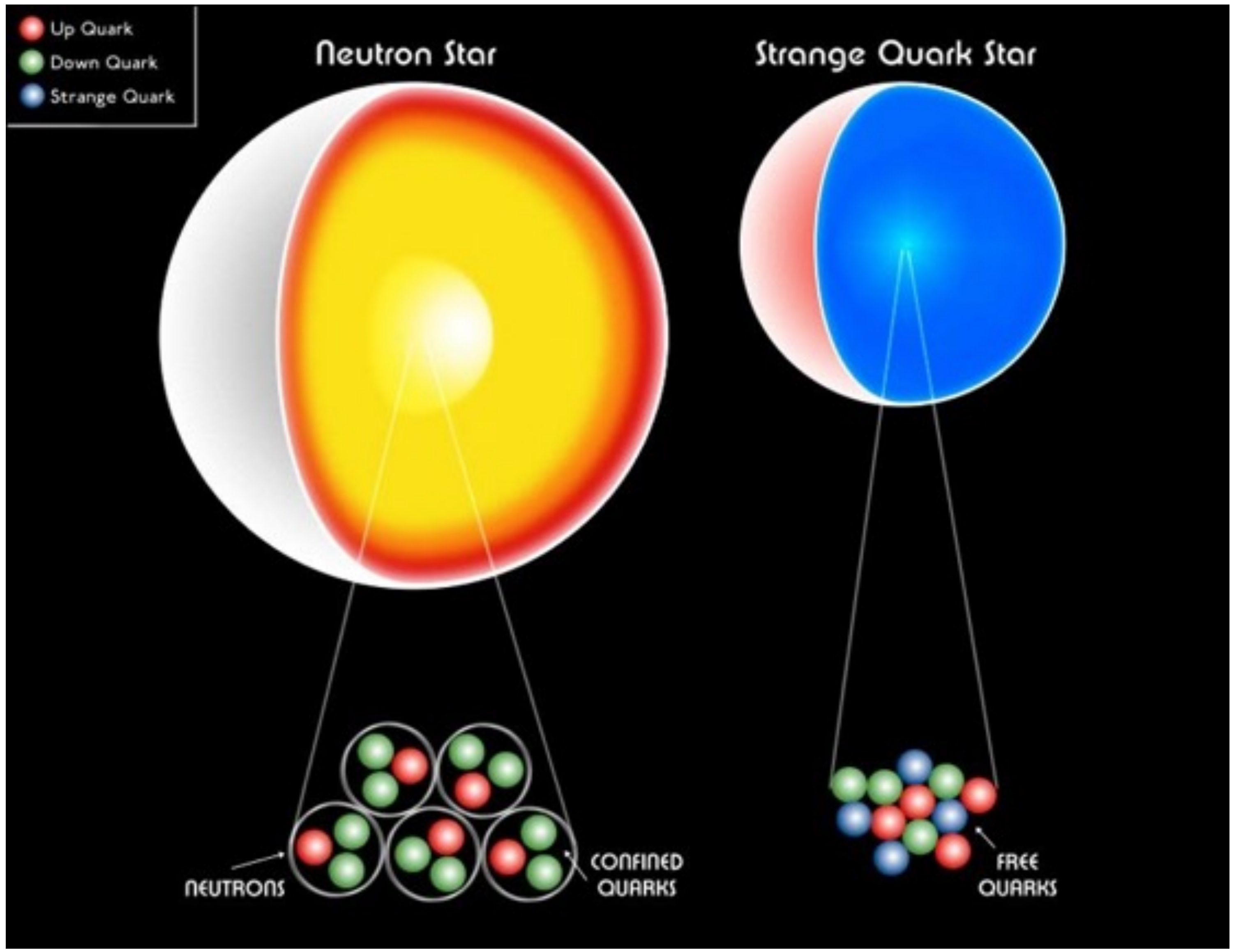
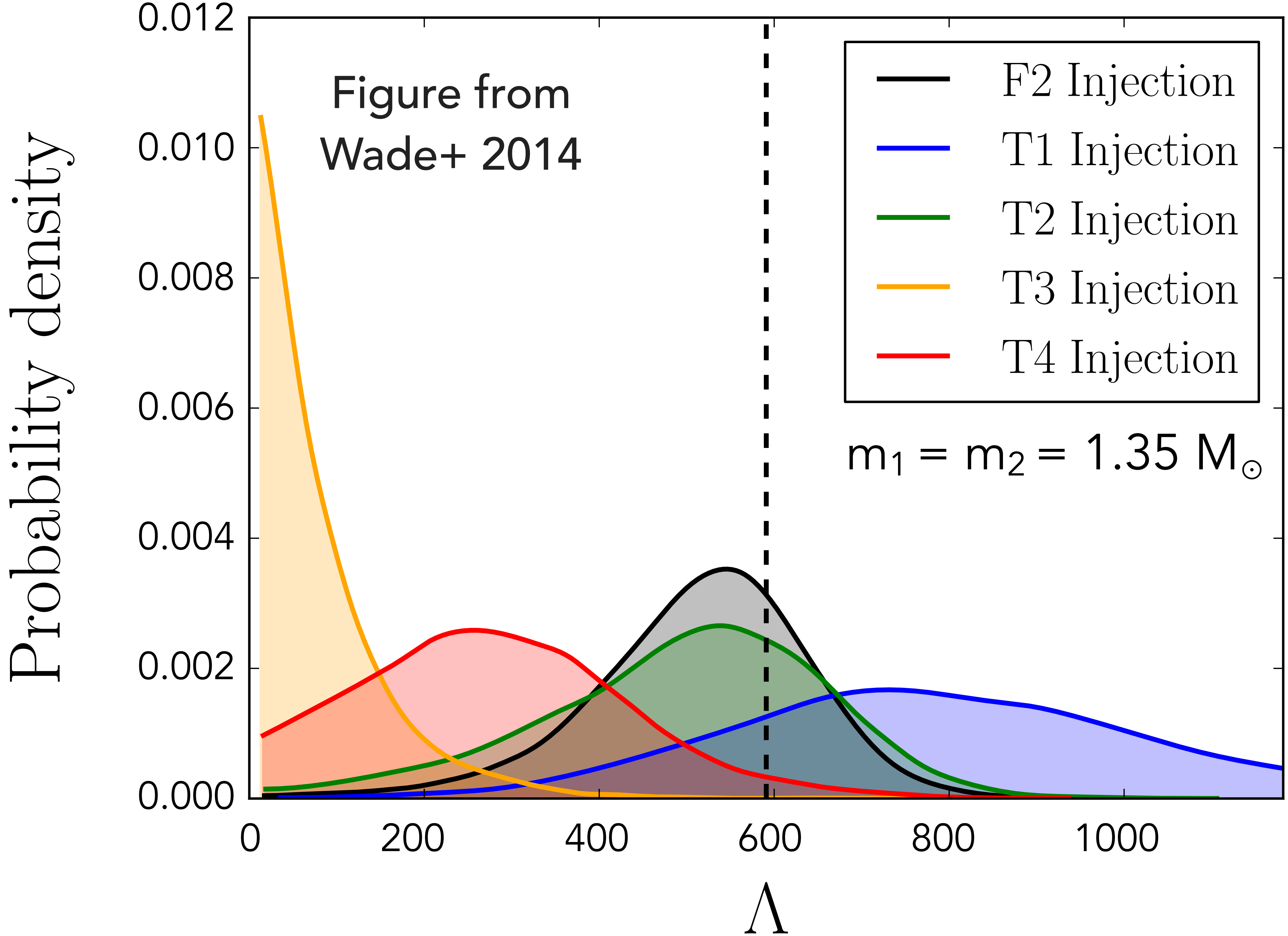
HOW WELL DO WE KNOW WAVEFORMS AND STATUS OF ADVANCED AND 3G DETECTORS



B.S. SATHYAPRAKASH
CARDIFF UNIVERSITY, CARDIFF, UK

IF OUR WAVEFORMS ARE WRONG WE ARE DOOMED!



Wade+ 2014, Favata 2013, Yagi+ 2013

HOW WELL DO WE KNOW OUR WAVEFORMS?
OR
IS THE SITUATION AS BAD AS THAT?

MODELLING COALESCING BINARIES

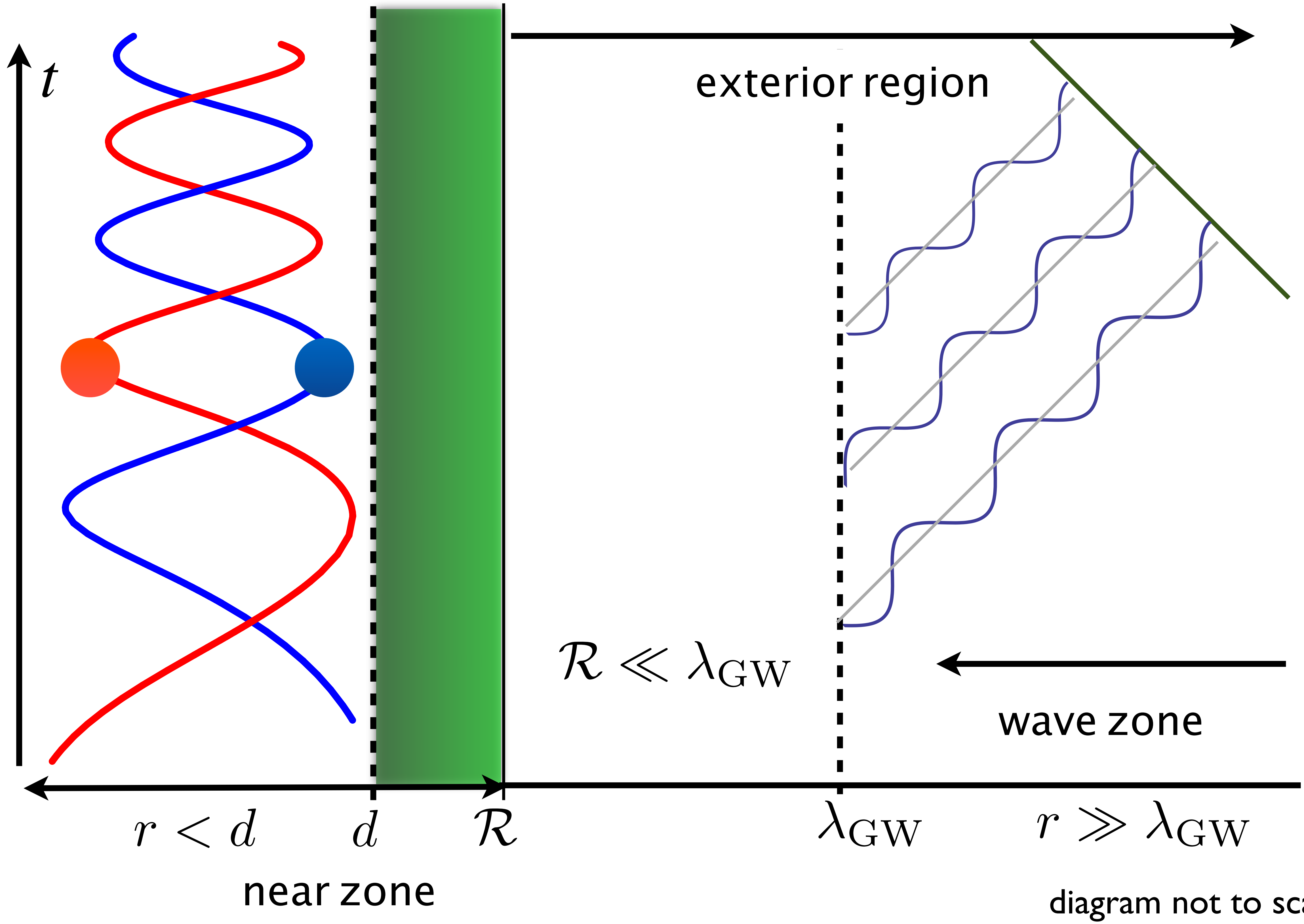
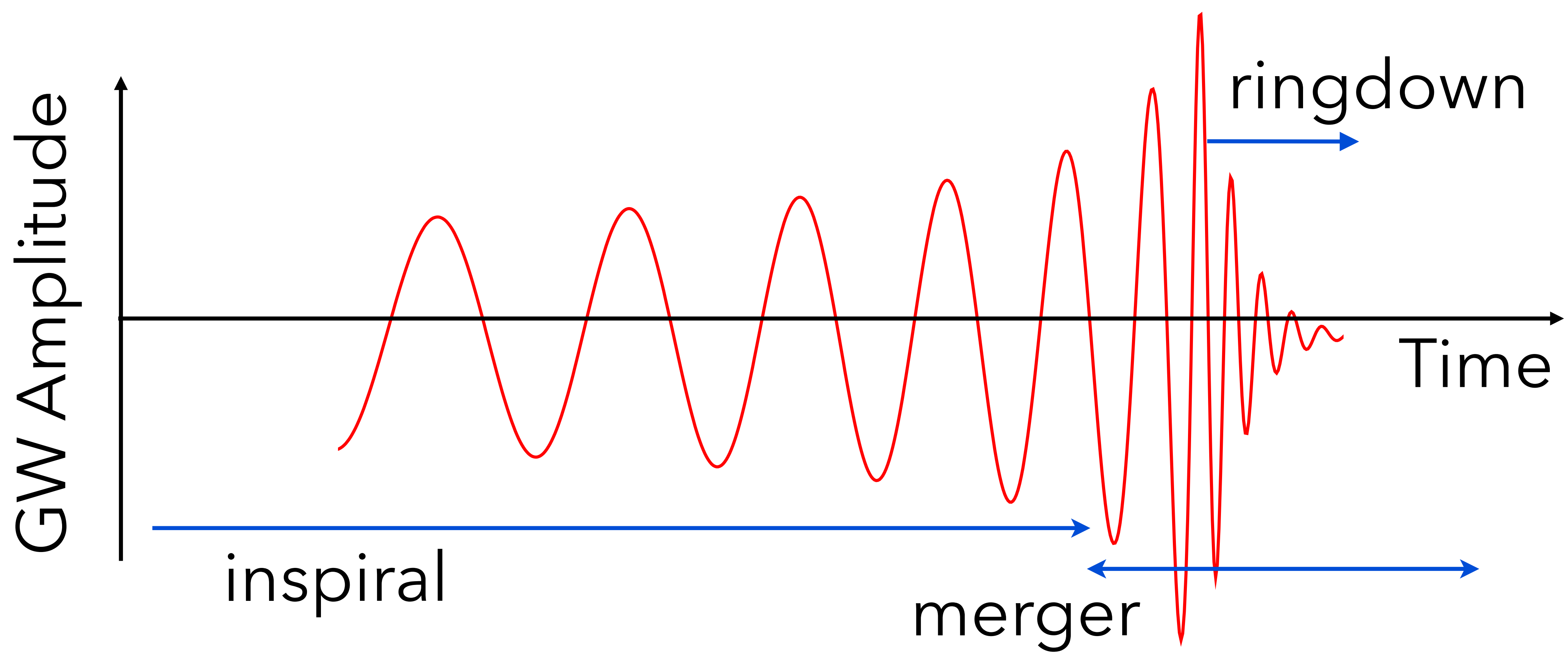


Figure from Buonanno and BSS 2014

BINARY BLACK HOLE DYNAMICS

- to understand dynamics of binary neutron stars we must first control binary black hole dynamics
- signal from a binary black hole is characterised by
  - slow adiabatic inspiral
  - fast and luminous merger
  - rapid ringdown phase
- shape of the signal contains information about the binary



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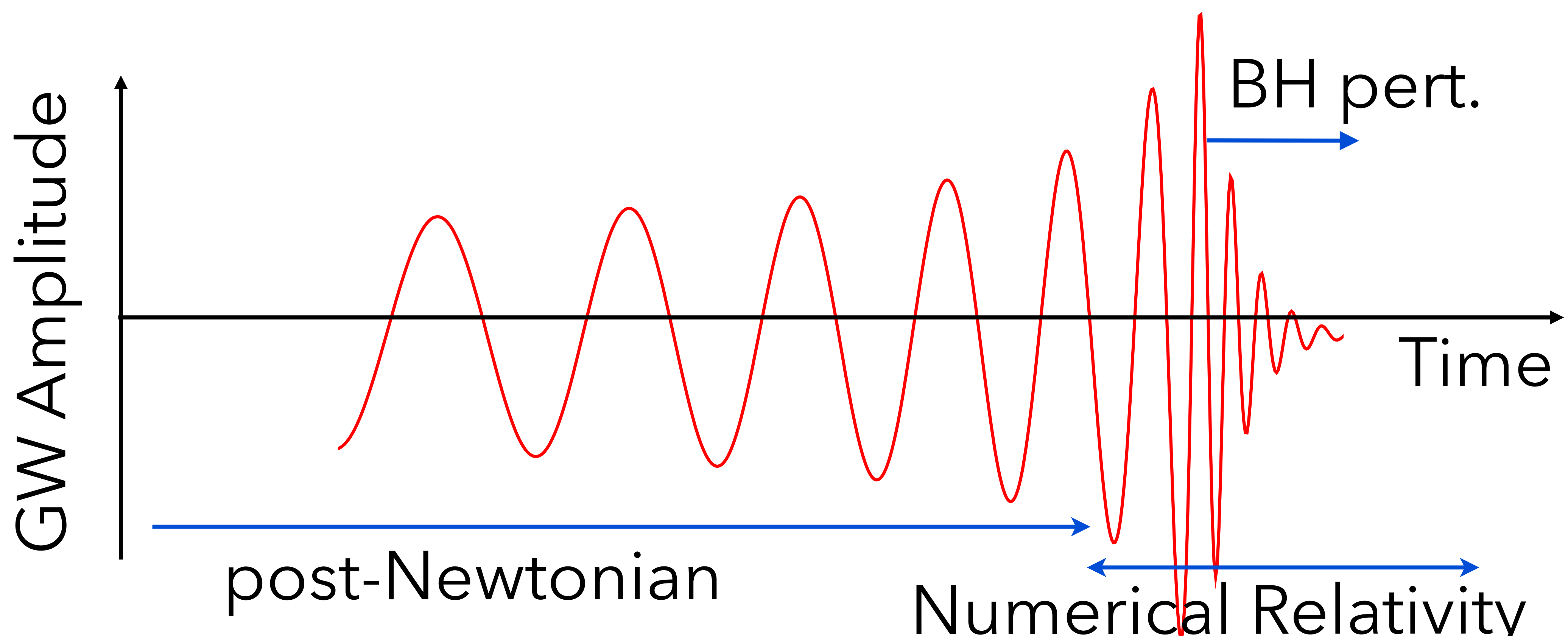
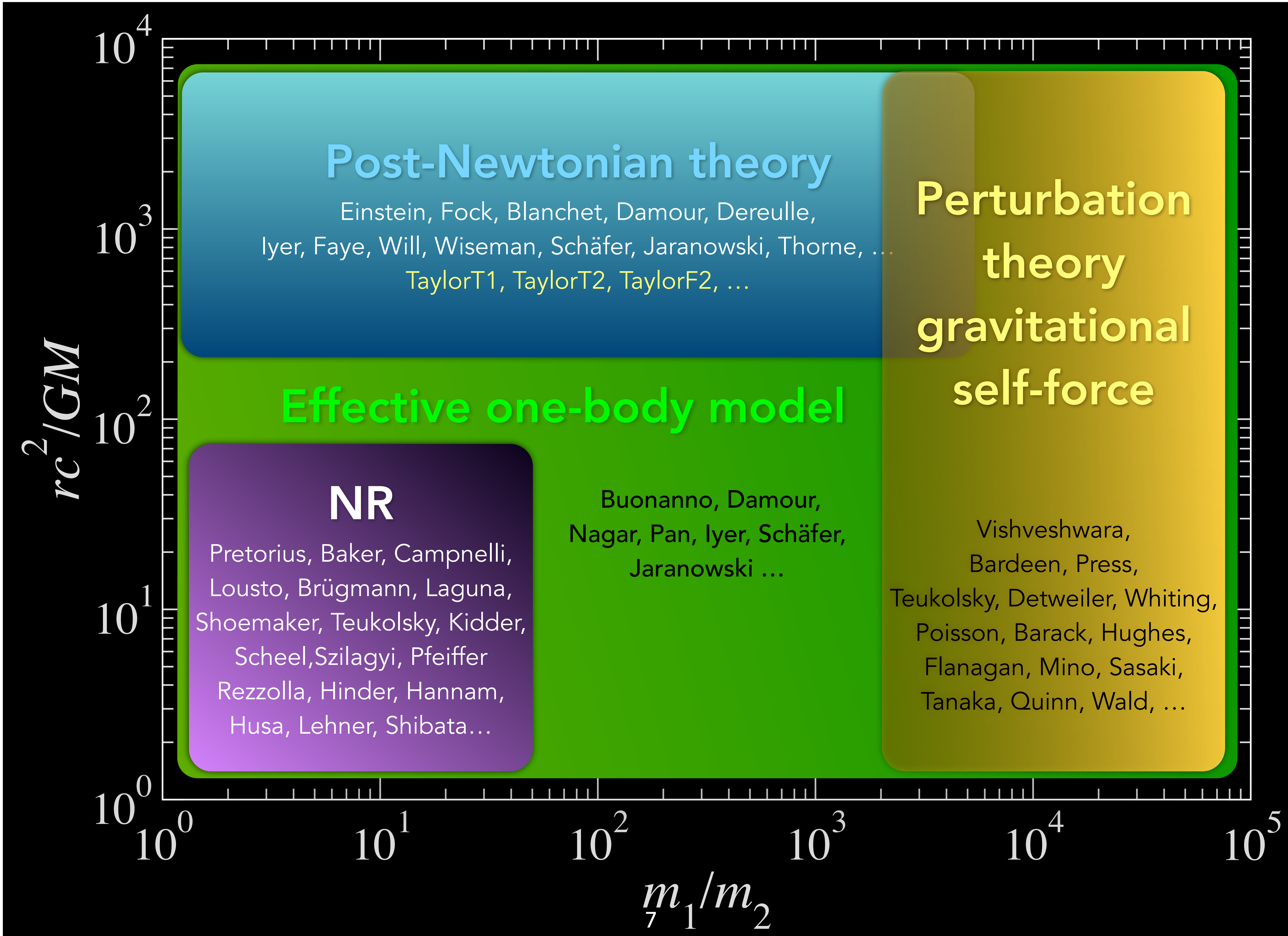




