Analytic characterisation of ideal infernal type instabilities in tokamaks with large edge pressure gradients

P2.2

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INTRODUCTION

■ **Tokamak H-mode**: High performance accompanied by ELMs (**sudden and violent**) [1] $\Longrightarrow$ High energy/particle loads $\Longrightarrow$ Material deterioration/Plasma contamination

■ **QH-mode**: High performance (large edge pressure gradients and high τ_E) with low heat loads. **ELMs replaced by low-n coherent edge MHD activity (Edge Harmonic Oscillations)**. EHOs always present in QH-mode discharges [2,3]

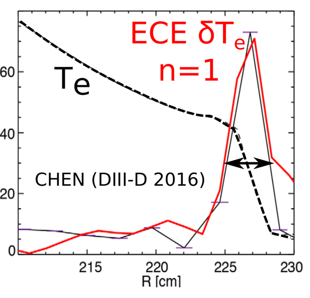
■ **EHO physics**: Numerical simulations (stability and 3D equilibrium) show unstable/equilibrium states with *infernal-like* characteristics localised near the edge (no core activity) [4,5]

■ **GOAL**: Analysis of edge stability with analytic tools focusing on **Edge Infernal Modes (EIM)**

EXPERIMENTAL EVIDENCE

■ **Peeling-Ballooning (P-B)** theory for large-n [6] cannot catch some EHOs features which are:

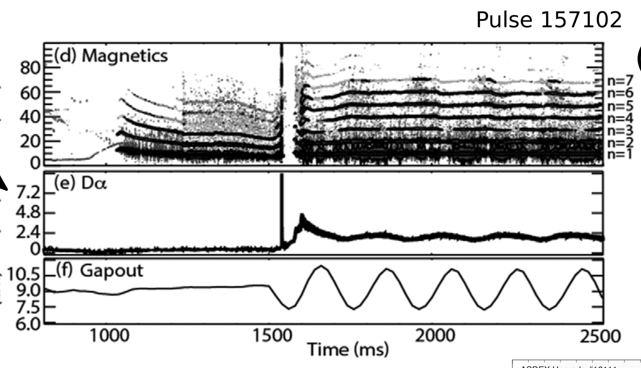
- **Below P-B stab. bound.**
- Rotation freq.$\propto$ $n\Omega^{\phi}_{Ped}$
- Radial width $\propto$ Δr_{Ped}

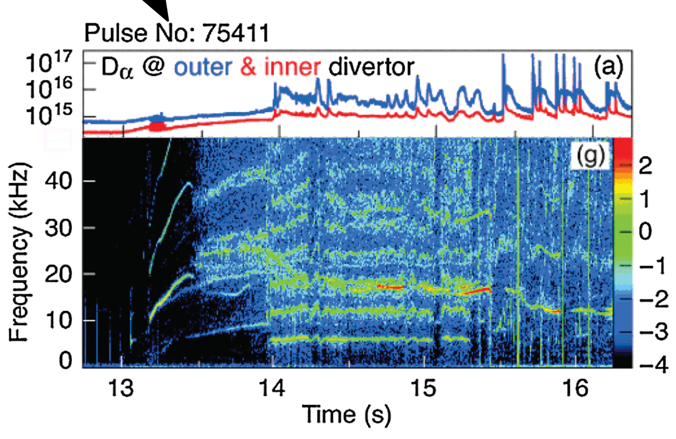


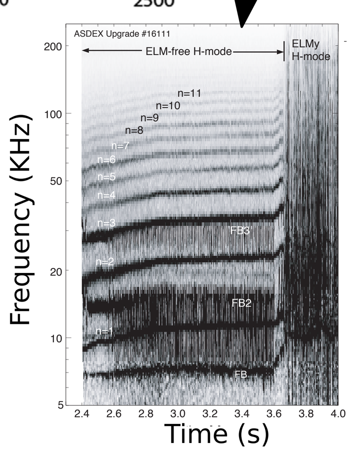
Chen (DIII-D 2016)

Suttrop (ASDEX-U 2003)

Solano (JET 2010)





