Scattering of radio frequency waves by cylindrical blobs in the plasma edge in tokamaks (*)

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Introduction

Radio frequency waves are routinely used in tokamak fusion plasmas for plasma heating, current control, and as well as in diagnostics. RF waves have to propagate through a turbulent layer known as scrape-off layer, before reaching the core plasma. This layer exhibits coherent density fluctuations in the form of blobs and filaments.
The scattering processes of RF plane waves and Gaussian beams by single cylindrical elongated blob is studied. The axis of the blob is not aligned with the externally applied magnetic field in the case of plane waves.
The investigation concerns the case of Electron cyclotron (EC) waves (f_0=170 GHz) for ITER-like and Medium Size Tokamak applications (such as TCV, ASDEX-U, DIII-D, etc). The study covers for a variety of density contrasts between the blobs and the ambient plasma and a wide range of blob radii.

Analytical theory

The axis of the cylindrical blob is not necessarily parallel to the externally imposed magnetic field $\mathbf{B}_0$, but in angle φ_0 (Figure 1). The magnetic field lines are parallel to $(z-x)$ plane, with z being the axis of the cylinder and z' parallel to the magnetic field lines.

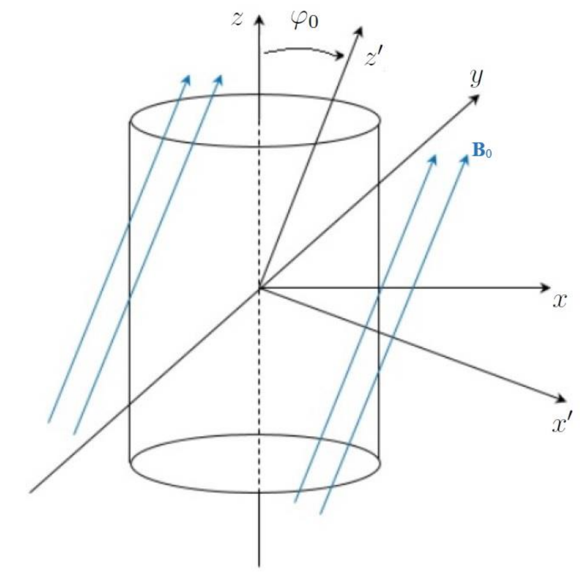

Figure 1: *Basic geometry considered*

The cylinder is considered to have infinite length. Plane waves (or Gaussian beams) propagating in the ambient plasma region (which has different density from the blob plasma region), are hitting the blob cylinder and are being scattered. It should be also mentioned, that that the incident propagating waves have their own orientation.
After making appropriate rotations and coordinate transformations, Faraday-Ampere equation in the Fourier domain is used

$$\varepsilon_0 \nabla\times\nabla\times\mathbf{E}(\mathbf{r}) - \left(\frac{\omega}{c}\right)^2 \mathbf{D}(\mathbf{r}) = 0$$

where the electric field is analyzed in the cylindrical vector functions. Dispersion relation

$$\det\left(\overleftrightarrow{\Delta}\right) = 0$$

is calculated by using the permittivity tensor and then solved. Boundary conditions (*SC:* scattered field, *BL:* field inside the blob and *0:* incident field)

$$\hat{\mathbf{r}}\times(\mathbf{e}_{SC}+\mathbf{e}_0-\mathbf{e}_{BL}) = 0, \quad \hat{\mathbf{r}}\times(\mathbf{h}_{SC}+\mathbf{h}_0-\mathbf{h}_{BL}) = 0$$

as well as the fact that the parallel to the cylinder axis wave vector component stays the same for all regions, are taken into account.
Finally, for the normalized with respect to the incident fields, dimensionless electric field and magnetic field respectively, one obtains:

$$\tilde{\mathbf{e}}(\rho,\varphi)_{(BL,SC)} = \sum_{M=O,X}\sum_{m=-\infty}^{m=\infty} i^m e^{im\varphi}\left[\mathcal{E}^M_{mr}(\rho)\hat{\mathbf{r}} + \mathcal{E}^M_{m\varphi}(\rho)\hat{\boldsymbol{\varphi}} + \mathcal{E}^M_{mz}(\rho)\hat{\mathbf{z}}\right]_{(BL,SC)}$$

$$\tilde{\mathbf{h}}(\rho,\varphi)_{(BL,SC)} = \frac{E_0}{H_0}\sqrt{\frac{\varepsilon_0}{\mu_0}}\sum_{M=O,X}\sum_{m=-\infty}^{m=\infty} i^m e^{im\varphi}\left[\mathcal{H}^M_{mr}(\rho)\hat{\mathbf{r}} + \mathcal{H}^M_{m\varphi}(\rho)\hat{\boldsymbol{\varphi}} + \mathcal{H}^M_{mz}(\rho)\hat{\mathbf{z}}\right]_{(BL,SC)}$$

where $\mathcal{E}^M_{mr}(\rho)$, $\mathcal{E}^M_{m\varphi}(\rho)$, $\mathcal{E}^M_{mz}(\rho), \mathcal{H}^M_{mr}(\rho), \mathcal{H}^M_{m\varphi}(\rho), \mathcal{H}^M_{mz}(\rho)$ are functions only of ρ and are calculated from equations of the form

$$\begin{pmatrix} \mathcal{E}^M_{mr}(\rho) \\ \mathcal{E}^M_{m\varphi}(\rho) \\ \mathcal{E}^M_{mz}(\rho) \\ \mathcal{H}^M_{mr}(\rho) \\ \mathcal{H}^M_{m\varphi}(\rho) \\ \mathcal{H}^M_{mz}(\rho) \end{pmatrix}_{(BL,SC)} = \sum_n \varepsilon_n^{M,(BL,SC)} \int_0^{2\pi} \frac{d\varphi_k}{2\pi} e^{i(n-m)\varphi_k} \begin{pmatrix} \dots \\ \dots \\ \dots \\ \dots \\ \dots \\ \dots \end{pmatrix}_{(BL,SC)}$$

where $\varepsilon_n^{M,(BL,SC)}$ are calculated from a linear system $4(2n_{max}+1)$ by $4(2n_{max}+1)$ (n_{max} is numerically estimated sufficient number) of the following form:

$$\sum_{n=-n_{max}}^{n=n_{max}} \left(a^{O,BL}_{j,mn}\varepsilon_n^{O,BL} + a^{X,BL}_{j,mn}\varepsilon_n^{X,BL} - a^{O,SC}_{j,mn}\varepsilon_n^{O,SC} - a^{X,SC}_{j,mn}\varepsilon_n^{X,SC}\right) = a^0_{j,m}, \quad j=1,2,3,4$$

(which is for every m from $m=-n_{max}$ to $m=n_{max}$).
As expected, since the cylinder blob has infinite length, the results for the electric field and magnetic field are z independent. After these, the Poynting vector is easily calculated from the well-known form:

$$\tilde{\mathbf{s}} = \frac{1}{2}Re\{\tilde{\mathbf{e}}\times\tilde{\mathbf{h}}^*\}$$

Plane wave scattering

In all cases ambient density is 10^{19} m^{-3} and blob density is expressed as a percentage of the ambient density.

