Neutron star constraints from theory, experiment and observations

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Workshop on Binary Neutron Star Merges

Outline

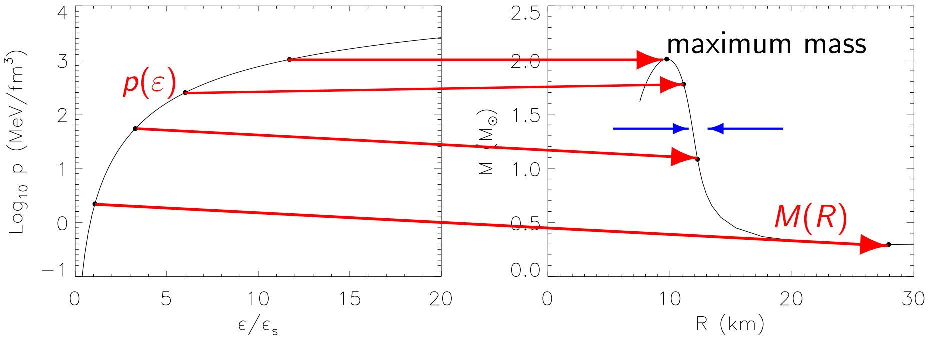
- General Relativity Constraints on Neutron Star Structure
- The Neutron Star Radius and the Nuclear Symmetry Energy
- Nuclear Experimental Constraints on the Symmetry Energy
- Constraints from Pure Neutron Matter Theory
- Quark Matter in Neutron Stars
- Astrophysical Constraints on Masses and Radii

Neutron Star Structure

Tolman-Oppenheimer-Volkov equations

$$\frac{dp}{dr} = -\frac{G}{c^4}\frac{(mc^2 + 4\pi pr^3)(\varepsilon + p)}{r(r - 2Gm/c^2)}$$

$$\frac{dm}{dr} = 4\pi\frac{\varepsilon}{c^2}r^2$$



Equation of State ← Observations

Neutron Star Structure

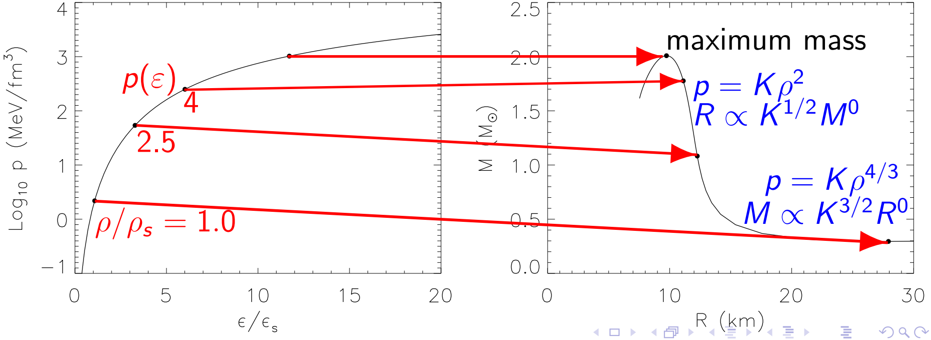
Newtonian Gravity:

$$\frac{dp}{dr} = -\frac{Gm\rho}{r^2}; \qquad \frac{dm}{dr} = 4\pi\rho r^2; \qquad \rho c^2 = \varepsilon$$

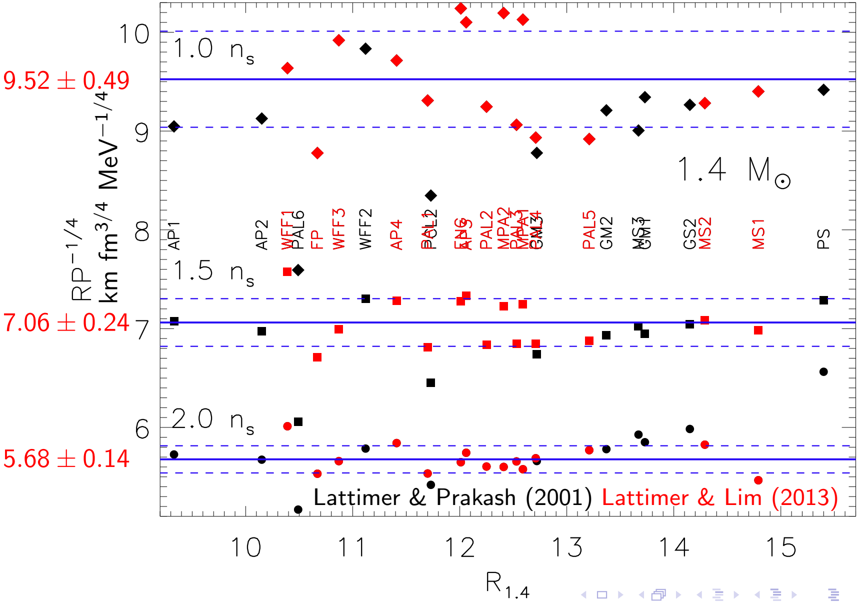
Newtonian Polytrope:

$$p = K\rho^\gamma; \qquad M \propto K^{1/(2-\gamma)} R^{(4-3\gamma)/(2-\gamma)}$$

$$\rho < \rho_s: \ \gamma \simeq \frac{4}{3}; \qquad \rho > \rho_s: \ \gamma \simeq 2$$



The Radius – Pressure Correlation



Nuclear Symmetry Energy and Pressure

Defined as the difference between energies of pure neutron matter ($x = 0$) and symmetric ($x = 1/2$) nuclear matter.

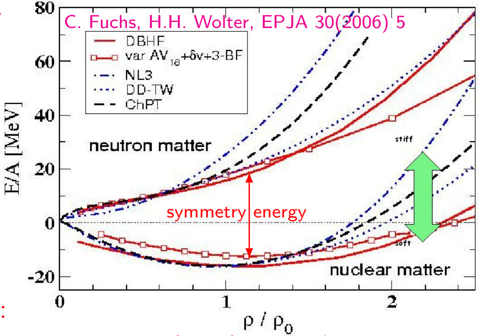
$$S(\rho) = E(\rho, x = 0) - E(\rho, x = 1/2)$$

Expanding around the saturation density (ρ_s) and symmetric matter ($x = 1/2$)

$$E(\rho, x) = E(\rho, 1/2) + (1-2x)^2 S_2(\rho) + \ldots$$

$$S_2(\rho) = \mathbf{S_v} + \frac{\mathbf{L}}{3}\frac{\rho - \rho_s}{\rho_s} + \ldots$$

$$\mathbf{S_v} \simeq 31 \text{ MeV}, \quad \mathbf{L} \simeq 50 \text{ MeV}$$



Connections to pure neutron matter:

$$E(\rho_s, 0) \approx S_v + E(\rho_s, 1/2) \equiv S_v - B, \qquad p(\rho_s, 0) = L\rho_s/3$$

Neutron star matter (in beta equilibrium):

$$\frac{\partial(E + E_e)}{\partial x} = 0, \quad p(\rho_s, x_\beta) \simeq \frac{L\rho_s}{3}\left[1 - \left(\frac{4S_v}{\hbar c}\right)^3 \frac{4 - 3S_v/L}{3\pi^2\rho_s}\right]$$