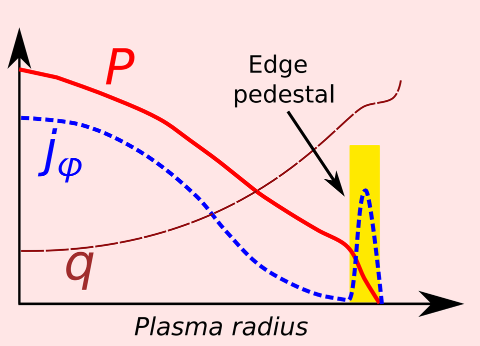


■ **Infernal modes:** $m \leftrightarrow m \pm 1$ **Coupling** (due to Jacobian θ oscillation) low-n (fixed) Fourier modes with nearly resonant flat q/large pressure gradient [7]. Purely toroidal mechanism (common in the core)

Does it work at the edge?

- Large edge p' ✔
- $\Longrightarrow$ large J_{BS} $\Longrightarrow$
- q flattening ✔

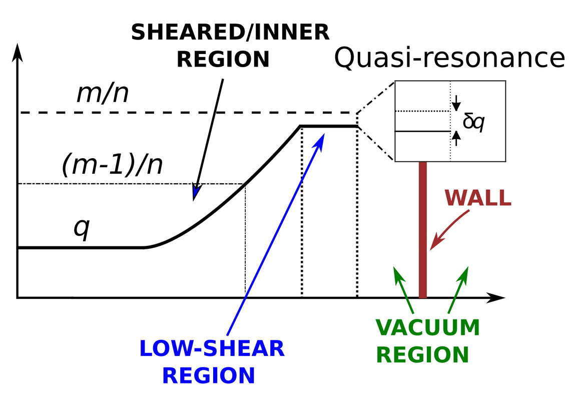
Infernal instability conditions met

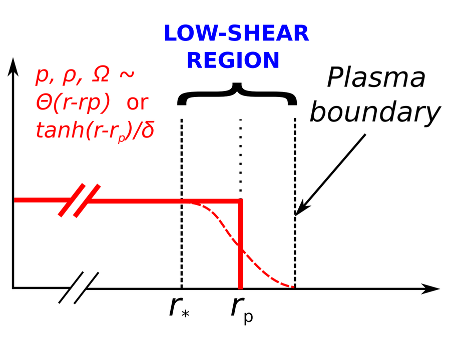


PHYSICAL MODEL

■ Ideal MHD, large aspect ratio approximation, shifted circular toroidal surfaces framework

■ Equilibrium modelling identifies three regions. Various choices (step-like or *tanh*) for equilibrium p and ρ





- Low-shear/shered/vacuum treated separately
- Mass density gradients required for explaining p' ⚠
- Subsonic toroidal rotation (step-like model, simpler)

■ **Sheared/Vacuum regions** ($Q = k_{\|}^2/n^2 + A_1$ with $A_1 \sim \rho\omega_D^2(1+2q^2)$, $\omega_D = \omega + n\Omega$, $\alpha \propto p'$):

$$L_m X_m = 0$$

$$L_m f = \left[r^3 Q f'\right]' + r\Big[(1-m^2)Q + rA_1' - \frac{\alpha^2}{2} + \underbrace{\frac{\alpha r}{R_0}\left(\frac{1}{q^2}-1\right)}_{\text{MERCIER}}\Big]f$$

No coupling/inertia: $\alpha \to 0$, $A_1 \to 0$

■ **Low-shear region** infernal **coupled** equations:

$$L_m X_m + \frac{\alpha}{2}\sum \frac{r^{\mp m}}{1 \pm m}\left(r^{2\pm m}X_{m\pm1}\right)' = 0$$

$$\left[r^{-1\mp 2m}\left(r^{2\pm m}X_{m\pm1}\right)'\right]' = \frac{1 \pm m}{2}\left[\alpha r^{\mp m} X_m\right]'$$

DISPERSION RELATION DERIVATION

■ **Sheared/Vacuum regions**: $\mu_* \approx n/m$, q monotonic

Mikhailovskii (**plasma**)	Step (**plasma**)
$(m-1)\mu - n = S(1-(r/r_s)^{\lambda})$ S such that $\mu(r_*) = \mu_*$	$\mu(r<r_0) = \mu_{ax}$ $\mu(r>r_0) = \mu_*(r_*/r)^2$

