Multiscale Modelling For Tokamak Pedestals

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Overview

- **The pedestal is crucial for tokamak reactor operation**
- **Multiscale modelling is extremely succesful for core transport**
- **We develop a multiscale model for ELMs, inter-ELM transport, and residual turbulence in the pedestal.**

Orderings for Pedestal Physics

- The system is strongly anisotropic $L_\perp/L_\parallel \ll 1$,
- The scale lengths are larger than the gyroradius $\rho_i/L_\perp \ll 1$,
- But frequencies are low $\omega/\Omega_i \ll 1$.

Key Assumptions

- Full diamagnetic effects $\omega \sim \omega^*$, for all species.
- Fully nonlinear $\omega \sim \boldsymbol{u_E}/L_\perp$.
- Finite amplitude perturbations $\delta f/f \sim 1$. Small magnetic perturbations $\delta\boldsymbol{B}/B \ll 1$.
- At most marginally ballooning unstable $\beta \lesssim L_\perp/L_\parallel$ i.e. $\alpha_{MHD} \sim 1$.
- Marginally Collisional – $\lambda_{\mathrm{mfp}} \sim L_\parallel$.

Orderings for ELMs

Assembling these assumptions, we produce the maximal ordering for fast dynamics in the pedestal:

$$\epsilon \equiv \frac{L_\perp}{L_\parallel} \sim \frac{m_e}{m_i} \sim \frac{\delta\boldsymbol{B}}{B} \sim \beta \sim \frac{\omega}{\Omega_i} \sim \left(\frac{\rho_i}{L_\perp}\right)^2 \tag{1}$$

Equations for ELMs

From our ordering we can obtain dynamical equations for sharp-gradient regions. The total magnetic field is

$$\boldsymbol{B} = \boldsymbol{B}_0 + \boldsymbol{b}_0 \times \nabla A_\parallel. \tag{2}$$

The kinetic equation for electrons:

$$\frac{\partial f_e}{\partial t} + \boldsymbol{u_E}\cdot\nabla f_e + v_\parallel \boldsymbol{b}\cdot\nabla f_e + \frac{\partial}{\partial t}\left(\varphi - \frac{v_\parallel A_\parallel}{c}\right)\frac{\partial f_e}{\partial \varepsilon_e} = C\left[f_e\right], \tag{3}$$

and ions:

$$\frac{\partial f_i}{\partial t} + \boldsymbol{u_E}\cdot\nabla f_i + \frac{\partial\varphi}{\partial t}\frac{\partial f_i}{\partial \varepsilon_i} = 0. \tag{4}$$

The electron kinetic equation above could give rise to a large parallel electron flow; this would violate Ampére's law. To maintain a small electron flow, we have the constraint:

$$\frac{\partial A_\parallel}{\partial t} = \frac{c}{en_e}\boldsymbol{b}\cdot\left(\nabla p_{\perp e} - en_e\nabla\varphi\right) + \boldsymbol{B}\cdot\nabla\left[\left(p_{\parallel e} - p_{\perp e}\right)/B\right] + \boldsymbol{R}_e, \tag{5}$$

where $\boldsymbol{R}_e$ is the collisional friction force on electrons.
The final equation for the field is a vorticity equation for φ.

$$\begin{aligned}\nabla\cdot\left\{\sum_s \frac{n_s m_s}{B}\left[\frac{\partial}{\partial t} + \boldsymbol{u_E}\cdot\nabla\right]\left(\frac{\nabla p_{\perp s}}{n_s m_s \Omega_s} + \frac{c\nabla\varphi}{B}\right)\right\} \\ = \nabla\cdot\left[-\frac{j_\parallel \boldsymbol{b}}{c}\left(1 - 4\pi\frac{p_\parallel - p_\perp}{B^2}\right) - p_\perp\nabla\times\left(\frac{\boldsymbol{b}}{B}\right) - \frac{\nabla\times\boldsymbol{b}}{B}\left(p_\parallel - p_\perp\right)\right],\end{aligned} \tag{6}$$

The parallel current in (6) is found from Ampére's law:

$$j_\parallel = \frac{c}{4\pi}\boldsymbol{b}\cdot\nabla\times\boldsymbol{B} = \frac{c}{4\pi}\left(\boldsymbol{b}_0\cdot\nabla\times\boldsymbol{B}_0 - \nabla_\perp^2 A_\parallel\right). \tag{7}$$

These equations contain all the physics that ELMs may entail:

- Fully nonlinear filamentary physics
- Linear Peeling-Ballooning modes [Connor *et al.*(1998)]
- Kinetic effects on all modes (electron Landau damping, trapped particles)
- Diamagnetic stabilization on ballooning modes [Rogers & Drake(1999)]

Inter-ELM Orderings

To slow down the dynamics to handle the Inter-ELM timescale, we introduce a new length scale:

$$L_\wedge \sim \sqrt{L_\parallel L_\perp}, \tag{8}$$

and insist that one of the perpendicular length scales is $L_\wedge$ not the shorter $L_\perp$. In a pedestal, only the radial extent is narrow! Under this assumption, we see that all the nonlinearities can be writen in the form

$$\boldsymbol{b}\times\nabla g\cdot\nabla f \sim \frac{gf}{L_\perp L_\wedge}, \tag{9}$$

which slows down the nonlinear dynamics compared to the ELMs.