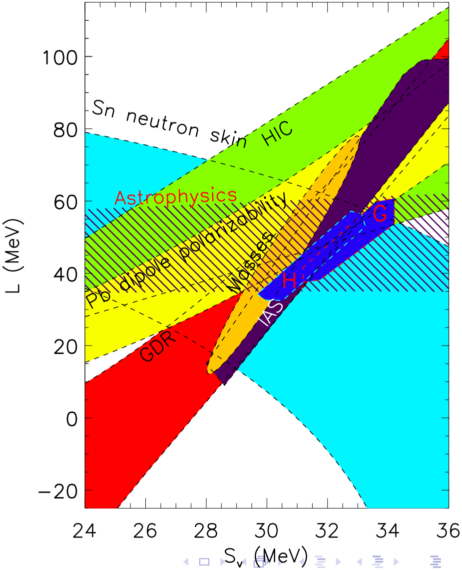
Experimental and Neutron Matter Constraints

H&S: Chiral Lagrangian

GC&R: Quantum Monte Carlo

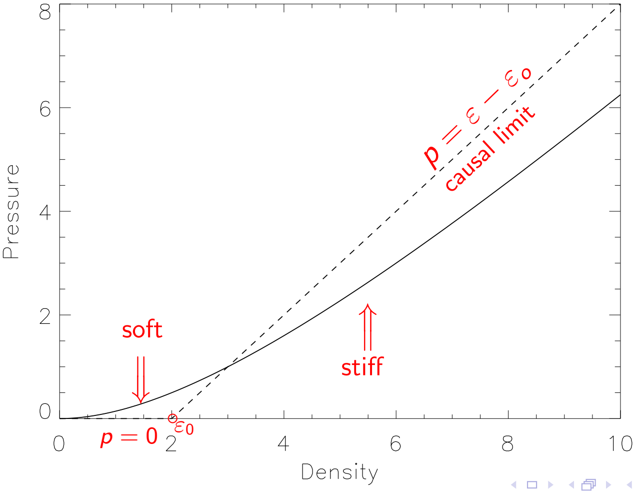
$S_v - L$ constraints from
Hebeler et al. (2012)

Gandolfi, Carlson & Reddy (2011);
Hebeler & Schwenk (2011)



Extremes of Compaction of Neutron Stars

- The most compact and massive configurations occur when the low-density equation of state is "soft" and the high-density equation of state is "stiff" (Koranda, Stergioulas & Friedman 1997).



ε_0 is the only EOS parameter

The TOV solutions scale with ε_0

$w = \varepsilon/\varepsilon_0$
$y = p/\varepsilon_0$
$x = r\sqrt{G\varepsilon_0}/c^2$
$z = m\sqrt{G^3\varepsilon_0}/c^2$

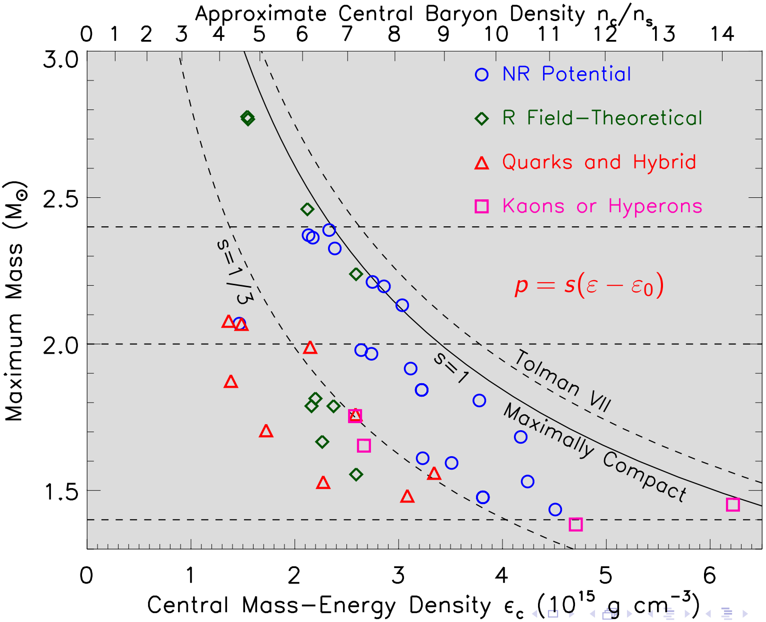
Extremal Properties of Neutron Stars

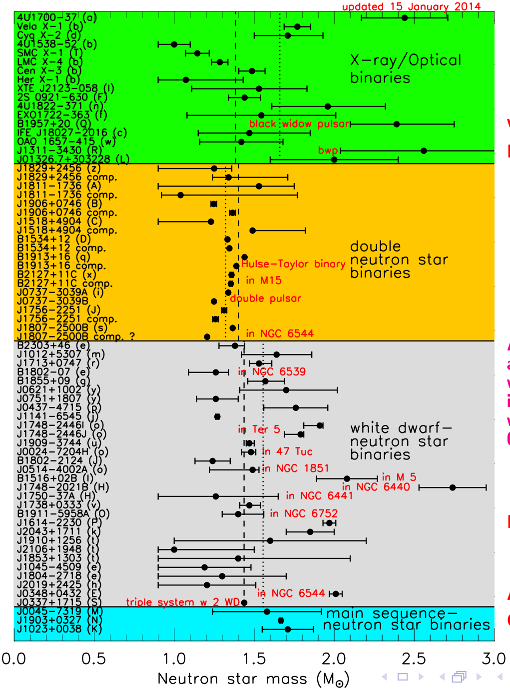
The maximum mass configuration is achieved when
$x_R = 0.2404$, $w_c = 3.034$, $y_c = 2.034$, $z_R = 0.08513$.

A useful reference density is the nuclear saturation density (interior density of normal nuclei):
$\rho_s = 2.7 \times 10^{14}$ g cm^{-3}, $n_s = 0.16$ baryons fm^{-3}, $\varepsilon_s = 150$ MeV fm^{-3}

- $M_{\rm max} = 4.1\ (\varepsilon_s/\varepsilon_0)^{1/2} M_\odot$ (Rhoades & Ruffini 1974)
- $M_{B,\rm max} = 5.41\ (m_B c^2/\mu_o)(\varepsilon_s/\varepsilon_0)^{1/2} M_\odot$
- $R_{\rm min} = 2.82\ GM/c^2 = 4.3\ (M/M_\odot)\ {\rm km}$
- $\mu_{b,\rm max} = 2.09$ GeV
- $\varepsilon_{c,\rm max} = 3.034\ \varepsilon_0 \simeq 51\ (M_\odot/M_{\rm largest})^2\ \varepsilon_s$
- $p_{c,\rm max} = 2.034\ \varepsilon_0 \simeq 34\ (M_\odot/M_{\rm largest})^2\ \varepsilon_s$
- $n_{B,\rm max} \simeq 38\ (M_\odot/M_{\rm largest})^2\ n_s$
- $\mathrm{BE}_{\rm max} = 0.34\ M$
- $P_{\rm min} = 0.74\ (M_\odot/M_{\rm sph})^{1/2}(R_{\rm sph}/10\ {\rm km})^{3/2}\ {\rm ms} = 0.20\ (M_{\rm sph,max}/M_\odot)\ {\rm ms}$

Maximum Energy Density in Neutron Stars





vanKerkwijk 2010

Romani et al. 2012

Although simple average mass of w.d. companions is 0.23 $M_\odot$ larger, weighted average is 0.04 $M_\odot$ smaller

Demorest et al. 2010

Antoniadis et al. 2013

Champion et al. 2008

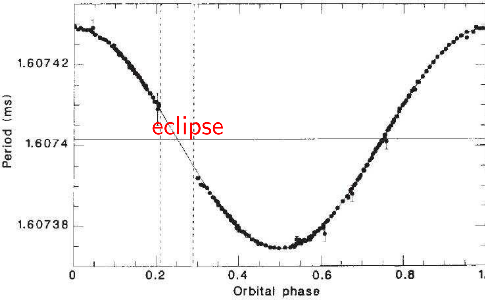
What is the Maximum Mass?