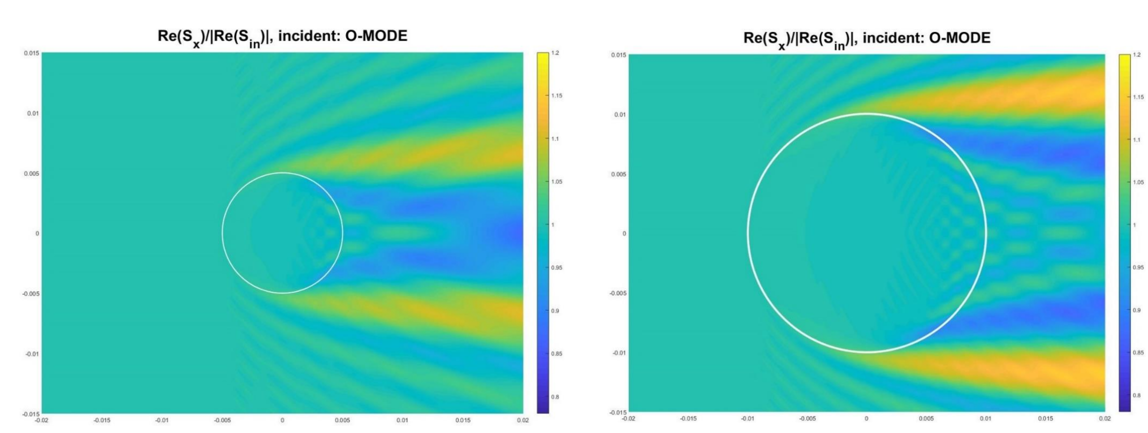

Figure 2: *Poynting flux in the forward direction (up to 1.2) for blob radius 5 mm and 10 mm, blob density 150%, magnetic field inclination 0°*

- Scattering process for r_{BL}=5mm and 10mm has many similarities.

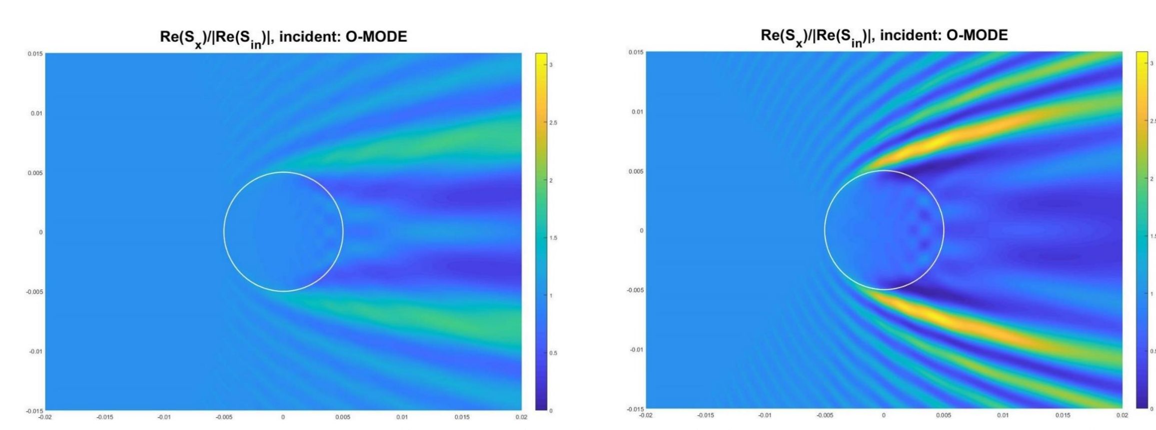

Figure 3: *Poynting flux in the forward direction (up to 3.10) for blob density 500% and 1500%, blob radius 5 mm, magnetic field inclination 0°*

- For r_{BL}=5mm, scattering is stronger for higher density contrast between the blob and the ambient plasma.

