ECCD magnetic island suppression as converse of a forced reconnection problem

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The problem of island control

- Classical and neoclassical tearing modes are a serious cause of degradation of plasma confinement and an important open issue is to find the appropriate means of controlling them.

- One of the most promising methods is based on the injection of an external control current within the magnetic island.

- Conventional approaches are based on the generalized Rutherford equation which describes the time evolution of the island width w

$$\frac{g_1}{\eta}\frac{dw}{dt} = \Delta'_{eq} + \Delta'_{bs} + \Delta'_{p} + \Delta'_{cd} + \ldots$$

where: w = amplitude island width depending on the magnetic perturbation

g_1 = constant

Δ'_{eq} = standard tearing instability parameter of the unperturbed equilibrium

$\Delta'_{bs,p,cd}$ = modifications of the instability parameter due the boostrap, polarization,

current driven currents

The problem of island control

- Here the effect of ECCD is investigated as a converse of the Ham Kulsrud Taylor (HKT) [1,2], forced reconnection problem, where the current drive plays the role of the drive force.
- We present analytical calculations which show the relation between the peaking value of the ECCD on the rational surface and the final island width.
- The application of intense external coercive means may lead to the violation the constant ψ regime on which the governing equation for the island evolution is based.
- Entering the non-constant ψ regime implies the possibility of driving instabilities growing faster than the current diffusion process out of the reconnecting region. This leads to expect a limit on the amplitude and localization of the externally applied perturbation as also found in [3].
- Numerical simulations aimed at analyzing the transition between the constant ψ and non-constant ψ regimes under the effect of a ECCD control are presented.
- It is shown that if the deposition width is below a critical value this transition occurs generating the *X*-point collapse and entering a plasmoid formation phase [4].

ECCD as a converse of the HKT problem

- RRMHD framework in slab geometry. Dimensionless equations are:

$$\frac{\partial\psi}{\partial t} + \mathbf{v}_\perp \cdot \nabla\psi = -\eta(J - J^{(0)} - J_{ec}) \qquad \mathbf{B} = B_0\mathbf{e}_z + \mathbf{B}_\perp$$

$$(\partial_t + \mathbf{v}\cdot\nabla)\omega_z = \mathbf{B}\cdot\nabla j_z + \nu\nabla^2\omega_z \qquad \mathbf{B}_\perp = \nabla\psi \times \mathbf{e}_z$$

$$\omega_z = \nabla^2\phi \qquad J = -\nabla^2\psi \qquad \mathbf{v}_\perp = -\nabla\varphi \times \mathbf{e}_z$$

- The forcing is represented by J_{ec} and the initial state is replaced with one with a finite magnetic island:

$$J_{ec}(x,y,t) = J_m(t)\exp\left(-\frac{(\psi(x,y,t) - \psi_O(t))^2}{\delta^2}\right)$$

ECCD as a converse of the HKT problem

- Following [5] we choose a static equilibrium with a flux function given by $\Psi_\Sigma \frac{\cosh(kx)}{\cosh(k)} cos(ky)$, where $\Psi_\Sigma \propto {w_0}^2$ the initial island width.

- The linearized problem gives:

$$\partial_t \psi_1 + kx\phi_1 = \eta[\psi_1'' - J_0 D(\psi)]$$

$$\partial_t \phi_1'' = kx\psi_1'' + \nu\phi_1''''$$

- where

$$J_0 D(\psi) = \frac{2J_0}{\delta_{CD}\sqrt{\pi}} \exp[-\frac{4(x - x_{dep})^2]}{\delta_{CD}^2}]$$

- With a change of variable $\bar{x}=kx$ and time-Laplace and space-Fourier trasforming, with straightforward algebra we obtain:

$$\partial_\theta(\frac{\theta^2}{s + \eta k^2\theta^2}\partial_\theta \phi_F) - (s\theta^2 + \nu k^2\theta^4)\phi_F = 0$$

ECCD as a converse of the HKT problem

- The expression of the Laplace transform on the *x=0* layer is:

$$\psi_L(0,s) = \frac{2i\eta k^2}{s}\int_0^\infty d\theta \frac{\theta^2}{s+\eta k^2\theta^2}\partial_\theta \phi_F - \frac{\eta J_0 D(0,s)}{s}$$