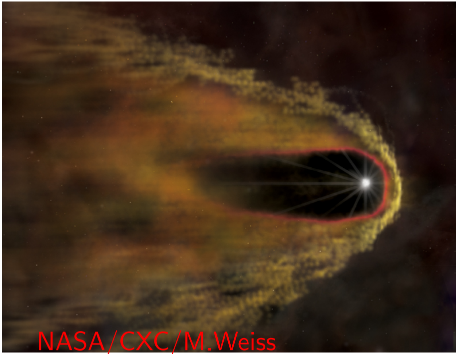
- PSR J1614+2230 (Demorest et al. 2010)
  $M = 1.97 \pm 0.04\ M_\odot$; a nearly edge-on system with well-measured Shapiro time delay.
- PSRJ0548+0432 (Antoniadis et al. 2013)
  $M = 2.01 \pm 0.04\ M_\odot$; measured using optical data and theoretical properties of companion white dwarf.
- B1957+20 (van Kerkwijk 2010) $M = 2.4 \pm 0.3\ M_\odot$; black widow pulsar (BWP).
- PSR J1311-3430 (Romani et al. 2012)
  $M = 2.55 \pm 0.50\ M_\odot$; BWP.
- PSR J1544+4937 (Tang et al. 2014)
  $M = 2.06 \pm 0.56 M_\odot$; BWP.
- PSR 2FGL J1653.6-0159 (Romani et al. 2014)
  $M > f(M_2)/\sin^3 i \gtrsim 1.96 M_\odot$; largest $f(M_2)$.
- PSR J1227-4859 (de Martino et al. 2014)
  $M = 2.2 \pm 0.8 M_\odot$; redback pulsar.

Black Widow Pulsar PSR B1957+20

1.6ms pulsar in circular 9.17h orbit with $\sim 0.03\ M_\odot$ companion.
Pulsar is eclipsed for 50-60 minutes each orbit; eclipsing object has a volume much larger than the companion or its Roche lobe.
It is believed the pulsar is ablating the companion leading to mass loss and an eclipsing plasma cloud. Companion nearly fills its Roche lobe.
Ablation by pulsar leads to eventual disappearance of companion.
The optical light curve does not represent the center of mass of the companion, but the motion of its irradiated hot spot.

pulsar radial velocity





NASA/CXC/M.Weiss

Black Widow Pulsar PSR B1957+20

$K_i = 2\pi \frac{a_i \sin i}{P}$

$q = \frac{M_P}{M_*} = \frac{a_*}{a_P} = \frac{K_*}{K_P}$

$K_* = K_{obs}\left(1 + \frac{R_*}{a_*}\right)$

Radiated area of companion has a smaller orbit than the center of mass.

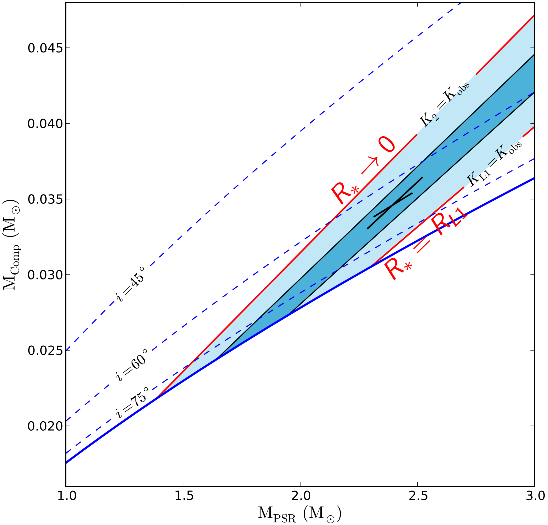
$M_P = q(1+q)^2 \frac{P}{2\pi G}\left(\frac{K_P}{\sin i}\right)^3$

Modeling of light curve shape suggests that

$M_P > 1.8\ \mathrm{M}_\odot$, $\sin i < 66^\circ$

Most probable values:

$2.20 M_\odot < M_P < 2.55 M_\odot$



van Kerkwijk 2010

Causality + GR Limits and the Maximum Mass

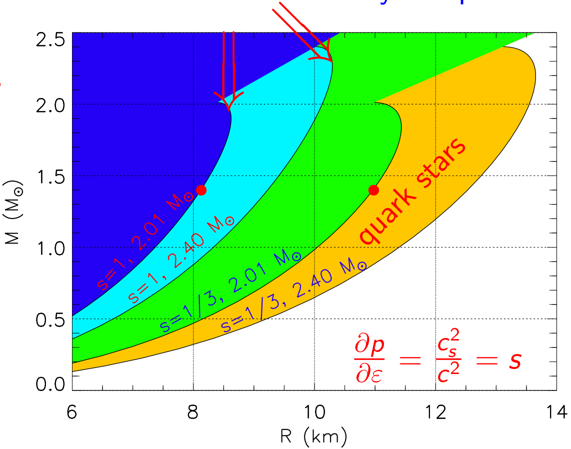
A lower limit to the maximum mass sets a lower limit to the radius for a given mass.

Similarly, a precise (M, R) measurement sets an upper limit to the maximum mass.

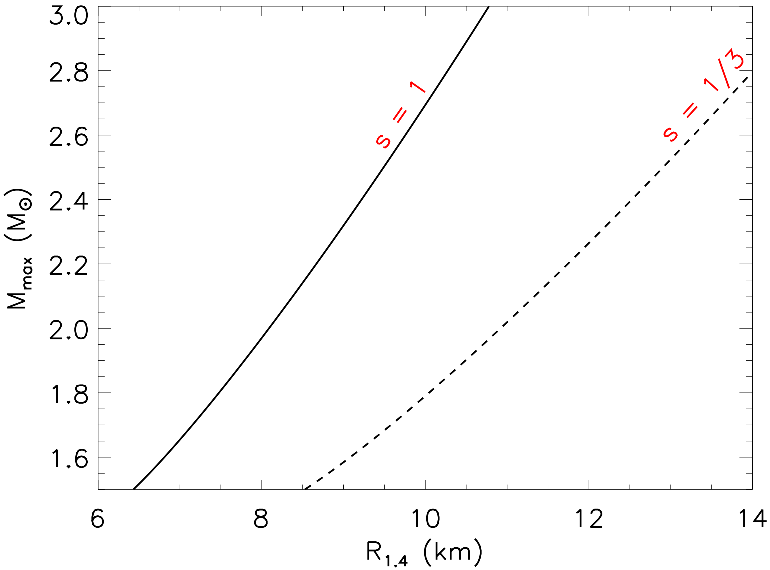
$1.4 M_\odot$ stars must have $R > 8.15 M_\odot$.

$1.4 M_\odot$ strange quark matter stars (and likely hybrid quark/hadron stars) must have $R > 11$ km.

$M - R$ curves for maximally compact EOS



Maximum Mass and Neutron Star Radii



What About Realistic EOSs?

It has been proposed that the effective sound speed limit is $c/\sqrt{3}$ (Bedaque & Steiner 2015), in which case $1.4 M_\odot$ stars must have $R_{1.4} > 11$ km.