Vacuum ($\mu = \mu_*(a/r)^2$) & **Wall** ($\tilde{\psi}'' = \sigma\gamma\tilde{\psi}$)
$\sigma \to \infty / \sigma < \infty$ (Ideal/Resistive) $\Longrightarrow$ BC modification

- Main harmonic X_m vanishing (low-shear reg. localised)
- Exaxct expression for $rX'_{m\pm1}/X_{m\pm1} = \mathbb{C}_{\pm}$ (SHEARED REGION), $\mathbb{B}_{\pm}$ (VACUUM REGION) $\Longrightarrow$ Information about the wall

■ **Low-shear region**: $\mu_* = \mu_{edge}$ flat q

Three harmonic equations combine in a single one

$$(\bigstar)\quad L_m X_m + \frac{\alpha}{2}\sum \frac{r^{1\pm m}L_{\pm}}{1 \mp m} = 0 \quad \xrightarrow{\text{Exact solution}}$$

$$X_m = c_1 + \left(c_2 + \frac{H\delta}{c}\right)\frac{x}{\delta} + c_2 c \ln[\cosh(\tfrac{x}{\delta}) - c\sinh(\tfrac{x}{\delta})]$$

$H \Longleftrightarrow$ **Coupling** $\qquad c \Longleftrightarrow$ **Inertia**

- $c_{1,2} \Longrightarrow X_m(\pm h) = 0$ $(x = r - r_p,\ h = (a - r_*)/2 = \Delta/2)$
- *tanh* model collapses to *step-like* if $\delta \to 0$

Step model	***tanh* model**
Integrate ($\bigstar$) across r_p	$\int_{r_*}^{a}\alpha X_m dr = 1$

⚠The two approaches are equivalent

Smooth matching across low-shear region interfaces

■ **Infernal-type dispersion relation** [8,9]

$$\overbrace{\frac{(\gamma - in\Omega)^2(1+2q^2)}{2\omega_A^2}}^{\text{INERTIA}} + \overbrace{\left(\frac{\delta q}{q}\right)^2}^{\text{FIELD LINE BENDING}} = \underbrace{\left(\frac{\hat{\beta}}{2\varepsilon}\right)^2}_{\text{PRESSURE DRIVE}} \times \underbrace{\left[\frac{L_+}{1+m} + \frac{L_-}{1-m}\right]}_{\text{COUPLING TERM WALL EFFECTS}} \times \underbrace{A_\gamma}_{\text{LAYER WIDTH}}$$

- Step model $\Longrightarrow$ Doppler shift, no Ω in the coupling term (similar behaviour inferred in the *tanh* model)
- Stability determined by the coupling term ($A_\gamma > 0$)
- Mercier term (stabilising) retained/neglected in the step/*tanh* model (**weak effect**)
- Supersonic flow $\rightarrow$ α (stability) modification (not treated)

