A fully implicit code for parallel electron transport in the Scrape-Off Layer

Imperial College London

S. Mijin[1],R. Kingham[1], F. Militello[2]

[1]Imperial College London, London, SW7 2BW, UK
[2]CCFE, Culham Science Centre, Abingdon, OX14 3DB, UK

ABSTRACT

In order to model plasma regimes of highly varying collisionality, a new 1D electron kinetic code is being developed, combining an arbitrarily anisotropic spherical harmonic decomposition of the distribution function with an extensive suite of collision operators for both the plasma and the neutrals. Presented here is the current state of development of the code, including both the 0D collision model benchmarking, as well as more integrated 1D tests.

SPHERICAL HARMONIC DECOMPOSITION

Using the spherical harmonic formalism, the distribution function can be written as[1]:

$$f(v,\theta,\varphi) = \sum_{l=0}^{\infty}\sum_{m=-l}^{l} f_l^m(v) P_l^{|m|}(\cos\theta) e^{im\varphi}, \qquad f_l^{-m}(v) = (f_l^m)^*$$

The kinetic equation can then be separated into equations for the harmonic components $f_l^m(v)$:

$$\frac{\partial f_l^m(v)}{\partial t} = A_l^m + E_l^m + B_l^m + C_l^m,$$

where the RHS terms are respectively: spatial advection, velocity space advection due to electric and magnetic fields[2], and collision operators. For the 0D and 1D tests here $B_l^m = 0$.

Two main reasons for using spherical harmonics:

- Eigenfunctions of pitch-angle scattering operator – main type of electron-ion collisions
- Natural representation of transport quantities (moments):

$$f_0^0 \rightarrow n, T; f_1^m \rightarrow \text{fluxes}; f_2^m \rightarrow \text{stresses (e.g. pressure tensor)}$$

CODE FEATURES

Fully implicit 1D3V finite difference code for solving the electron Vlasov-Fokker-Planck-Boltzmann equation with arbitrary number of harmonics

Includes the following collisional processes:

- Electron-electron collisions for all f_l^m
- Electron-ion collisions for $f_{l>0}^m$ assuming cold stationary ions
- Electron impact excitation of hydrogen atoms for all f_l^m
- Electron impact ionization of hydrogen atoms for all f_l^m
- Electron-neutral elastic collisions for all f_l^m

Follows neutral (including excited states) density using a Collisional-Radiative model including:

- Excitation and ionization
- Spontaneous de-excitation
- Radiative and 3-body recombination

Includes **neutral diffusion** and calculates electric field using **Ampere-Maxwell's law**. In 1D, without magnetic fields this reduces to

$$\frac{\partial E}{\partial t} = -\frac{j}{\epsilon_0}$$

Periodic, **fixed**, and **reflective boundary conditions** have been implemented, as well as a plasma sink and neutral source due to recycling at the divertor.

MODEL DETAILS AND PRELIMINARY BENCHMARKING

The current build of the code is a MATLAB prototype designed to test the numerics.

Electron-electron collision operator for f_0^0:

$$C_{ee-00} = \frac{\Gamma_{ee}}{v^2}\frac{\partial}{\partial v}\left[C(f_0^0)f_0^0 + D(f_0^0)\frac{\partial f_0^0}{\partial v}\right]$$

$$C(f_0^0) = 4\pi\int_0^v f_0^0(u)u^2\,du; D(f_0^0) = 4\pi\int_0^v u^2\left[\int_u^\infty f_0^0(u')u'du'\right]du$$

Discretization performed using Chang-Cooper-Langdon scheme in the same way as in IMPACT[3]. This scheme is conservative both in particle density and energy.

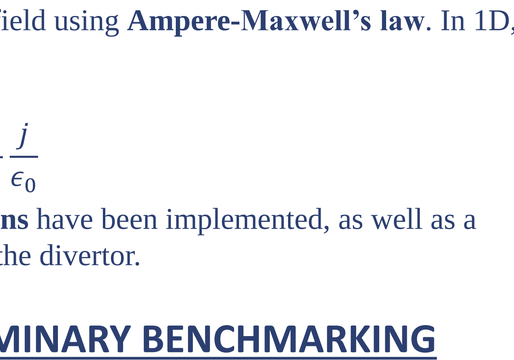

Figure 1: Relaxation of initial "bump-on-tail" distribution towards a Maxwellian through electron-electron collisions. τ_{ee} – electron-electron collision time. Timesteps taken were $dt = 0.1\tau_{ee}$.

Electron-electron collisions for higher harmonics much more complicated[1]– no momentum conserving scheme.

Spherical harmonics are eigenfunctions of pitch-angle scattering – simple electron-ion collision operator (proportional to $-l(l+1)f_l^m$ - provide natural cut-off for high l).

Electron-neutral collisions implemented using Boltzmann collision integral assuming cold neutrals and azimuthally symmetric collision cross-sections. For particle-conserving inelastic collisions[4] (e.g. electron impact excitation with excitation energy ϵ):

$$\left(\frac{\partial f_l^m}{\partial t}\right)_{ex-en} = -n_n v\left(\sigma^{TOT}(v)f_l^m(v) - f_l^m(\alpha_p v)\alpha_p^2\left(\sigma^{TOT}(\alpha_p v) - \sigma^{(l)}(\alpha_p v)\right)\right)$$

$$\alpha_p = v'/v = (1 + 2\epsilon/mv^2)^{1/2}$$

The code uses a particle density conserving discretization scheme for excitation collisions. Ionization collisions are approximated as excitation collisions (with proper cross-section) that leave an electron in the lowest velocity cell.