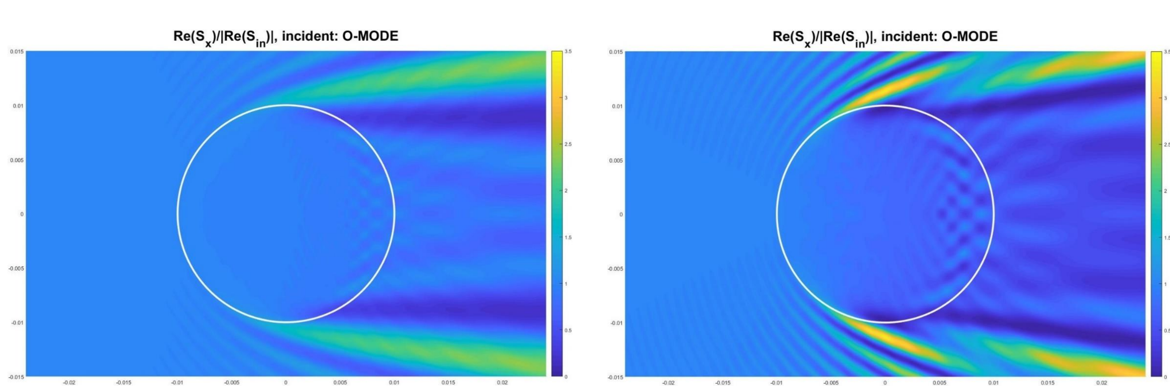

Figure 4: *Poynting flux in the forward direction (up to 3.50) for blob density 500% and 1500%, blob radius 10 mm, magnetic field inclination 0°*

- For r_{BL}=10mm, scattering is stronger for higher density contrast between the blob and the ambient plasma.

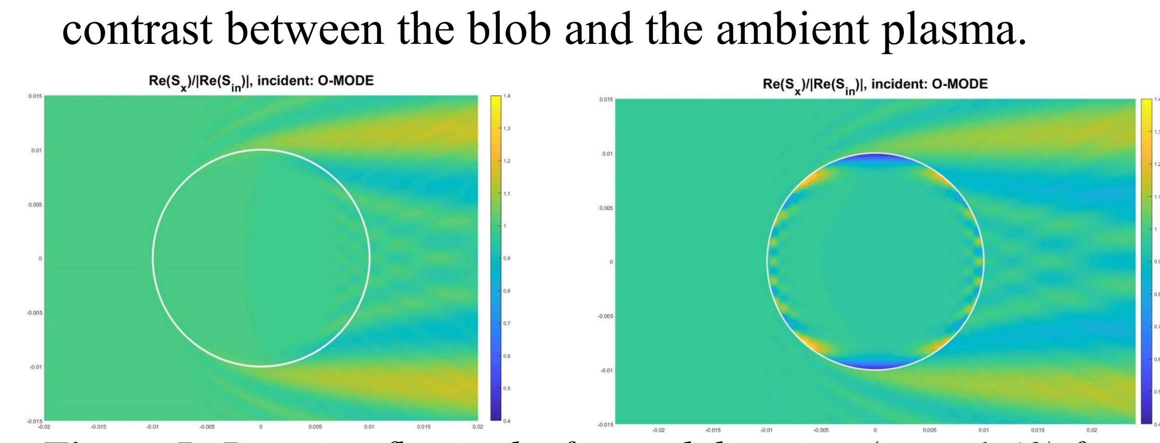

Figure 5: *Poynting flux in the forward direction (up to 1.40) for magnetic field inclination 0° and 15° for blob radius 10 mm and blob density 150%*

- Magnetic field inclination, for small angles (0° and 15°) affects the Poynting flux, mostly inside the blob.

