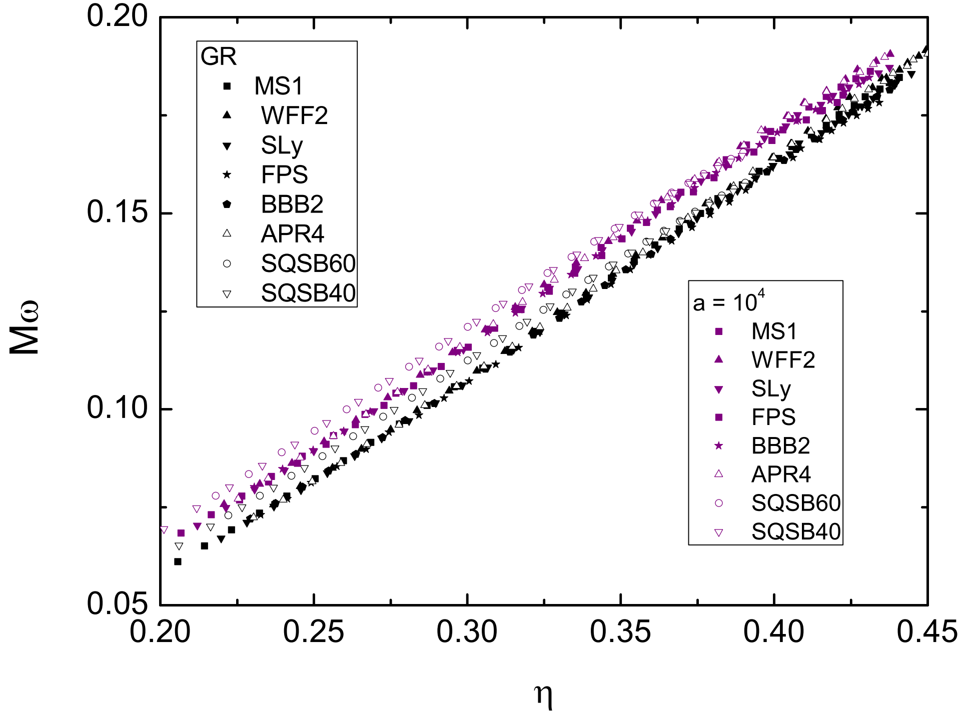
Doneva-KK 2015

Asteroseismology:

Alternative Theories of Gravity







$$\eta = \sqrt{M^3 / I}$$

- **The maximum deviation between the f-mode frequencies in GR and R^2 gravity is up to 10% and depends on the value of the R^2 gravity parameter a.**
- **Alternative normalizations show nicer relations**

The CFS instability

Chandrasekhar 1970: Gravitational waves lead to a secular instability

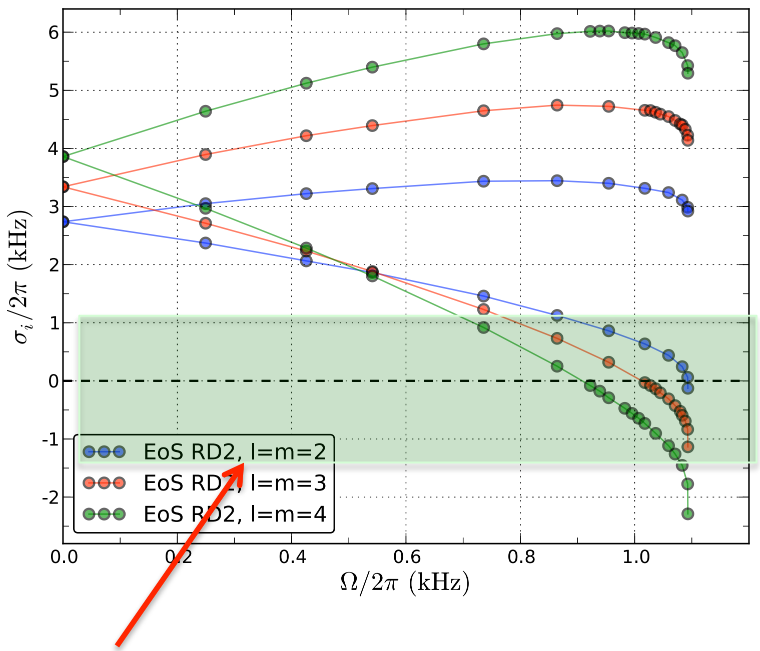
Friedman & **S**chutz 1978: The instability is generic, modes with sufficiently large *m* are unstable.

