Fast Secondary Reconnection and the Sawtooth Crash

Maurizio Ottaviani [1], Daniele Del Sarto [2]

1 CEA-IRFM, Saint-Paul-lez-Durance (France)

2 Université de Lorraine, Institut Jean Lamour UMR-CNRS 7198, Nancy (France)

(EFTC-17, Athens, 9 October 2017)

Problem: why is the sawtooth crash so “fast” at large Lundquist number $S=\tau_R/\tau_A$?

$\tau_R = a^2/\eta$ Resistive time
$\tau_A = a/V_{Alfven}$ Alfven time

The m=1 mode and the sawtooth crash

- The m=1 mode: an essential element of the sawtooth cycle [Von Goeler et al.(1974), Kadomtsev(1975)]
  Process mediated by the restivity η. Becomes slow at large S, small η.

- **The sawtooth crash time in JET is shorter than the collision time.** [Edwards et al., (1986)]
  This process is too fast to be explained by resistive MHD

- Non-collisional physics (electron inertia and other effects) gives the right time scale (100 microseconds) for JET:
  $\gamma \tau_A \sim (d_e/L)$ both in the linear and in the nonlinear phase.
  (d_e = skin depth, $L \sim a \sim R$ = machine scale length. τ_A = Alfven time).
  Typically $d_e/L \sim 10^{-3}$
  [Ottaviani and Porcelli, (1993)].

- **Probably not sufficient to explain fast crashes in medium size tokamaks** and in some simulations at moderate Lundquist number

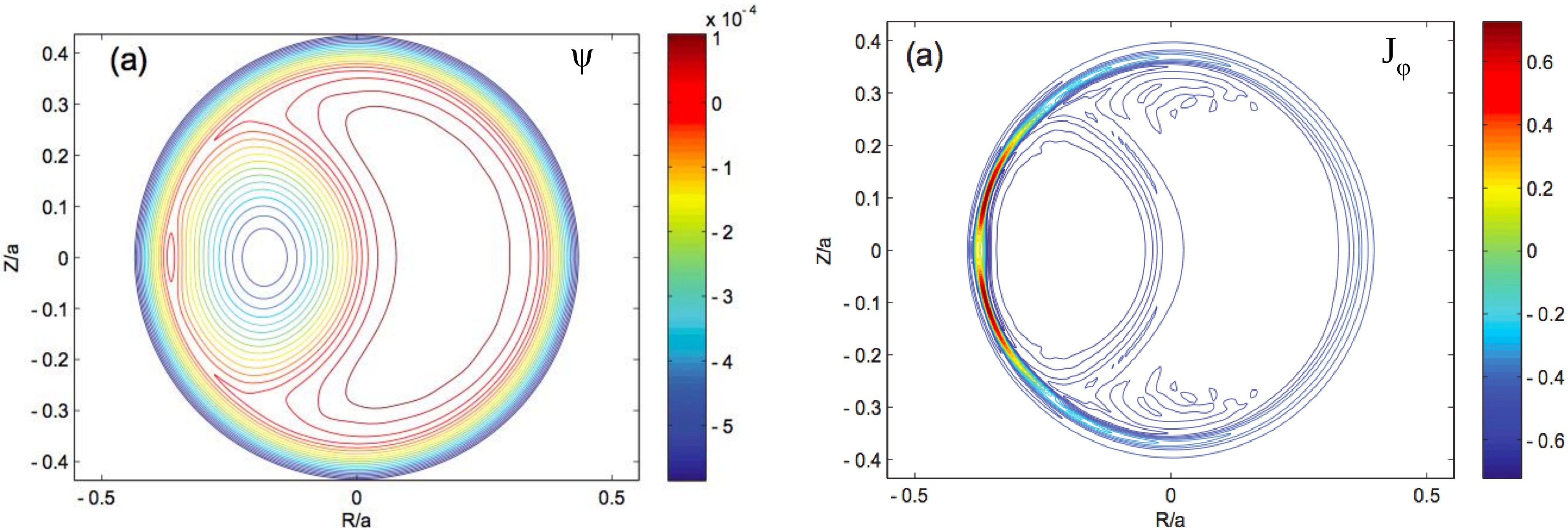
Look for a new explanation

What about the stability of m=1 islands?

• ***Current sheets of large aspect ratio are developed during the nonlinear stage of primary internal-kink modes. How does their instability to secondary reconnecting modes relate to fast sawtooth crashes?***

$$\Delta' = \infty$$

Ex :



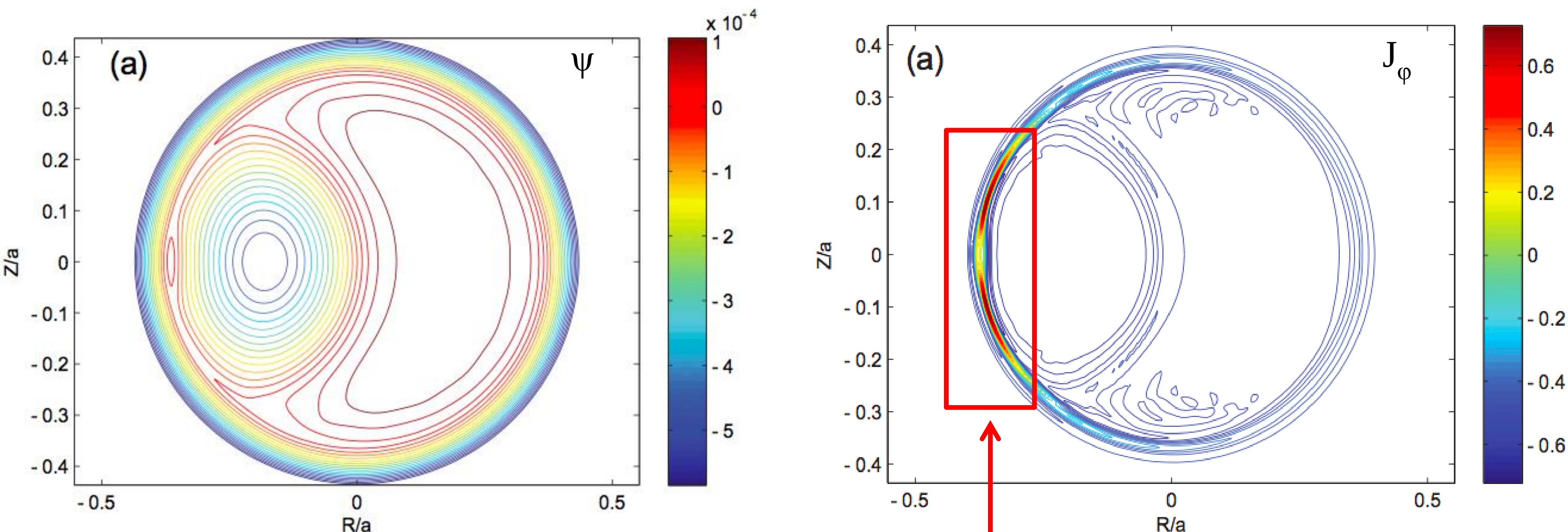
(Figures taken from [*Q.Yu et al.*, *Nucl. Fusion* ***54****, 072005 (2014)*])

What about the stability of m=1 islands?