Post-Newtonian theory
Einstein, Fock, Blanchet, Damour, Dereulle,
Iyer, Faye, Will, Wiseman, Schäfer, Jaranowski, Thorne, …
TaylorT1, TaylorT2, TaylorF2, …
Perturbation theory gravitational self-force
Effective one-body model
NR
Pretorius, Baker, Campnelli, Lousto, Brügmann, Laguna, Shoemaker, Teukolsky, Kidder, Scheel,Szilagyi, Pfeiffer Rezzolla, Hinder, Hannam, Husa, Lehner, Shibata…
Buonanno, Damour, Nagar, Pan, Iyer, Schäfer, Jaranowski …
Vishveshwara, Bardeen, Press, Teukolsky, Detweiler, Whiting, Poisson, Barack, Hughes, Flanagan, Mino, Sasaki, Tanaka, Quinn, Wald, …
rc^2/GM
m_1/m_2
10^0
10^1
10^2
10^3
10^4
10^5

ORBITAL DYNAMICS AND METRIC PERTURBATION

- **orbital dynamics** is needed to obtain an accurate formula for evolution of phase $\varphi(t)$ and frequency $\omega(t)$
  - post-Newtonian theory gives energy and flux of gravitational waves
  - phase and frequency evolution computed from either energy balance formula or a Hamiltonian description that uses a RR force
- **metric perturbation** to compute the detector response
  - post-Newtonian theory is used to iteratively solve for metric perturbation h^{lm} which is then used to compute the plus and cross polarizations: $h_+ + i\, h_x = \sum {}_{-2}Y_{lm}\, h^{lm}$
  - detector response is a linear combination of the two polarisatios: $h(t) = h_+ F_+ + i\, h_x F_x$, where F_+ and F_x are antenna response functions

POST-NEWTONIAN (PN) SOLUTION TO ORBITAL DYNAMICS

- end product of post-Newtonian approximation is the computation of orbital binding energy E and gravitational wave luminosity L
- E and L are derived as asymptotic series in the small parameter v - the orbital velocity:

$$E(v) = \frac{mv^2}{2}\left(1 + e_2 v^2 + e_4 v^4 + e_6 v^6 + \ldots\right)$$

$$L(v) = \frac{32\,\nu^2\,v^{10}}{5}\left(1 + l_2 v^2 + l_3 v^3 + l_4 v^4 + l_5 v^5 + l_6 v^6 + l_7 v^7 \ldots\right)$$

- a phasing formula is derived using energy balance:

ENERGY BALANCE EQUATION

- energy lost to gravitational waves comes from the (negative) time rate of change of binding energy:

$$L(v) = -\frac{\mathrm{d}E(v)}{\mathrm{d}t} = -\frac{\mathrm{d}E(v)}{\mathrm{d}v}\frac{\mathrm{d}v}{\mathrm{d}t} \quad \Rightarrow \quad \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{L(v)}{E'(v)}$$

- to obtain an expression for the orbital phase as a function of time one must supplement the above energy balance equation with Kepler's third law:

$$\omega \equiv \frac{\mathrm{d}\varphi}{\mathrm{d}t} = \frac{v^3}{GM}, \quad \frac{\mathrm{d}\varphi}{\mathrm{d}t} = \frac{\mathrm{d}\varphi}{\mathrm{d}v}\frac{\mathrm{d}v}{\mathrm{d}t} \quad \Rightarrow \quad \frac{\mathrm{d}\varphi}{\mathrm{d}v} = \frac{v^3}{GM}\frac{E'(v)}{L(v)}$$

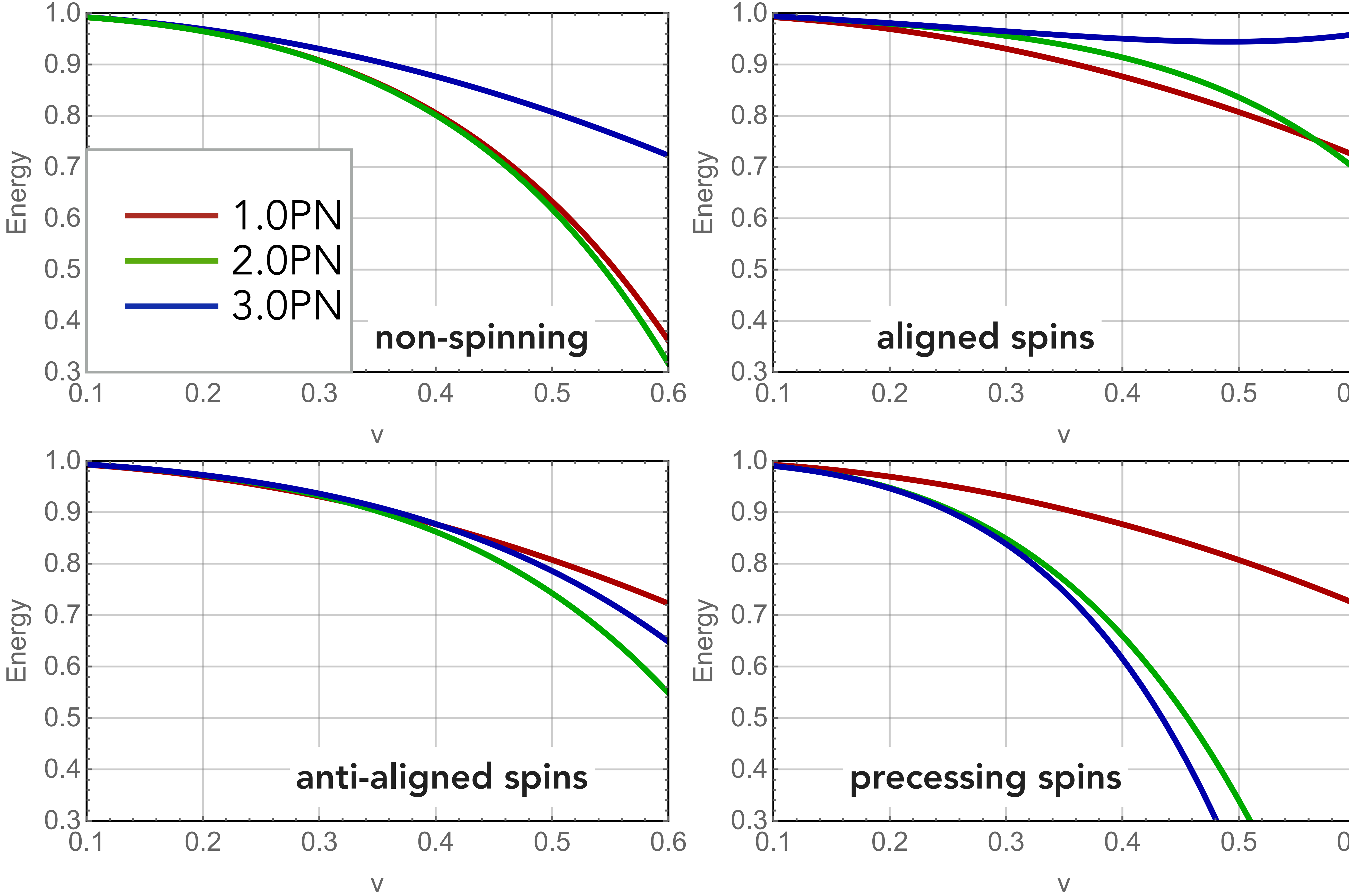
WHY DO WE HAVE SO MANY POSTNEWTONIAN WAVEFORM MODELS?

- solves a pair of ODEs to obtain a phasing formula $\varphi(t)$:

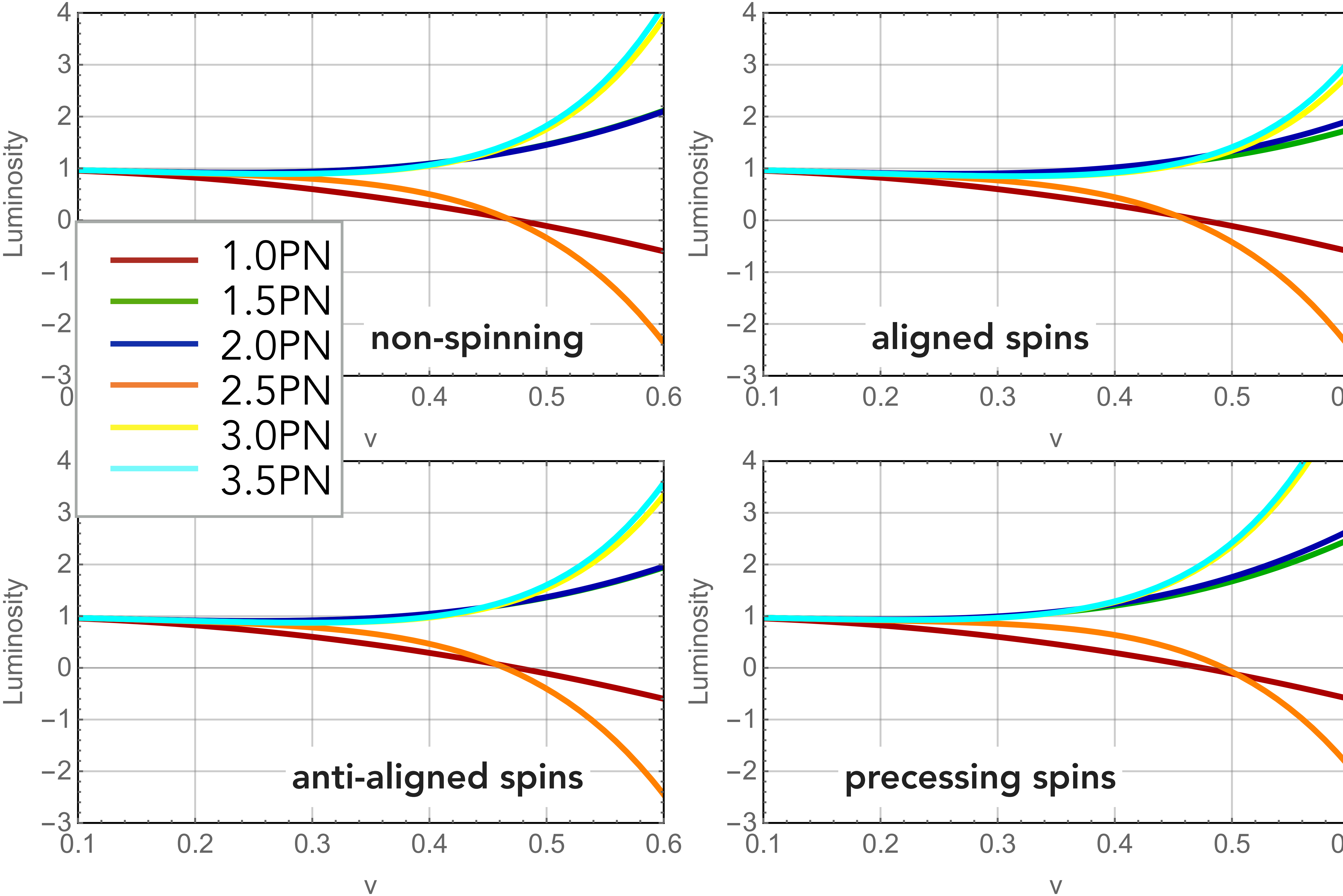
$$\frac{\mathrm{d}\varphi}{\mathrm{d}t} = \frac{v^3}{GM} \quad \text{or} \quad \frac{\mathrm{d}\varphi}{\mathrm{d}v} = \frac{v^3}{GM}\frac{E'(v)}{L(v)} \quad \text{and} \quad \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{L(v)}{E'(v)}$$

- recall that L and E are both asymptotic series or Taylor series with a finite number of turns
- they can be approximated in different ways leading to numerically different, but perturbatively equivalent, formulas for the phasing of the binary orbit

ENERGY AT DIFFERENT PN ORDERS



LUMINOSITY AT DIFFERENT PN ORDERS



PROBLEMS WITH PN AND CURES

- PN being a Taylor series, quantities of interest cannot have poles
  - poles can be artificially introduced by approximating a Taylor series as a rational polynomial
- energy is expected to have an extrema but PN series might not have any in the region of interest
  - one can introduce extrema at desired points by factorising the zeroes
- PN series is poorly convergent
  - re-summation techniques can be used to accelerate the convergence of PN series (but has converged to the right value?)

EXAMPLES OF TRANSFORMATIONS

$$E(v) = E_0(v)(1 + e_2 v^2 + e_3 v^3 + \ldots)$$

a zero

$$= E_0(v)(1 - v/v_0)(1 + f_1 v + f_2 v^2 + f_3 v^3 + \ldots)$$

$$\Rightarrow f1 = 1/v_0,\ f_2 = e_2 + (f_1/v_0),\ f_3 = e_3 + (f_2/v_0)$$

$$F(v) = F_0(v)(1 + a_2 v^2 + a_3 v^3 + \ldots)$$

a pole

$$= \frac{F_0(v)}{(1 - v/v_p)}(1 + b_1 v + b_2 v^2 + b_3 v^3 + \ldots)$$

$$\Rightarrow b_1 = -1/v_p,\ b_2 = a_2,\ b_3 = a_3 - a_2/v_p$$

- Reformed expressions are perturbatively equivalent but numerically different and have different asymptotic structure

LANDSCAPE OF WAVEFORMS AND WHY THEY ARE NOT ALL RIGHT

PROLIFERATION OF PN APPROXIMATES

$$\frac{\mathrm{d}\varphi}{\mathrm{d}t} = \frac{v^3}{GM} \quad \text{or} \quad \frac{\mathrm{d}\varphi}{\mathrm{d}v} = \frac{v^3}{GM}\frac{E'(v)}{L(v)} \quad \text{and} \quad \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{L(v)}{E'(v)}$$

- There are many Taylor models
  - **TaylorT1**: numerical solve the ODEs to get φ(t) and v(t)
  - **TaylorT4**: expand L(v)/E'(v) in Taylor series and then solve
  - **TaylorT2**: solve φ'(v) and t'(v) ODEs to get φ(v), t(v)
  - **TaylorT3**: invert t(v) PN series to get v(t) and use in φ(v)
  - **TaylorF2**: stationary phase approximation to TaylorT2
- Could also have introduced many more
  - TaylorF1, TaylorF3, TaylorF4, even more Taylor models and their frequency domain equivalents

BEFORE MATURE NR SIMULATIONS ALL WE COULD DO WAS

COMPARISON OF TAYLOR MODELS

- figures-of-merit used for comparison
- **overlap:** scalar product of two waveforms w_1 and w_2:

$$\langle w_1, w_2 \rangle \equiv 4\,\Re \int_{f_{\rm lo}}^{f_{\rm hi}} \frac{W_1(f)\, W_2^*(f)}{S_h(f)} {\rm d}f$$

- **faithfulness:** overlap maximised over phase and time:

$$\mathcal{F}_{w_1,w_2} \equiv \max_{\phi_0, t_C} \langle w_1, w_2 \rangle$$

- **effectualness:** faithfulness maximised also over masses and spins

$$\mathcal{E}_{w_1,w_2} \equiv \max_{\phi_0, t_C, M_1, \nu_1, \ldots} \langle w_1, w_2 \rangle$$