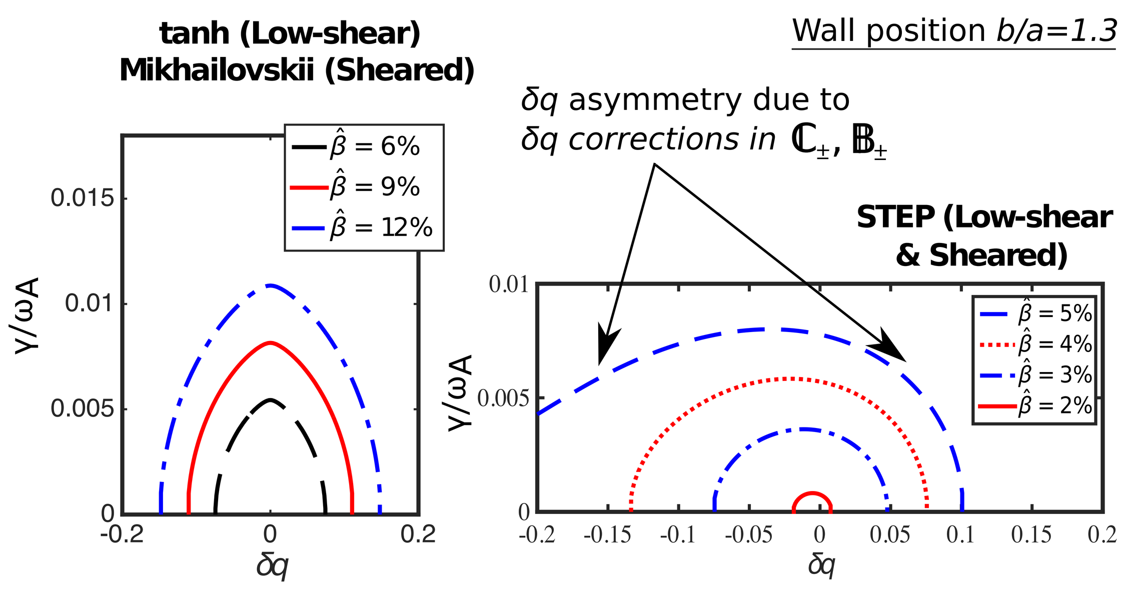
STABILITY ANALYSIS

■ **Ideal wall**: Simplified stability criterion (**step model** low-shear/sheared)

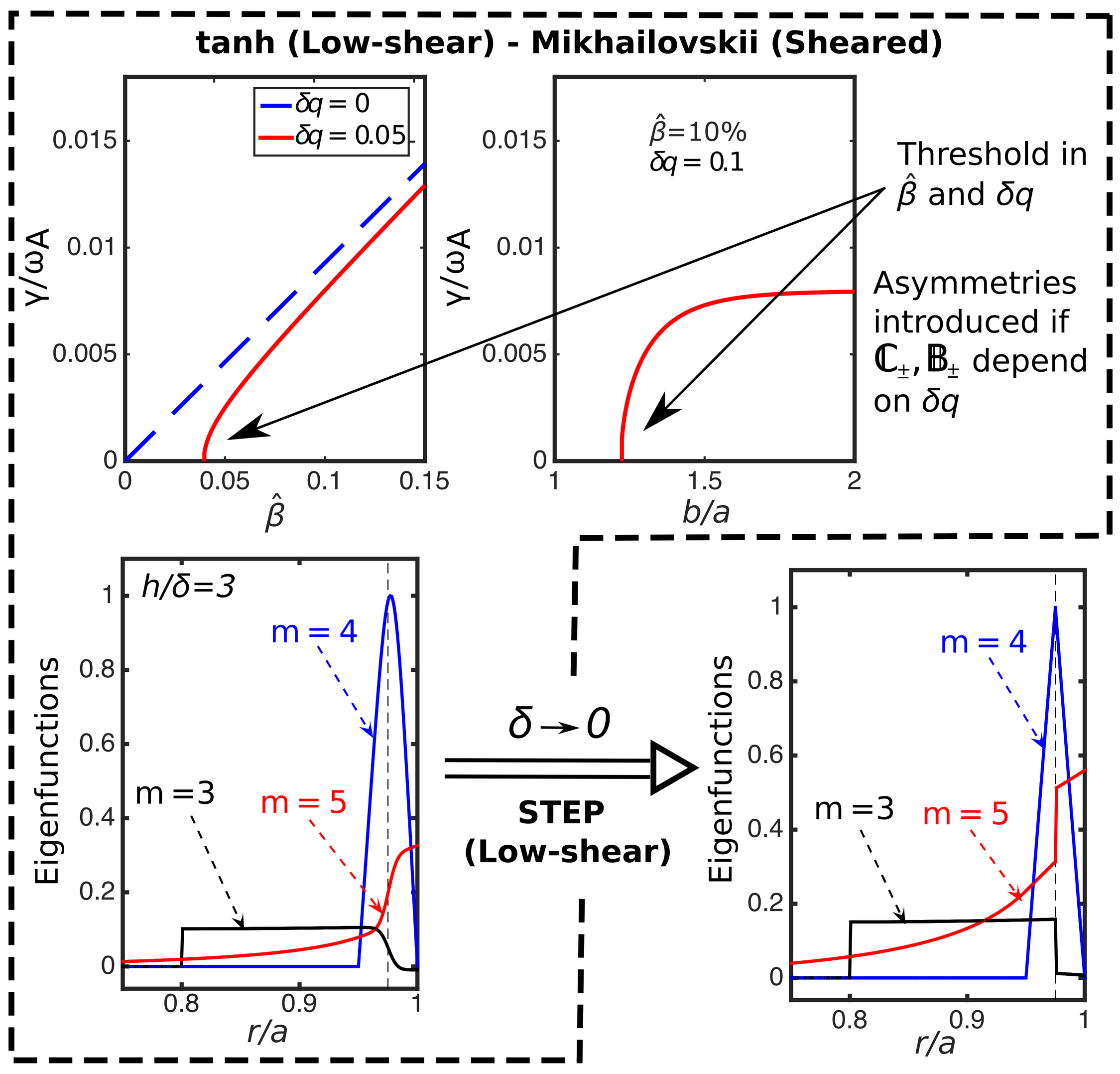
$$\frac{h}{a} \times \left(\frac{\hat{\beta}}{\varepsilon}\right)^2 \times m\left[\frac{5}{8} - \left(\frac{a}{b}\right)^{2m}\right] > \left(\frac{\delta q}{q}\right)^2$$