- ***Current sheets of large aspect ratio are developed during the nonlinear stage of primary internal-kink modes. How does their instability to secondary reconnecting modes relate to fast sawtooth crashes?***

$$\Delta' = \infty$$

Ex :



(Figures taken from [*Q.Yu et al., Nucl. Fusion* **54**, *072005 (2014)*])

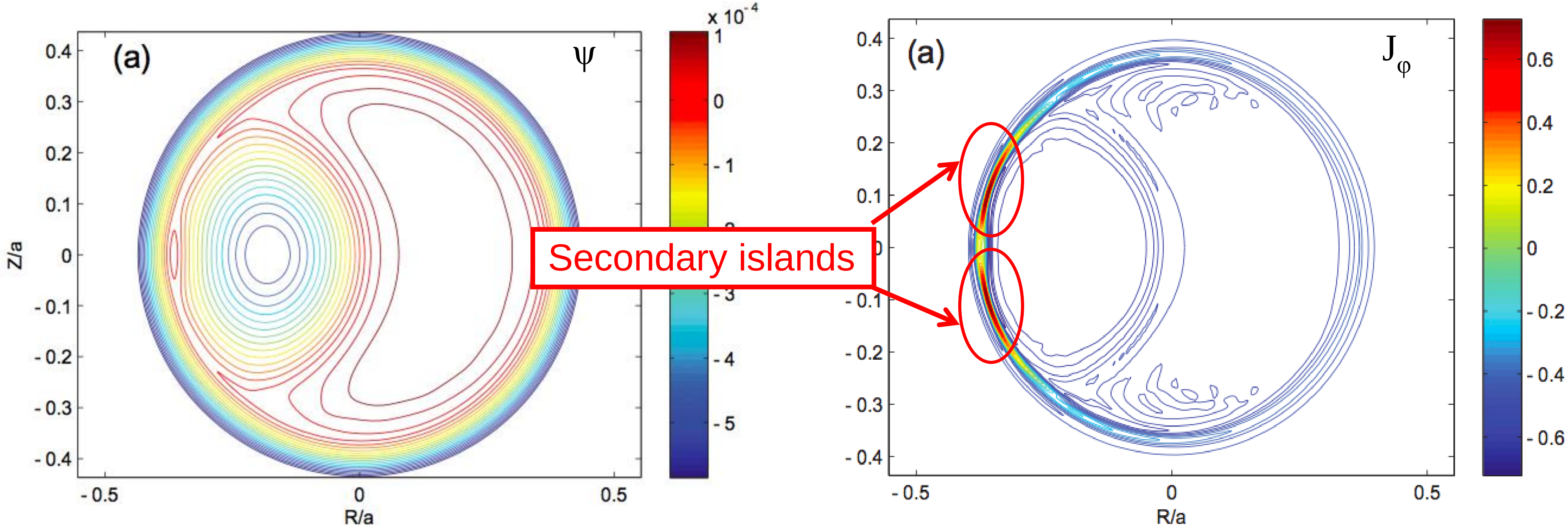
CURRENT SHEET

What about the stability of m=1 islands?

• ***Current sheets of large aspect ratio are developed during the nonlinear stage of primary internal-kink modes. How does their instability to secondary reconnecting modes relate to fast sawtooth crashes?***

$$\Delta' = \infty$$

Ex :



(Figures taken from [*Q.Yu et al., Nucl. Fusion* ***54****, 072005 (2014)*])

Large Δ' regime in slab geometry

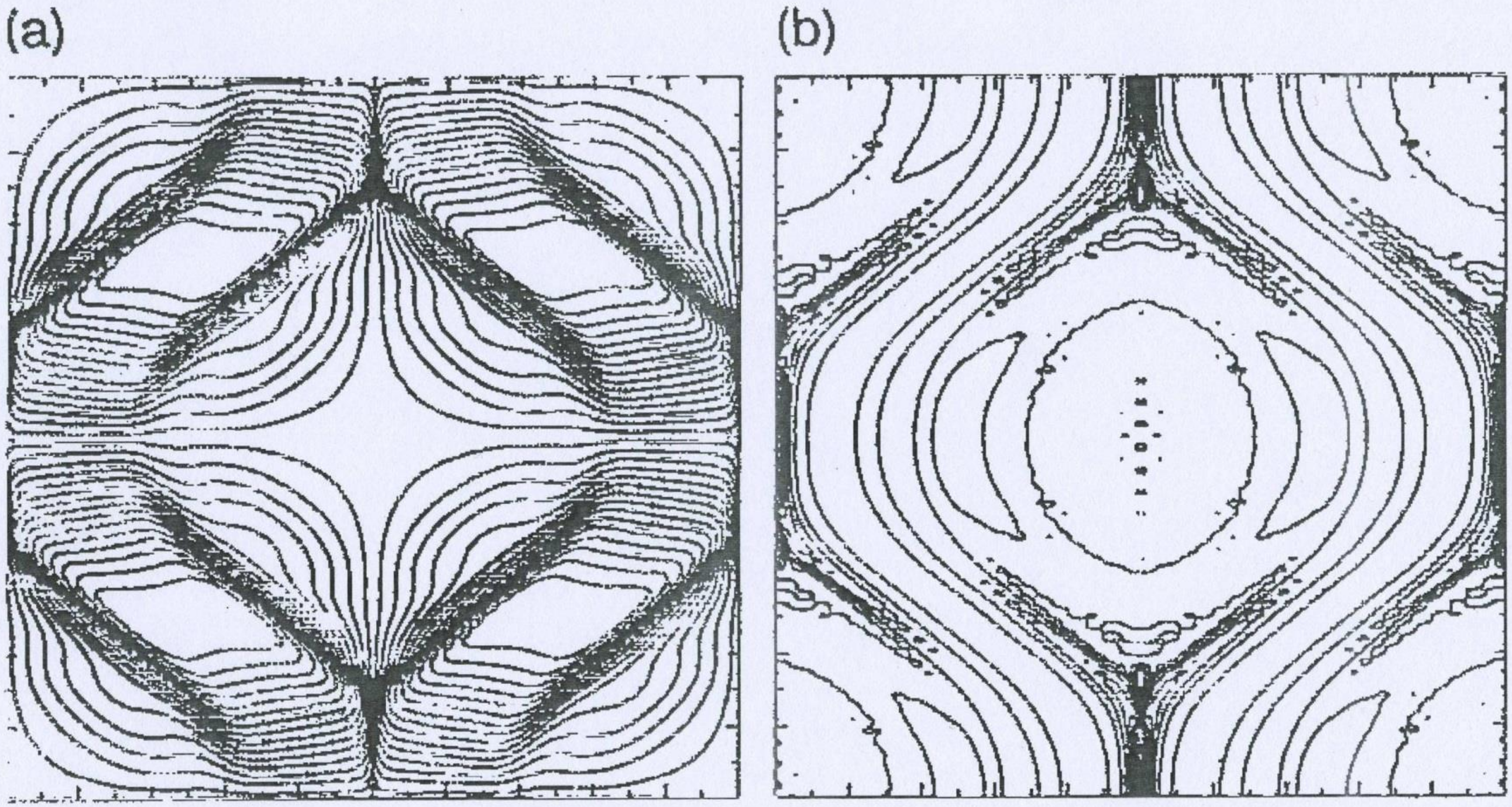


FIG. 13. Contour plots of (a) the streamfunction φ and (b) the current density J in the purely resistive case.

(Figures taken from [M. Ottaviani *et al.*, *Phys. Plasmas* **2**, 4104 *(1995)*])