EFFECTUALNESS: BINARY NEUTRON STARS

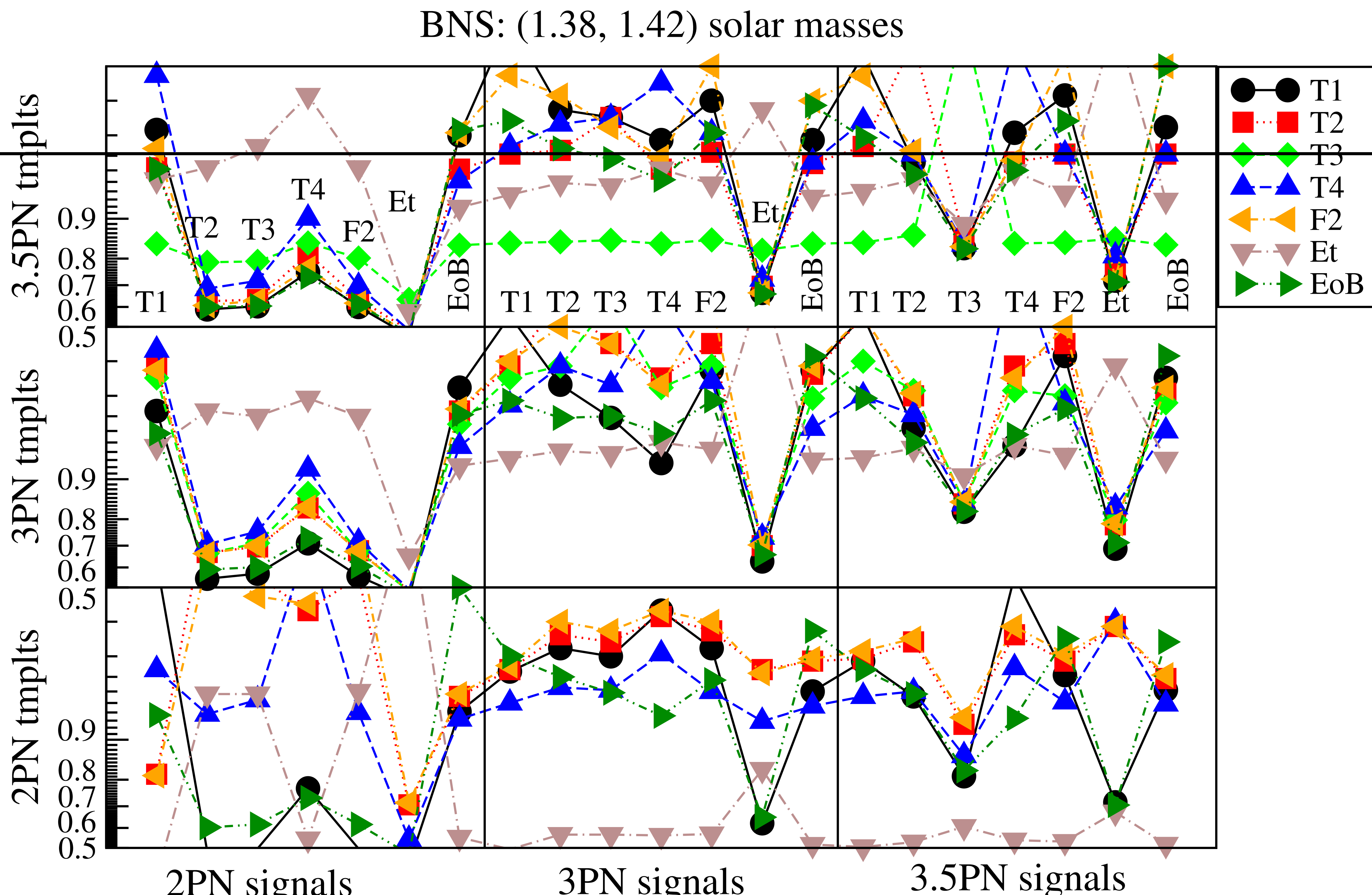


Figure from Buonanno+ 2009

EFFECTUALNESS: BINARY BLACK HOLES

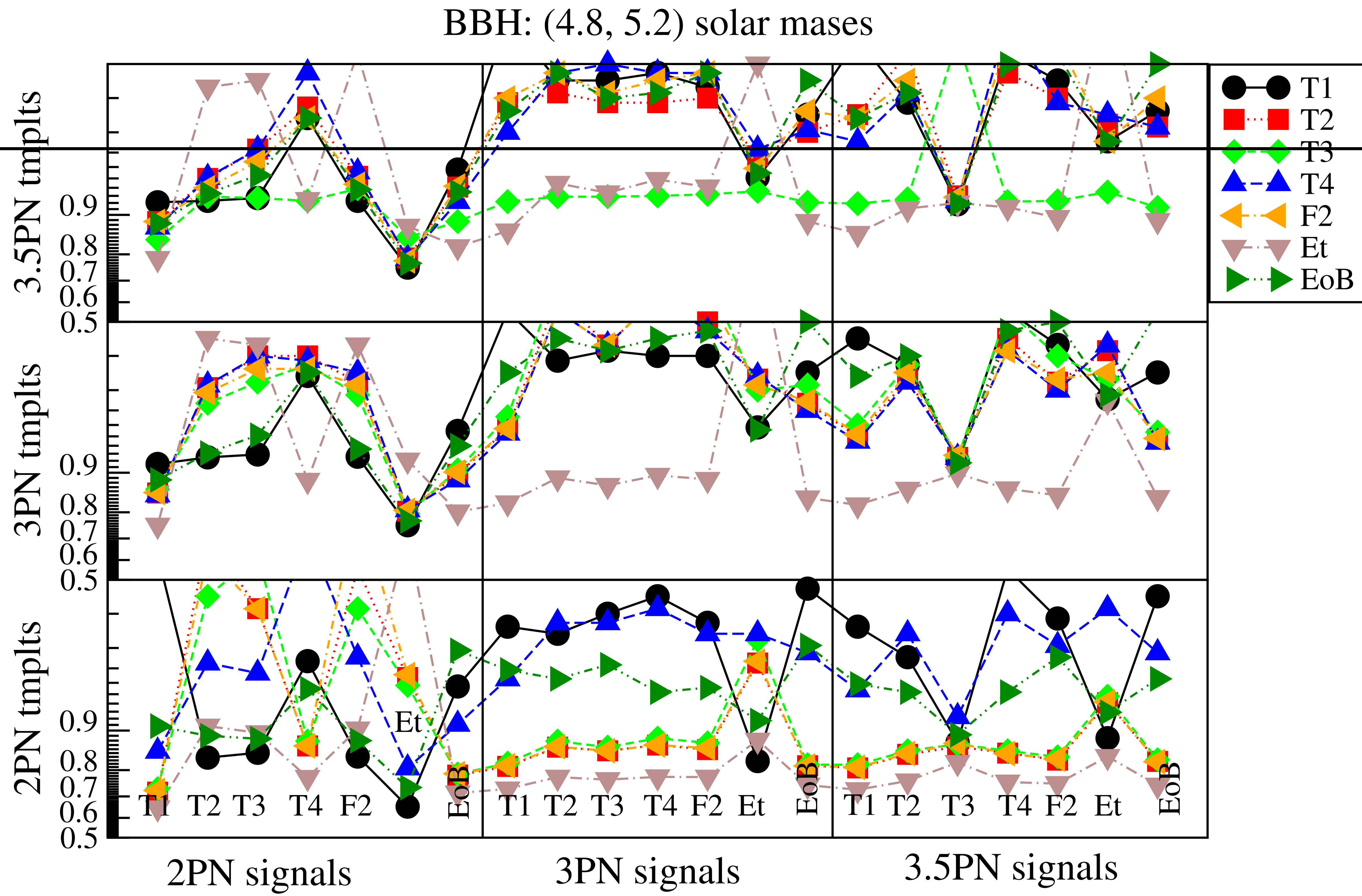


Figure from Buonanno+ 2009

EFFECTUALNESS: NEUTRON STAR-BLACK HOLE BINARY

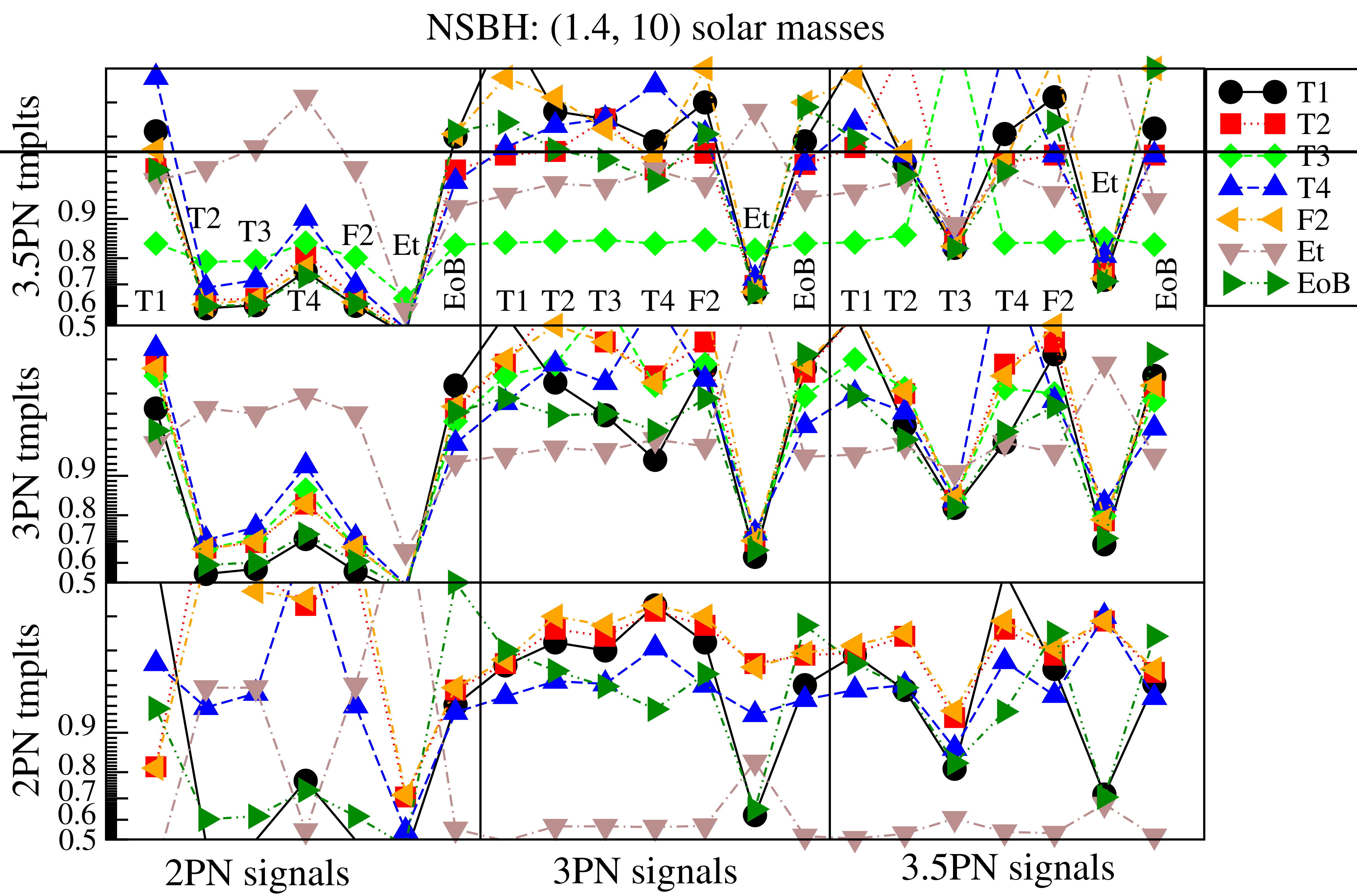


Figure from Buonanno+ 2009

EFFECTUALNESS: BINARY BLACK HOLES

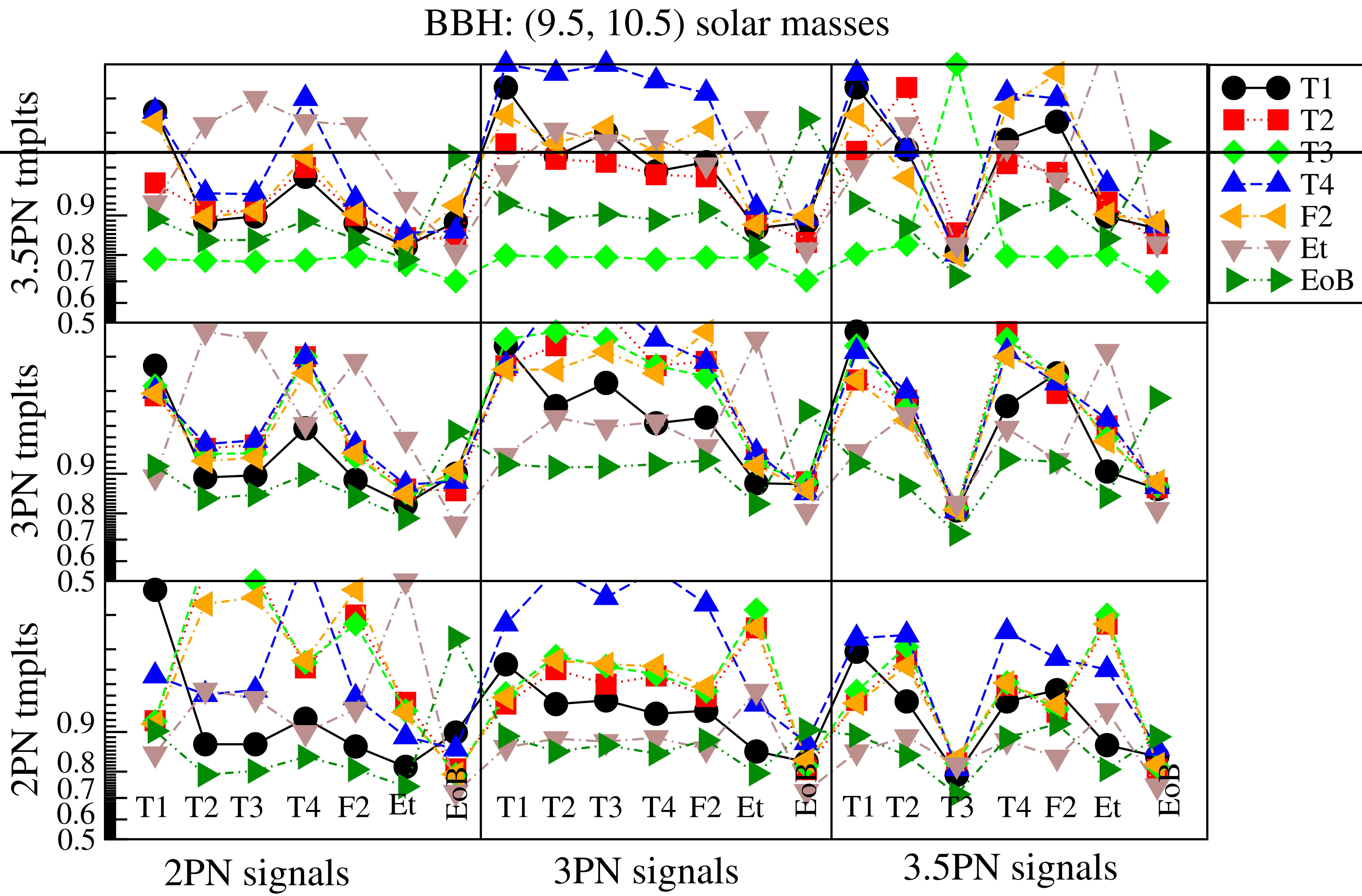
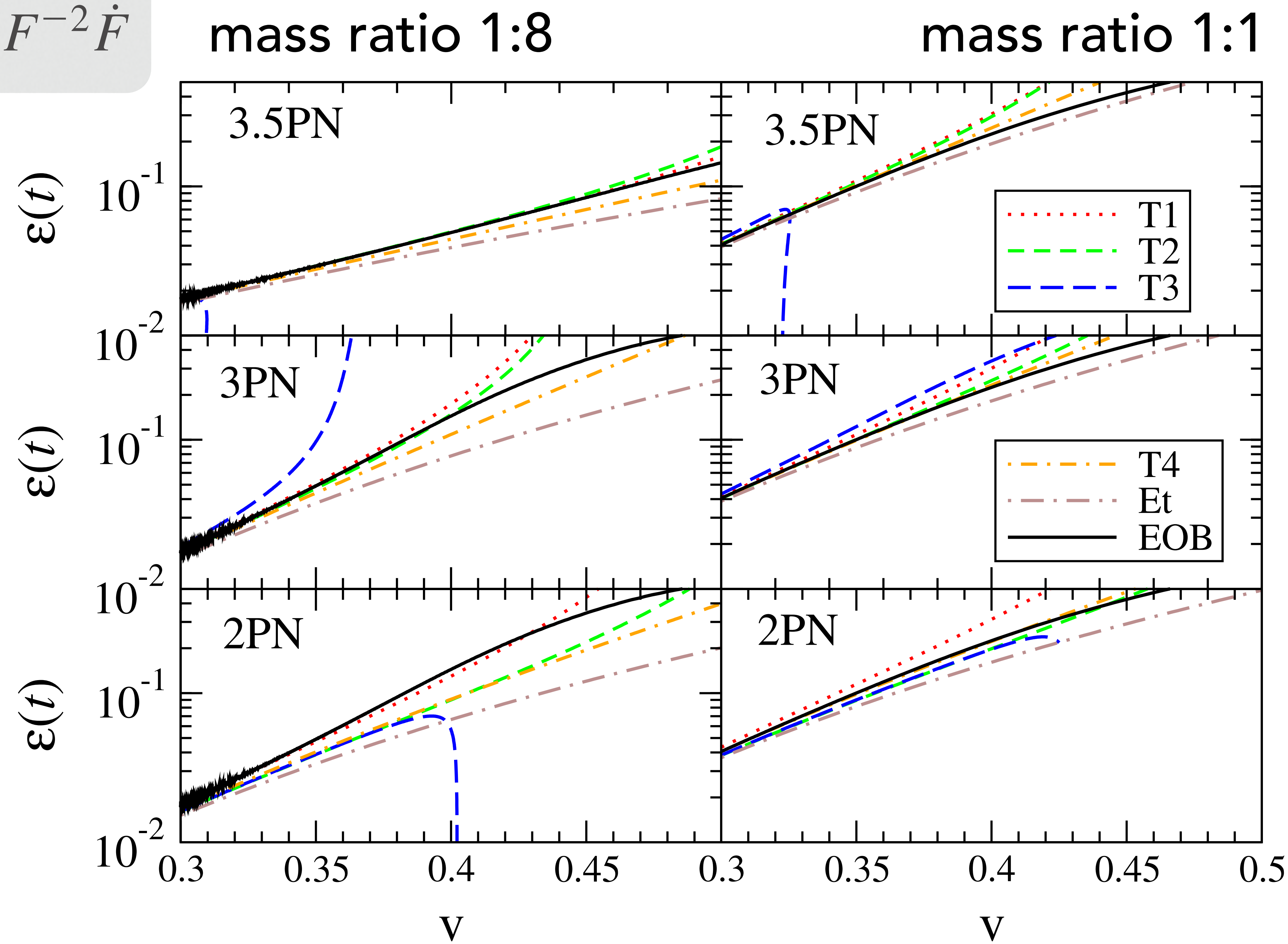


Figure from Buonanno+ 2009

WHY IS TAYLORT3 SO BAD?

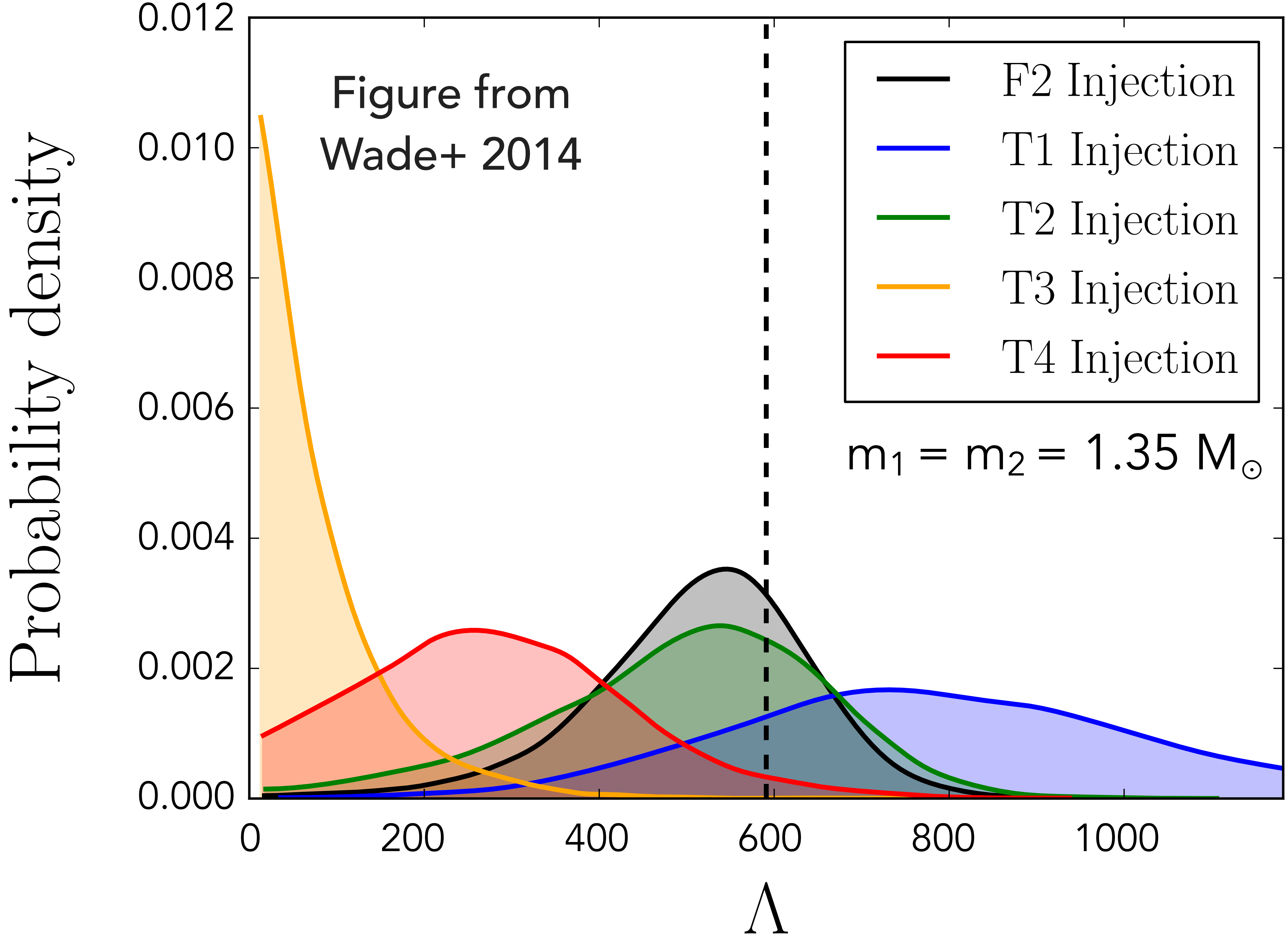
$\epsilon(t) \equiv F^{-2}\dot{F}$



Plots essentially show frequency evolution

Figure from Buonanno+ 2009

PN MODELS OK FOR DETECTION, BUT THEY SHOULD NEVER BE USED FOR PARAMETER ESTIMATION

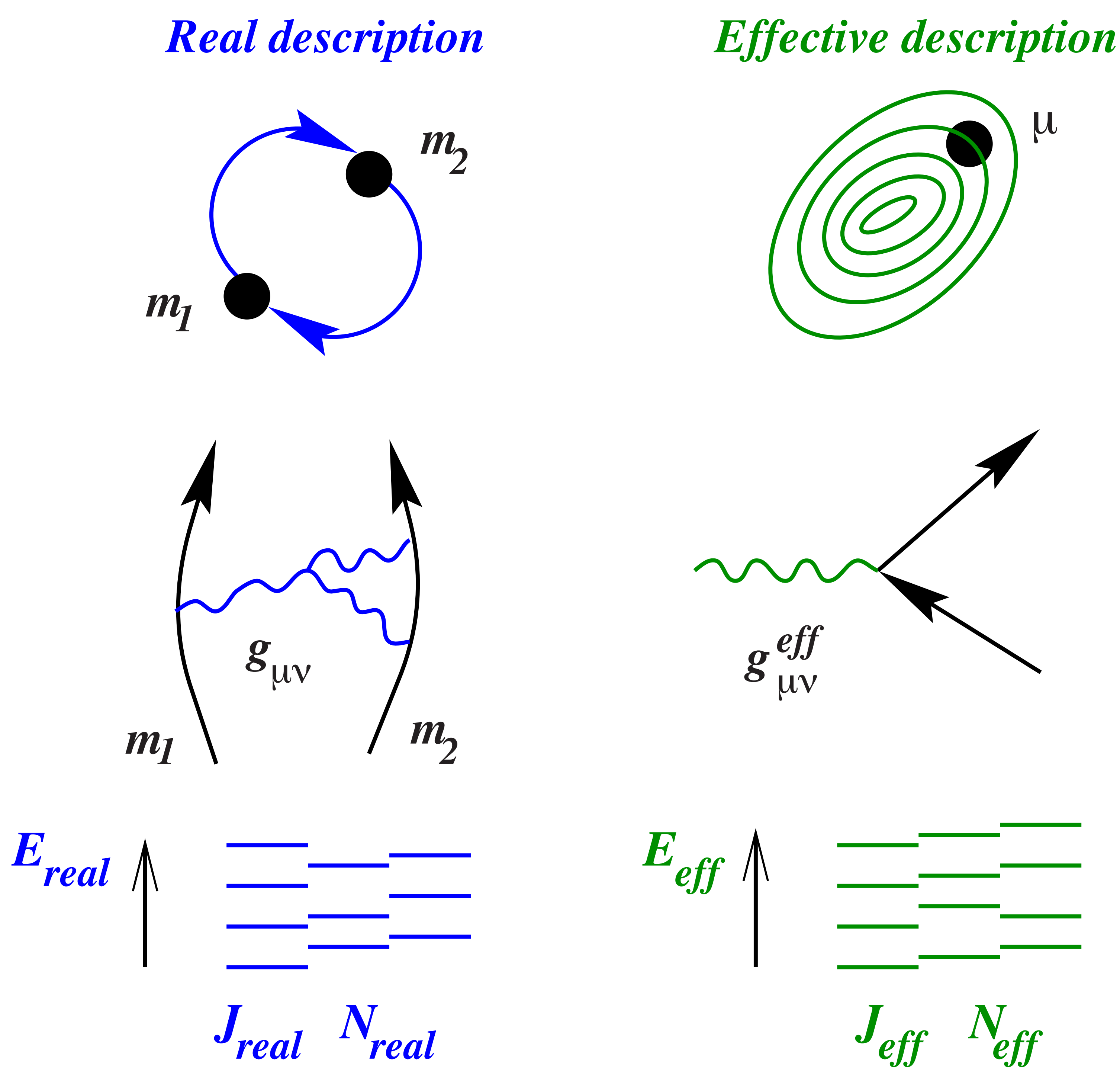


Wade+ 2014, Favata 2013, Yagi+ 2013

WHY AREN'T MISSING POST-NEWTONIAN TERMS IMPORTANT

- starting point of analytical waveform models, such as EOB, is PN equations
  - additionally, convergence techniques are used in EOB
- analytical models introduce additional functions with adjustable parameters to tune the waveforms to numerical relativity
  - typically parameters are tuned or calibrated at a small number of points in the parameter space
  - models are then tested at other points in the parameter space
- so missing post-Newtonian terms are of no consequence

