Evolving the ITG driven turbulence with test modes

Virgil Băran, Mădălina Vlad, Florin Spineanu

National Insitute of Plasma, Laser and Radiation Physics, Bucharest-Magurele, Romania

The Idea

- We extract the evolution tendencies of the turbulence from the growth rates and frequencies of the test modes, which, in turn, depend on the statistical properties of the turbulent background on which they develop.

Starting Equations

The GK equation for electrons and ions ($\rho_L << \xi_{\mathrm{corr},\phi}$):

$$\left(\partial_t - \frac{\nabla\phi\times\mathbf{b}}{B}\cdot\nabla + v_z\partial_z - \frac{q_\alpha}{m_\alpha}\partial_z\phi\frac{\partial}{\partial v_z} + \nabla_\perp\cdot\mathbf{u}_p\right) f^\alpha = 0$$

The polatization $\mathbf{u}_p = \dfrac{m_\alpha \dot{\mathbf{E}}_\perp}{eB^2}$ drift gives the finite ρ_L effects.

Short Time equilibrium

Electron density

$$n_e = n_0(x)\exp\left(\frac{e\phi_0}{T_e}\right) \ .$$

Ion distribution function:

$$f_0^i = n_0(x) F_M^i \exp\left(\frac{e\phi_0}{T_e}\right) \ .$$

with F_M^i the Maxwell distribution corresponding to $T_i(x)$.

Perturbing the background potential

In a given background field $\phi_0(x_i,t)$, the system responds to the change of the potential with

$$\delta\widetilde{\phi} = \delta\phi\exp\left(ik_x x + ik_y y + ik_z z - i\omega t\right) \ .$$

- For electrons:

$$f^e = n_0(x) F_M^e \exp\left(e\left(\phi_0 + \delta\widetilde{\phi}\right)/T_e\right) \ .$$

- For ions:

$$f_1 = n_0(x) F_M^i \exp\left(e\left(\phi_0 + \delta\widetilde{\phi}\right)/T_e\right) + h \ ,$$

where h is the nonadiabatic part of the ion distribution function.

By a linearization procedure for the ion perturbation, we obtain:

$$\mathcal{O}[\phi_0]\ h = f_0\,\frac{e\delta\widetilde{\phi}}{T_e}\,\mathcal{T}(\omega, k_i; v_z; \phi_0) \ .$$

Solution

The solution is of the form:

$$h = \frac{e\delta\widetilde{\phi}}{T_e} f_0 \int_{-\infty}^{t} d\tau\ \mathcal{T}\exp\left[ik_i x_i(\tau;t) - i\omega(\tau - t)\right] \ ,$$

where the integral is performed along the trajectories in the background field:

$$\frac{d\mathbf{x}_\perp}{d\tau} = -\frac{\nabla\phi_0(\mathbf{x}_\perp(\tau),\tau)\times\mathbf{b}}{B} \quad , \quad \frac{dz}{d\tau} = v_z \ .$$

We then:

- Find the ion density by $\delta n_i = \int dv\ h$.
- Average over the statistical realizations of the background fluctuating potential.

We obtain, with $\mathcal{T}' = \mathcal{T}'(\omega, k_i; \phi_0)$ now:

$$\delta n_i = \frac{e\delta\phi}{T_e} n_0 \exp\left(\frac{e\phi_0}{T_e}\right)\int_{-\infty}^{t} d\tau\ \left\langle \mathcal{T}'\exp\left[i\mathbf{k}_\perp\cdot\mathbf{x}_\perp(\tau;t)\right]\right\rangle$$

$$\exp\left[-i\omega(\tau - t) - \frac{k_z^2 v_{T_i}^2(\tau - t)^2}{2}\right] \ ,$$

The dispersion relation is the quasineutrality condition:

$$\frac{e\delta\phi}{T_e} = \frac{e\delta\phi}{T_e} + \delta n_i,$$

where the electron contribution is on the l.h.s. and the ion term is on the r.h.s.