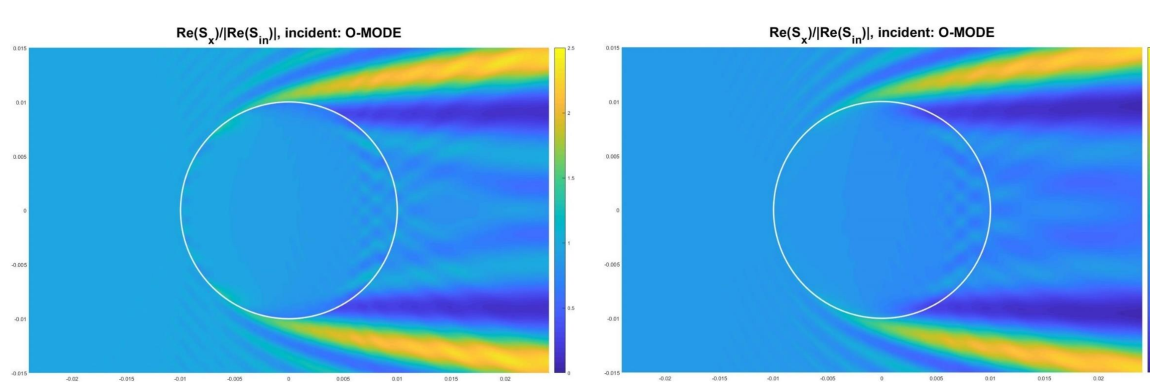

Figure 6: *Poynting flux in the forward direction (up to 2.50) for magnetic field inclination 15° and 30° for blob radius 10 mm and blob density 500%*

- From 15° to 30° magnetic field inclination, scattering process has many similarities

Note that in all numerical results on this poster, only the real part of the normalized dimensionless Poynting flux in the forward direction vector appears.

Gaussian beam scattering

In the figures below, three density cases appear for both O-mode (left column) and X-mode (right column) incident waves. Blob radius is 10 mm.

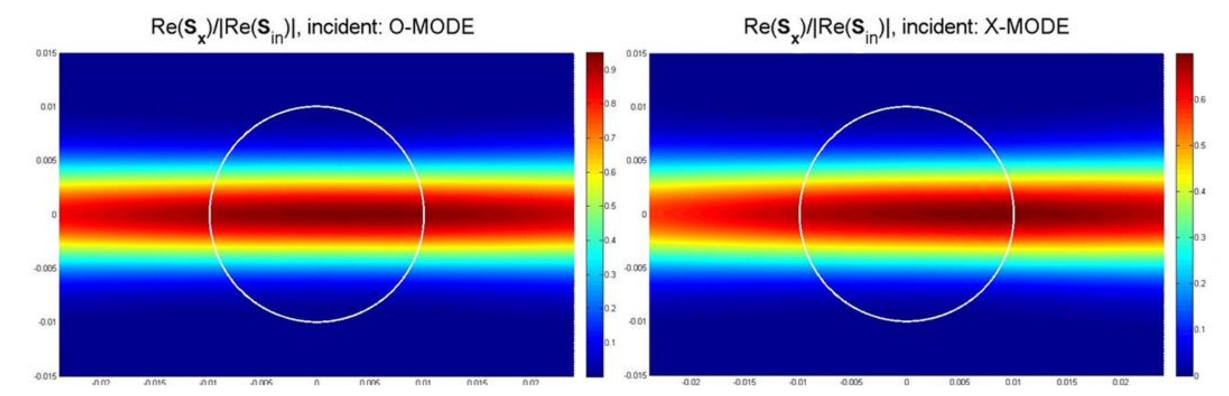

Figure 7: *Poynting flux in the forward direction (up to 1.0) for ambient density 10^{19} m^{-3} and blob density 1.50*10^{19} m^{-3} magnetic field inclination 0°*

- Gaussian beams in both cases (for O-mode and X-mode incident waves) propagate through the blob almost like it is not there.