A neutral mode of oscillation signals the onset of CFS instability

- ✓ Radiation drives a mode unstable if the mode pattern moves backwards according to an observer on the star *($J_{rot}<0$)*, but forwards according to someone far away *($J_{rot}>0$)*.
- ✓ They radiate positive angular momentum, thus in the rotating frame the angular momentum of the mode increases leading to an increase in mode's amplitude.

$$\frac{\omega_{\text{in}}}{m} = -\frac{\omega_{\text{rot}}}{m} + \Omega$$

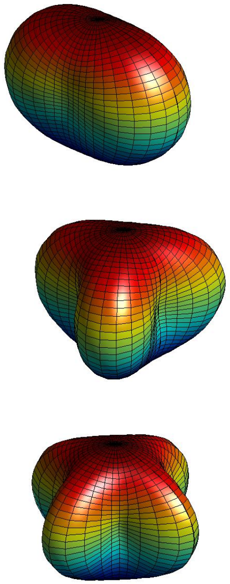
Gaertig+Kokkotas 2008

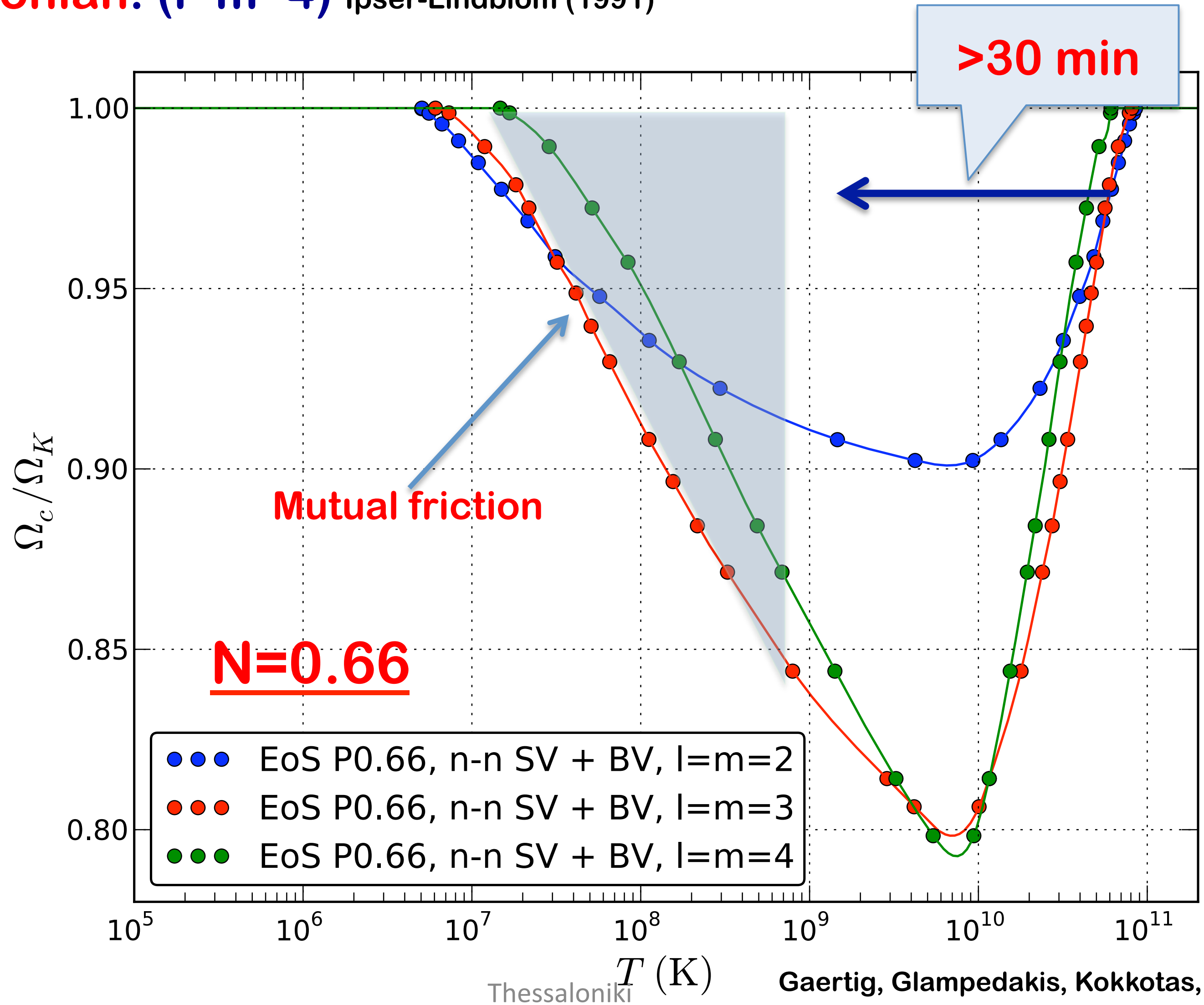


LIGO/Virgo/GEO/KAGRA/ET band

Instability Window

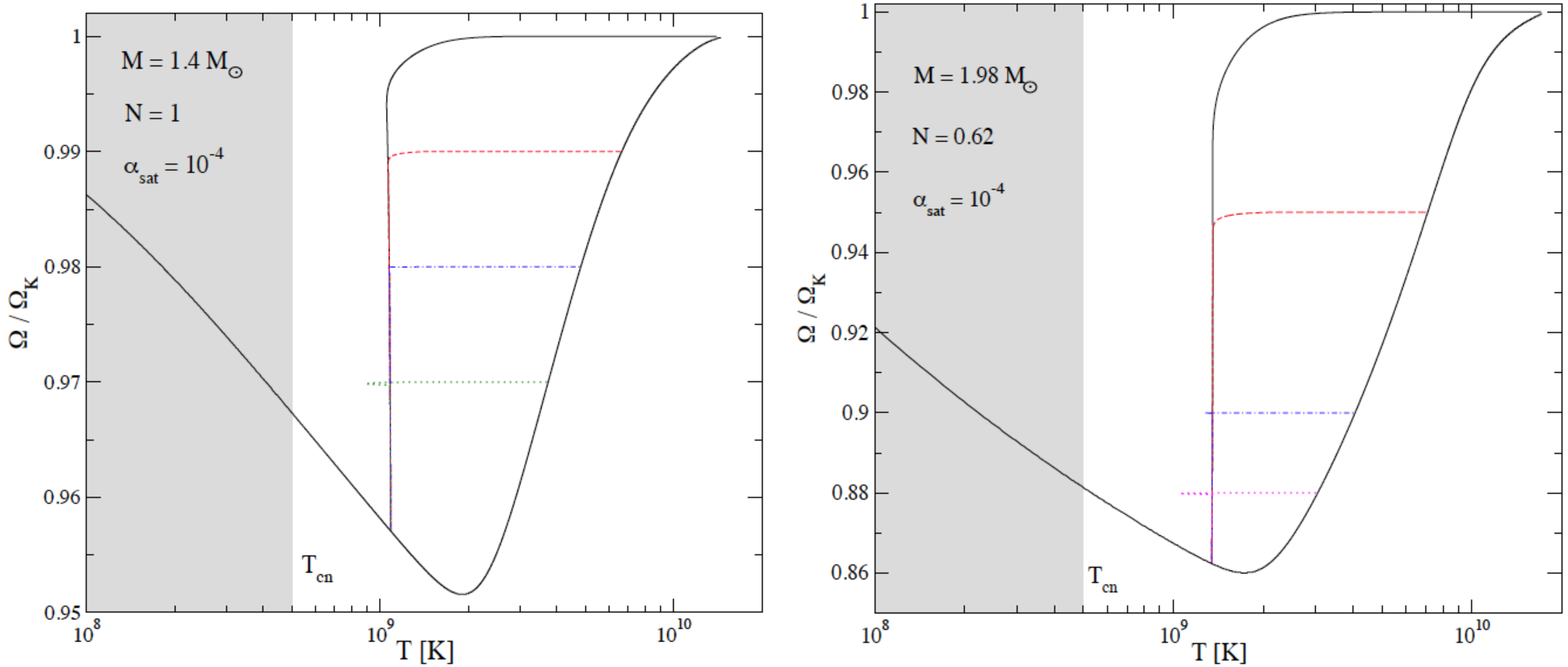
- ✓ For the first time we have the window of f-mode instability in GR
- ✓ Newtonian: (l=m=4) Ipser-Lindblom (1991)





Gaertig, Glampedakis, Kokkotas, Zink (2011)