Hybrid quark/hadron stars are realistically at least 1-2 km larger (Alford et al. 2015).

What additional constraints are imposed by our knowledge of the low-density equation of state?

Chiral Lagrangian Neutron Matter Calculations

The study of Hebeler & Schwenk (2010) suggested moderate values 40 MeV $< L <$ 60 MeV, consistent with but at the lower boundary of the range favored by nuclear experiments.

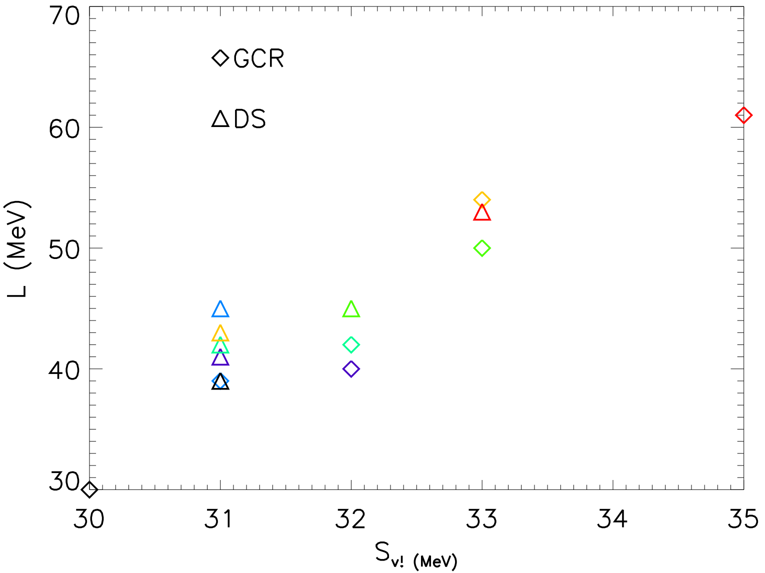
These results were in substantial agreement with the quantum Monte Carlo neutron matter calculations of Gandolfi, Carlson & Reddy (2012).

The chiral Lagrangian calculations have been refined and extended to matter with proton fractions up to and including symmetric matter (Drischler & Schwenk 2014).

The symmetry energy coefficients are found to be correlated with the saturation properties for a given parameter set.

There is a small quartic contribution to the symmetry energy.

Neutron Matter Comparisons



Extrapolation of Neutron Matter EOS to Higher Densities

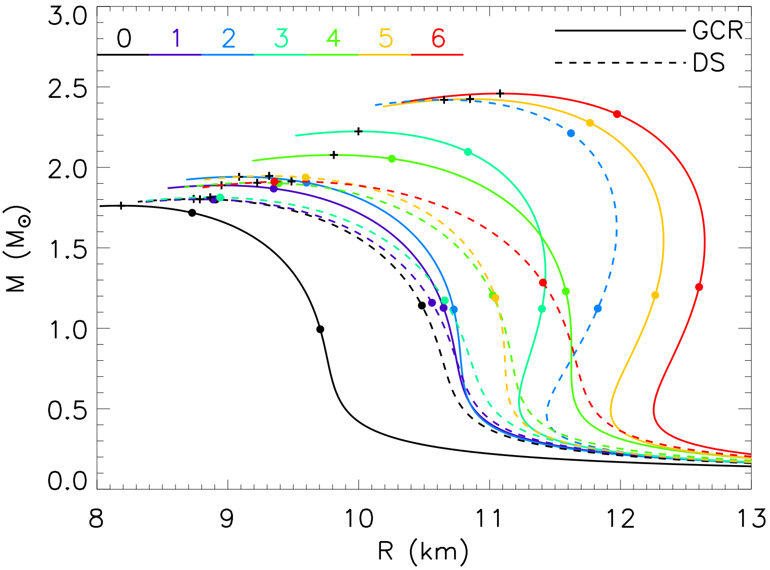
Gandolfi, Carlson & Reddy fit their QMC neutron matter equations of state to the 4-parameter fit:

$$E_n(u) = au^\alpha + bu^\beta$$

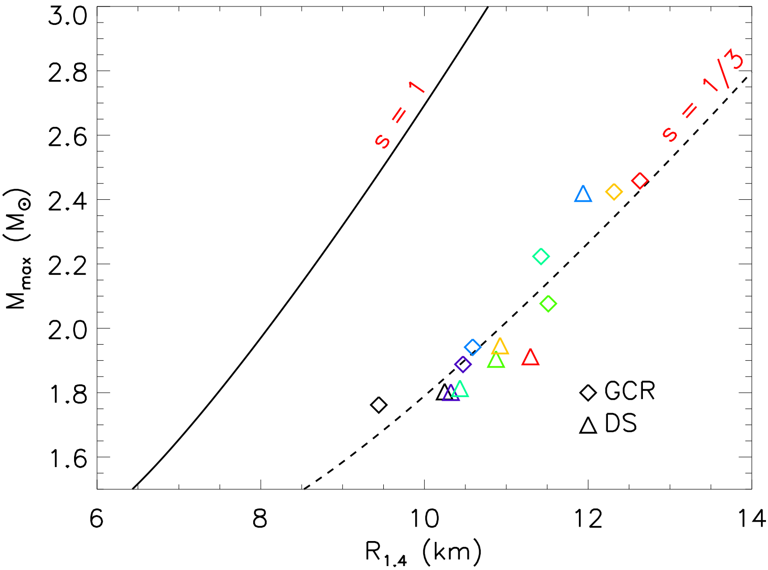
with $u = n/n_s$ and $n_s = 0.16$ fm^{-3}.

This can also be done with the chiral Lagrangian neutron matter equations of state computed by Drischler, Hebeler & Schwenk.

Neutron Matter Extrapolations and $M - R$



Neutron Matter Extrapolations and $M_{max} - R_{1.4}$

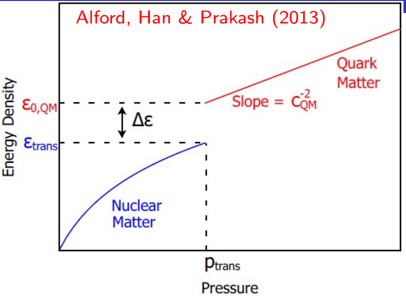


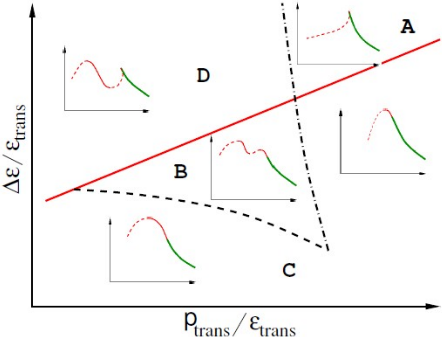
First Order Phase Transition in Neutron Stars

- Generic first order phase transiton with 3 parameters: $\Delta\varepsilon$, ε_t and P_t.
- Make 2 dimensionless parameter combinations: $\Delta\varepsilon/\varepsilon_t$ and P_t/ε_t.
- Critical condition for existence of stable hybrid core connected to normal branch (**A, D**):

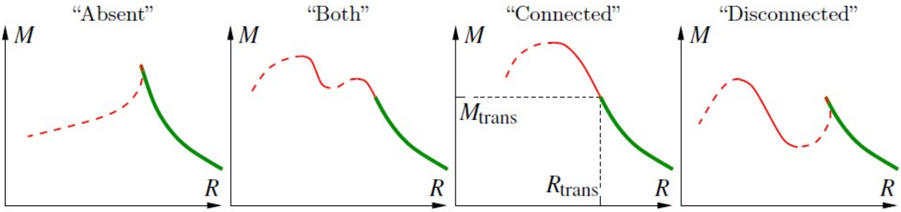
$$\frac{\Delta\varepsilon}{\varepsilon_t} \leq \frac{1}{2} + \frac{3}{2}\frac{P_t}{\varepsilon_t}.$$

- It is also possible to have a stable hybrid core disconnected from normal barnch (**B, D**).
- Parametrize high-density phase with a constant sound speed $c^2_{\rm QM} = dp/d\varepsilon \sim 1/3$.