BEYOND INSPIRAL: EFFECTIVE ONE BODY FORMALISM



Buonanno and Damour 1999

Summed PN conservative dynamics in the EOB formalism

"Effective" description

"Real" description

$$H_{\rm real}^{\rm PN} = H_{\rm Newt} + \frac{1}{c^2}\, H_{\rm 1PN} + \frac{1}{c^4} H_{\rm 2PN} + \cdots$$

$$H_{\rm eff}^{\nu} = \mu \sqrt{A_\nu(r)\, \left[1 + \frac{\mathbf{p}^2}{\mu^2} + \left(\frac{1}{B_\nu(r)} - 1\right)\, \frac{p_r^2}{\mu^2}\right]}$$

$$\boxed{H_{\rm real}^{\rm EOB} = M \sqrt{1 + 2\nu\, \left(\frac{H_{\rm eff}^{\nu}}{\mu} - 1\right)}}$$

$$ds_{\rm eff}^2 = -A_\nu(r)\, dt^2 + B_\nu(r)\, dr^2 + r^2\, d\Omega^2$$

- **Dynamic condensed in** $A_\nu(r)$ **and** $B_\nu(r)$
- $A_\nu(r)$**, which encodes the energetics for circular orbits, is rather** *simple*

$$A_\nu(r) = 1 - \frac{2M}{r} + \frac{2M^3\nu}{r^3} + \left(\frac{94}{3} - \frac{41}{32}\pi^2\right) \frac{M^4\,\nu}{r^4} + \frac{a_5(\nu)}{r^5} + \frac{a_6(\nu)}{r^6} + \cdots$$

Buonanno and Damour 1999+, many papers since

EOB dynamics and waveforms

- **EOB dynamics**

$$\dot{\mathbf{r}} = \frac{\partial H^{\rm EOB}}{\partial \mathbf{p}}\,, \qquad \dot{\mathbf{p}} = -\frac{\partial H^{\rm EOB}}{\partial \mathbf{r}} + \mathbf{F}\,, \qquad \mathbf{F} \propto \frac{dE}{dt}\,, \qquad \frac{dE}{dt} = \frac{1}{16\pi}\sum_{\ell,m} |\dot{h}_{\ell m}|^2$$

$$\dot{\mathbf{S}}_1 = \frac{\partial H^{\rm EOB}}{\partial \mathbf{S}_1} \times \mathbf{S}_1\,, \qquad \dot{\mathbf{S}}_2 = \frac{\partial H^{\rm EOB}}{\partial \mathbf{S}_2} \times \mathbf{S}_2$$

[AB & Damour 00; Damour et al. 98; AB et al. 05; Damour et al. 07-09; AB et al. 09; Pan et al. 09]

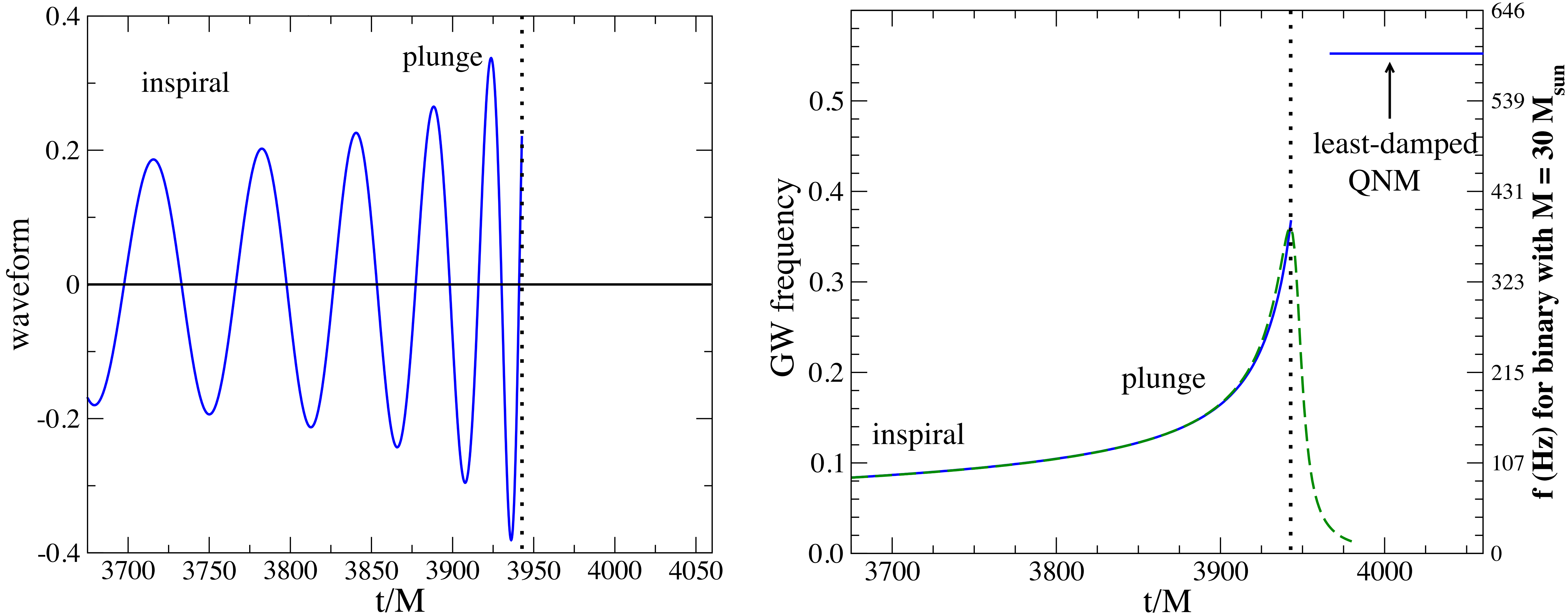
- **EOB (factorized) waveforms**

$$h_{22}(t) = -\frac{8\,\pi}{5}\,\frac{\nu\,M}{R}\,v^2\,e^{-2i\,\Phi}\left\{1 - \left(\frac{107}{42} - \frac{55}{42}\,\nu\right)\,v^2 + \left[2\pi + 12i\,\log\left(\frac{v}{v_0}\right)\right]\,v^3 + \ldots\right\}$$

$$h_{\ell m}^{\rm insp-plunge}(t) = \hat{h}_{\ell m}^{N}\,e^{-im\Phi}\,\mathcal{S}_{\rm eff}\,T_{\ell m}\,e^{i\delta_{\ell m}}\,(\rho_{\ell m})^{\ell}\,h_{\ell m}^{\rm NQC}(a_i, b_i)$$

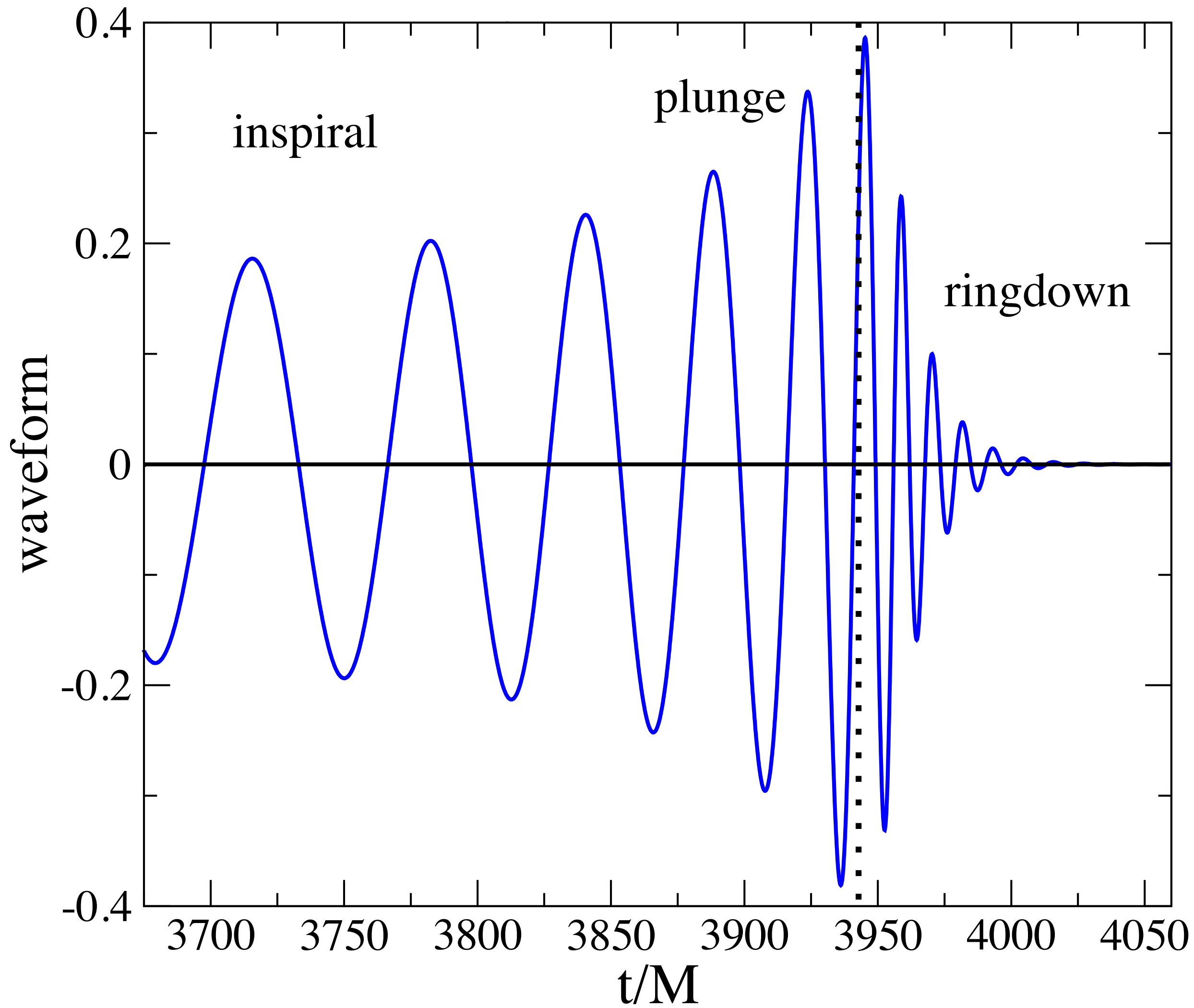
[Damour, Iyer & Nagar 09; Fujita & Iyer 10; Pan, AB, Fujita, Racine & Tagoshi 10]