Large Δ' regime in slab geometry

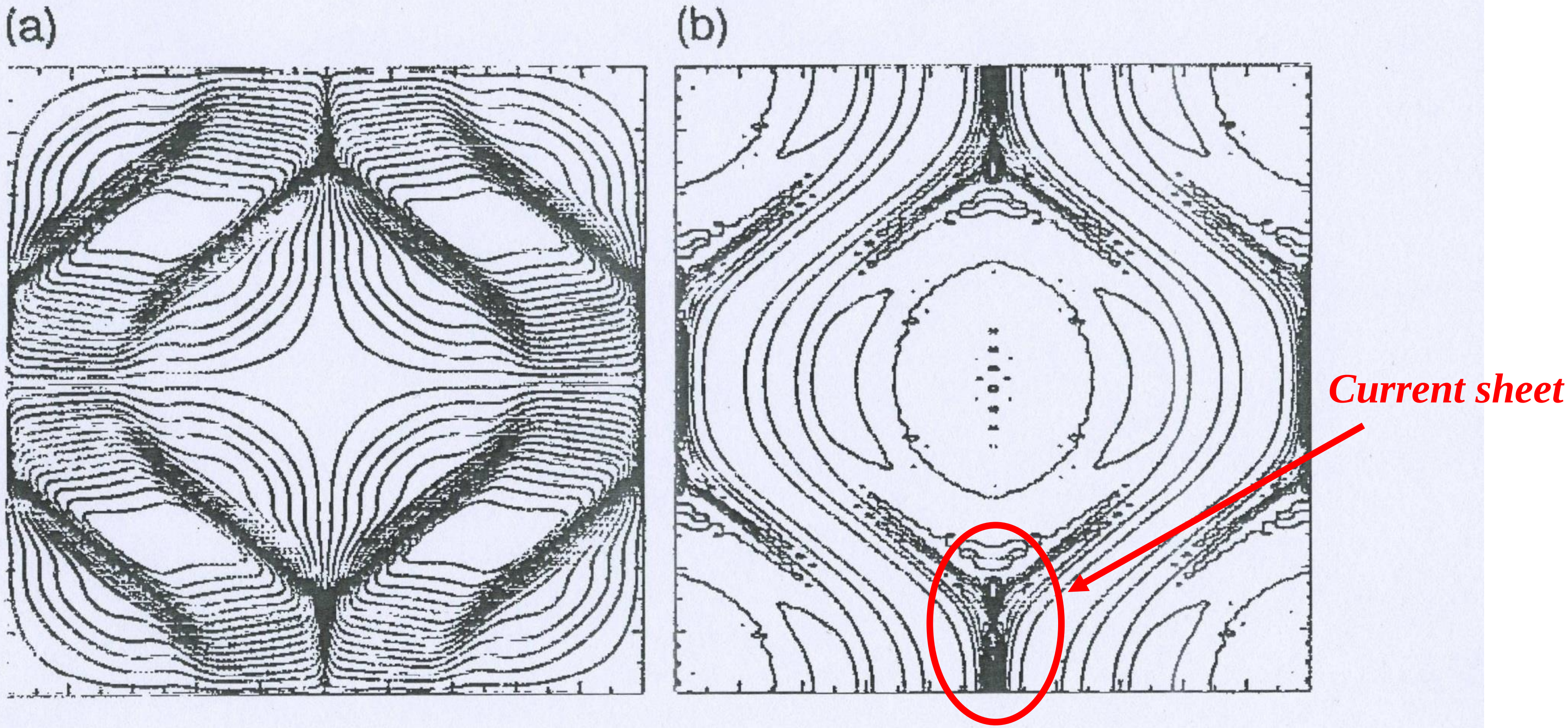
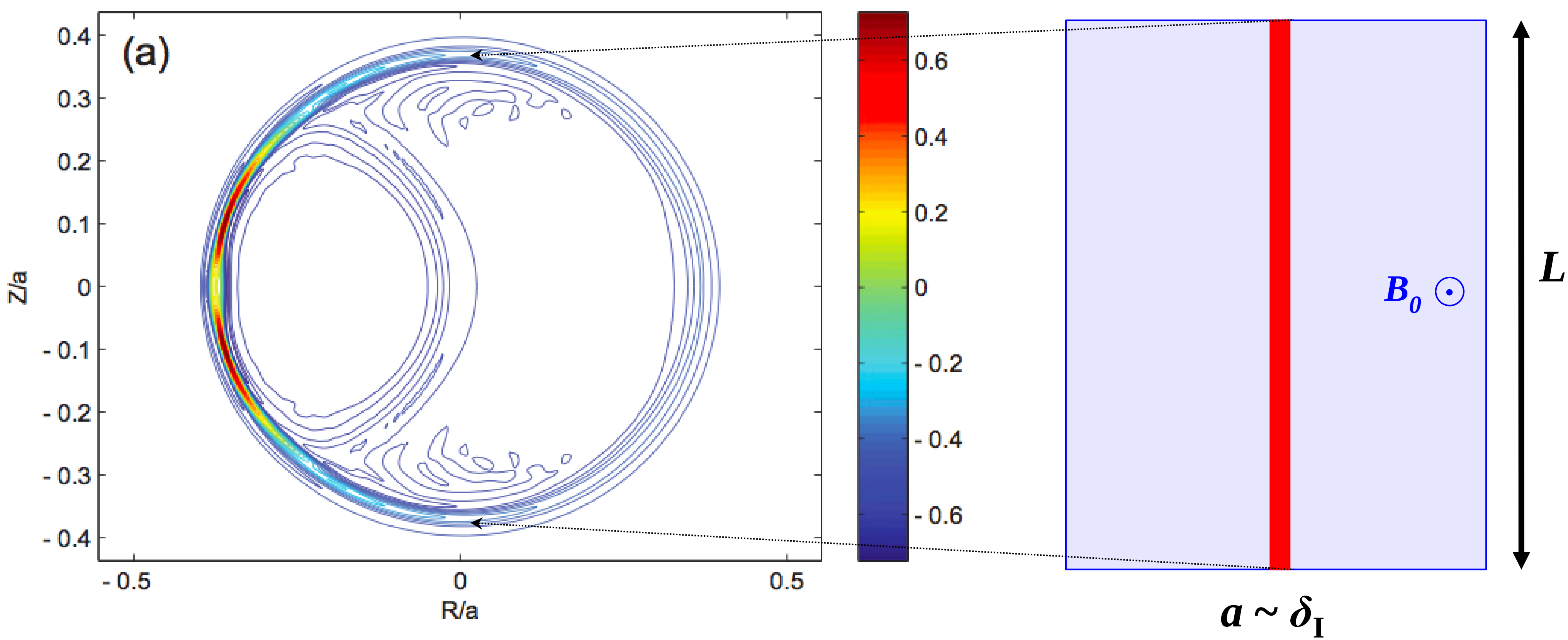


FIG. 13. Contour plots of (a) the streamfunction φ and (b) the current density J in the purely resistive case.

(Figures taken from [M. Ottaviani *et al.*, *Phys. Plasmas* **2**, 4104 *(1995)*])

Slab approximation for the current sheet



⅄ Due to the current sheet's large aspect ratio $L/\delta_I >> 1$ a quasi-continuum spectrum of unstable wavenumbers can be assumed.

⅄ **The maximum growth rate occurs at short wavelength. This justifies the local assumption about the geometry**

Key assumptions

1) In the current sheet, the current density is comparable to the equilibrium current density which is what one gets at the end of the linear phase of the m=1 mode:

$$J_{cs} \sim J_0$$

2) The above is reasonable if the instability of the current sheet (what we call the secondary instability) is fast enough that its growth rate exceeds the evolution rate of the “equilibrium” (i.e. the primary reconnecting mode):

$$\gamma_{II} > \tau_{cs}^{-1} \sim \gamma_I$$

This is verified *a posteriori*. The m=1 island can be considered as *static*.

3) *The width of the current sheet is assumed to be of the order of the m=1 layer width.*

This is justified at the end of the linear phase/beginning of the nonlinear phase of the m=1 primary island

4) We perform an *asymptotic analysis* (i.e. Lundquist number $S \to \infty$, etc.)