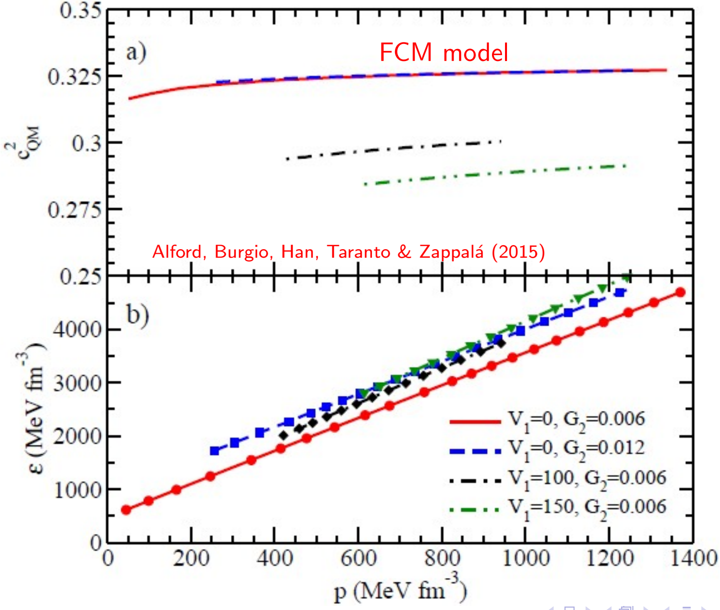
Possible Hybrid Configurations



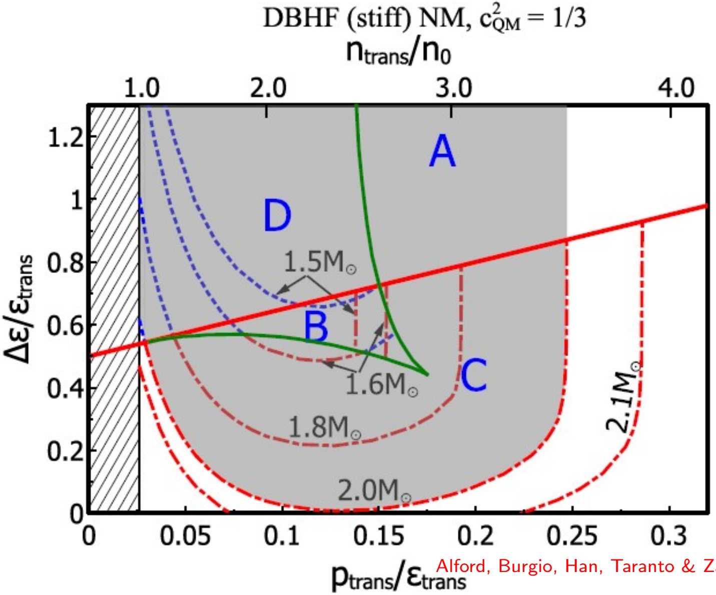
A **B** **C** **D**

Alford, Han & Prakash (2013)

Sound Speed in Quark Matter

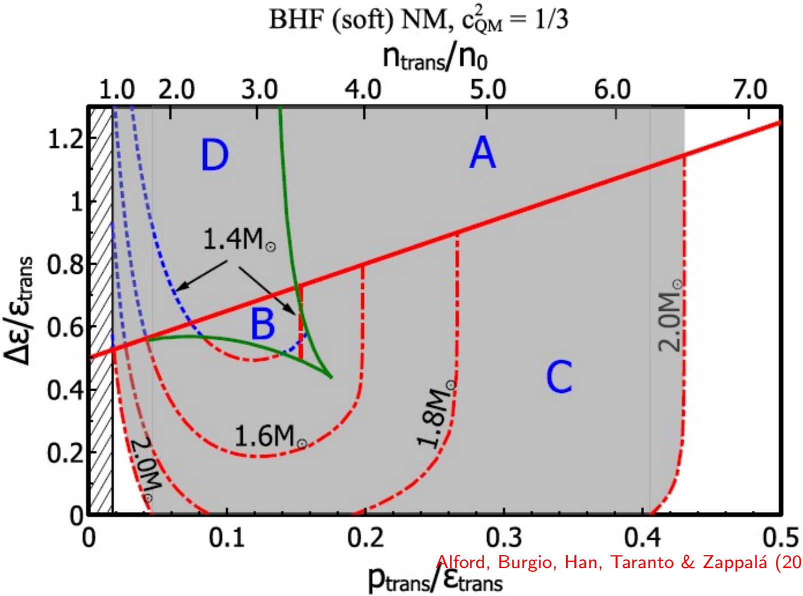


Mass Constraint



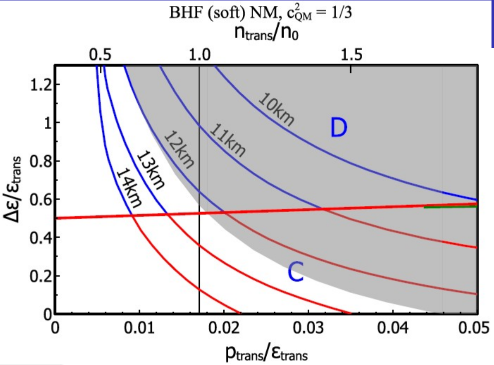
Alford, Burgio, Han, Taranto & Zappalá (2015)

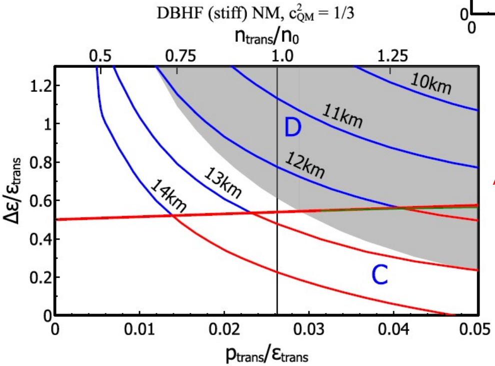
Mass Constraint



Alford, Burgio, Han, Taranto & Zappalá (2015)

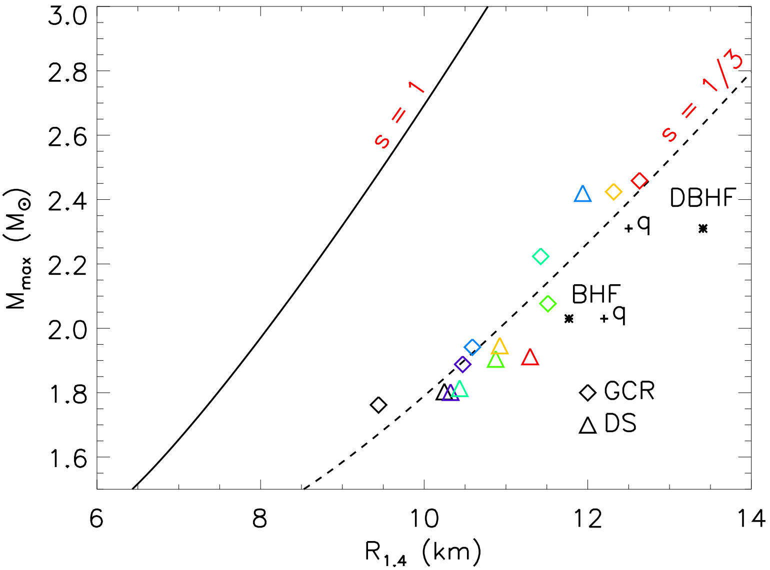
Radius Constraint





Alford, Burgio, Han, Taranto & Zappalá (2015)