Evolution of a nascent (unstable) NS

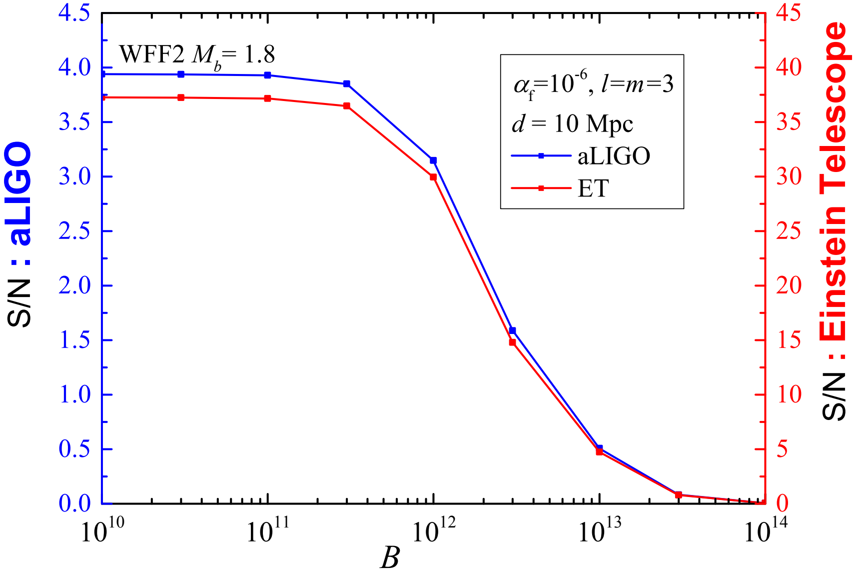


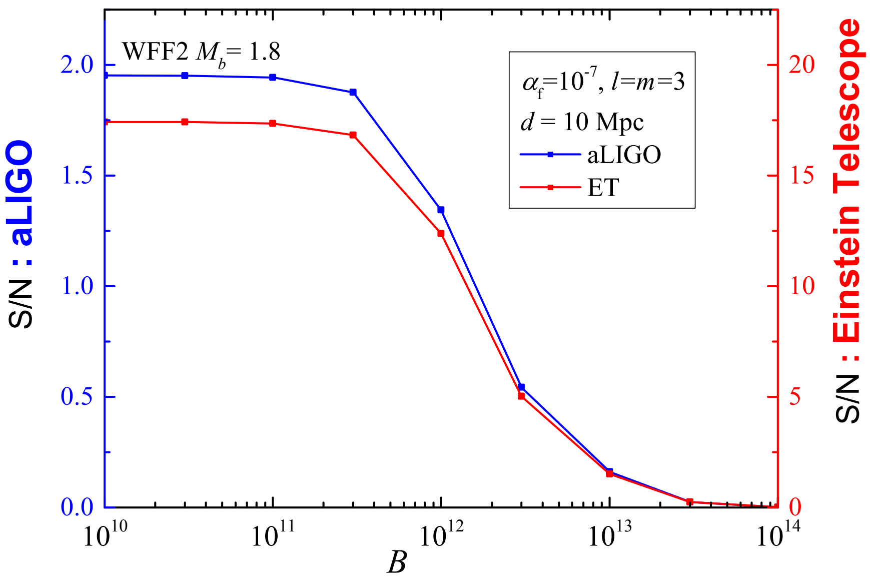
Mutual Friction plays NO ROLE for the f-mode instability

Procedure as described in Owen etal 1998 & Anderson, Jones, KK 2002

Passamonti-Gaertig-KK-Doneva (2013)

Evolution of a nascent (unstable) NS





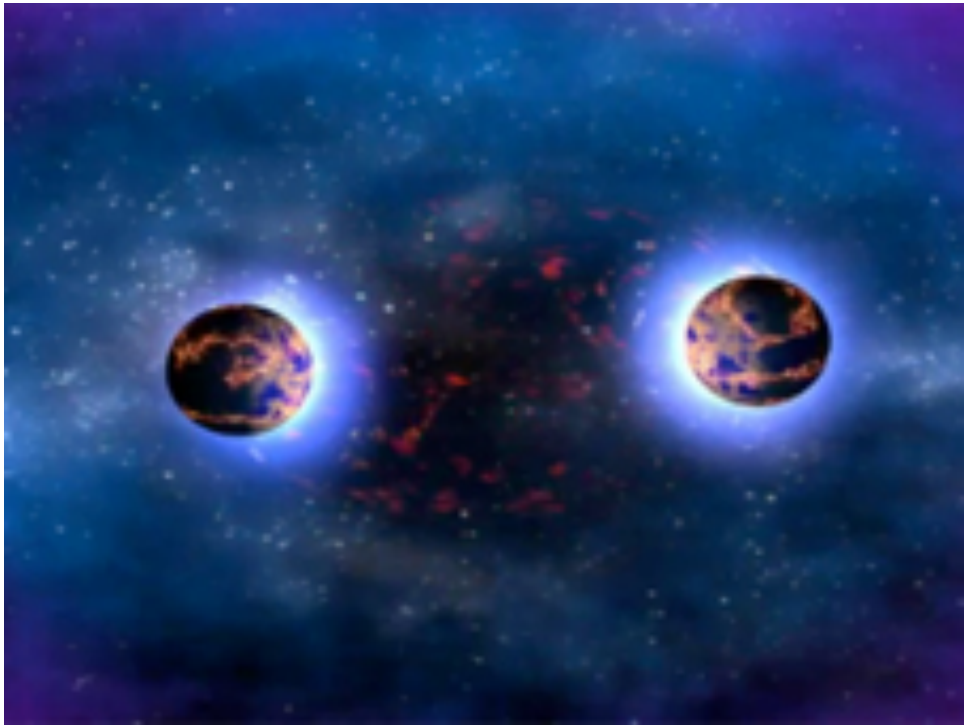
The instability can be potentially observed by events in Virgo cluster

BUT

- Event rate is unknown
- Saturation amplitude is varying during the procces

Passamonti-Gaertig-Kokkotas-Doneva (2013)

A GRAVITATIONAL WAVE AFTERGLOW IN BINARY NEUTRON STAR MERGERS



Binary Neutron Star Mergers
the standard scenario

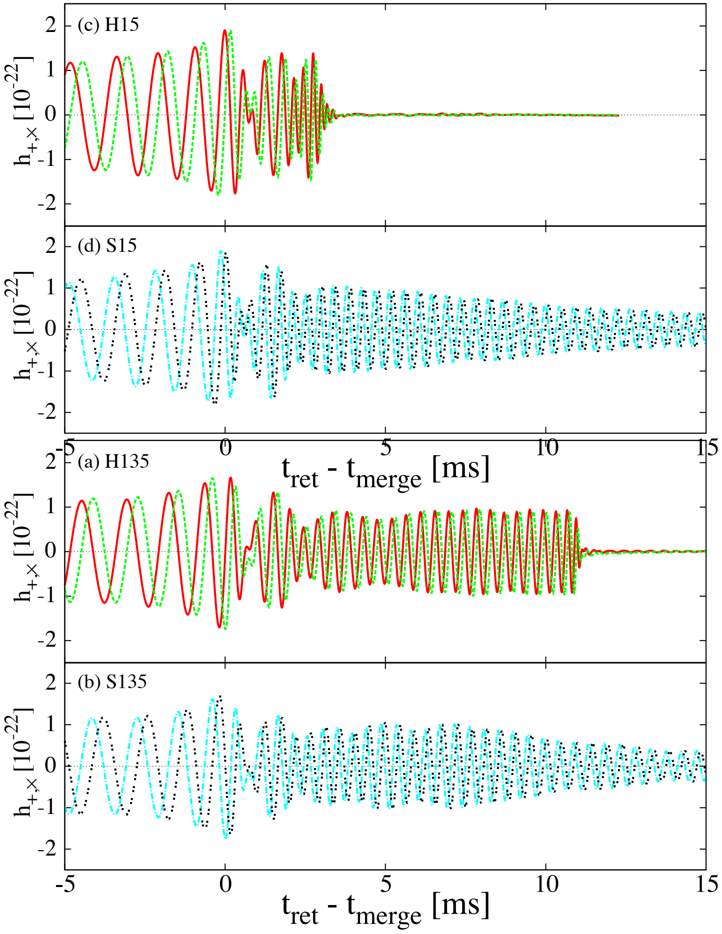
I. After the merging the final body most probably will be a supramassive NS (2.5-3 $M_\odot$)