Size of the transverse magnetic field in the current sheet

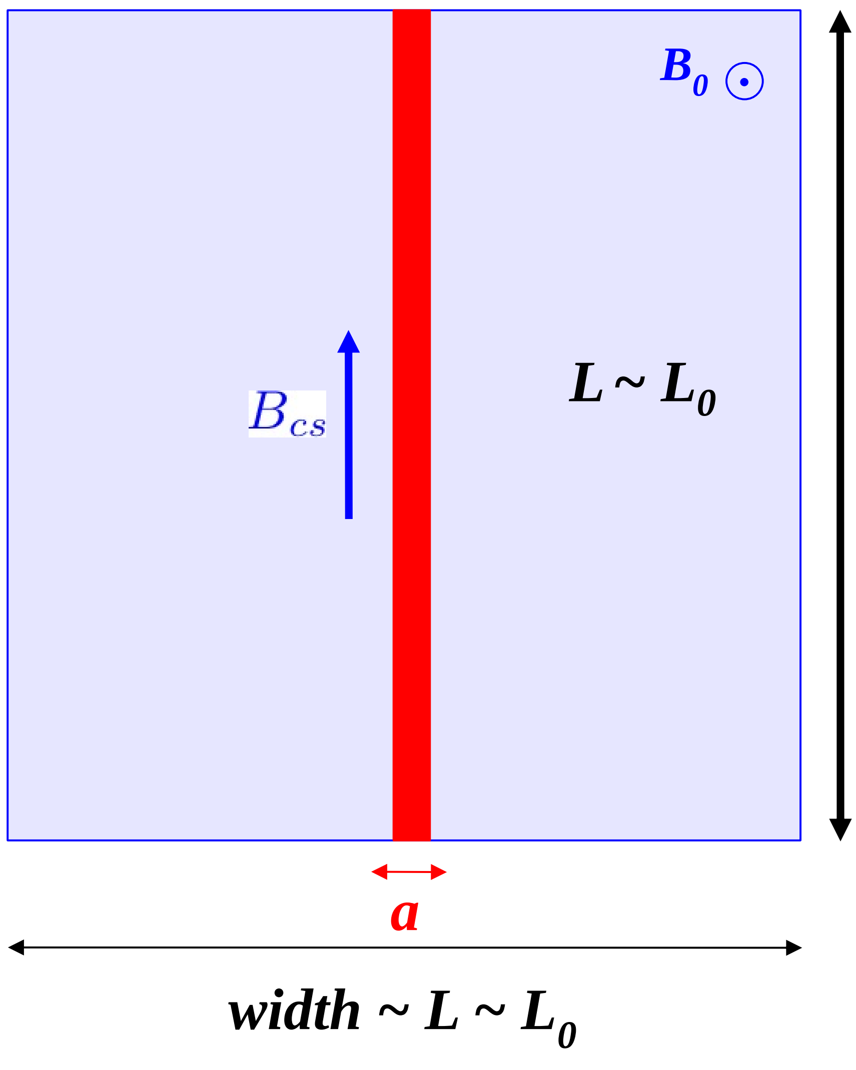
In a tokamak $L_0 \sim R$ and $L \sim R \sim L_0$.
And the end of the internal-kink linear phase $a \sim \delta_I = S^{-1/3} L$

$$\boxed{J_{cs} \sim J_0 \quad \Rightarrow \quad \frac{B_{cs}}{B_0} = \frac{a}{L}}$$

$$\Delta' \simeq \frac{1}{ka^2}$$

Example of Harris-pinch equilibrium, typical for $a / L \leq ka << 1$
(thus valid for the tearing at small *ka* and for the dominant mode at $\Delta'\delta \sim 1$)

Note: in several papers $B_{cs} \sim B_0$ and $J_{cs} >> J_0$
NOT JUSTIFIED



Scaling of the reconnection rates (n/m=1/1 case of a tokamak in cylindrical approximation)

Estimation : $$\frac{B^I_{cs,\theta}}{B_0} \simeq \frac{x}{R}, \quad \text{for} \quad x \le \delta_I$$

Tokamak geometry : $$L_0 = R/\hat{s}_1 \qquad \hat{s}_1 \equiv r_1 q'(r_1)/q_1 \qquad k = 1/r_1$$

Primary internal-kink : $\Delta'\delta_I = \infty$

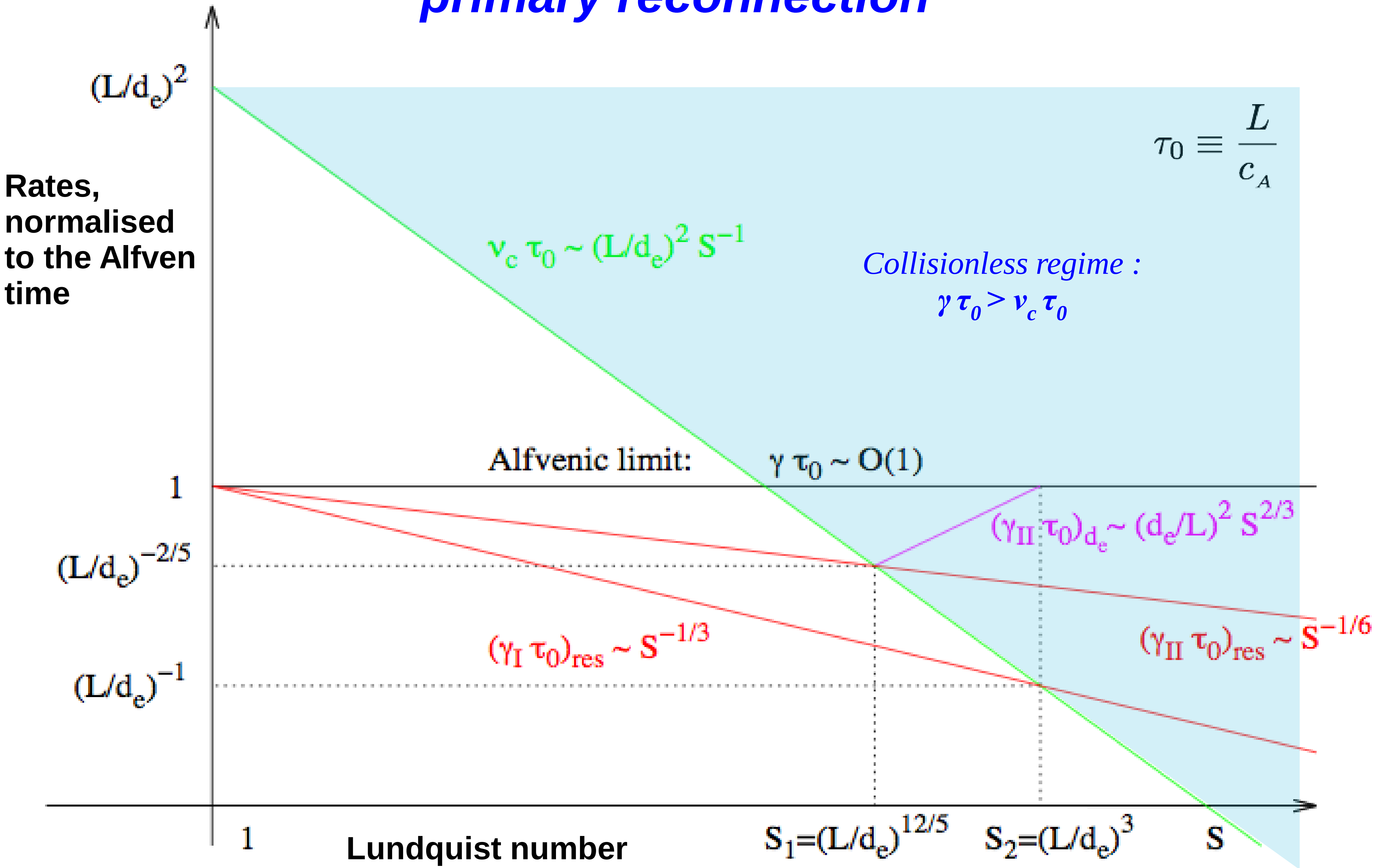
$$\gamma_I \tau_0 \sim S_R^{-\frac{1}{3}} \left(\frac{r_1}{R\hat{s}_1}\right)^{-\frac{2}{3}} \qquad \frac{\delta_I}{R} \sim S_R^{-\frac{1}{3}} \left(\frac{r_1}{R\hat{s}_1}\right)^{-\frac{1}{3}}$$

A broad range of unstable secondary modes *with a broad range of wavenumbers k and a varying Δ'* and layer width δ_{II}