$M_{max} - R_{1.4}$



3.0
2.8
2.6
2.4
2.2
2.0
1.8
1.6
M_{max} ($M_\odot$)
6
8
10
12
14
$R_{1.4}$ (km)
s = 1
s = 1/3
DBHF
q
BHF
q
◇ GCR
△ DS

Mass-Radius Diagram and Theoretical Constraints

GR:

$R > 2GM/c^2$

$P < \infty$:

$R > (9/4)GM/c^2$

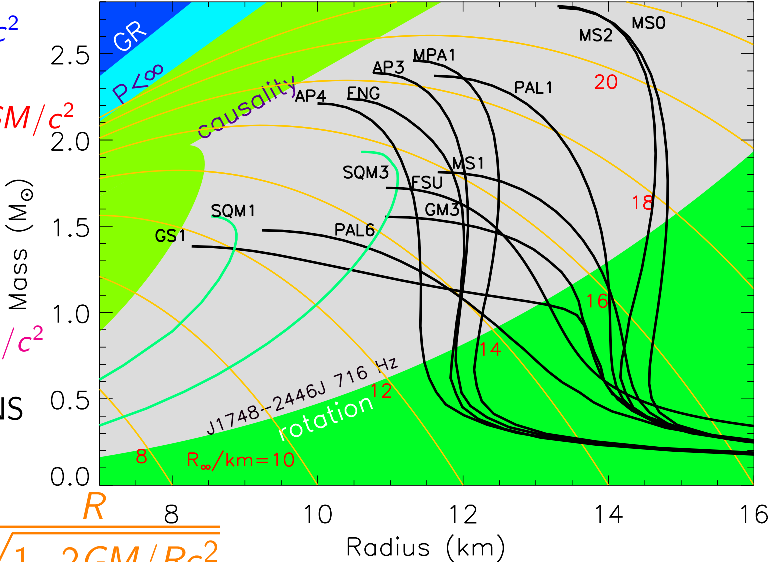
causality:

$R \gtrsim 2.9GM/c^2$

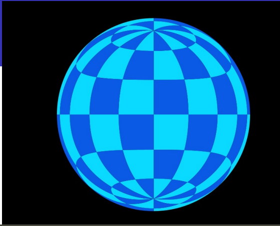
— normal NS

— SQS

— $R_\infty = \dfrac{R}{\sqrt{1-2GM/Rc^2}}$



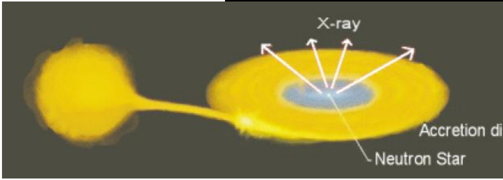
Simultaneous Mass/Radius Measurements



- Measurements of flux $F_\infty = (R_\infty/D)^2 \sigma T_{\rm eff}^4$ and color temperature $T_c \propto \lambda_{\rm max}^{-1}$ yield an apparent angular size (pseudo-BB):

$$\frac{R_\infty}{D} = \frac{R}{D} \frac{1}{\sqrt{1 - 2GM/Rc^2}}$$

- Observational uncertainties include distance D, interstellar absorption N_H, atmospheric composition

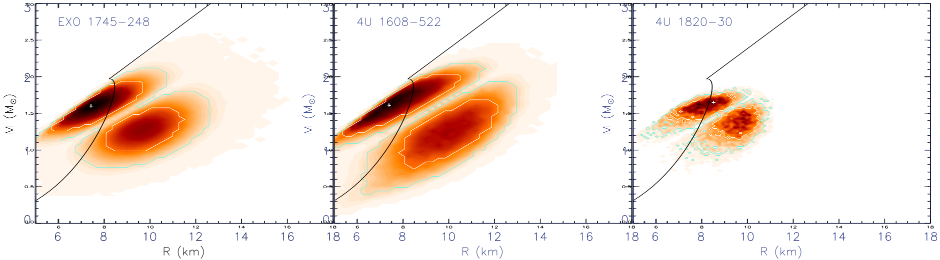


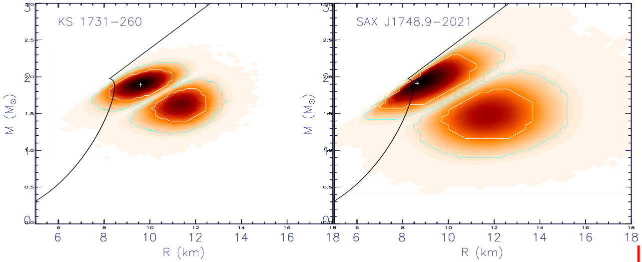
Best chances for accurate radius measurement:

- Nearby isolated neutron stars with parallax (uncertain atmosphere)
- Quiescent low-mass X-ray binaries (QLMXBs) in globular clusters (reliable distances, low B H-atmosperes)
- Bursting sources (XRBs) with peak fluxes close to Eddington limit (where gravity balances radiation pressure)

$$F_{\rm Edd} = \frac{cGM}{\kappa D^2} \sqrt{1 - 2GM/Rc^2}$$

$M - R$ PRE Burst Estimates



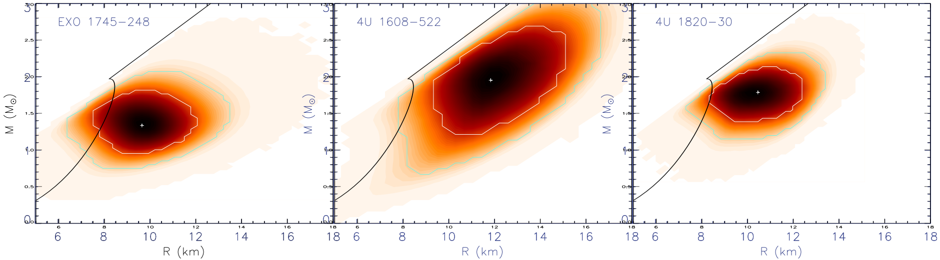


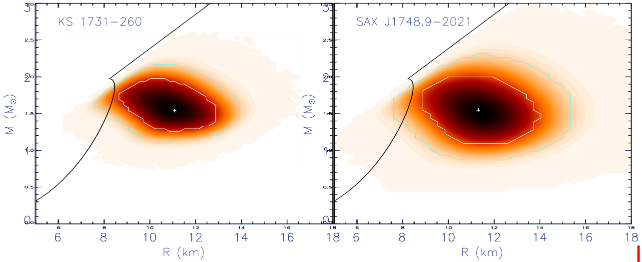
$F_{Edd,\infty}, (R_\infty/D)^2 f_c^{-4}, D,$
f_c from Ozel et al.