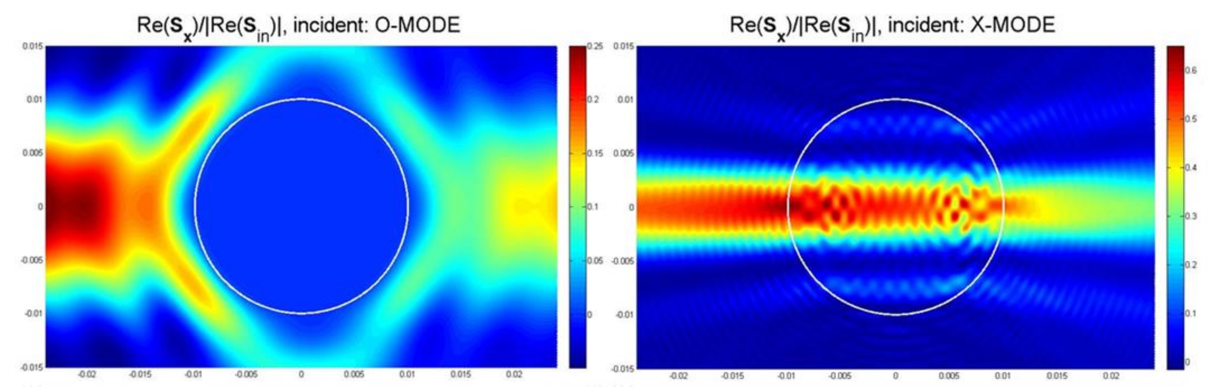

Figure 8: *Poynting flux in the forward direction (up to 0.7) for ambient density 3.50* 10^{20} m^{-3} and blob density 4.50* 10^{20} m^{-3}, magnetic field inclination 0°*

- It can be seen that the blob density, is such that the cut-off frequency for the O-mode beam is below EC frequency range, while for the X-mode beam the cut-off frequency is a little higher than 170 GHz. As a result, the O-mode beam cannot propagate through the blob and the X-mode can (hardly) get through it.

- In addition, one observes that a kind of tunneling effect appears in the O-mode case (parts of the incident beams that do not propagate inside the blob, exist after the blob).

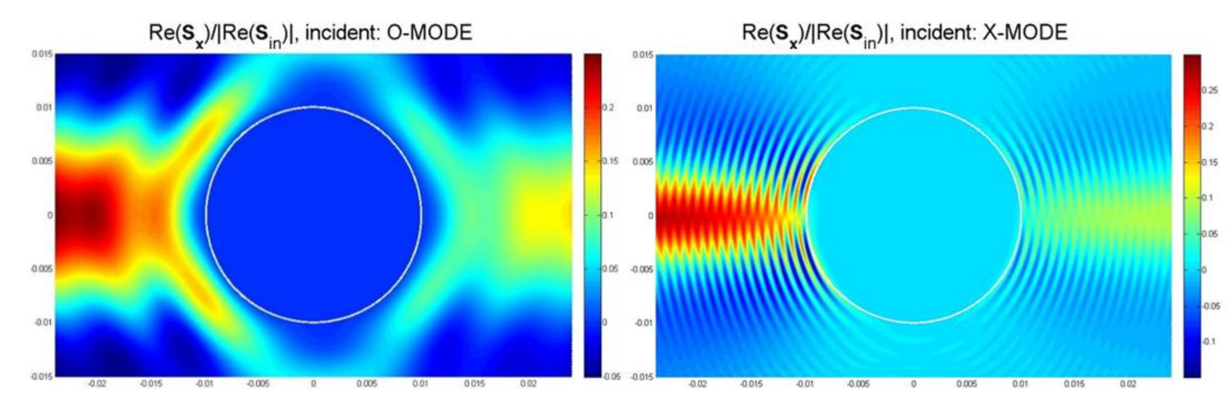

Figure 9: *Poynting flux in the forward direction (up to 0.32) for ambient density 3.50* 10^{20} m^{-3} and blob density 6.50* 10^{20} m^{-3} , magnetic field inclination 0°*

- It's clear that in this case, none of the O-mode and X-mode beams can propagate through the blob.

- Also, note that tunneling effect appears this time in both O-mode and X-mode cases.

References

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[2] Z. C. Ioannidis, A. K Ram, K. Hizanidis, I. G. Tigelis, "Computational studies on scattering of radio frequency waves by density filaments in fusion plasmas", to be published in PoP