- The boundary conditions to be applied are:

$$\lim_{\theta\to\infty} \phi_F(\theta,s) = 0$$

- In the viscous-resistive range $s \ll \eta k^2\theta^2$ the asymptotic matching of the small and large inner solutions with the outer (ideal) solution gives:

$$\Psi_L(0,s) = \psi_{out}(0,s) = \frac{\Delta'_s \Psi_\Sigma}{(\Delta'(s) - \Delta'_0)s}$$

- wnere :

$$\Delta'_0 = -\frac{2k}{\tanh k} \quad \Delta'_s = \frac{2k}{\sinh k} \quad \Delta'(s) = s\tau_{\nu\eta}$$

$$\tau_{\nu\eta} \propto \nu^{1/6}\eta^{-5/6}k^{-1/3}$$

ECCD as a converse of the HKT problem

- The boundary value which in HKT is a driving term here corresponds to the flux label of the separatrix proportional to the square of the initial magnetic island.

- Now following [5] we get:

$$\Psi_{ext}(s) = [\Psi_\Sigma - \frac{(\Delta'(s) - \Delta'_0)}{S\Delta'_s} J_0 D(0,s)]$$

- A good choice of $J_0(t)$ can control the reconnected flux amplitude. This condition can be expressed in terms of a partial suppression parameter *G* $(0<G<1)$ *as:*

$$\frac{(\Delta'(s) - \Delta'_0)}{S\Delta'_s} J_0 D(0,s) = G\Psi_\Sigma$$

- The combination of this equation with the solution of the ideal region eventually leads to the solution of the KHT problem as:

$$\psi(x,s) = \Psi_{ext}(s)[\cosh(kx) - \frac{\sinh|kx|}{\tanh(k)}]cos(ky) + \Psi_\Sigma \frac{\sinh|kx|}{\sinh(k)} cos(ky)$$

ECCD as a converse of the HKT problem

- When the island in the constant ψ regime shrinks below a critical value, if the driven current has a deposition depth comparable with this critical value , it can drive the perturbation into a non-constant ψ regime, giving rise to secondary instabilities.

- According to [5] in the viscoresistive regime a nonlinear instability condition is reached when under the ECCD the island approaches $w_{crit} = 4\sqrt{\Psi_{crit}}$ where:

$$\Psi_{crit} = C\frac{k}{\Delta'_s}\frac{\sqrt{(1+P)}}{S} \qquad P=\nu/\eta \quad S=1/\eta$$

  and C is a parameter related to the marginally stable current sheet aspect ratio.

- The critical condition is then transferred to J_0/δ_{CD} (here dimensionless)

$$\frac{2J_0}{\delta_{CD}\sqrt{\pi}} \gtrsim \frac{S\Delta'_s}{(\Delta'(s)-\Delta'_0)s}[\Psi_{crit} - \Psi_\Sigma]$$

- which using previous relations turns out to be:

$$\frac{J_0}{\delta_{CD}} \gtrsim \frac{\sqrt{\pi}}{2}\Big(\frac{G}{1+G}\Big)\frac{Ck\sqrt{1+P}}{(-\Delta'_0)}$$

ECCD as a converse of the HKT problem

- If this aspect ratio is exceeded the non-constant ψ regime is entered where the collapse of the X-point is possible.

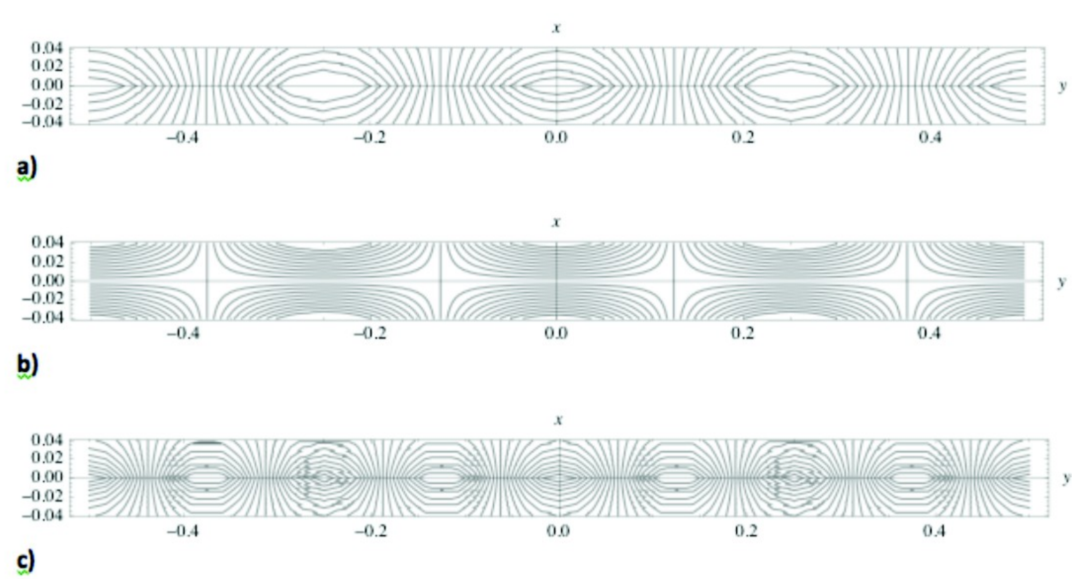
- This unstable regime will eventually saturate into a controlled equilibrium condition dominated by the rf driven current. In such condition the constant C can be taken as:

$$C = \frac{2}{\sqrt{\pi}}\left(\frac{1+G}{G}\right)\frac{(-\Delta_0')}{\sqrt{1+P}}\frac{J_0}{m_p}$$

- where $k=m_p/\delta_{CD}$. This value of C is compatible with the formation of current sheets of aspect ratio $\varepsilon_c>C^{-1/2}$.

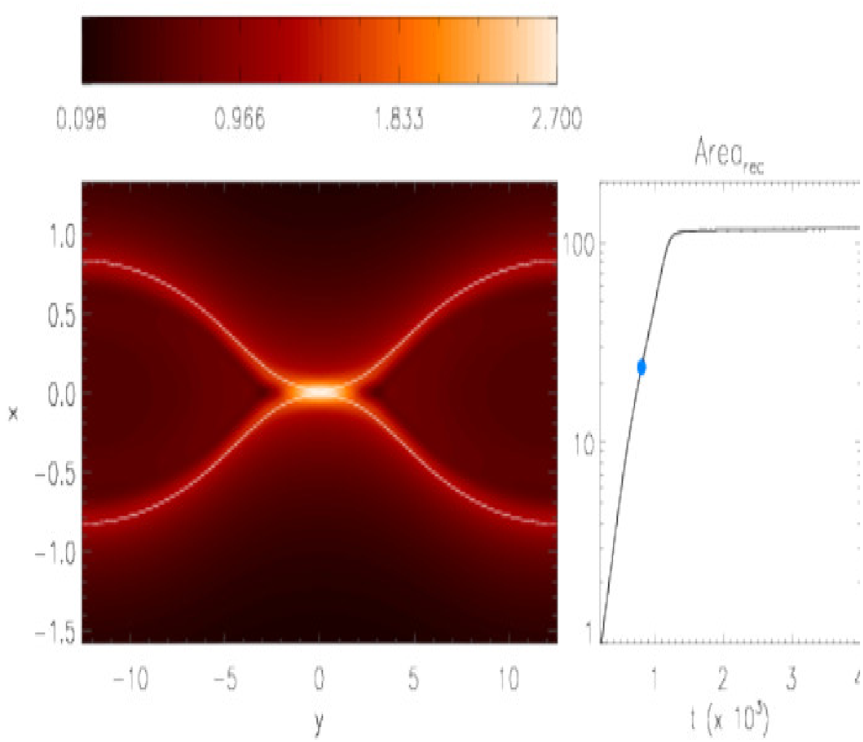
- For small J_0 k results from a consistency condition imposed by matching the transition from the constant to the non-constant ψ regime:

$$k = P^{1/8}S^{1/2}(1+P)^{-1/2}C^{-3/4}$$



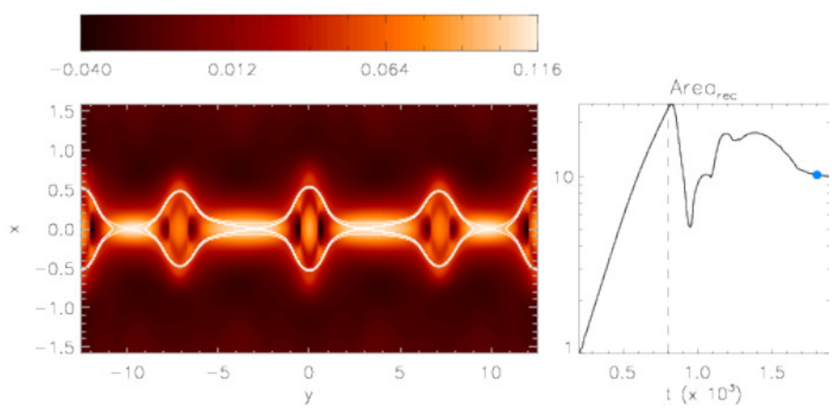
ECCD numerical epxeriment

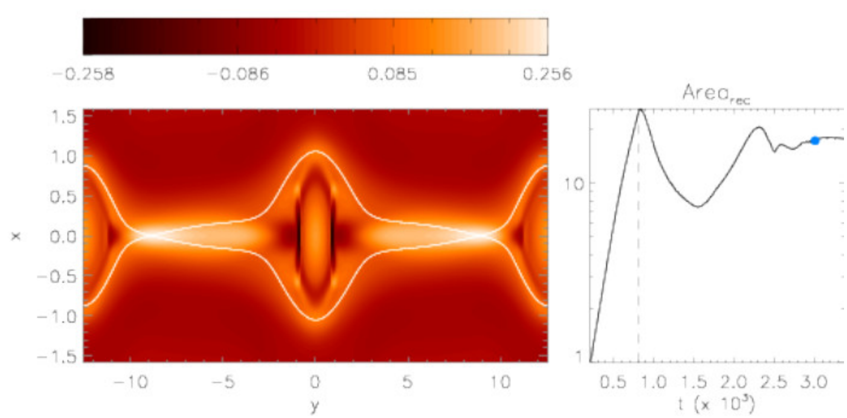
- Numerical experiments carried out applying a rf driven current in a spontaneous reconnection event. The current drive is applied continuously starting from a large nonlinear magnetic island.

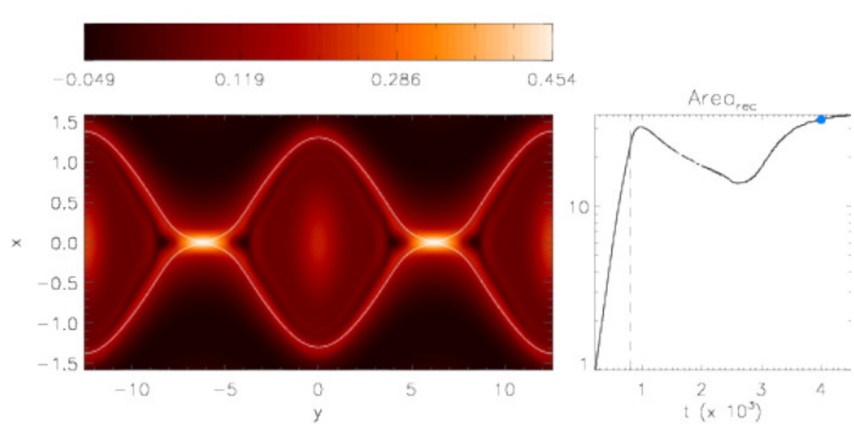


ECCD numerical experiment

- Here the effect of the ECCD beam for three different values of the *b* parameter are shown.







ECCD numerical experiment

- In all the cases considered the system moves towards a stationary configuration with the area of the reconnected region comparable with the area of the initial magnetic island. However the current control has a significant effect on the change of the magnetic topology compared with the initial magnetic island.

- Moreover this change appear to be strongly dependent on the value of the beam width.

- The numerical analysis shows that the new topology is the results of a complex dynamics induced by the continuous deposition of the J_{CD}. After an initial phase when the J_{CD} reduces effectively the magnetic island, in fact, the small scale current layers induced by the external control current along the null axis $x=0$ give rise to "plasmoid" like secondary structures. These structures grow and recombine on fast time scales, leading to a continuous change of the magnetic topology until the saturation is reached.

[1] Hahm T S and Kulsrud R M 1985, *Phys. Fluids,* **28**, 2412

[2] Lazzaro E and Comisso L 2011, *Plasma Physics and Controlled Fusion*, **53**, 054012

[3] Borgogno D *et al.*, 2014, *Phys. Plasmas*, **21**, 060704.

[4] Lazzaro E *et al.*, invietd paper at the *44th European Physical Society Conference on Plasma Physics*, accepted for publication on *Plasma Physics and Controlled Fusion.*

[5] Comisso L. et al., 2015, Phys. Plasmas, ***21****, 042109.*