Equations for the Inter-ELM pedestal

The electrons are an isothermal fluid, with simple evolution equations for the density

$$\left(\frac{\partial}{\partial t} + u_{\parallel e}\boldsymbol{b}\cdot\nabla + \boldsymbol{u_E}\cdot\nabla\right)n_e + \nabla\cdot\left\langle\boldsymbol{\Gamma}\right\rangle_{\mathrm{turb}} = 0, \tag{10}$$

and temperature

$$\left\langle\frac{3}{2}n_e\left(\frac{\partial}{\partial t} + \boldsymbol{u_E}\cdot\nabla\right)T_e\right\rangle_\psi + \left\langle\nabla\cdot\left\langle\boldsymbol{q}\right\rangle_{\mathrm{turb}}\right\rangle_\psi = -T_e\left\langle n_e\nabla\cdot\left(u_{\parallel e}\boldsymbol{b}\right)\right\rangle_\psi, \tag{11}$$

where $T_e = T_e(\psi)$.
The ions obey a drift-kinetic equation:

$$\begin{aligned}\left(\frac{\partial}{\partial t} + v_\parallel\boldsymbol{b}\cdot\nabla + \boldsymbol{u_E}\cdot\nabla\right)f_i + \left[\frac{\partial\varphi}{\partial t} - v_\parallel\boldsymbol{b}\cdot\nabla\left(\ln n_e\frac{T_e}{e} - \varphi\right)\right]\frac{\partial f_i}{\partial\varepsilon_i} \\ + \nabla\cdot\left\langle\frac{c}{B}\boldsymbol{b}\times\nabla\left\langle\delta\varphi\right\rangle_{\boldsymbol{R}} h_i\right\rangle_{\mathrm{turb}} = C\left[f_i\right].\end{aligned} \tag{12}$$

The fluctuating magnetic field is given by the lowest-order voriticity equation

$$\nabla\cdot\left[-\frac{j_\parallel\boldsymbol{b}}{c}\left(1 - 4\pi\frac{p_\parallel - p_\perp}{B^2}\right) - p_\perp\nabla\times\left(\frac{\boldsymbol{b}}{B}\right) - \frac{\nabla\times\boldsymbol{b}}{B}\left(p_\parallel - p_\perp\right)\right] = 0. \tag{13}$$

We can then use

$$\frac{1}{c}\boldsymbol{b}\cdot\frac{\partial\boldsymbol{A}}{\partial t} = \boldsymbol{b}\cdot\nabla\left(\frac{T_e}{e}\ln n_e - \varphi\right), \tag{14}$$

to find the electrostatic potential, up to a flux function $\overline{\varphi}(\psi)$.
These equations allow the turbulent transport effects to compete with sound-waves. This allows pedestals to build up, but also inhomogeneities in this transport to compete with coherent oscillations in the pedestal.

Pedestal Turbulence

The residual turbulence that is consistent with the above transport and The fluctuations are small, with

$$\frac{\delta f_s}{f_s} \sim \frac{e\delta\varphi}{T_e} \sim \sqrt{\epsilon} \quad \text{and} \quad \frac{\delta\boldsymbol{B}}{B} \sim \epsilon. \tag{15}$$

The fluctuations will have typical length scales,

$$k_\perp\rho_i \sim 1 \quad \text{and} \quad k_\parallel^{-1} \sim \sqrt{L_\perp L_\parallel} \tag{16}$$

and occur on the rapid timescale corresponding to high-$k_\parallel$ electron turbulence

$$\omega \sim k_\parallel v_{\mathrm{th}_e} \sim k_\parallel v_{\mathrm{A}}. \tag{17}$$

These orderings give rise to sheared-slab gyrokinetic equations, which are a generalisation of [Zocco & Schekochihin(2011)].
The physics contained in these orderings includes:

- Small-scale high-$k_\parallel$ electromagnetic turbulence
- Microtearing modes
- Strong slab-like ETG (both electromagnetic and electrostatic)

which comprise turbulence suggested to be important in pedestals [Hatch *et al.*(2017)].

Future Work: Stability Theory and Open Field Lines

- Analytic theory for a large-aspect-ratio pedestal region
- Detailed analysis of how the pedestal stabilises ITG
- Extend this approach to the open-field-line region

Summary

- **A fully multiscale approch is applied to the pedestal**
- **We provide a first-principles basis for EPED-like modelling**
- **We consistently include kinetic effects, and full diamagnetic effects**
- **Self-consistent inclusion of turbulence alongside ELMs, and inter-ELM transport**

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