II. The body will be differentially rotating

III. The “averaged” magnetic field will amplified due to MRI (up to 3-4 orders of magnitude)

IV. The strong magnetic field and the emission of GWs will drain rotational energy

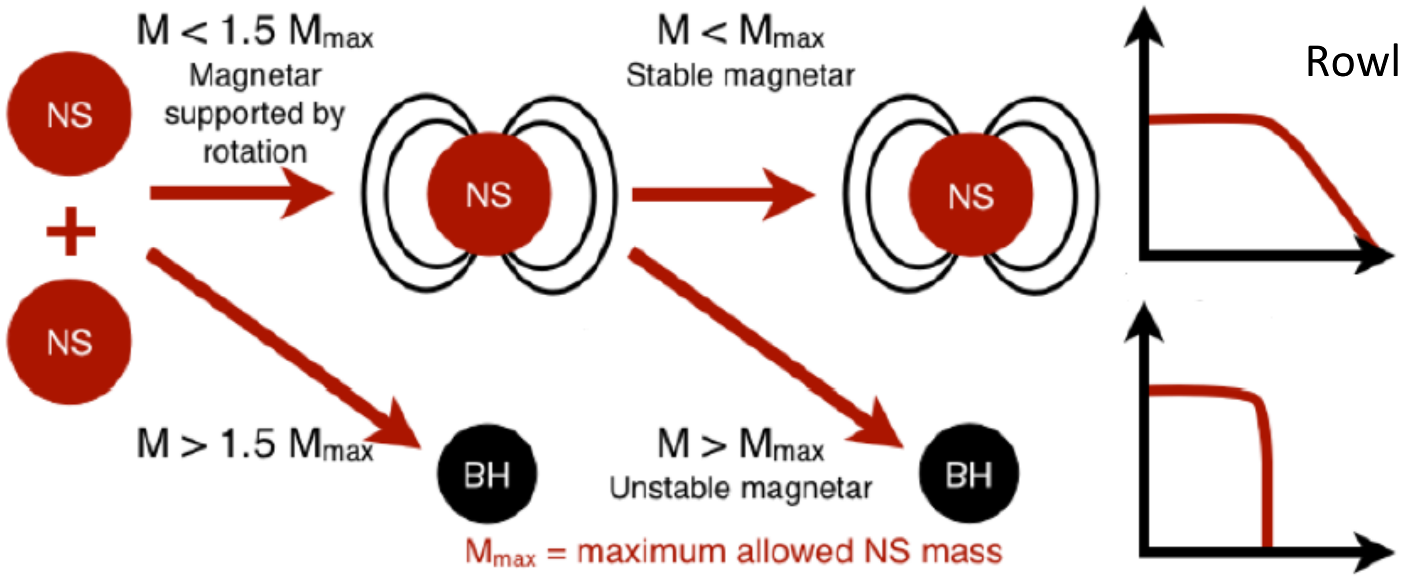
V. This phase will last only a few tenths of msecs and can potentially provide information for the Equation of State (EOS)