$z_{ph} = z$

Lattimer & Steiner (2013)

$M - R$ PRE Burst Estimates





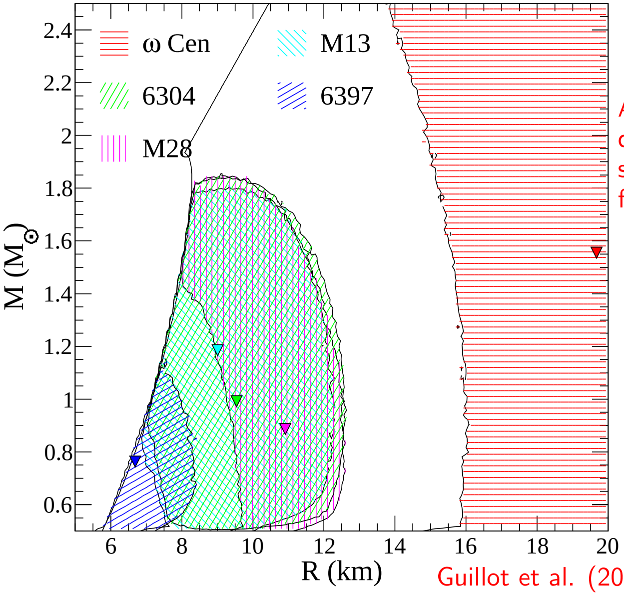
$F_{Edd,\infty}, (R_\infty/D)^2 f_c^{-4}, D$
from Ozel et al.

$z_{ph} = 0$

Altered uncertainties
for f_c, D

Lattimer & Steiner (2013)

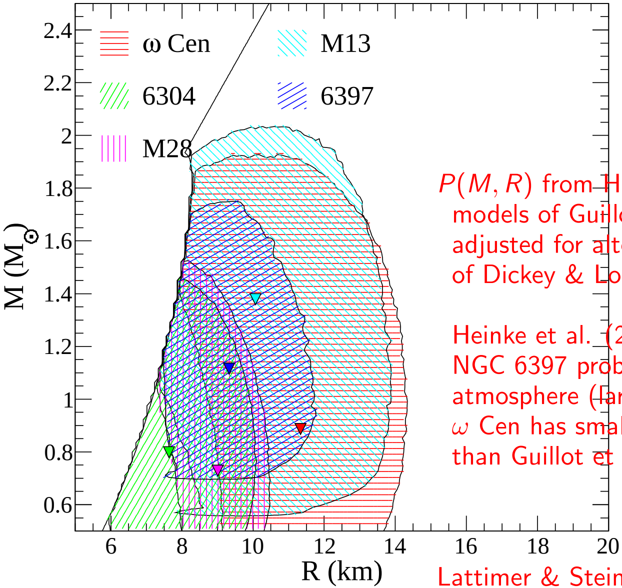
$M - R$ QLMXB Estimates



Absorption (N_H) determined self-consistently from spectra

Guillot et al. (2013)

$M - R$ QLMXB Estimates

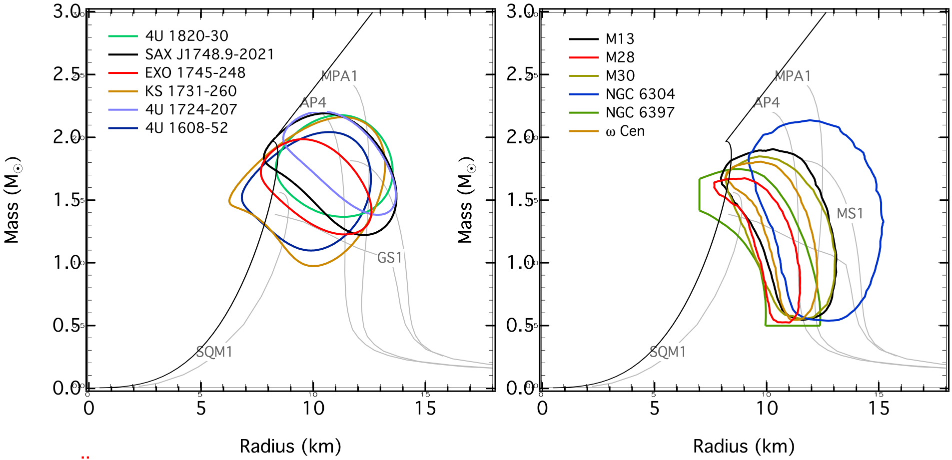


$P(M, R)$ from H atmosphere models of Guillot et al. (2013), adjusted for alternate N_H values of Dickey & Lockman (1990).

Heinke et al. (2014) found NGC 6397 probably has He atmosphere (larger R); ω Cen has smaller N_H (and R) than Guillot et al. (2013) found.

Lattimer & Steiner (2013)

Özel et al. (2015)

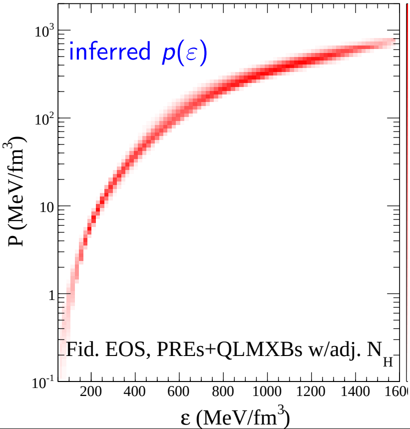


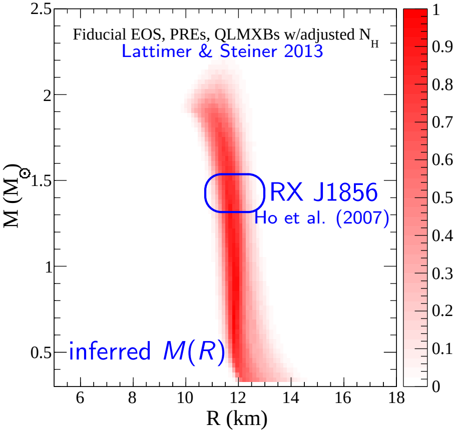
Özel et al. (2015) adopts Bayesian approach following Steiner et al. (2010).
Mean XRB radius changes from 9.74 ± 0.5 km to 10.6 ± 0.8 km.
Neglect of causality and TOV constraints underestimate radius.
Steiner et al. analysis is vindicated.

Bayesian TOV Inversion

- $\varepsilon < 0.5\varepsilon_0$: Known crustal EOS
- $0.5\varepsilon_0 < \varepsilon < \varepsilon_1$: EOS parametrized by K, K', S_v, γ
- Polytropic EOS: $\varepsilon_1 < \varepsilon < \varepsilon_2$: n_1; $\varepsilon > \varepsilon_2$: n_2

- EOS parameters $K, K', S_v, \gamma, \varepsilon_1, n_1, \varepsilon_2, n_2$ uniformly distributed
- $M_{\rm max} \geq 1.97$ $M_\odot$, causality enforced
- All 10 stars equally weighted





Astronomy vs. Astronomy vs. Physics

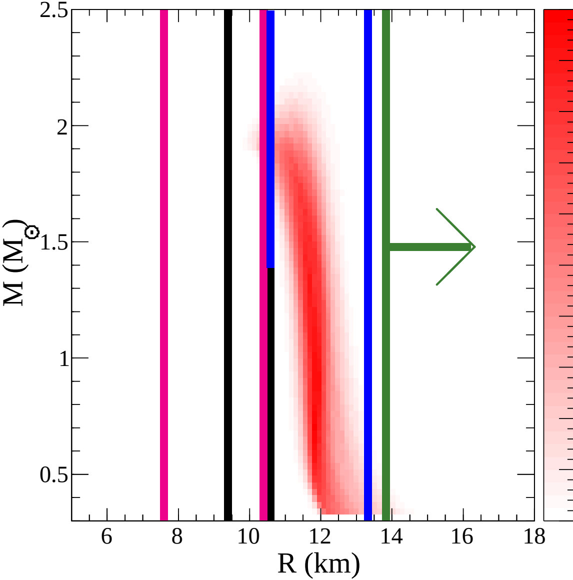
Ozel et al., XRB+QLMXB, $M_{max} > 2M_\odot$, crust, $z_{ph} = z$: $R = 10.6 \pm 0.6$ km.