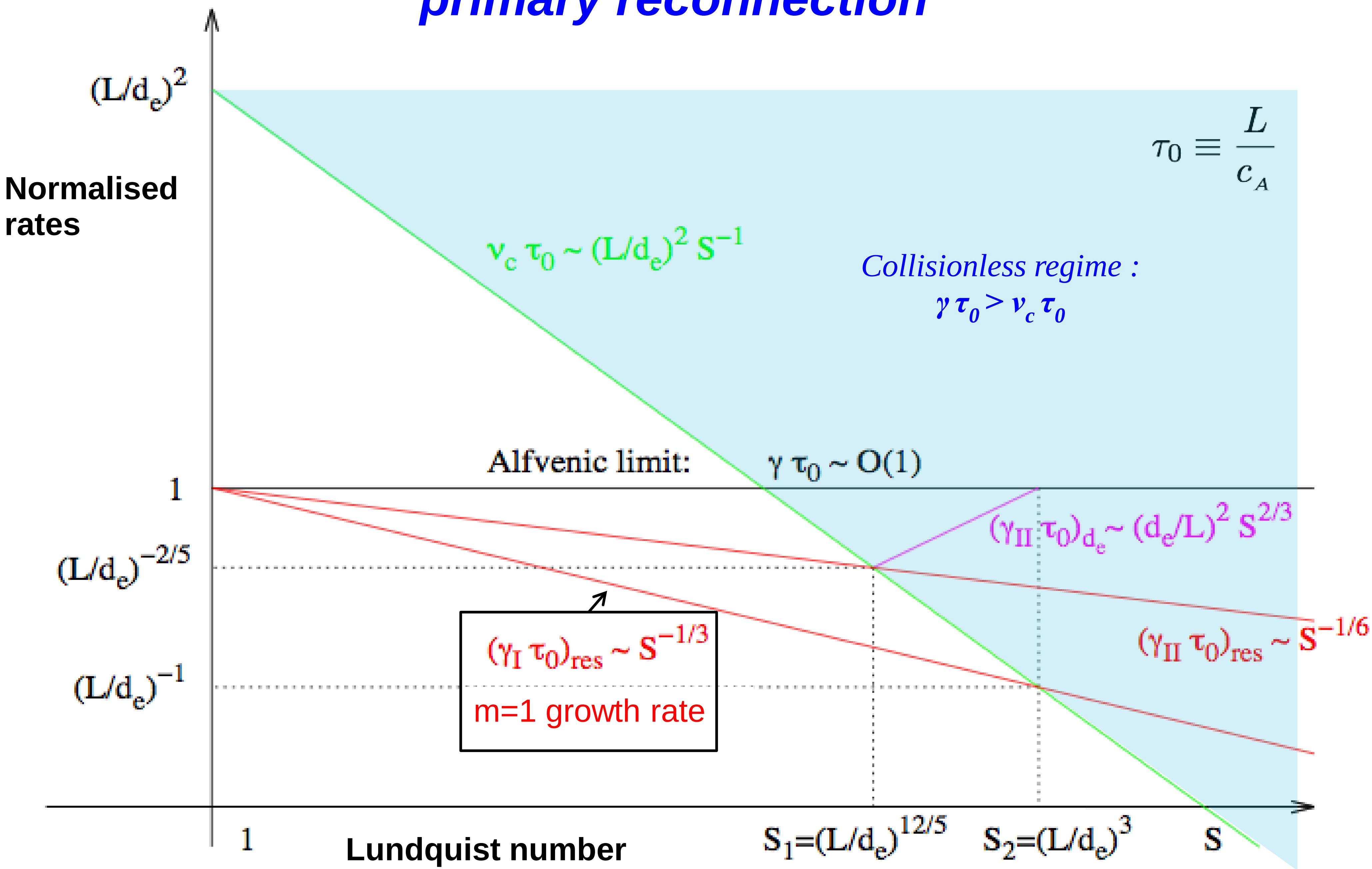
The fastest growing mode always occurs at $\Delta'\delta_{II} \sim 1$

$$k_M^{(II)} R \sim S_R^{\frac{1}{4}} \left(\frac{r_1}{\hat{s}_1 R}\right)^{-\frac{1}{2}} \qquad \gamma_{II}^{(M)} \tau_0 \sim S_R^{-\frac{1}{6}} \left(\frac{r_1}{\hat{s}_1 R}\right)^{-\frac{1}{3}}$$

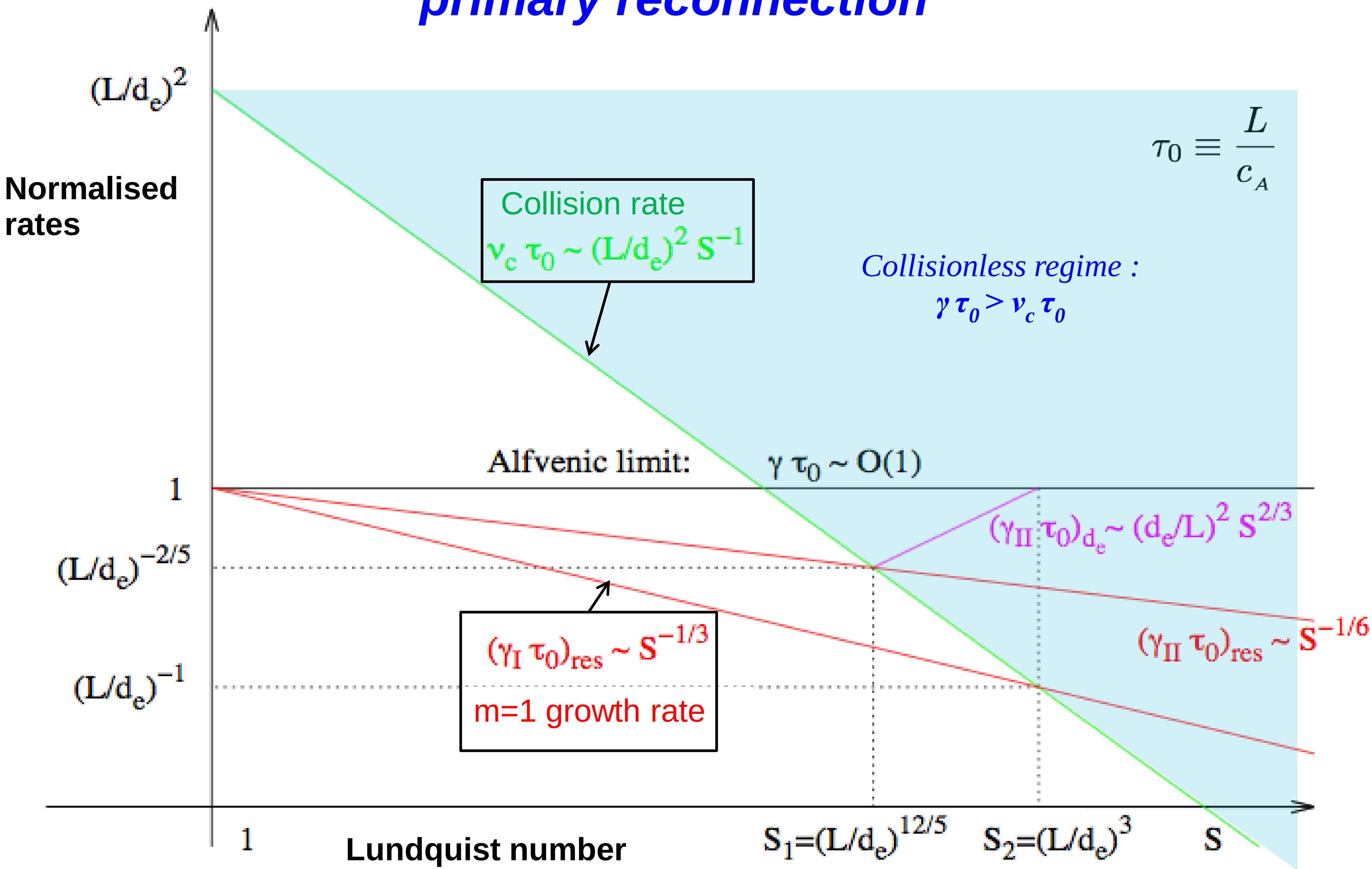
Results : secondary collisionless reconnection, possibly at near Alfvenic rate, in tokamak devices with resistive primary reconnection