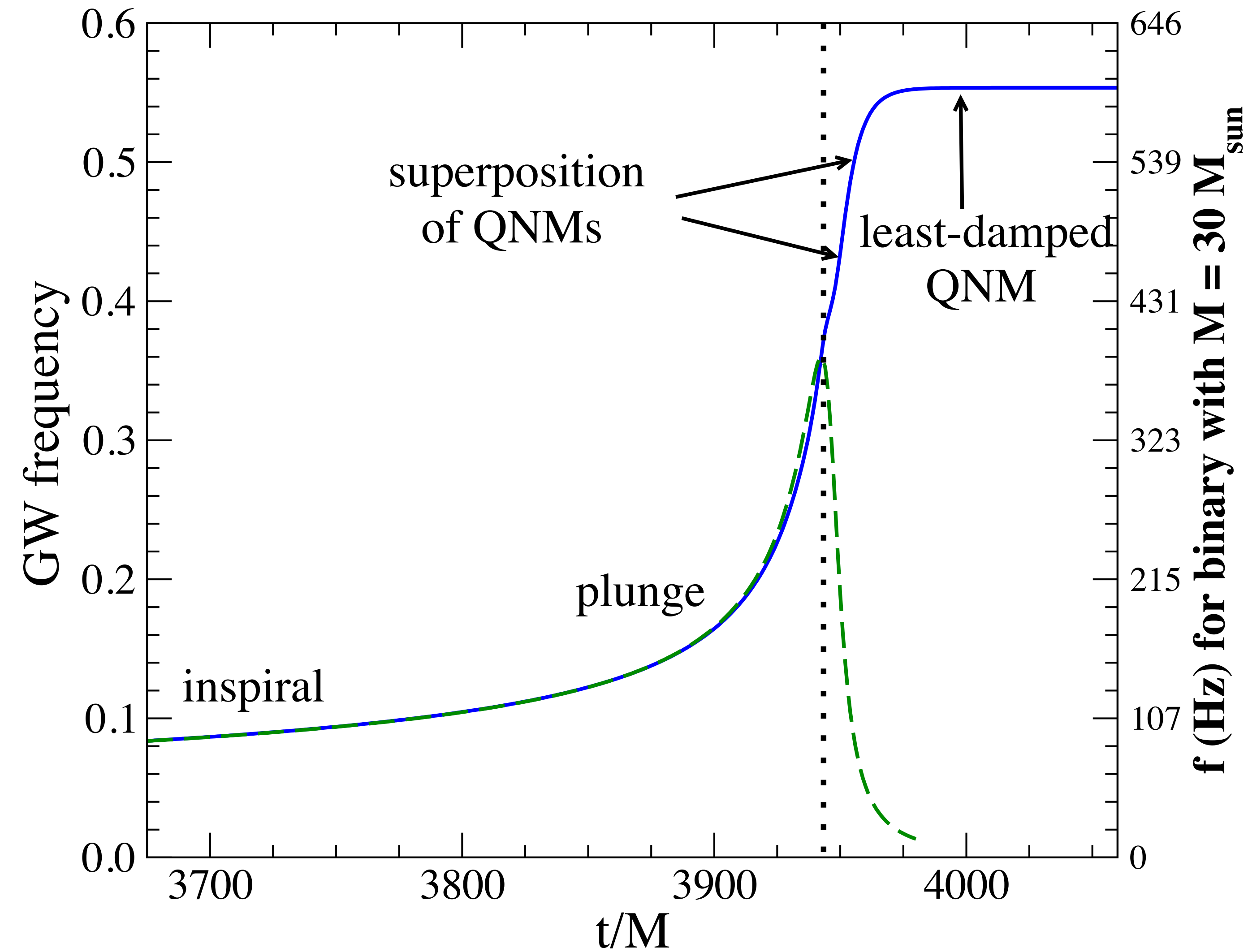
BUILDING AN EOB MODEL



Buonanno and Damour 1999

BUILDING AN EOB MODEL

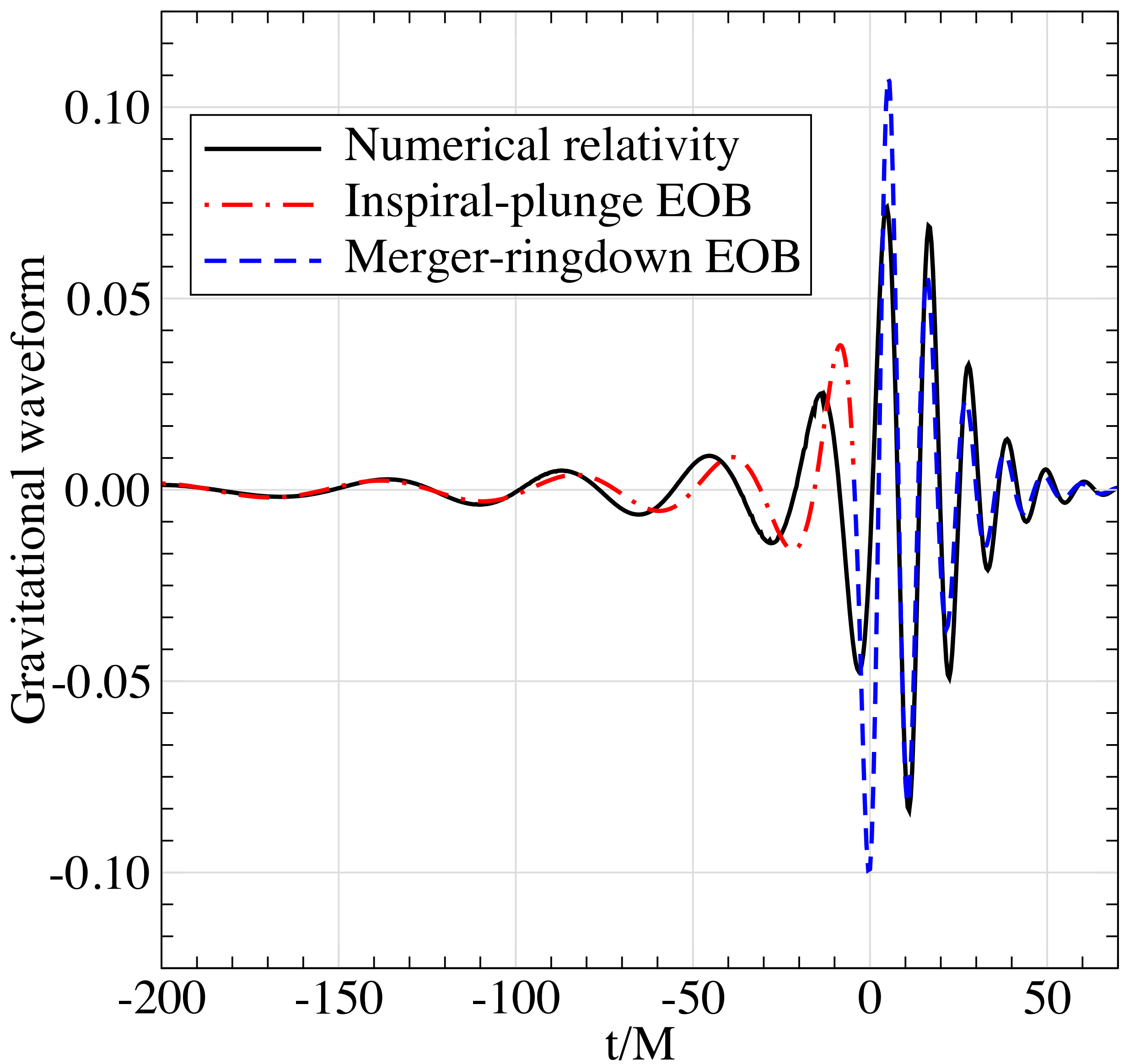




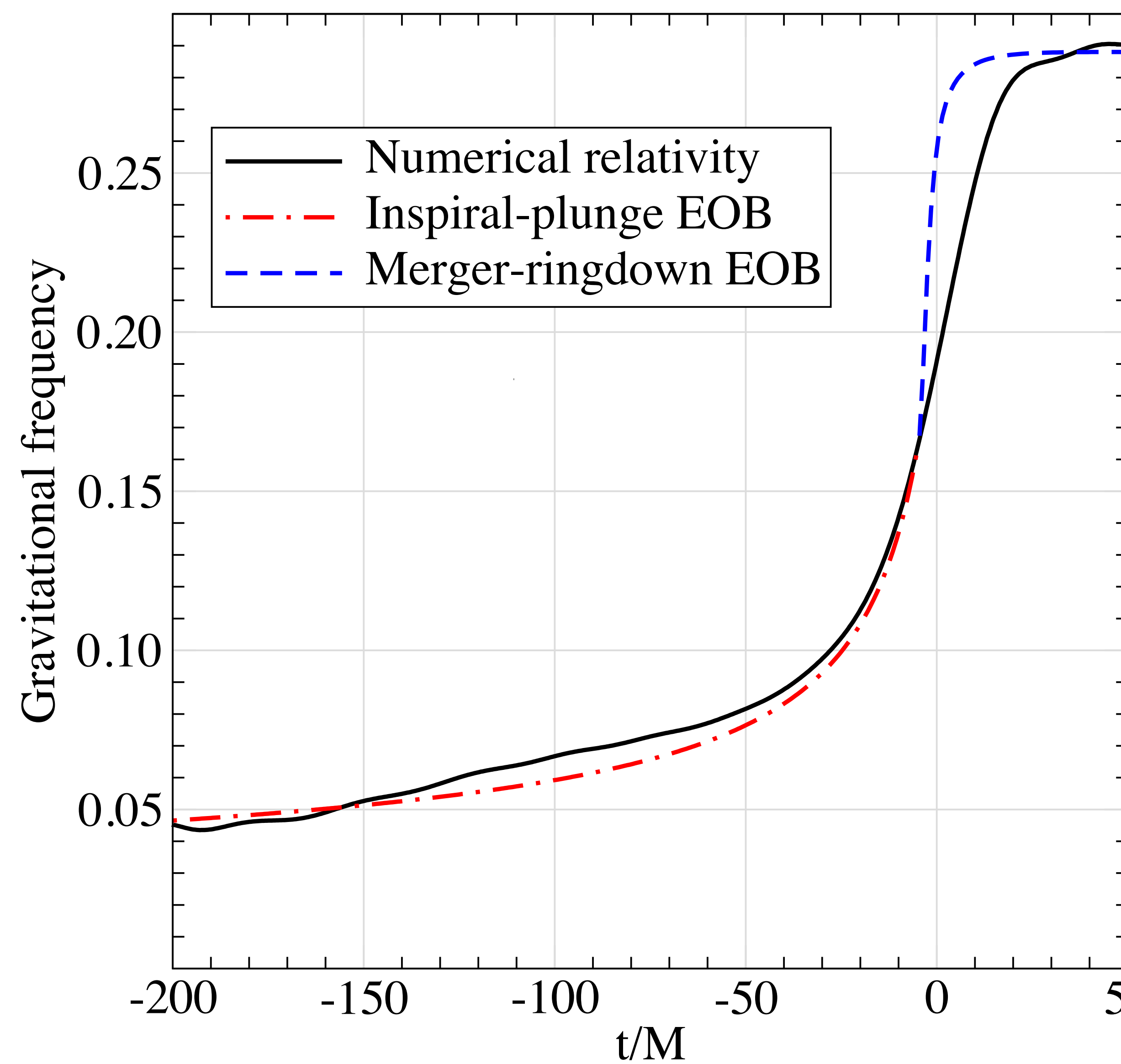
Buonanno and Damour 1999

FIRST EOB-NR COMPARISON

Waveform



Phase



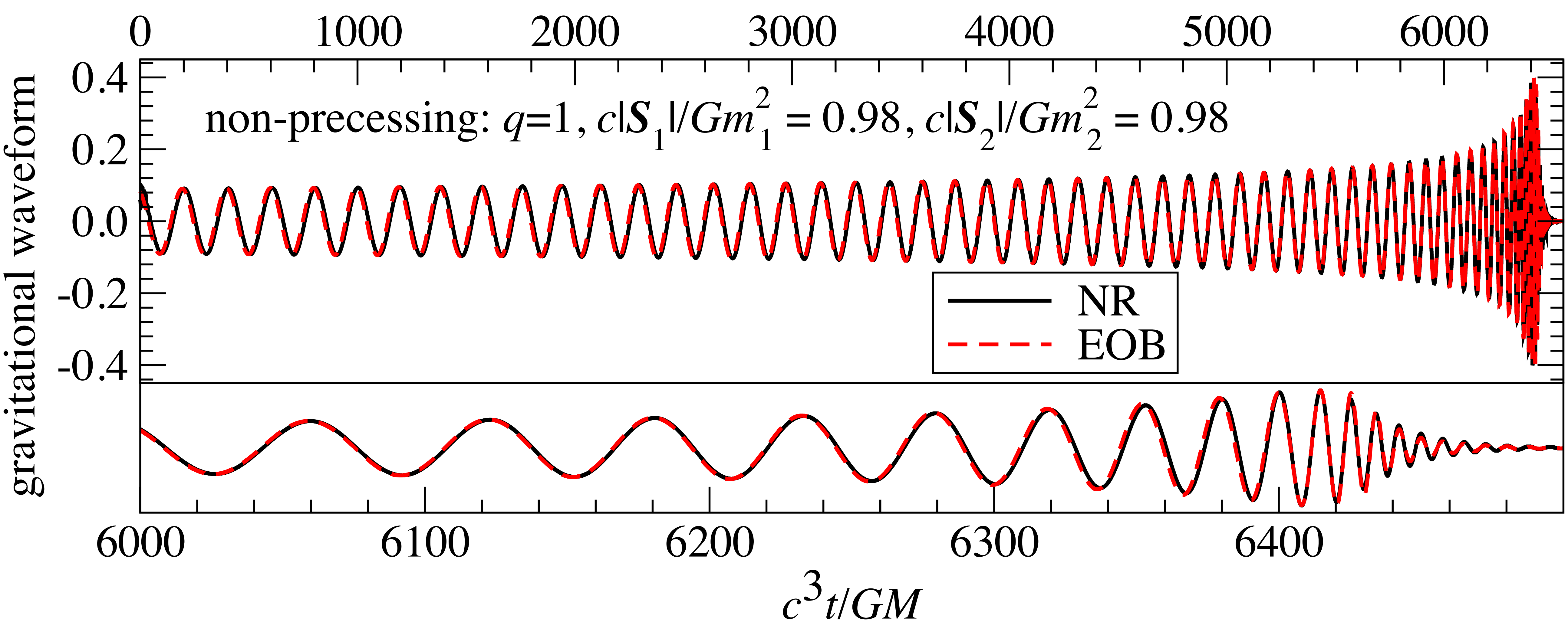
- **Very** *short* **transition merger–ringdown**
- **Energy quickly released during merger**

- $E_{\rm rad} \sim 2\%–12\%\ M\,c^2$

  $1 M_\odot\, c^2 \sim 10^{54}\,\text{erg} \sim 10^{56}\,\text{GeV!}$

COMPARISON OF EOB AND NR WAVEFORMS

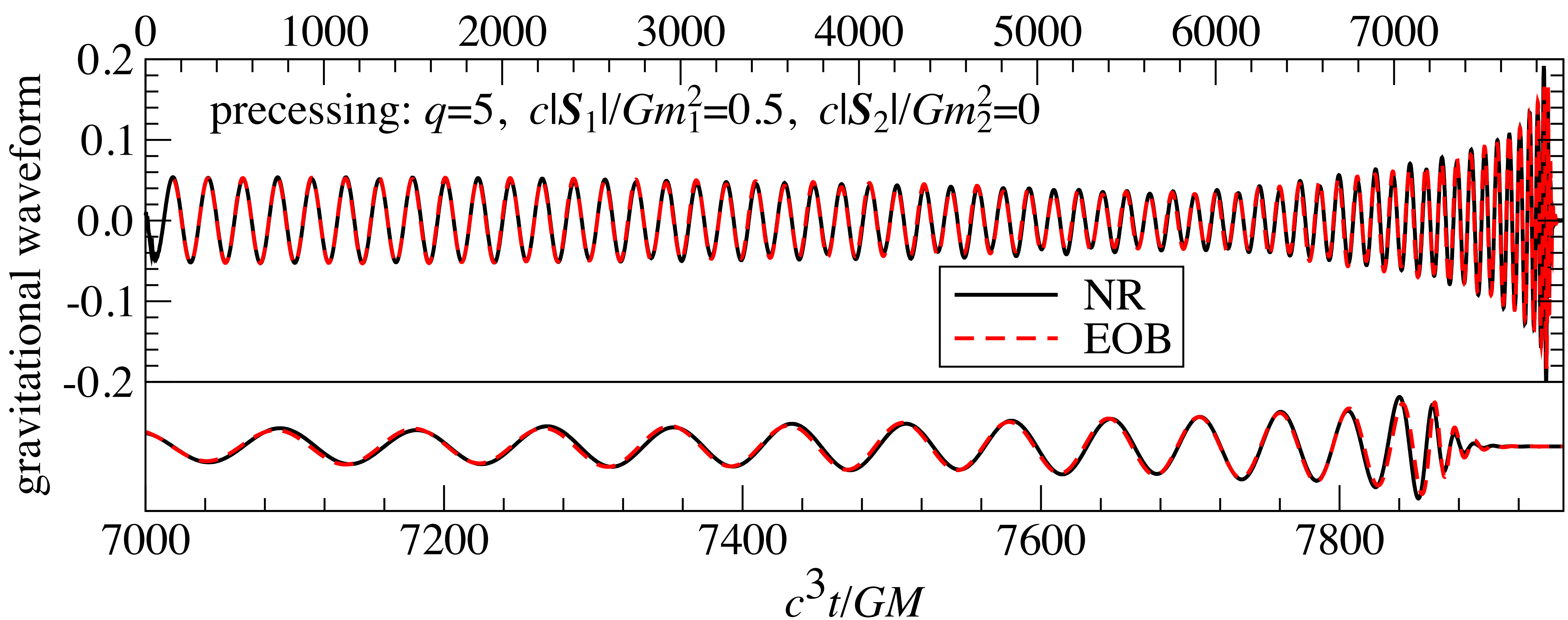
- aligned large spins (not expected in BNS), equal masses