Suleimanov et al., long XRB: $R_{1.4} \gtrsim 13.9$ km

Guillot et al. (2013), QLMXB, equal radii stars, self N_H: $R = 9.1^{+1.3}_{-1.5}$ km.

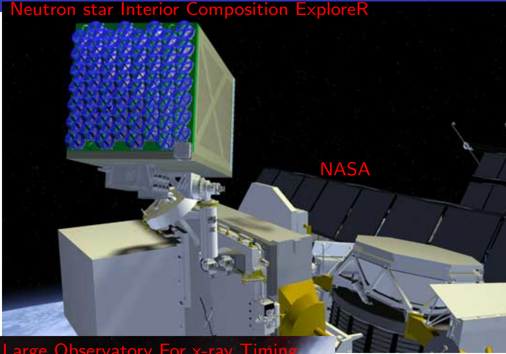
Lattimer & Steiner (2013), XRB+QLMXB, TOV, crust, causality, $M_{max} > 2M_\odot$, $z_{\rm ph} \neq z$, alt N_H.

Lattimer & Lim (2013), nuclear experiments: 29 MeV $< S_v <$ 33 MeV, 40 MeV $< L <$ 65 MeV: $R_{1.4} = 12.0 \pm 1.4$ km.



Additional Proposed Radius and Mass Constraints

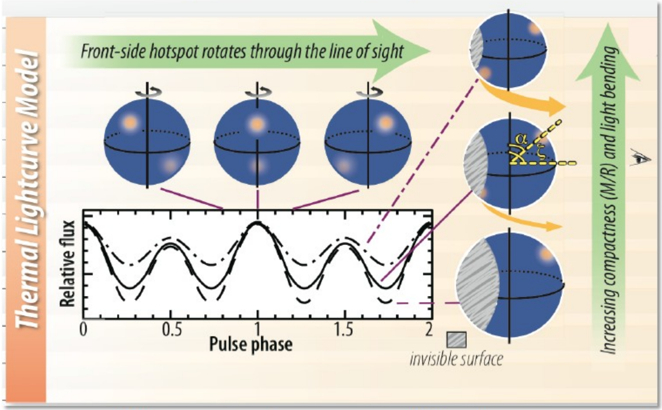
- Pulse profiles
  Hot or cold regions on rotating neutron stars alter pulse shapes: NICER and LOFT will enable timing and spectroscopy of thermal and non-thermal emissions. Light curve modeling $\to M/R$; phase-resolved spectroscopy $\to R$.
- Moment of inertia
  Spin-orbit coupling of ultra-relativistic binary pulsars (e.g., PSR 0737+3039) vary i and contribute to $\dot{\omega}$: $I \propto MR^2$.
- Supernova neutrinos
  Millions of neutrinos detected from a Galactic supernova will measure BE$= m_B N - M$, $< E_\nu >$, τ_ν.
- QPOs from accreting sources
  ISCO and crustal oscillations



Science Measurements

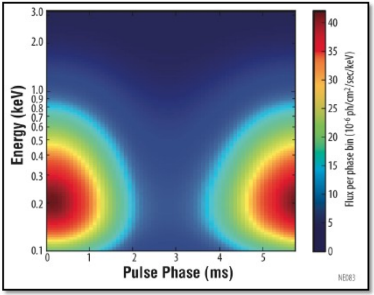
NICER

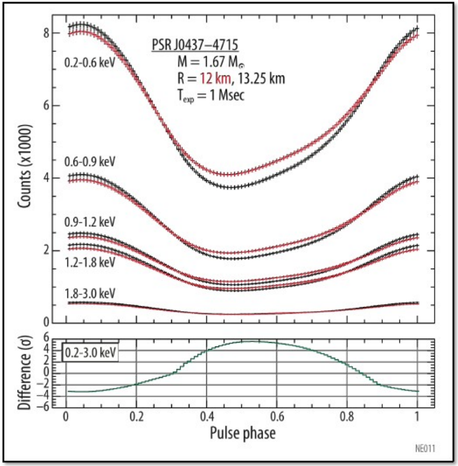
Reveal stellar structure through lightcurve modeling, long-term timing, and pulsation searches



Lightcurve modeling constrains the compactness (M/R) and viewing geometry of a non-accreting millisecond pulsar through the depth of modulation and harmonic content of emission from rotating hot-spots, thanks to gravitational light-bending…

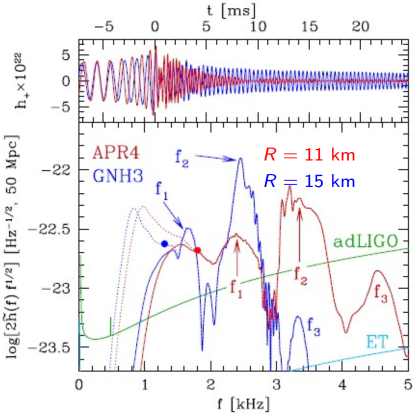
Science Measurements (cont.) NICER



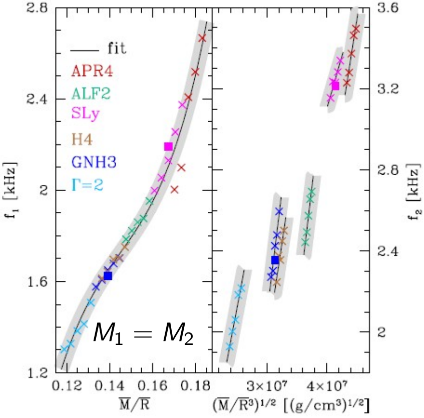


… while phase-resolved spectroscopy promises a direct constraint of radius *R*.

Constraints from Observations of Gravitational Radiation



Takami, Rezzolla and Baiotti (2014)



- Chirp mass and tidal deformability measurable during inspiral.
- Frequency peaks are tightly correlated with compactness.
- Mass determinations from prompt and delayed black hole formation.
- In neutron star-black hole mergers, disc mass depends on a/M_{BH} and on $M_{NS}M_{BH}/R^2$.
- R-mode instabilities in rotating neutron stars.

Conclusions

- Measured neutron star masses imply lower limits to radii of typical neutron stars.
- Symmetry energy determines typical neutron star radii.
- Nuclear experiments set reasonably tight constraints on symmetry energy parameters.
- Theoretical calculations of pure neutron matter predict very similar symmetry constraints.
- These constraints predict neutron star radii $R_{1.4} = 12.0 \pm 1.4$ km.
- Combined astronomical observations of photospheric radius expansion X-ray bursts and quiescent sources in globular clusters suggest $R_{1.4} \sim 12.1 \pm 0.6$ km.
- The properties of a high-density phase, such as quark matter, are tightly constrained by current mass measurements.
- A mass measurement above $2.4 M_{\odot}$ may be incompatible with other constraints, assuming GR is correct.