Dispersion Relations

In the absence of the turbulent background, i.e. $\phi_0 = 0$, we recover the standard ITG dispersion relation for slab geometry:

$$\omega = \frac{1 + i\sqrt{3}}{2}\left(\frac{k_y k_z^2}{1 + k_\perp^2}\right)^{1/3} \ .$$

In the weak turbulence regime the displacement probabilities are Gaussian, which leads to the trajectories' diffusion:

$$M[\phi_0] \equiv \left\langle \exp\left[i\mathbf{k}_\perp\cdot(\mathbf{x}_\perp(\tau) - \mathbf{x})\right]\right\rangle_{Gauss.} \sim$$

$$\sim \exp\left[-\frac{k_i^2\left\langle x_i^2(t-\tau)\right\rangle}{2}\right] = \exp\left[-k_i^2 D_i (t - \tau)\right] \ .$$

The dispersion relation is modified as:

$$W^2(W - ik_i^2 D_i) = \frac{T_i}{T_e}\left(k_y - \frac{k_x D_x}{L_{T_i}}\right)\frac{k_z^2}{1 + k_\perp^2} \ ,$$

written in terms of $W = \omega + ik_i^2 D_i$.

Test Particles Method

The equations of motion for test particles are:

$$\frac{dx_i}{dt} = -\frac{1}{B}\epsilon_{ij}\frac{\partial}{\partial x_j}\left(\varphi^{ITG} + \varphi^{ZFM}\right) \ , \ \frac{dz}{dt} = \eta_\parallel \ .$$

The collisional velocity is characterized by the correlation:

(1) $\langle \eta_\parallel(0)\eta_\parallel(t)\rangle = \chi_\parallel^c\ \nu\ \mathrm{e}^{-\nu|t|}$, where $\chi_\parallel^c = \lambda_{mfp}^2\nu$, $\lambda_{mfp} = \dfrac{v_{T_i}}{\nu}$.

We use analytical expression for the Eulerian correlation of the potential $E(\mathbf{r},t) = \left\langle \langle \phi(\mathbf{r}',t')\ \phi(\mathbf{r}+\mathbf{r}', t'+t)\rangle_\phi \right\rangle_\parallel$, defined as

$$E^{ITG} = \partial_y\left[\mathrm{e}^{-\frac{x^2}{2\lambda_x^2} - \frac{y^2}{2\lambda_y^2}}(\sin k_{0y} y)/k_{0y}\right]\partial_z\left[\mathrm{e}^{-\frac{z^2}{2\lambda_z^2}}(\sin k_{0z} z)/k_{0z}\right]$$

$$E^{ZFM} = \partial_x\left[\mathrm{e}^{-\frac{x^2}{2(\lambda_x^z)^2}}(\sin k_{0x}^z x)/k_{0x}^z\right]\partial_z\left[\mathrm{e}^{-\frac{z^2}{2(\lambda_z^z)^2}}(\sin k_{0z}^z z)/k_{0z}^z\right]$$

The diffusion coefficients are obtained by integrating the Lagrangian correlation of velocities $v_i = -\epsilon_{ij}\partial_j\varphi$, that is $L_{ii} = \langle v_i(0) v_i(t)\rangle$:

$$D_i = \int_0^\infty L_{ii}(\tau)d\tau = \int_0^\infty\left(-\frac{\partial^2}{\partial x_j^2}\right)E\ d\tau$$

- Within the present approach we uncover a new mechanism of generation of zonal flow modes ($k_y = 0$), originating already in the quasilinear regime due to trajectory diffusion along the temperature gradient.

The Iterated Self-consistent Method

Step 1

The evaluation of approximate (short time) equilibrium distribution functions in the presence of background turbulence and of the EC of the potential.

Step 2

The calculation of the statistical characteristics of the trajectories (diffusion coefficients, probability of displacements, characteristics of the quasi-coherent structures) as functions of the EC of the potential.