Taracchini+ 2012

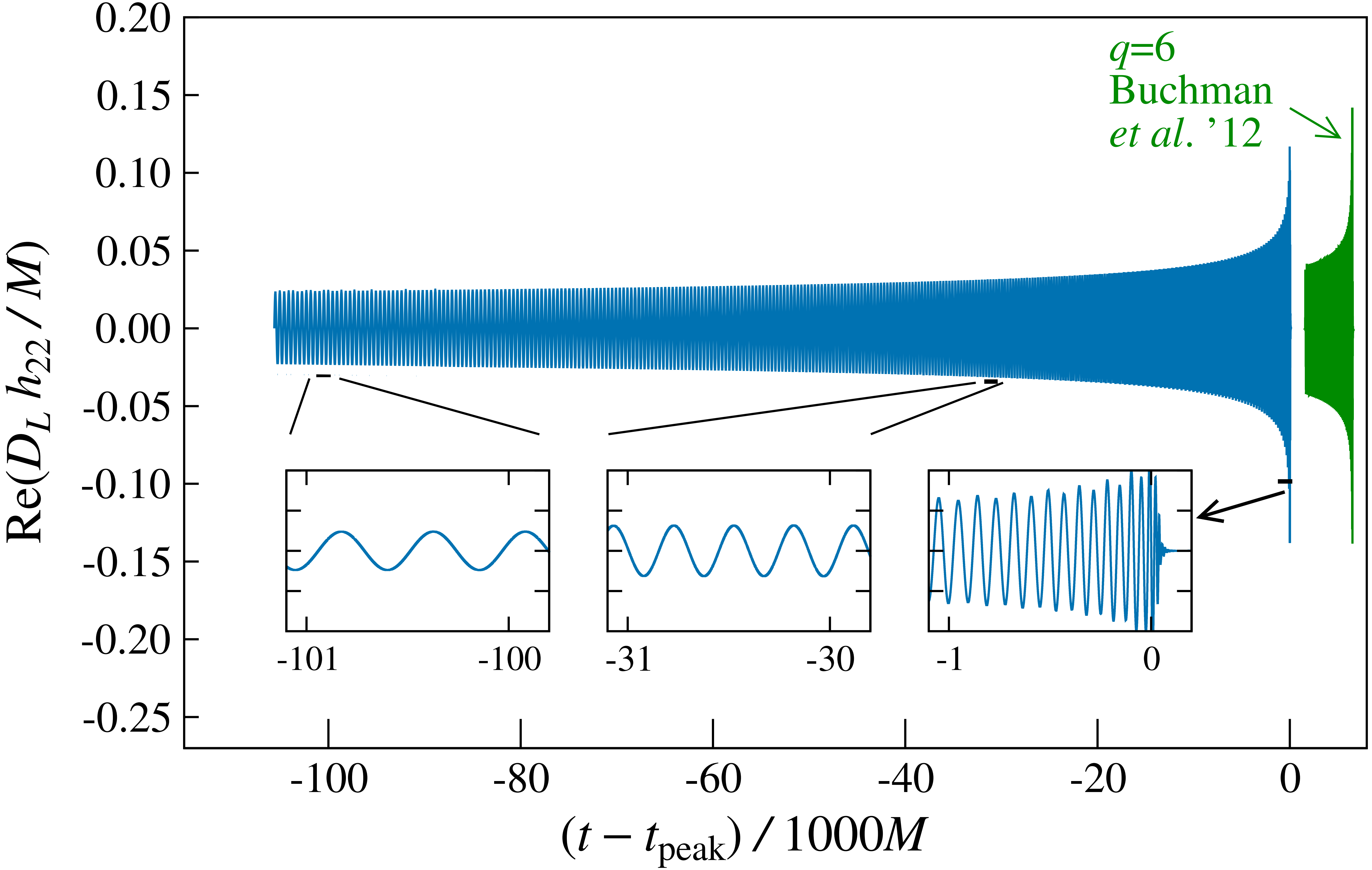
COMPARISON OF EOB AND NR WAVEFORMS

- precessing BH spin but non-spinning NS, unequal masses 1:5



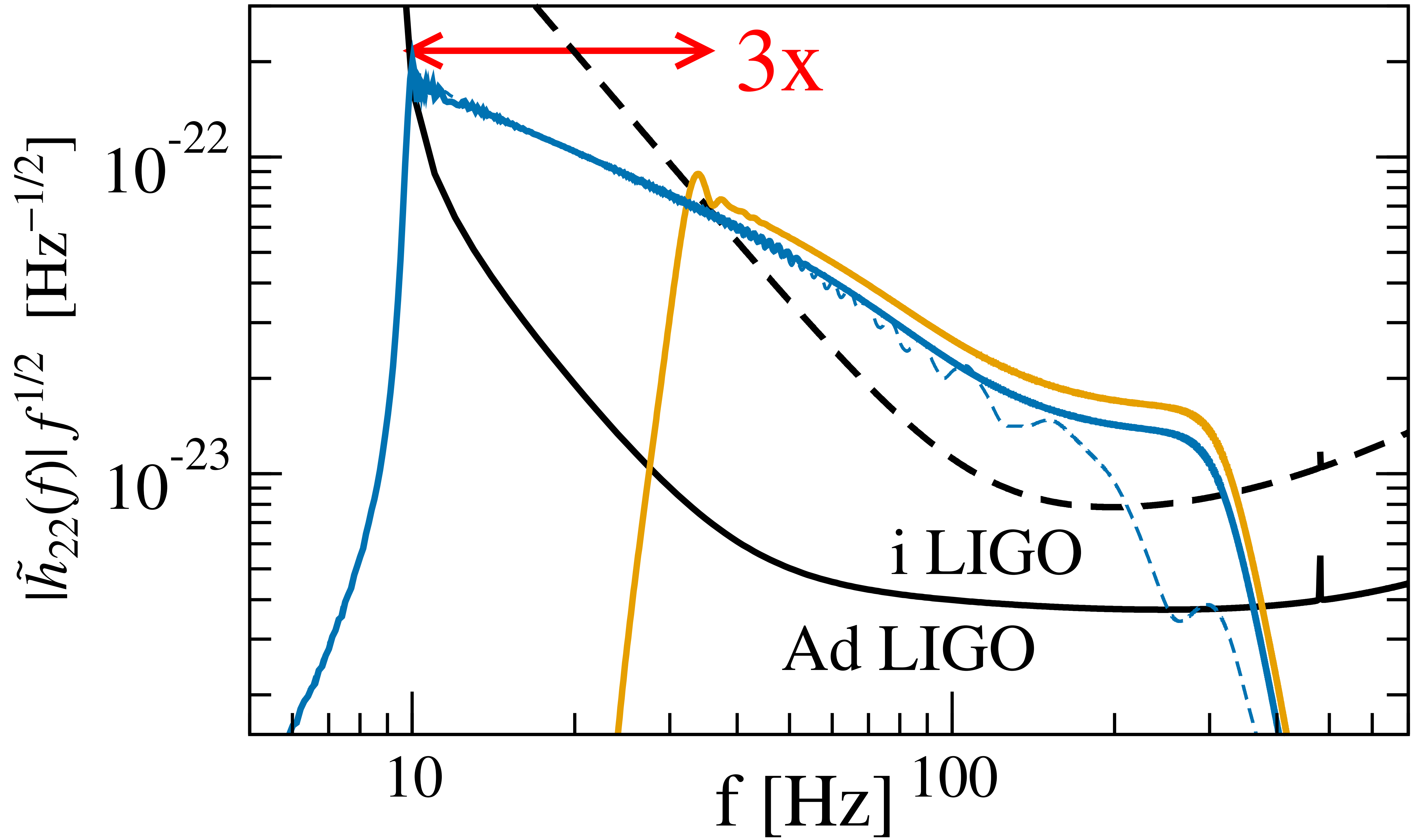
Taracchini+ 2012

170-ORBITS, MASS RATIO 1:7, NON-SPINNING



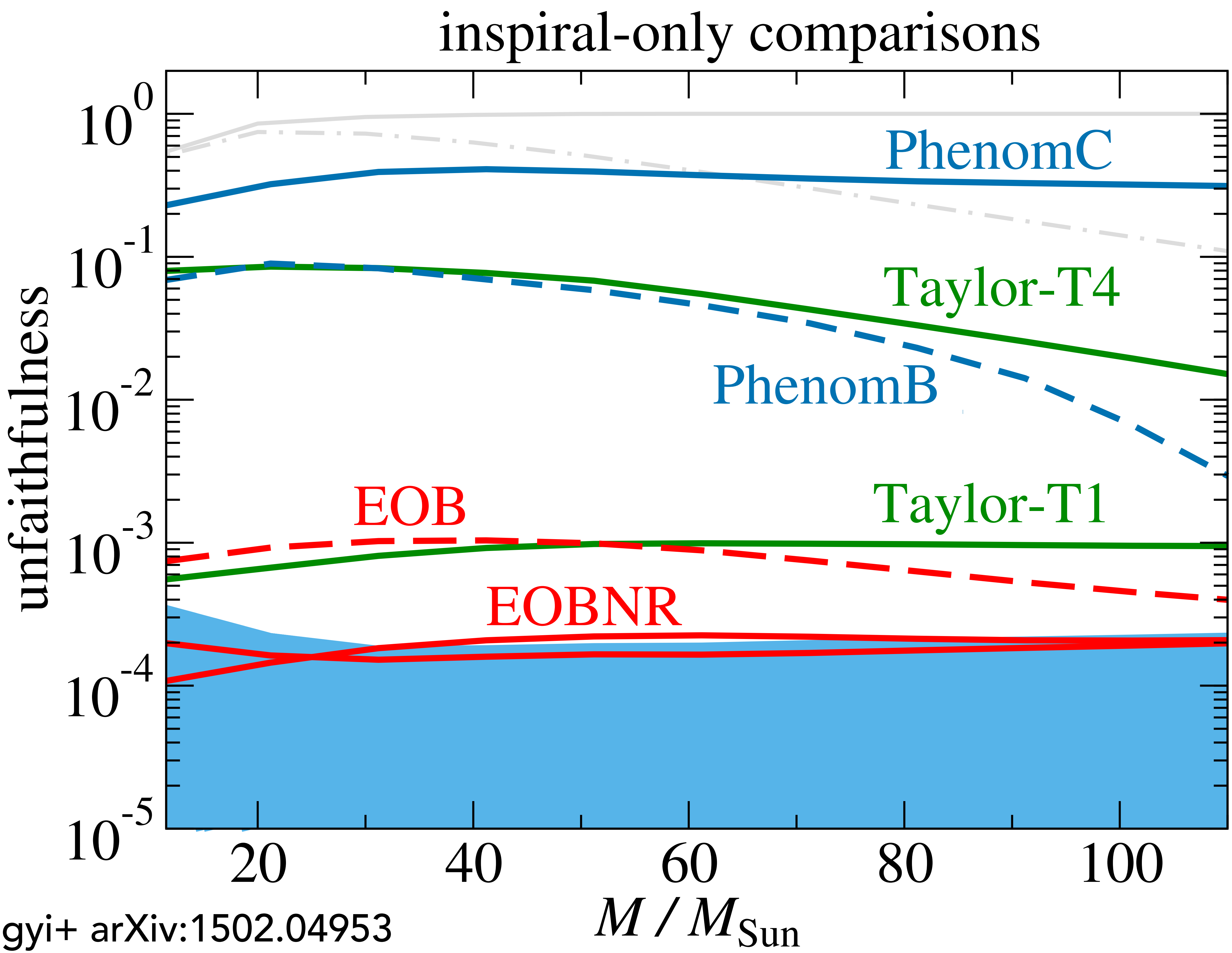
Szilagyi+ arXiv:1502.04953

THEY COVER QUITE A LARGE FREQUENCY RANGE



Szilagyi+ arXiv:1502.04953

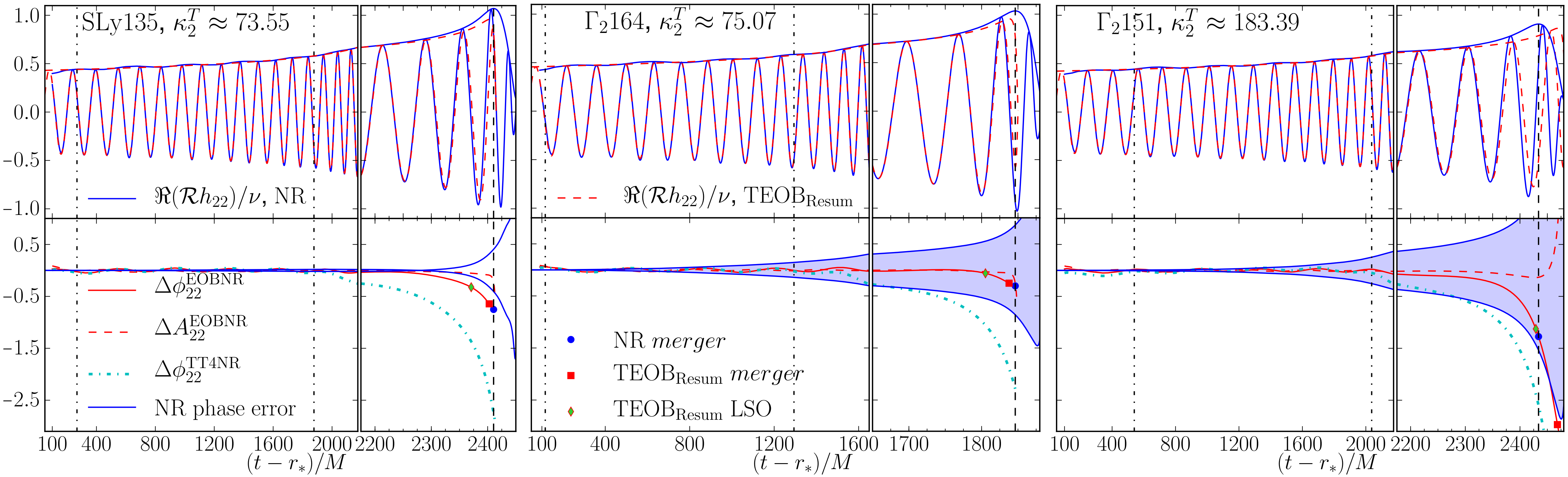
UNFAITHFULNESS OF EOB < 0.1%



Szilagyi+ arXiv:1502.04953

TIDAL EOB MODEL FOR BINARY NEUTRON STAR INSPIRALS

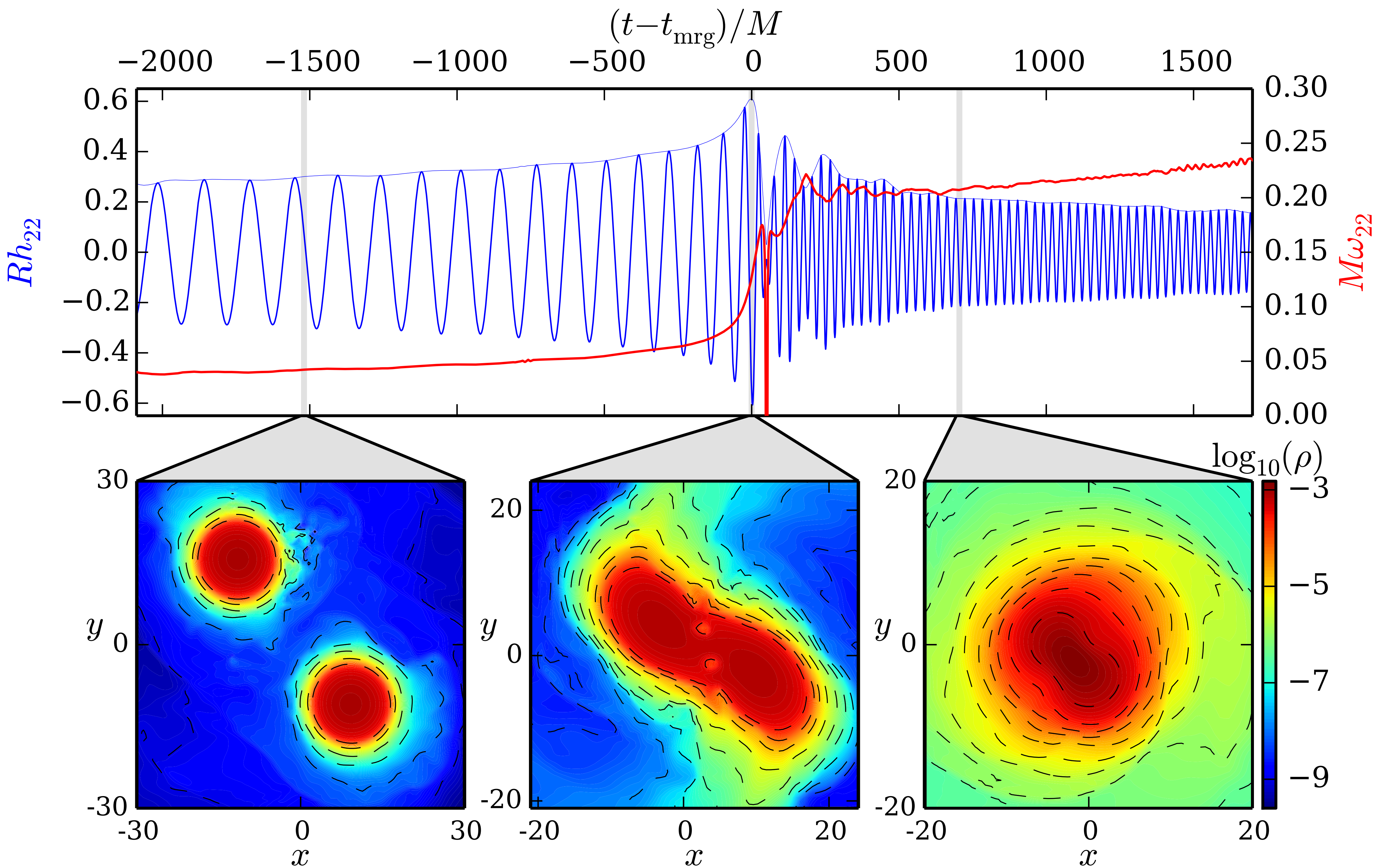
- EOB uses a single parametrisation but this may not be adequate for all EoS



Bernuzzi+ PRL 2015

BEYOND INSPIRAL

- many, many papers on understanding merger state

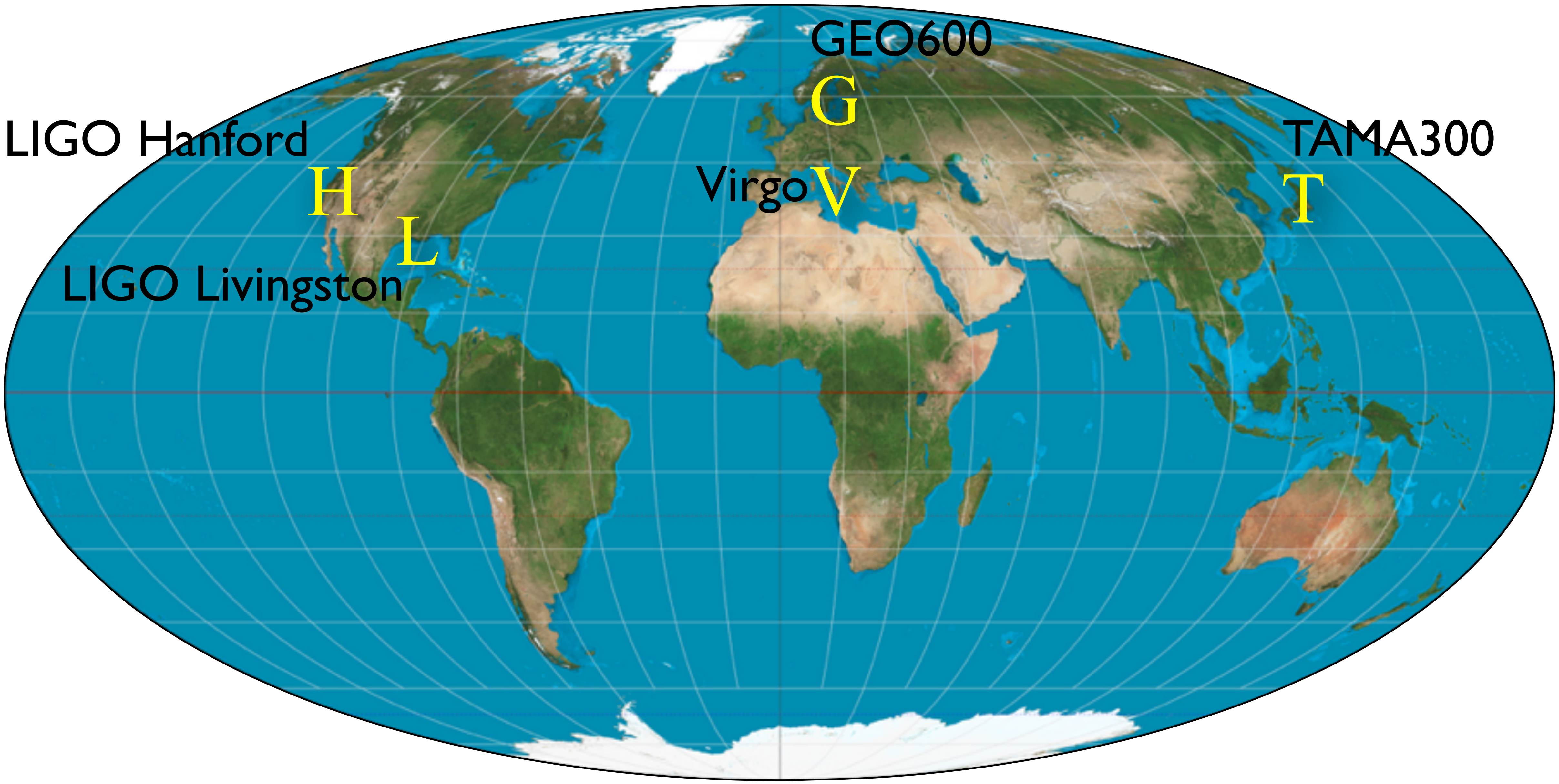


Bernuzzi+ arXiv:1504.0176v1

CURRENT STATUS AND FUTURE CHALLENGES

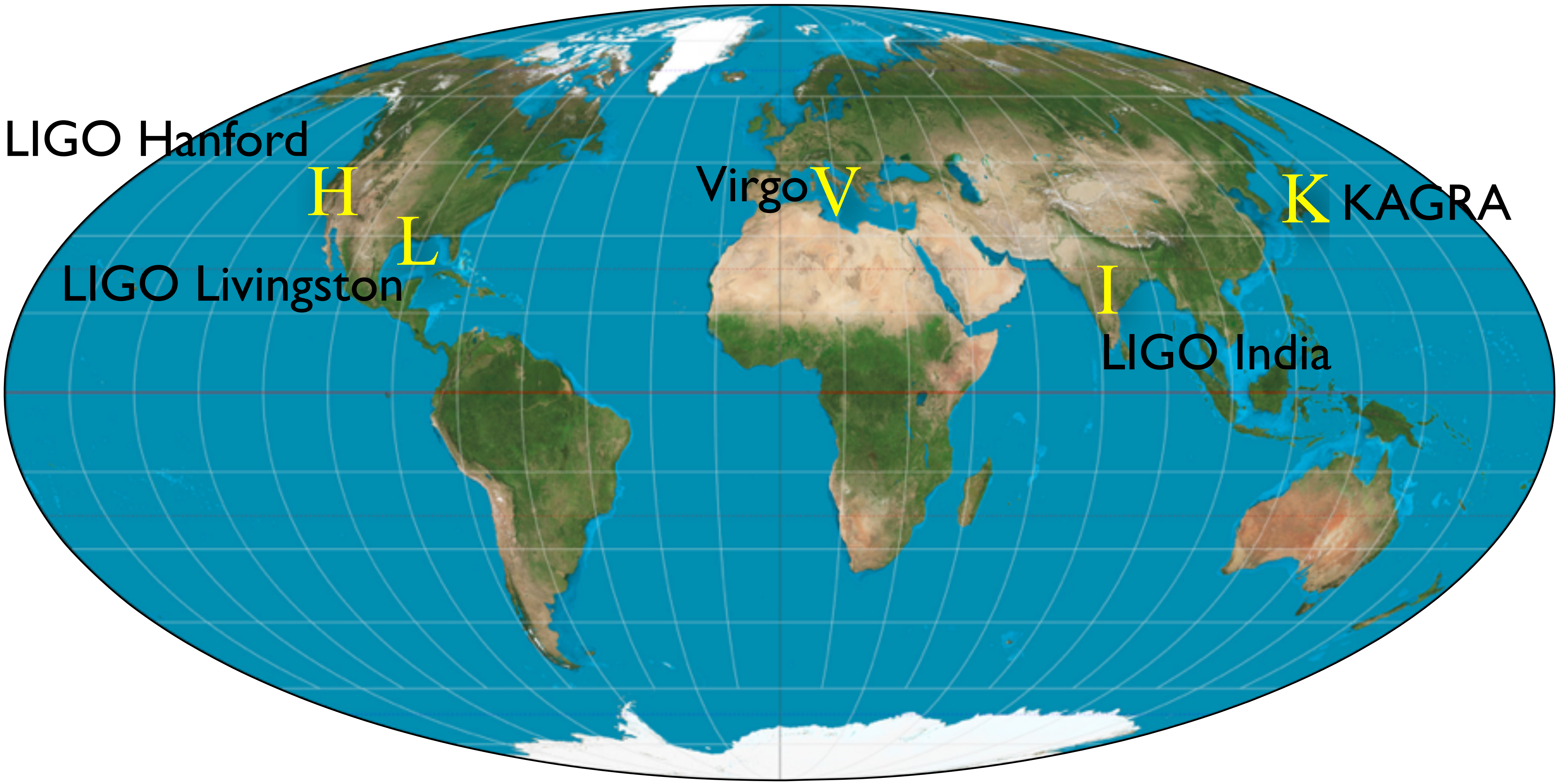
- Vacuum solution, binary black holes, known pretty well
  - good agreement between NR simulations and EOB over several hundred cycles
  - still it is necessary to confirm no de-phasing between NR and EOB over ~1000 cycles
  - spin effects (and possibly mass ratios) need to be controlled as well
- NR simulations with matter still at infancy
  - BNS merger simulations don't converge well
  - comparison between different groups is necessary
  - longer BNS simulations with ~100 cycles would be needed

INITIAL INTERFEROMETER NETWORK



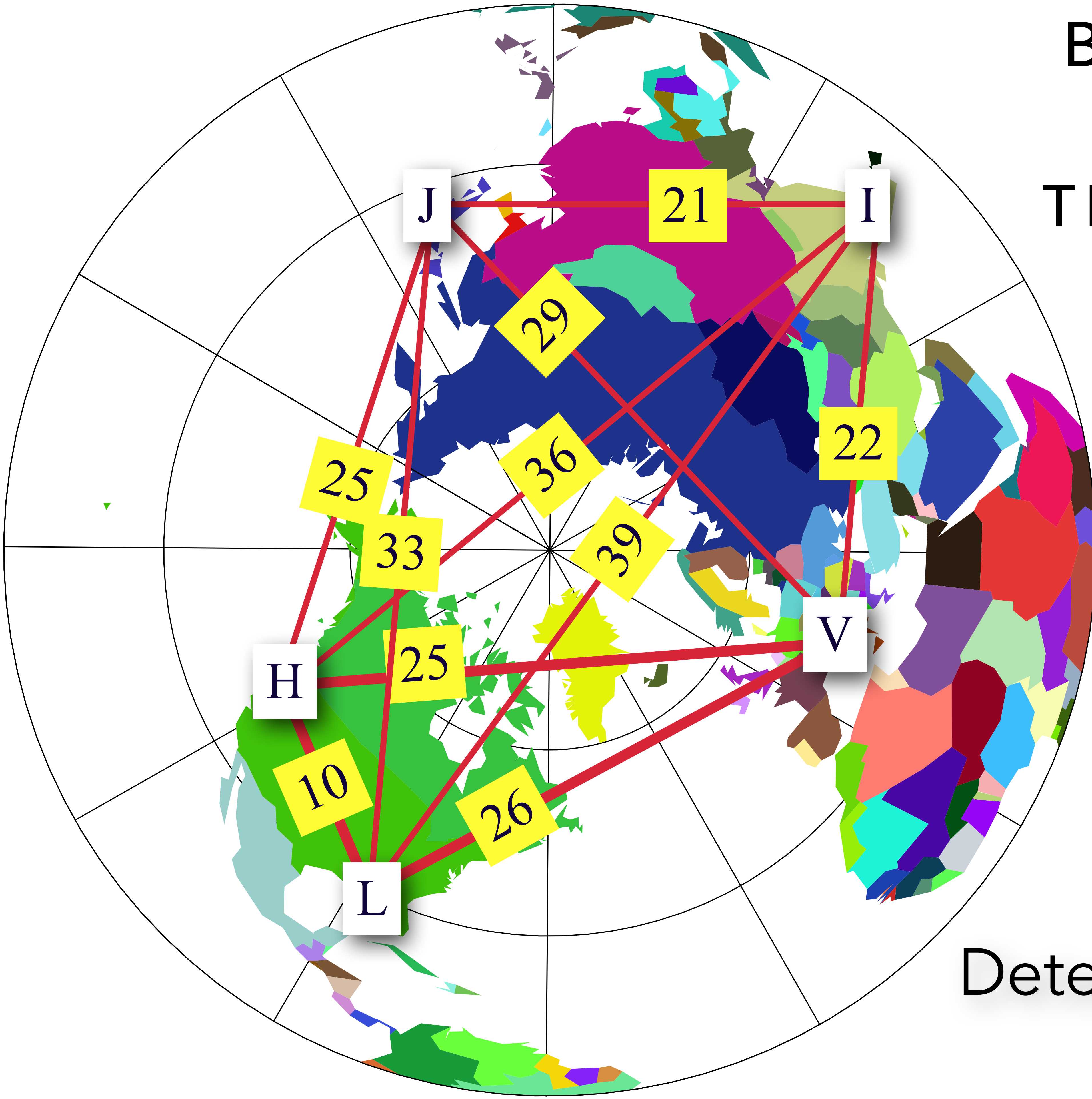
- Between 2006-2010 larger detectors took 2 years worth of data at unprecedented sensitivity levels
- No detections so far but beginning to impact astrophysics

ADVANCED DETECTOR NETWORK



- During 2015-2022 five large detectors will become operational
- Advanced LIGO detectors both installed and locked, commissioning over the next 3 years should see first detections

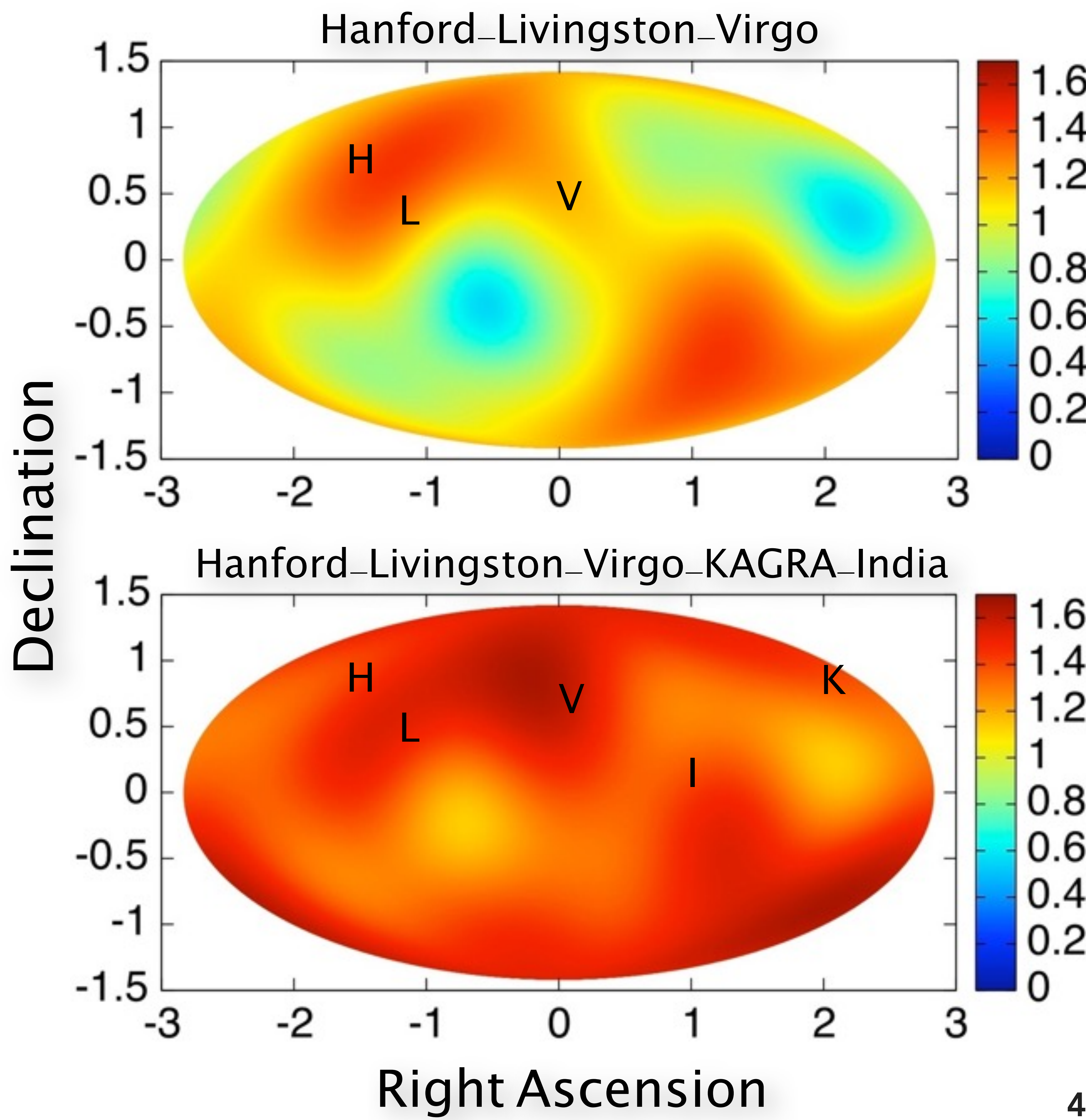
BASELINES IN LIGHT TRAVEL TIME (MS)