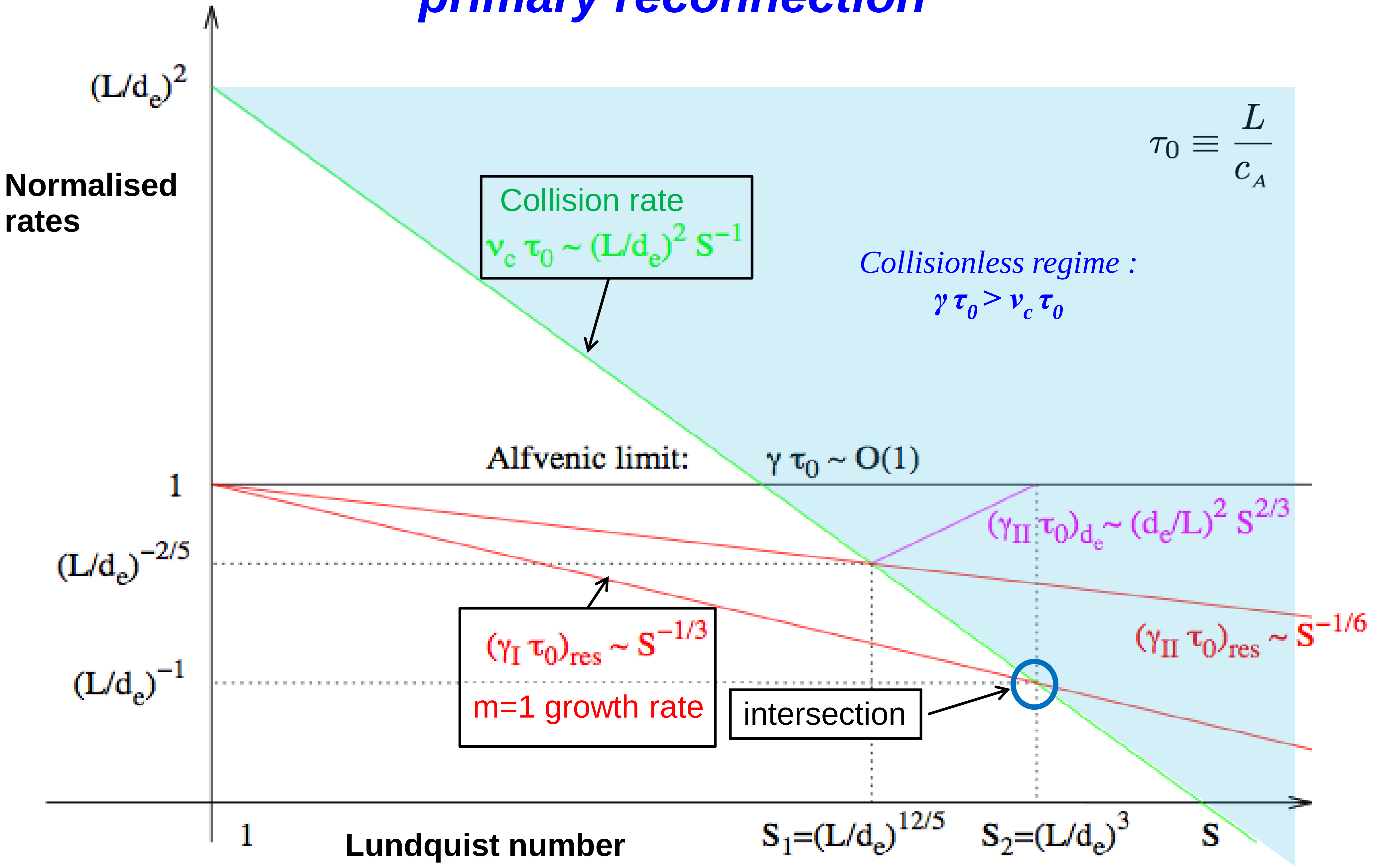
Results : secondary collisionless reconnection, possibly at near Alfvenic rate, in tokamak devices with resistive primary reconnection



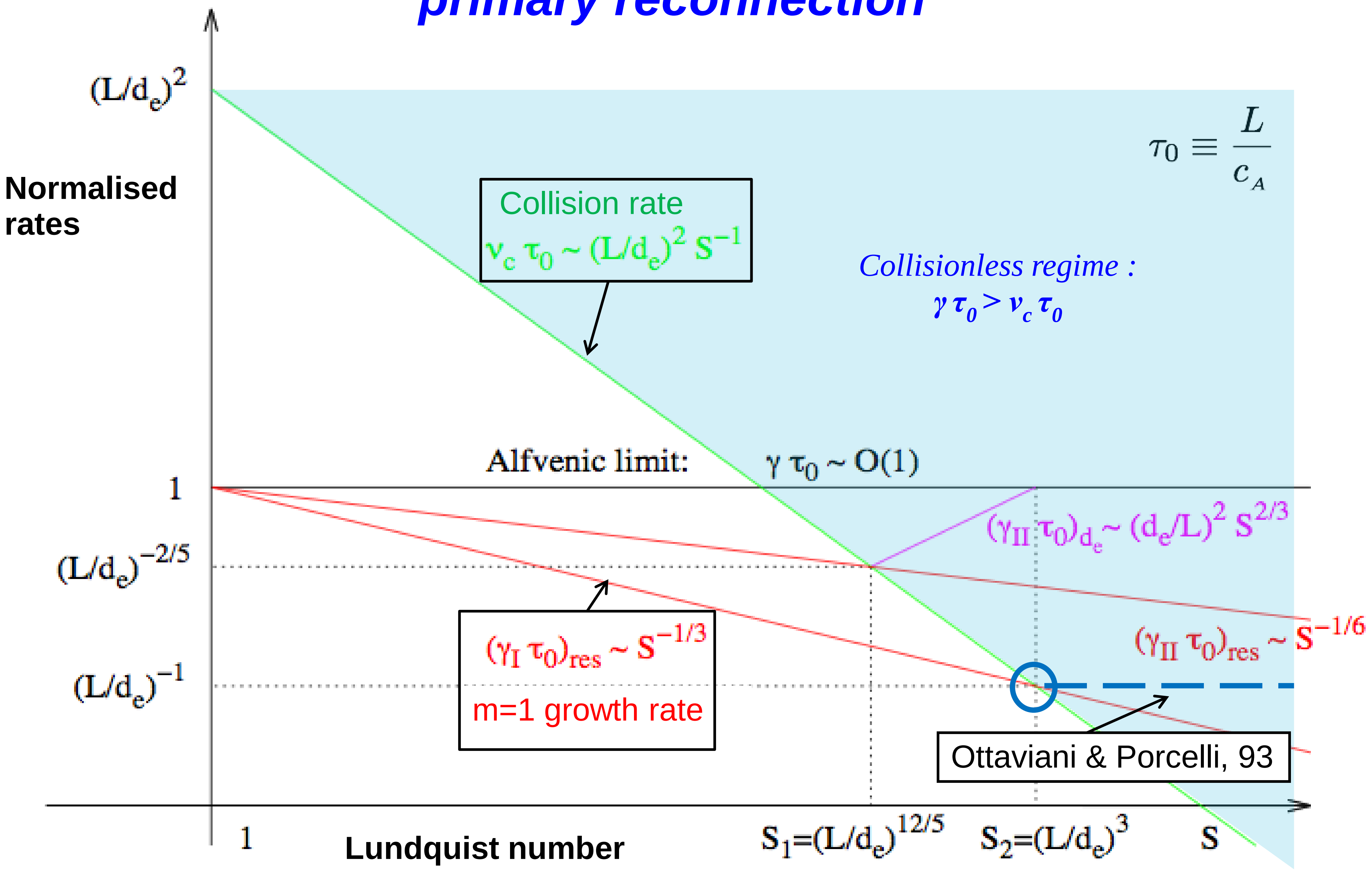
Results : secondary collisionless reconnection, possibly at near Alfvenic rate, in tokamak devices with resistive primary reconnection