Step 3

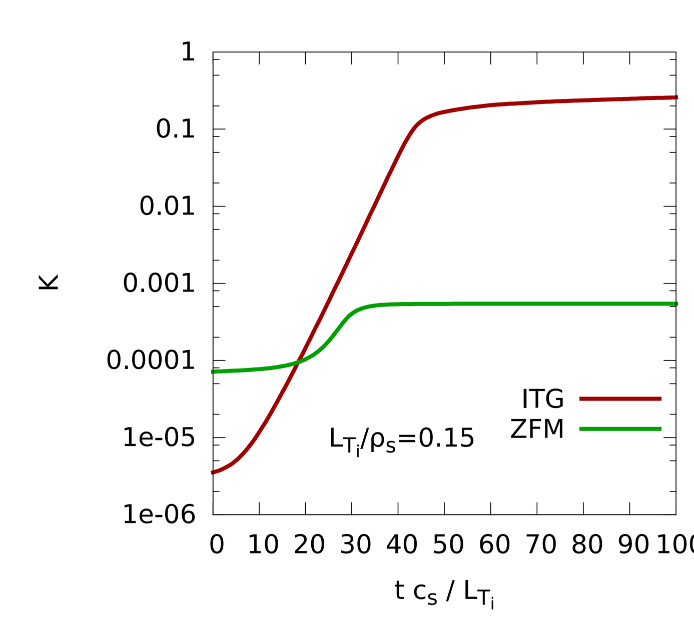
The calculation of the renormalized propagator (averaged over trajectories) and evaluation of the frequencies and the growth rates of the test modes.

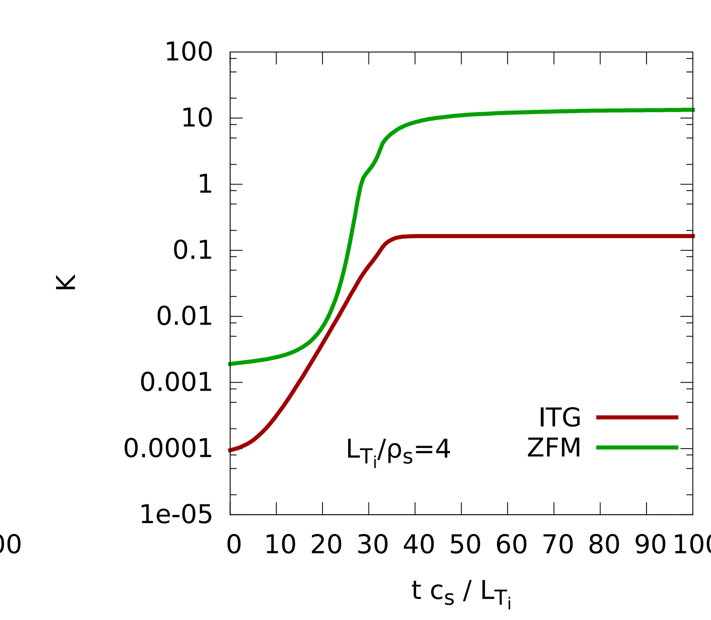
Step 4

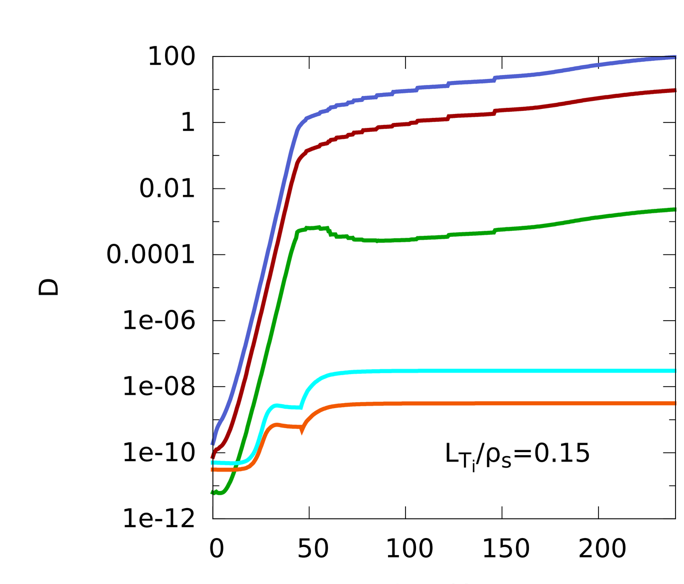
The evolution of the spectrum on a small time interval is obtained using the growth rates of all modes. It is the starting point of a new step in this iterated method.