Kiuchi, Sekiguchi, Kyutoku, Shibata2012

Post-Merger Scenario

Three different outcomes of the merger of a BNS merger

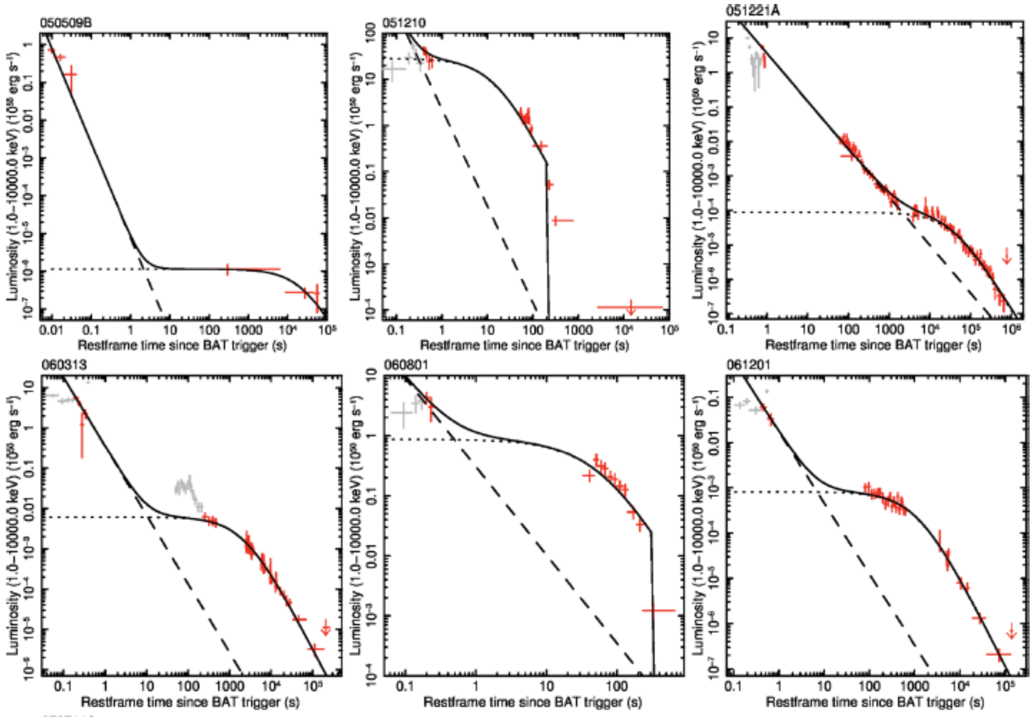


Rowlinson 2013

- ✓ The outcome is dependent upon the mass (M) of the central object formed and the maximum possible mass of a neutron star (M_{max}).
- ✓ On the right are sketches of the expected light-curves if a stable (top) or an unstable magnetar (bottom) is formed.

Short γ-ray light curves

- The favored progenitor model for SGRBs is the merger of two NSs that triggers an explosion with a burst of collimated γ-rays.
- Following the initial prompt emission, some SGRBs exhibit a plateau phase in their X-ray light curves that indicates additional energy injection from a central engine, believed to be a rapidly rotating, highly magnetized neutron star.
- The collapse of this “protomagnetar” to a black hole is likely to be responsible for a steep decay in X-ray flux observed at the end of the plateau.