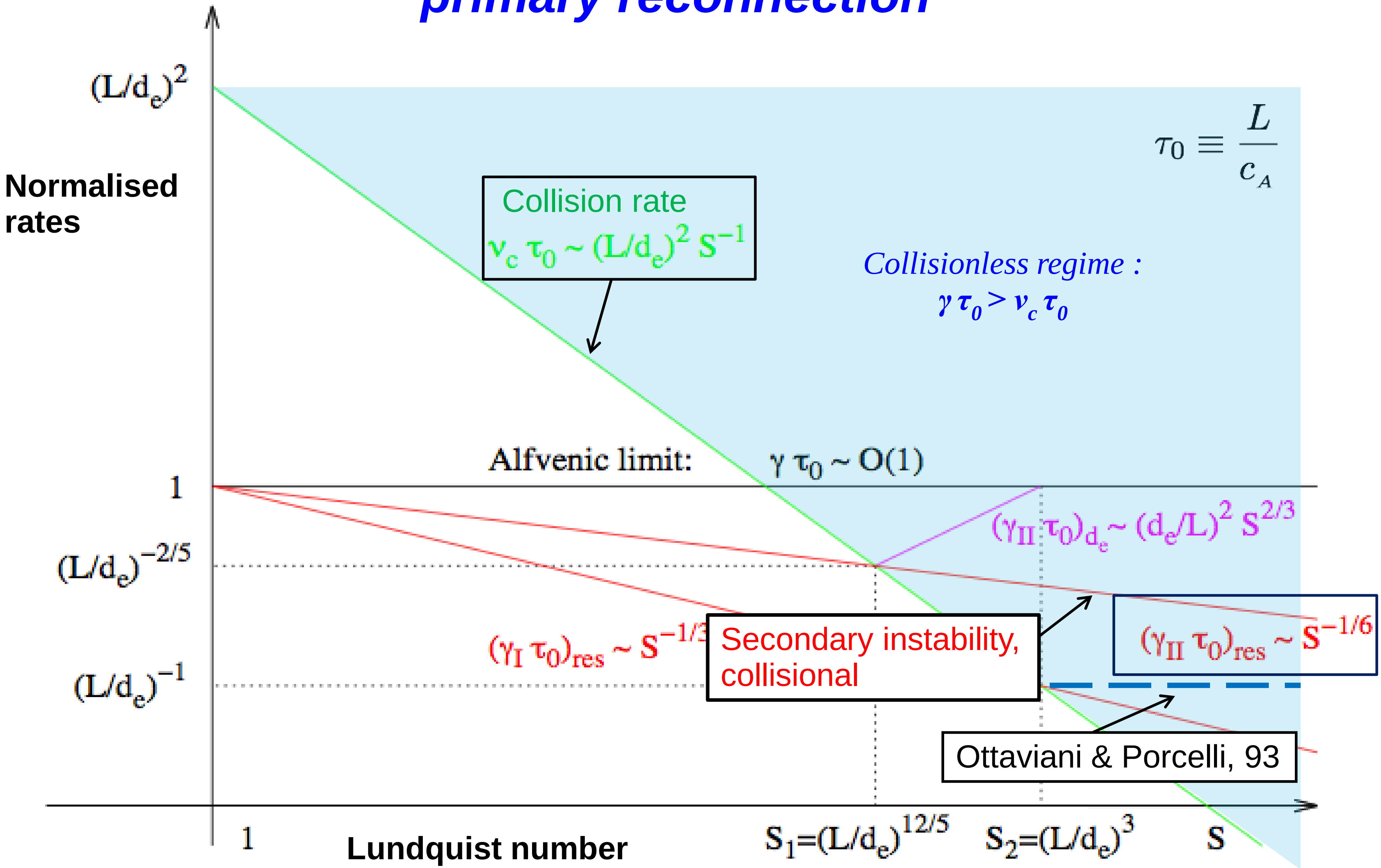
Results : secondary collisionless reconnection, possibly at near Alfvenic rate, in tokamak devices with resistive primary reconnection



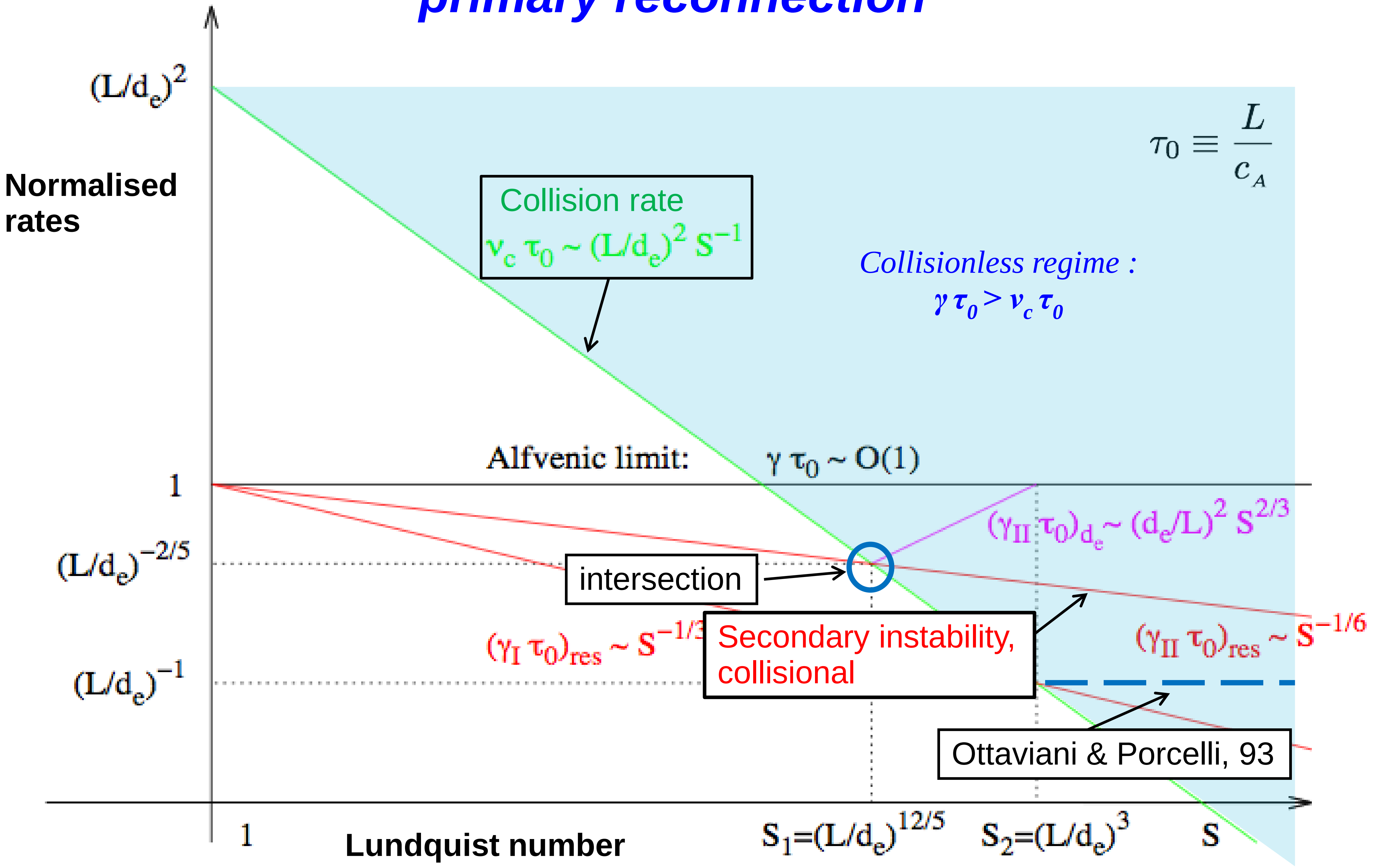
Results : secondary collisionless reconnection, possibly at near Alfvenic rate, in tokamak devices with resistive primary reconnection