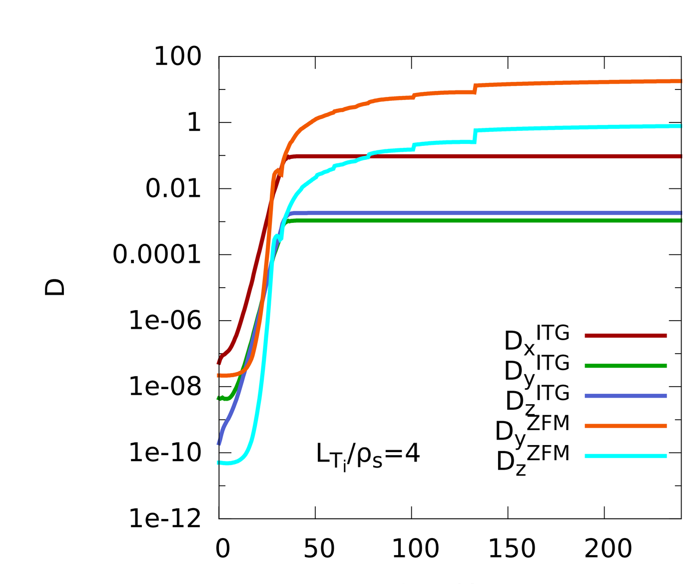
Results

The evolution of the normalized amplitude of the potential $K = \dfrac{e\Phi}{T_e}\dfrac{L_{T_i}}{\rho_s}$ and that of the diffusion coefficients are given below:

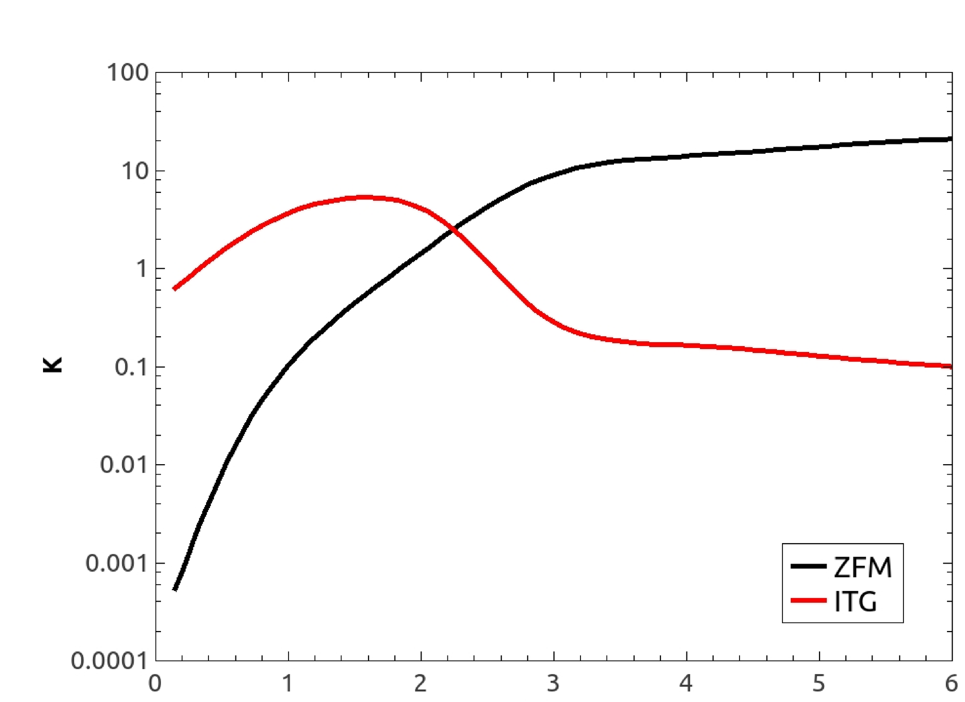








We find that the turbulence evolution at large times is dominated by either the ZFM or ITG modes, as a function of the parameter L_{T_i}/ρ_s:



References

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Acknowledgements

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