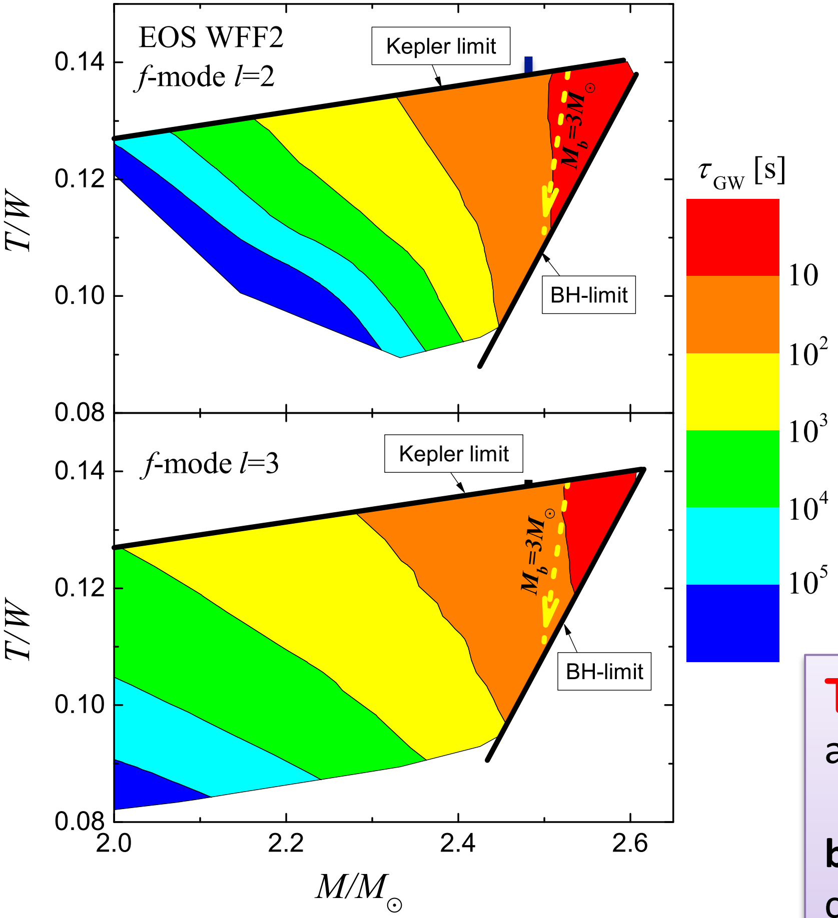


Rowlinson, O’Brien, Metger, Tanvir, Levan 2013

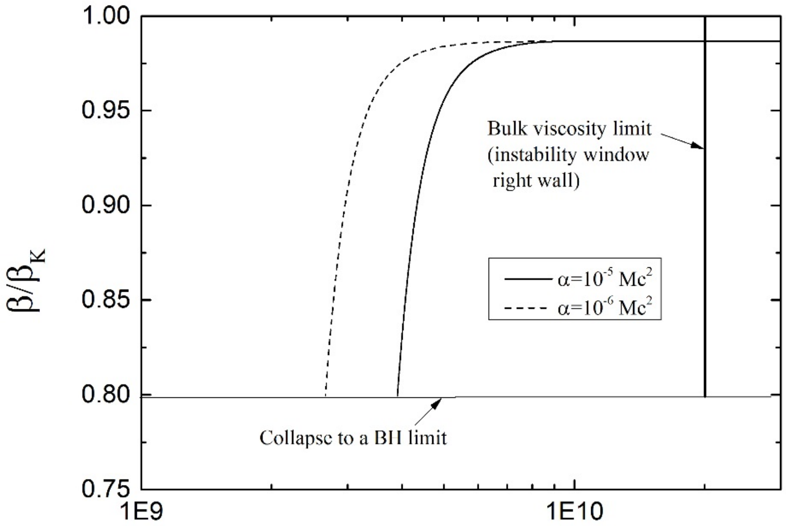
Post-Merger NS: secular instability

Doneva-KK-Pnigouras 2014

The post-merger object **is still stable** and rotates at nearly Kepler **periods < 1ms**



The evolution into the instability window



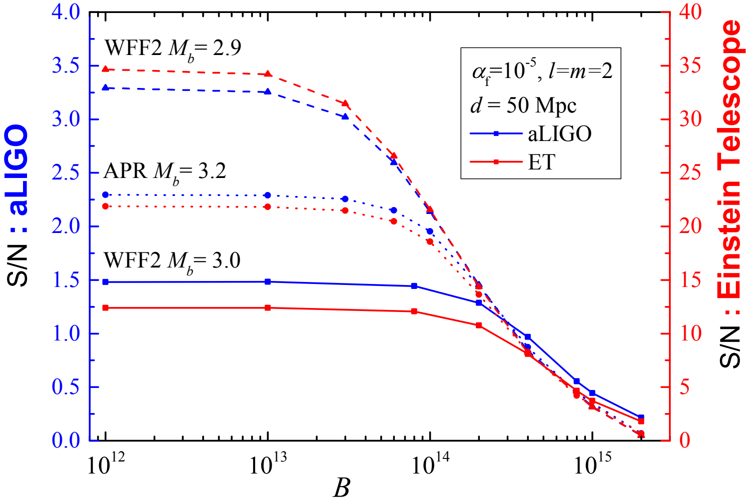
The detailed evolution depends:

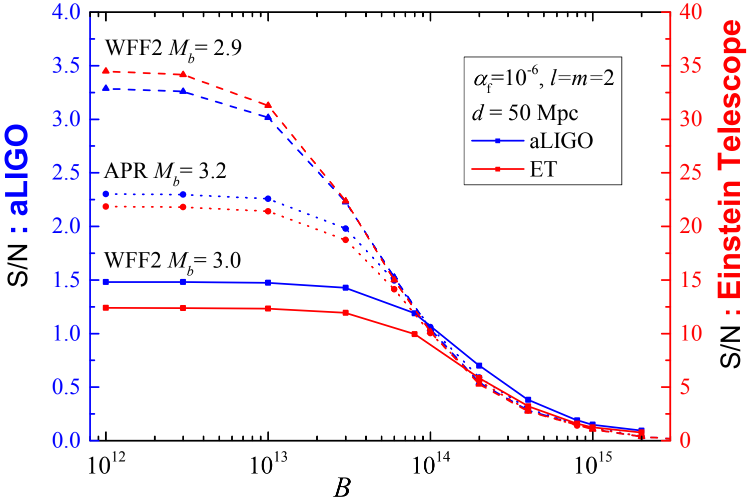
a) Strength of the **magnetic field** (averaged may reach $10^{15\text{-}16}$ G !)

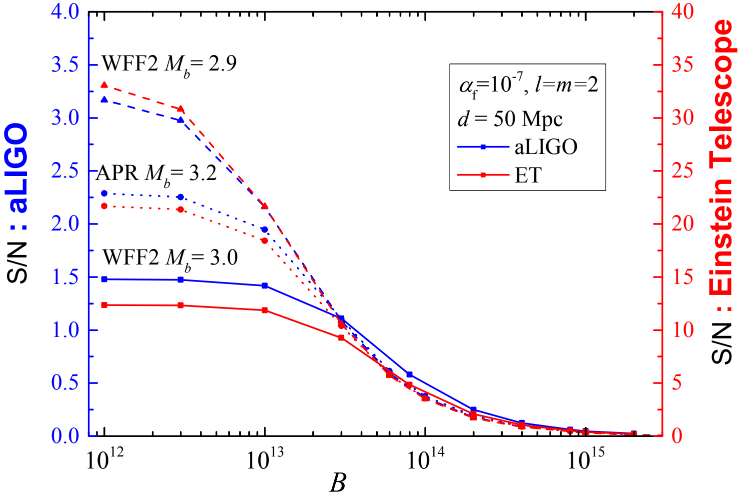
b) Equation of state of the post-merger neutron star

c) Fine details of **the non-linear dynamics** (three mode coupling, shock waves, wave breaking)

