Hamiltonian construction of translationally symmetric extended MHD with equilibrium applications

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The noncanonical Hamiltonian structure of translationally symmetric extended MHD (XMHD) [1-3] with barotropic ion and electron fluids, is obtained by employing a method of Hamiltonian reduction [4] on the three-dimensional noncanonical Poisson bracket of XMHD [2]. The existence of the continuous spatial translation symmetry allows the introduction of the so-called poloidal representation for the magnetic field and an analogous Clebsch-like representation for the velocity field, consistent with the Helmholtz decomposition theorem. Upon employing the chain rule for functional derivatives, the 3D Poisson bracket is reduced to its translationally symmetric counterpart. Using this symmetric version of the noncanonical Poisson bracket, the families of symmetric Hall, Inertial, and extended MHD Casimir invariants are identified and used to obtain Energy-Casimir variational principles for generalized XMHD equilibrium equations with arbitrary macroscopic flows. The obtained set of equilibrium equations is cast into one of the Grad-Shafranov-Bernoulli (GSB) type. Then the following special cases of equilibria are investigated: static plasmas, equilibria with flows parallel to the axis of symmetry, and Hall MHD equilibria with finite ion flow but neglected electron inertia. The barotropic Hall MHD equilibrium equations are derived as a limiting case of the XMHD GSB system and they are consistent with those derived for axisymmetric plasmas in [5] via direct projection of the 3D equilibrium equations. In addition, we present a numerically computed equilibrium with D-shaped boundary, that plausibly shows the separation of ion flow from electron-magnetic surfaces, since in the framework of Hall MHD the magnetic field is frozen into the electron fluid.

References

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