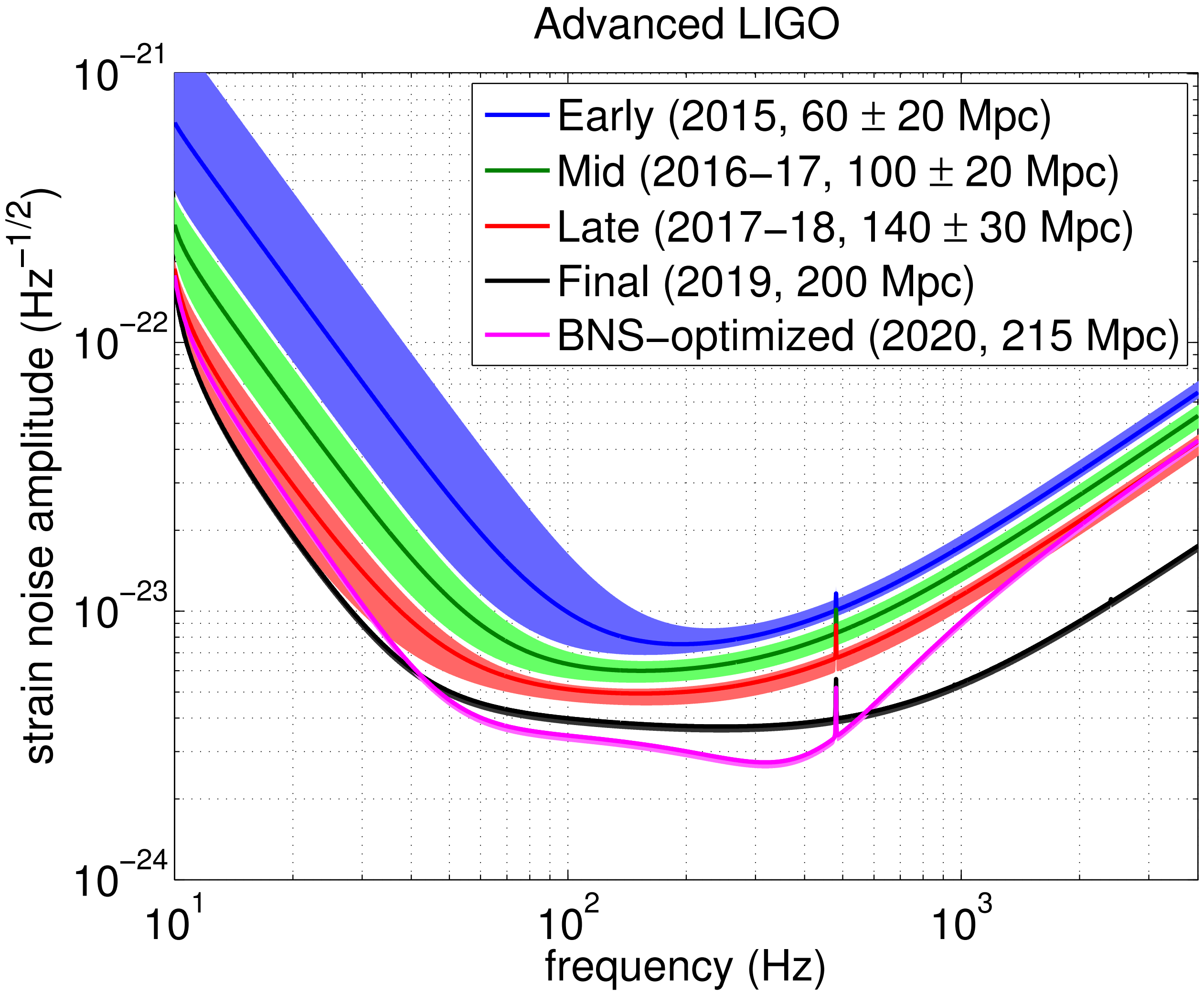
Detector Networks 2022+

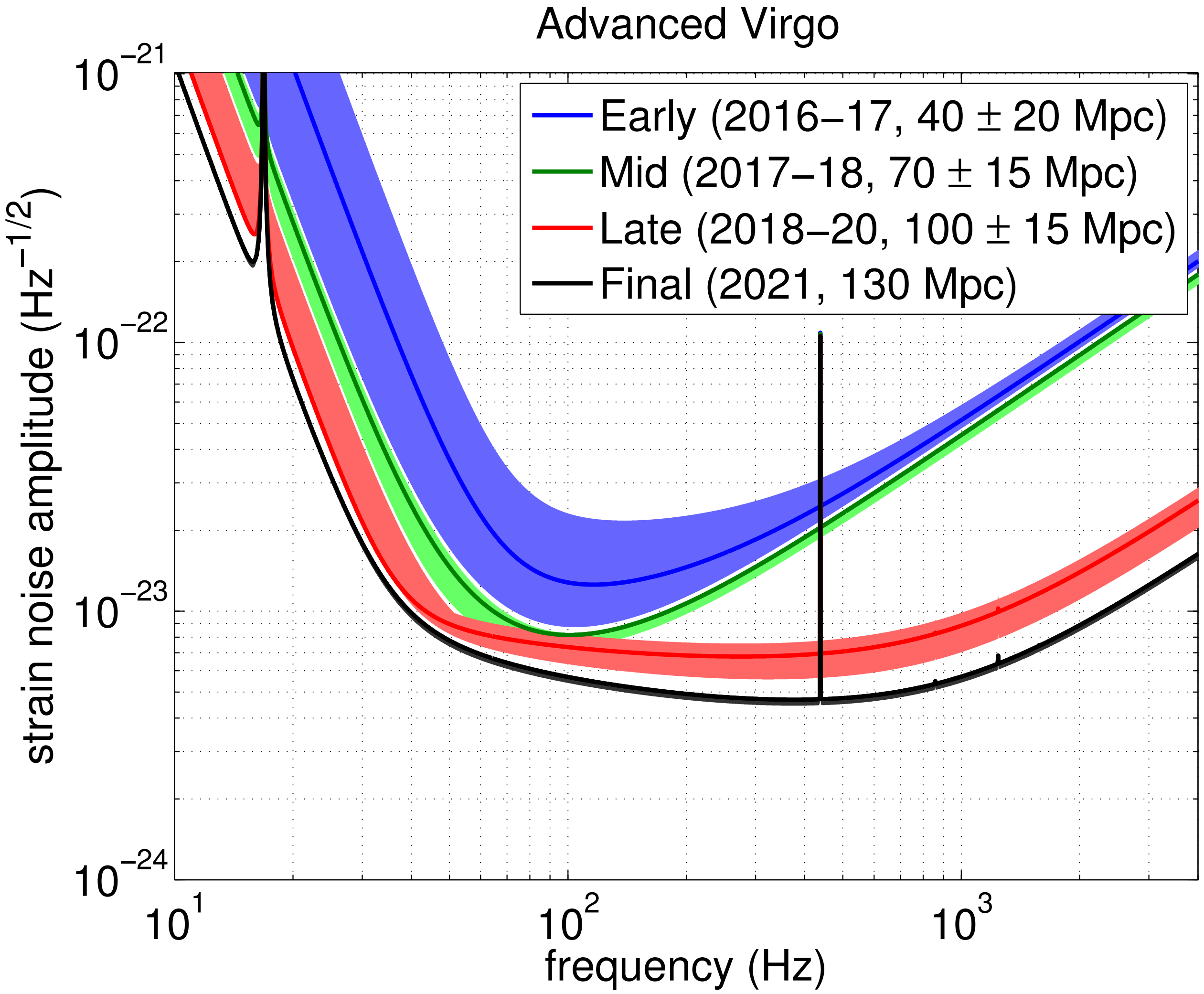
GW DETECTOR NETWORK - HIKLV

- A *network* of gravitational wave detectors is always on and sensitive to *most* of the sky
- We can integrate and build SNR by coherently tracking signals in phase



ADVANCED DETECTORS: SCHEDULE AND SENSITIVITY SHOWN LAST YEAR

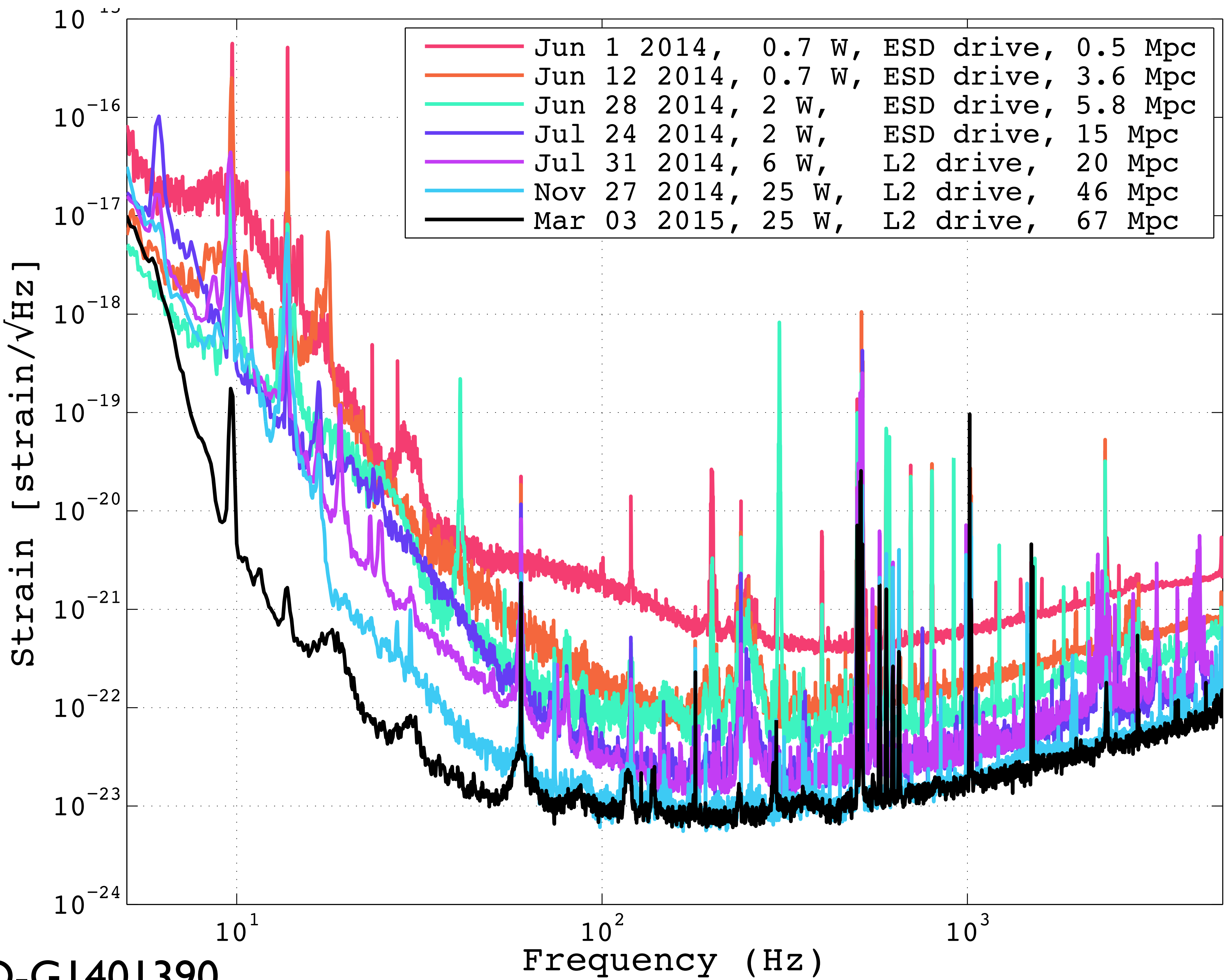




Epoch	Run Duration	BNS range (Mpc) LIGO	BNS range (Mpc) Virgo	Number of Detections	Median Area (deg^2)	% localized within $5\,deg^2$	% localized within $20\,deg^2$
2015	3 months	60 ± 20	—	0.0004 - 3	2000	-	-
2016–17	6 months	100 ± 20	40 ± 20	0.006 - 20	70	2	15
2017–18	6 months	140 ± 30	70 ± 15	0.02 - 70	84	1	12
2019+	(per year)	200	100 ± 15	0.2 - 200	31	5	37
2022+ (India)	(per year)	200	130	0.4 - 400	11	19	73

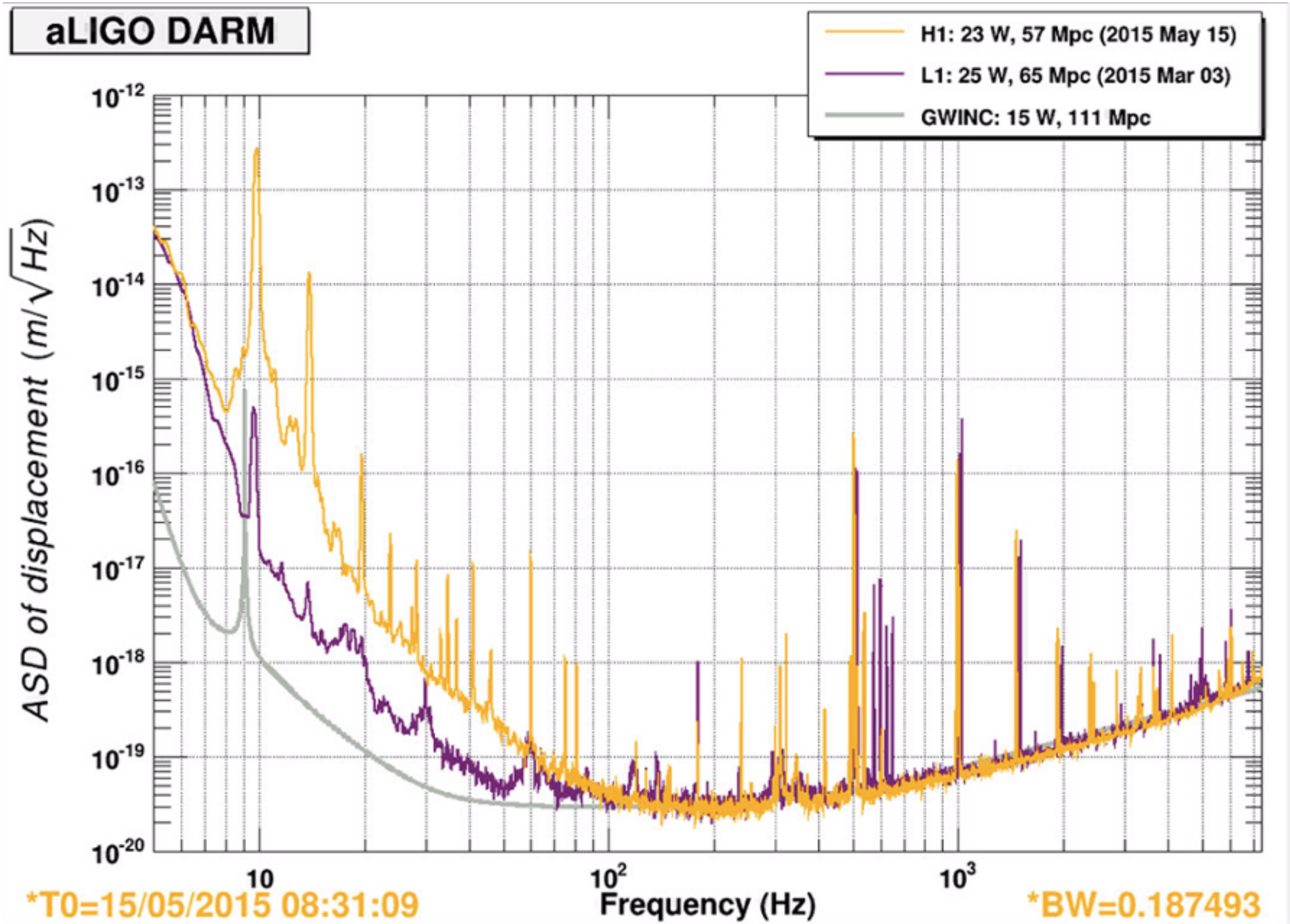
Aasi et al 2013 (arXiv:1304.0670)