Density of both ground and excited states of hydrogen evolved using **CR model**. For excitation and spontaneous de-excitation (ionization and recombination similar):

$$\left(\frac{\partial n_b}{\partial t}\right)_{ex} = \sum_{b'}\left[A_{b\rightarrow b'}n_b + A_{b'\rightarrow b}n_{b'} - K^e_{b\rightarrow b'}n_b + K^e_{b'\rightarrow b}n_{b'}\right]$$

$$K_{b\rightarrow b'} = 4\pi\int dv\, v^3 f_0^0(v)\sigma^{TOT}(v)$$

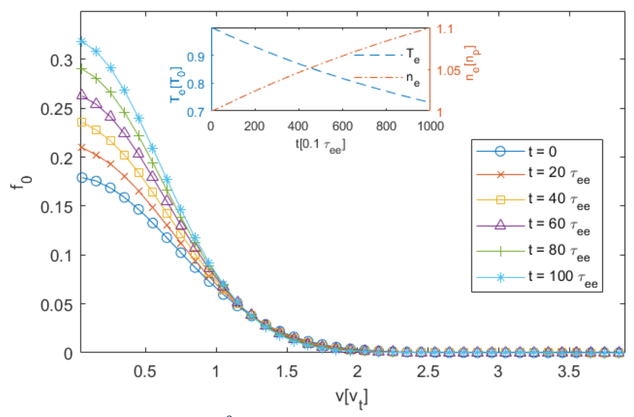

Figure 2: 0D isotropic (f_0^0 only) run with e-e collisions, excitation, de-excitation, ionization, and recombination. Sublot shows T_e drop due to collisional cooling, and n_e increase due to ionization.

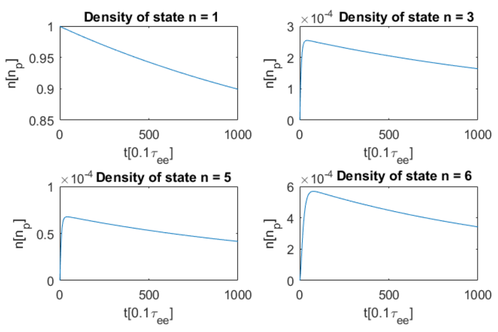

Figure 3: CR output for state densities for run in Figure 2. Current model tracks up to state n = 6. Cross-sections and recombination rates from Janev[5]. $T_0 = 10eV$, $n_p = 10^{19}\ m^{-3}$.

Preliminary 1D (using staggered grid and first two harmonics) **local transport** tests retrieve heat flow within ~10% of Spitzer-Härm value[6] (further tests required to increase accuracy).

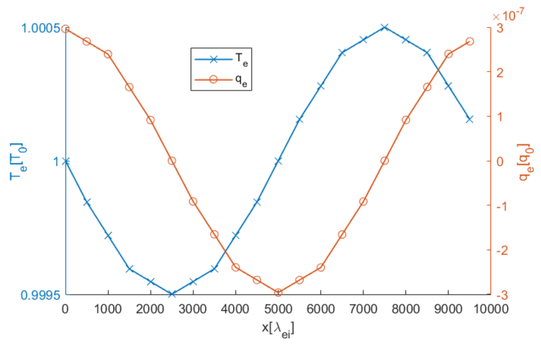

Figure 4: Local transport test. Scale length of system chosen to be many times the electron-ion mfp λ_{ei}. $T_0 = 10eV$, constant density $n_p = 10^{19}\ m^{-3}$ enforced up to numerical tolerance via E-field from Ampere-Maxwell law. Simulation ran for $30\tau_{ee}$, during which temperature evolution was negligible. Heat flow profile q_e in Figure taken at end of simulation, but converges suitably much earlier ($t \approx 15\tau_{ee}$).

CURRENT WORK AND FUTURE PLANS

- Testing approximations for divertor boundary condition: ion flux assumed from quasi-neutrality (density taken from last cell and velocity assumed to be sonic), neutral recycling calculated from ion flux and plasma sink term from

$$\left(\frac{\partial f_0^0}{\partial t}\right)_{sink} = -\frac{v}{3\Delta x_c}f_1^0(v)\frac{sgn\left(f_1(v)\right)+1}{2}\Theta(v - v_{cut}),$$

  where v_{cut} is calculated trying to balance ion and electron fluxes through sheath.
- Building full scale FORTRAN code
- Testing code on ELM burst and detachment simulations

REFERENCES

[1] Tzoufras M. et al. *Journal of Computational Physics* **230** (2011) 6475
[2] Bell A. R. et al. *Plasma Phys. Control. Fusion* **48** (2006) R37
[3] Kingham R. J. et al. *Journal of Computational Physics* **294** (2004) 1
[4] Makabe T. et al. *PLASMA ELECTRONICS Applications in Microelectronic Device Fabrication* 2nd ed. (2015) CRC Press
[5] Janev R. K. et al. *Collisional Processes in Low-Temperature Hydrogen Plasmas* www.eirene.de/report_4105.pdf
[6] Braginskii, S. I. *Reviews of Plasma Physics* **1** (1965) 205

ACKNOWLEDGEMENTS

This work was supported through the President's PhD Scholarship Scheme by Imperial College London.