Post-Merger NS: secular instability







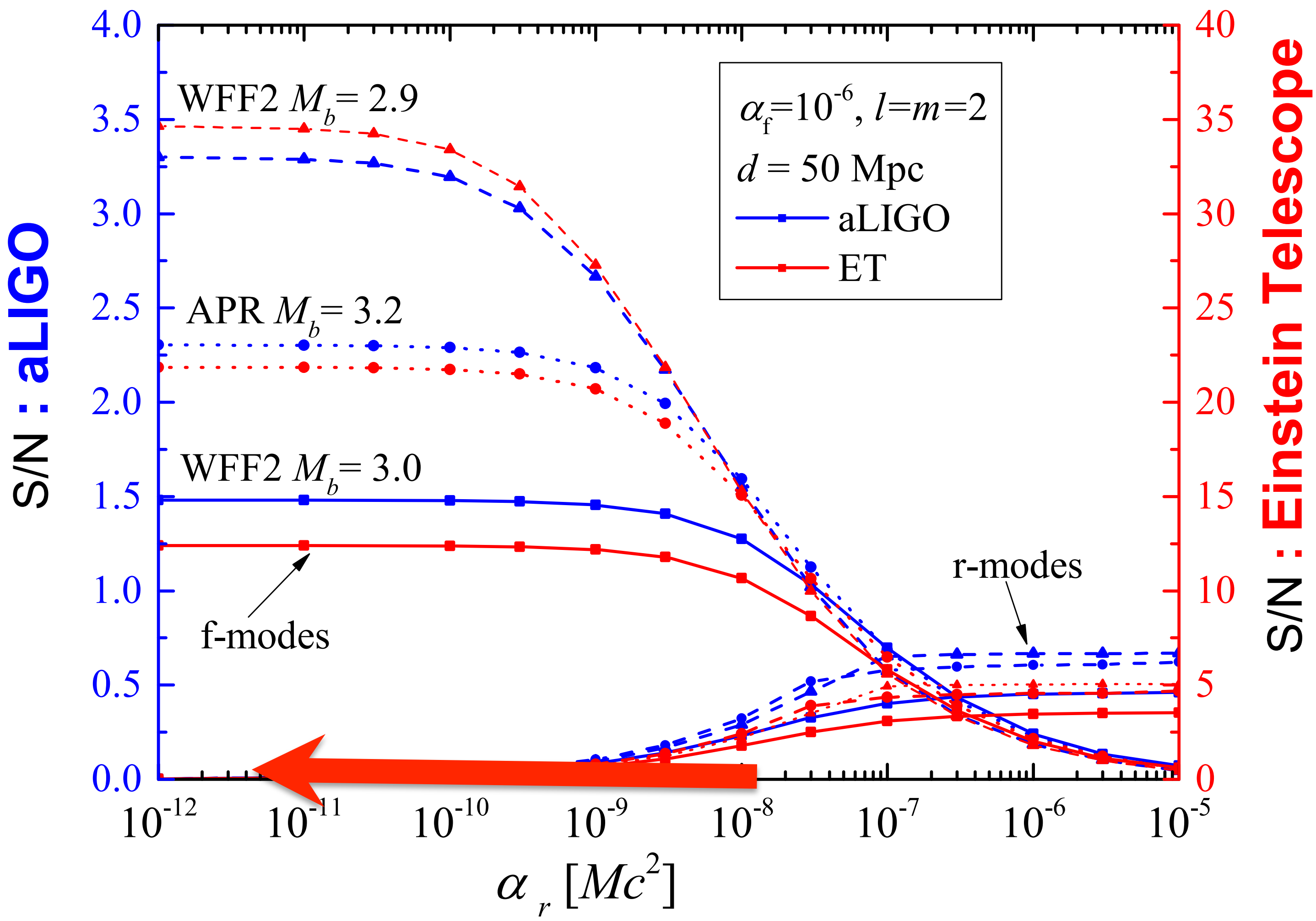
Competition between the B-field and the secular instability

GW frequencies:

WW2a: 920-1000 Hz
APR: 370–810 Hz
WFF2b: 600–780 Hz

Doneva-KK-Pnigouras 2014

f vs r-mode


S/N : aLIGO
S/N : Einstein Telescope
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
40
35
30
25
20
15
10
5
0
10^{-12}
10^{-11}
10^{-10}
10^{-9}
10^{-8}
10^{-7}
10^{-6}
10^{-5}
$\alpha_r\ [Mc^2]$
$\alpha_f=10^{-6}$, $l=m=2$
d = 50 Mpc
aLIGO
ET
WFF2 M_b= 2.9
APR M_b= 3.2
WFF2 M_b= 3.0
f-modes
r-modes

Conclusions

- ✓ The influence of the scalar field is much more pronounced for fast rotation.
- ✓ This is true also for f(R) gravity

- ✓ Asteroseismology for fast rotating stars is possible
- ✓ Asteroseismology for magnetars is possible

- ✓ f-mode instability can be potentially a good source for GWs for supramassive NS
- ✓ Saturation amplitude and strength of B-field are the key factors