Ideal wall (at $r = b$) stabilising effect evident also in the *tanh* model

• $q_{edge} \approx 4$ $(m=4, n=1)$ / Low-shear region $\Delta/a = 5\%$



- Instability favoured if q above m/n if δq corrections are retainedi in $\mathbb{C}_{\pm}$, $\mathbb{B}_{\pm}$
- No Ω effects on stability $\Longrightarrow$ Rotation freq $\propto$ $n\Omega_{Ped}$



■ **Resistive wall**: Series expansion in $1/\sigma$

$$\hat{\gamma}\Big[(\hat{\gamma} - in\hat{\Omega})^2 - \overbrace{\gamma_I^2}^{\text{ID. WALL}}\Big] = \omega_A^3 \times \overbrace{\gamma_w^3}^{\text{RES. WALL}}$$

■ $\gamma_I = 0$ & $\gamma_w, \hat{\Omega} \neq 0$

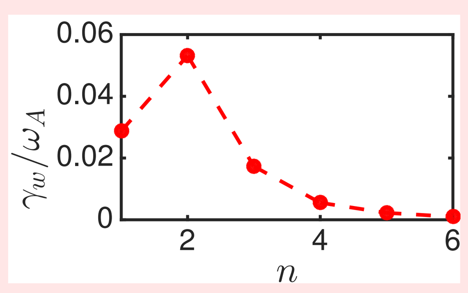
$$\hat{\gamma} = i\tfrac{2}{3}n\hat{\Omega} + \gamma_w \ (\tfrac{\gamma_w}{\hat{\Omega}} \gg 1), \quad \hat{\gamma} = in\hat{\Omega} + 0.7\sqrt{\frac{\gamma_w^3}{|n\hat{\Omega}|}}\ (\tfrac{\gamma_w}{\hat{\Omega}} \ll 1)$$

■ $\hat{\Omega} = 0$ & $\gamma_w, \gamma_I \neq 0$

$$\frac{\gamma_w}{\gamma_I} \ll 1 \Rightarrow \hat{\gamma} \approx \gamma_I + \frac{\gamma_w^3}{2\gamma_I^2}\ (\gamma_I^2 > 0), \quad \hat{\gamma} \approx \frac{\gamma_w^3}{\gamma_I^2}\ (\gamma_I^2 < 0)$$

$$\frac{\gamma_w}{\gamma_I} \gg 1 \Rightarrow \hat{\gamma} = \gamma_w + \frac{\gamma_I^2}{3\gamma_w}\ (\gamma_I^2 \lessgtr 0)$$

- **Small n peaking**
- $\gamma_w > 0$ **destabilising**
- Large Ω screens the resistive wall



CONCLUSIONS

■ Dispersion relation derivation for edge ideal infernal-like instabilities in QH-like regimes with edge local q flattening and sharp pressure gradients

■ Exact analytic treatment possible with simple classes (though general) of equilibrium profiles for q, p, ρ and Ω

■ Vacuum gap necessary for instability with ideal wall. Instability occcurs if $m/n - \delta q_{crit} < q_a < m/n + \delta q_{crit}$, δq_{crit} ($\ll 1$) depends on β. **EIM** have similar features typical of EHOs: eigenfunction structure, rotation frequency, growth rate independent of the sign of Ω

■ Outlook: more refined analysis of poloidal sheared flow effects, modifications to geometry e.g. elongation and X-point

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