ADVANCED LIGO DETECTORS HAVE MADE RAPID PROGRESS

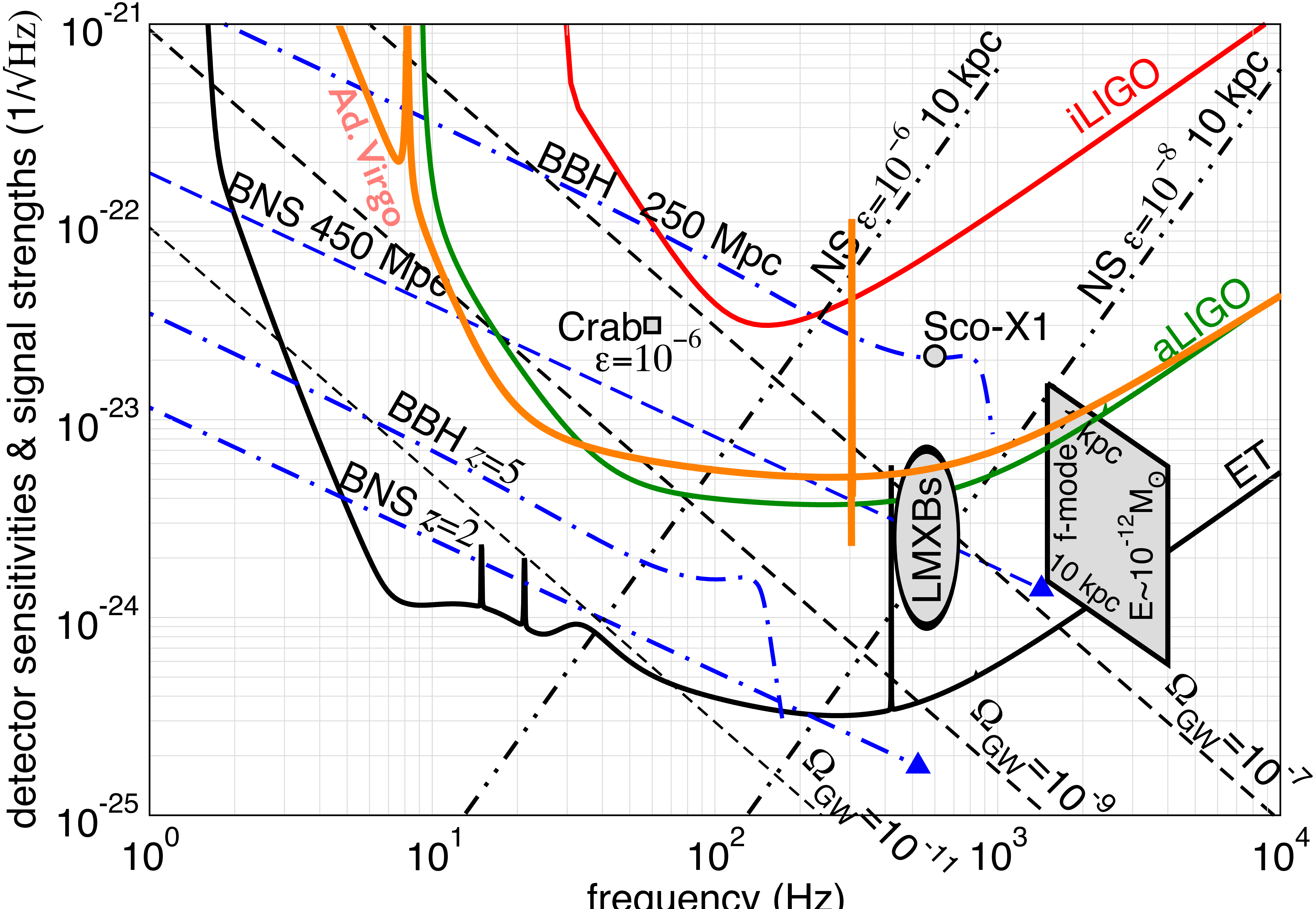


LIGO-G1401390

BOTH HANFORD AND LIVINGSTON ON TARGET

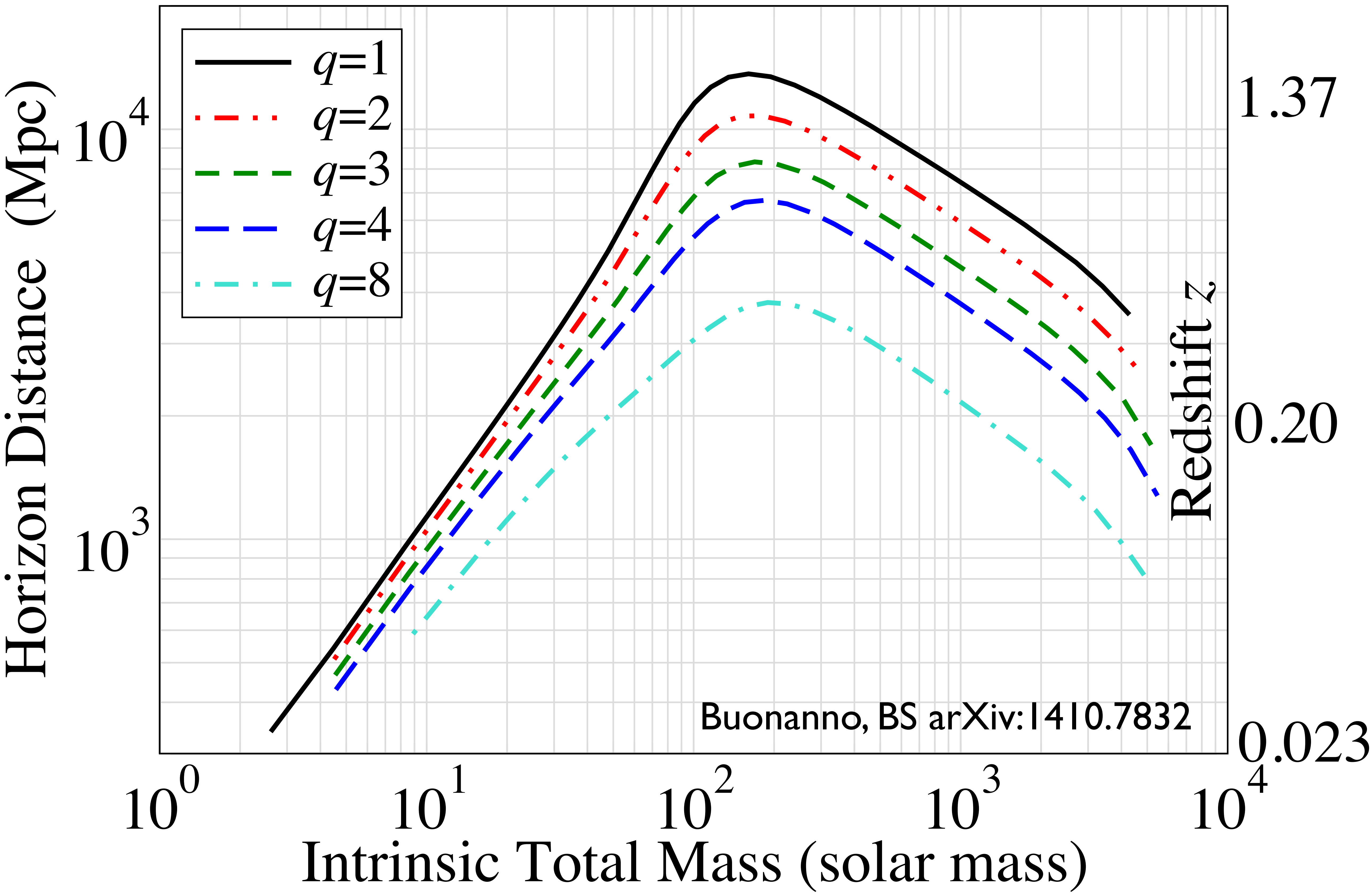


DESIGN SENSITIVITY BY 2018

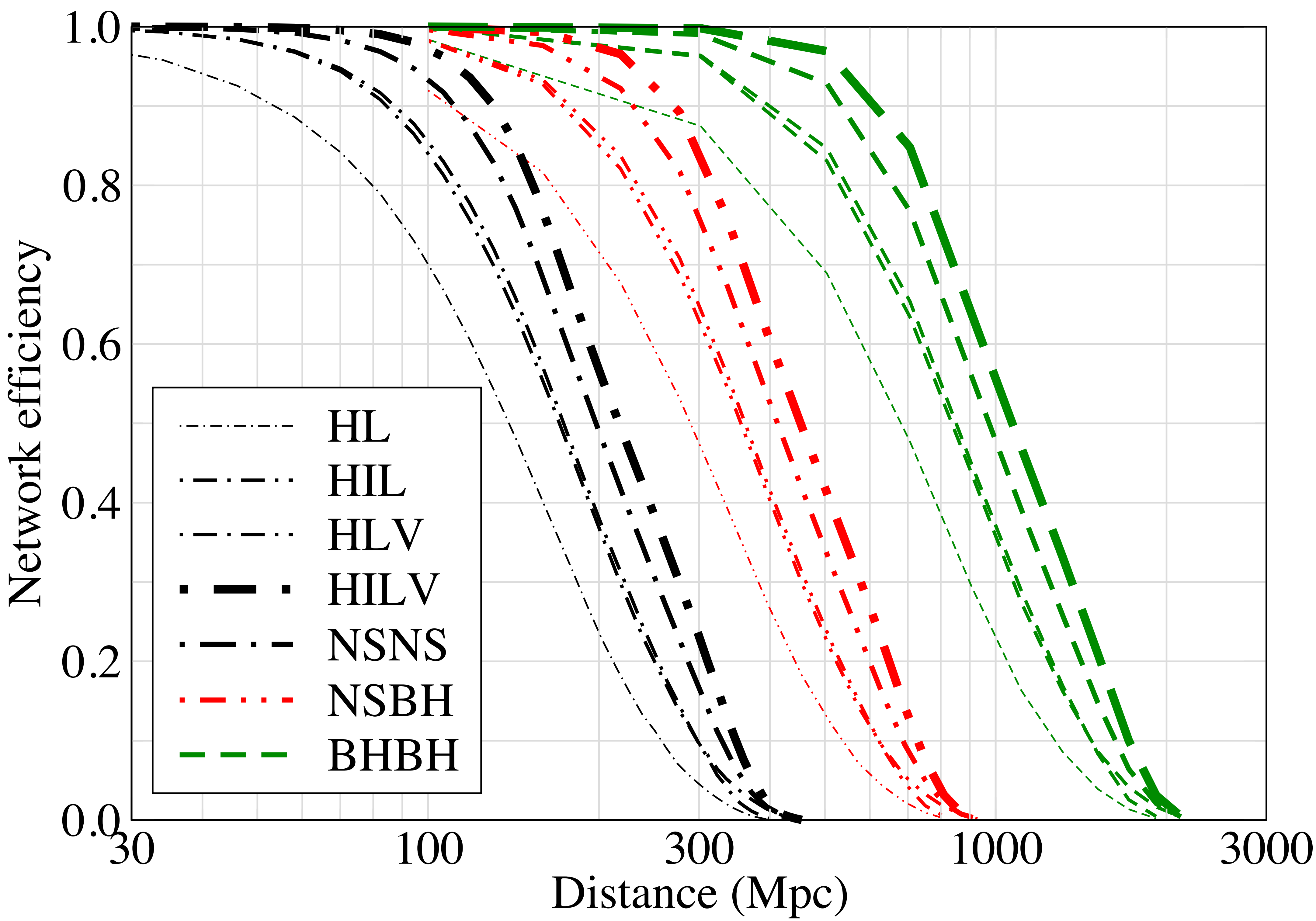


EXPECTED HORIZON DISTANCE OF ALIGO

DISTANCE TO A FACE-ON OVERHEAD SNR 8 BINARY



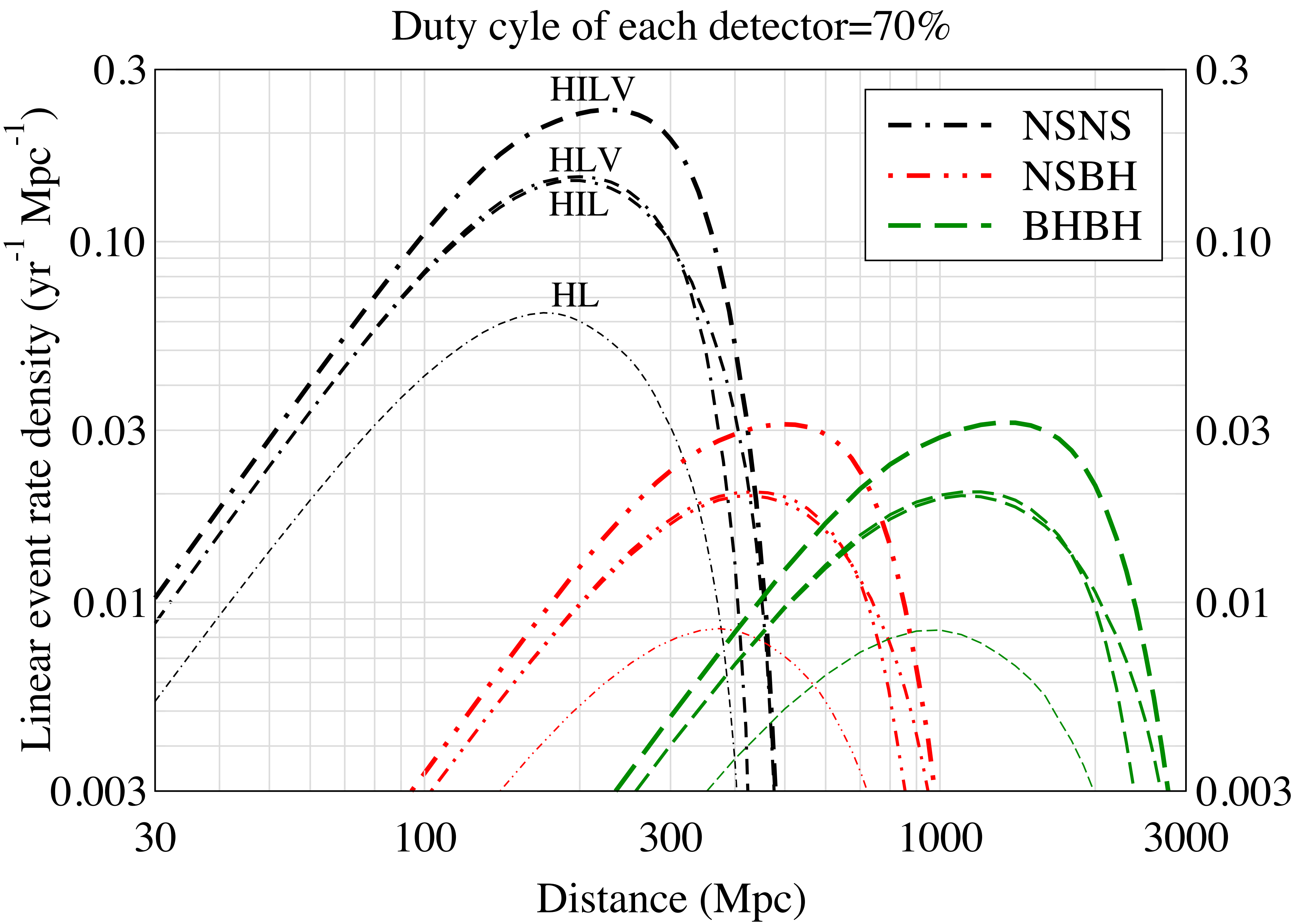
COMPLETENESS OF SURVEYS



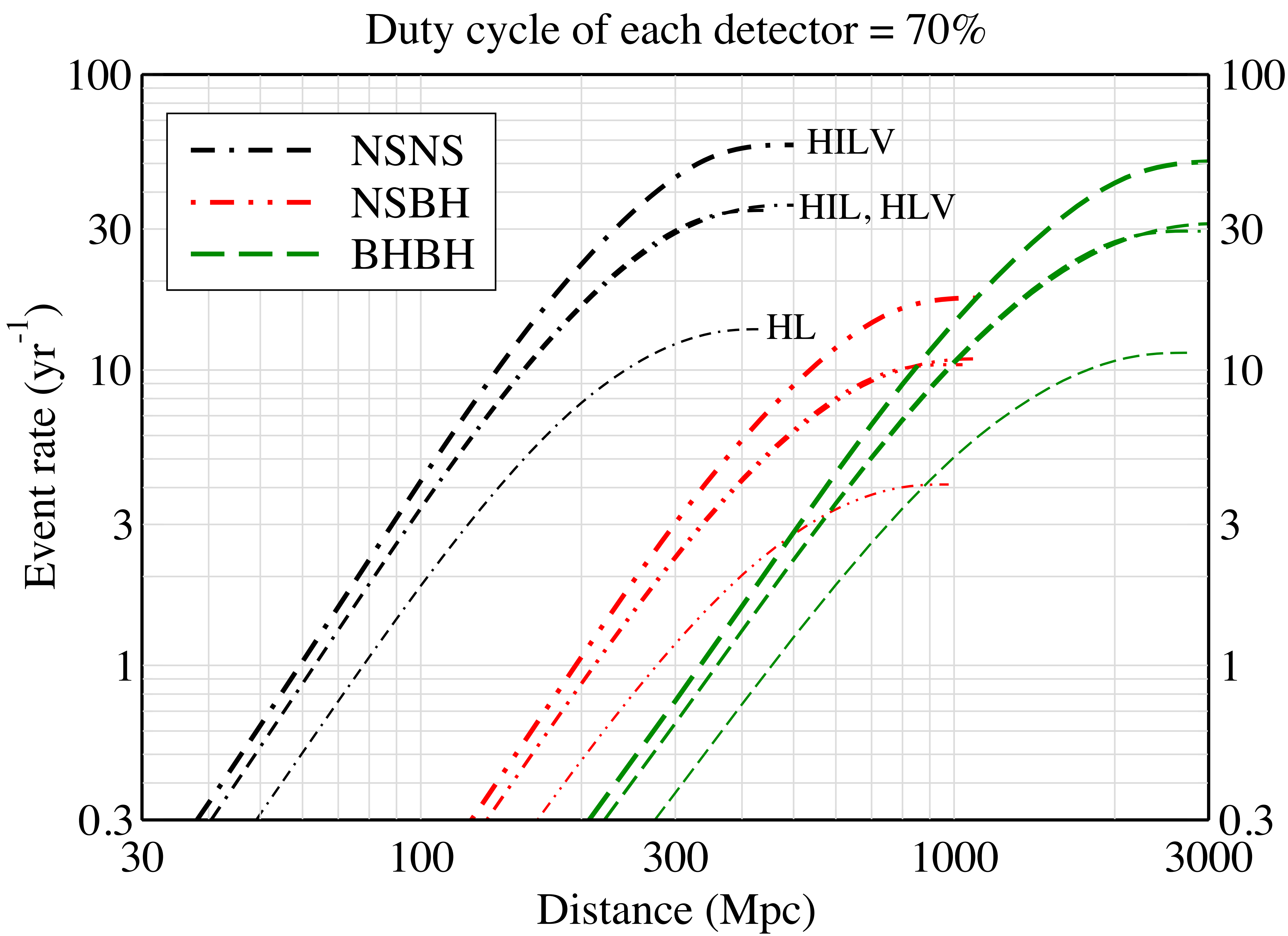
SHORT GRBS AND ALIGO EVENT RATES

- Observed short GRB rate ~ 10 yr^{-1} Gpc^{-3}
  - known for a while and has not changed much since SWIFT or Fermi
- We won't observe all GRBs because
  - most GRB satellites are not sensitive to the whole sky
    - SWIFT is typically covers between 10-25 %
  - gamma emission is not expected to be isotropic
    - half opening angle could be anywhere from a few degrees to isotropic
- Comoving volume rate depends on the beaming angle
  - Smaller the beaming angle, less likely we will observe them and so greater the rate
- A half beam open angle of 5° gives a rate of ~2,000 yr^{-1} Gpc^{-3}
  - This implies a detection rate of ~ 50 yr^{-1} at design sensitivity

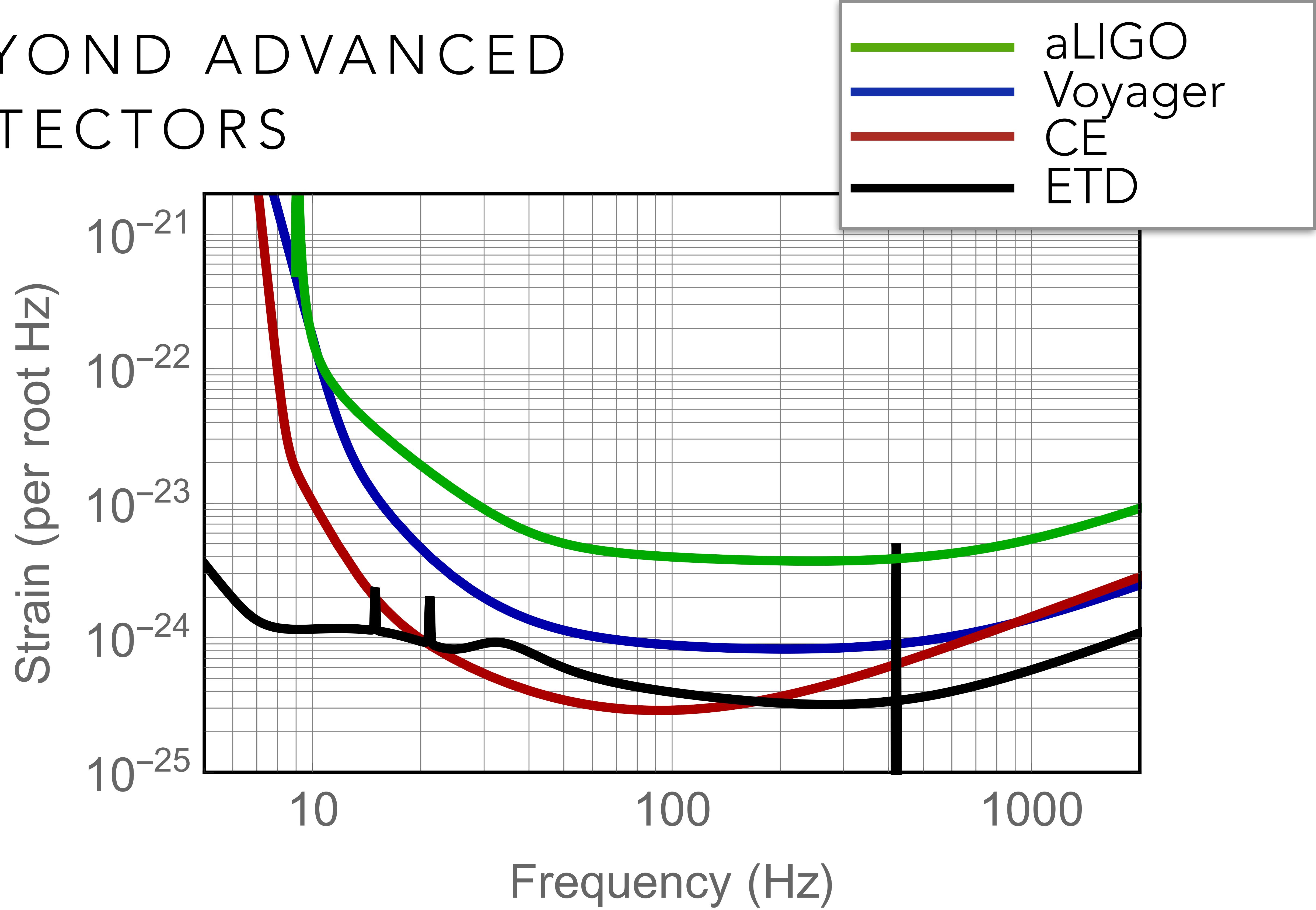
EXPECTED LINEAR RATE DENSITY



CUMULATIVE RATE AS A FUNC. OF DIST.

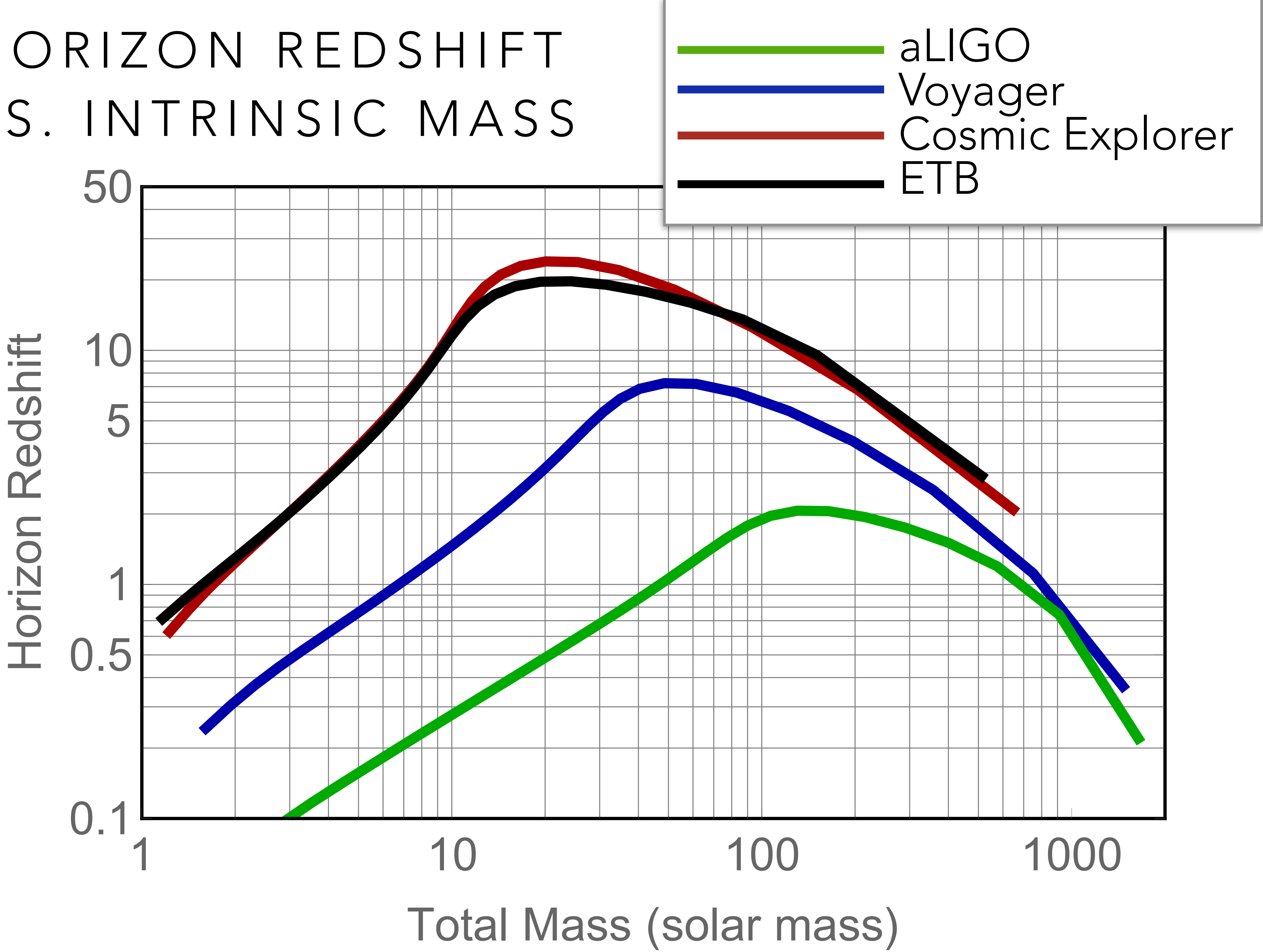


BEYOND ADVANCED DETECTORS



- Cosmic Explorer (CE) is a new concept studied by colleagues in LIGO; sensitivity here for a 40 km arm length detector

HORIZON REDSHIFT VS. INTRINSIC MASS



CONCLUSIONS

- LIGO and Virgo on track, on budget, on time
  - Engineering run next week, first observing run starting second week of September
  - KAGRA construction in good shape and provides a 3rd generation facility
  - awaiting the final word on LIGO India
- We are thinking and planning next generation of detectors
  - enhancements within current facilities will take us a factor 2-3 better in strain sensitivity (10-30 in volume)
  - new facilities will be necessary to detect binary black holes from the edge of the Universe