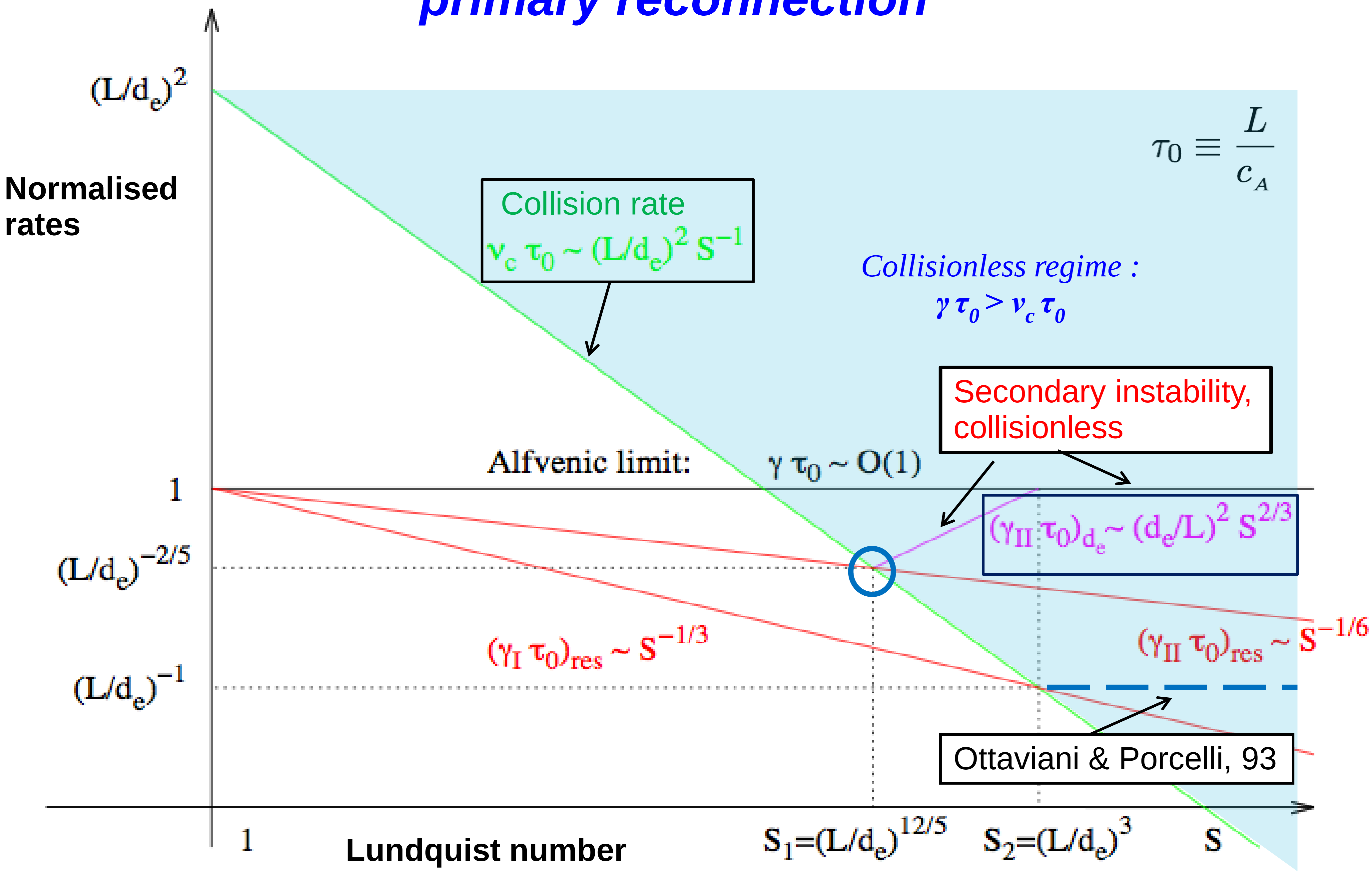
Results : secondary collisionless reconnection, possibly at near Alfvenic rate, in tokamak devices with resistive primary reconnection



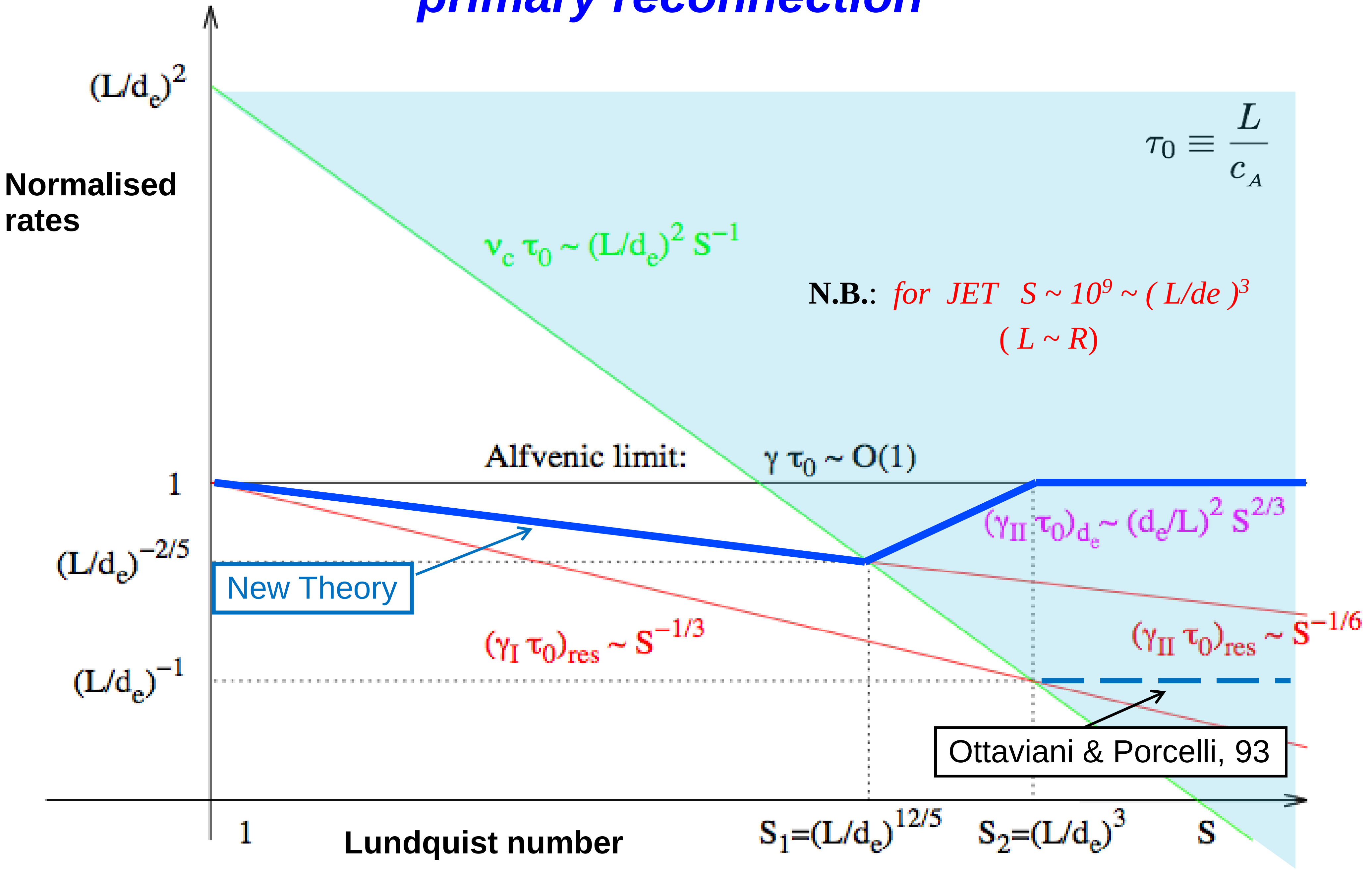
Results : secondary collisionless reconnection, possibly at near Alfvenic rate, in tokamak devices with resistive primary reconnection



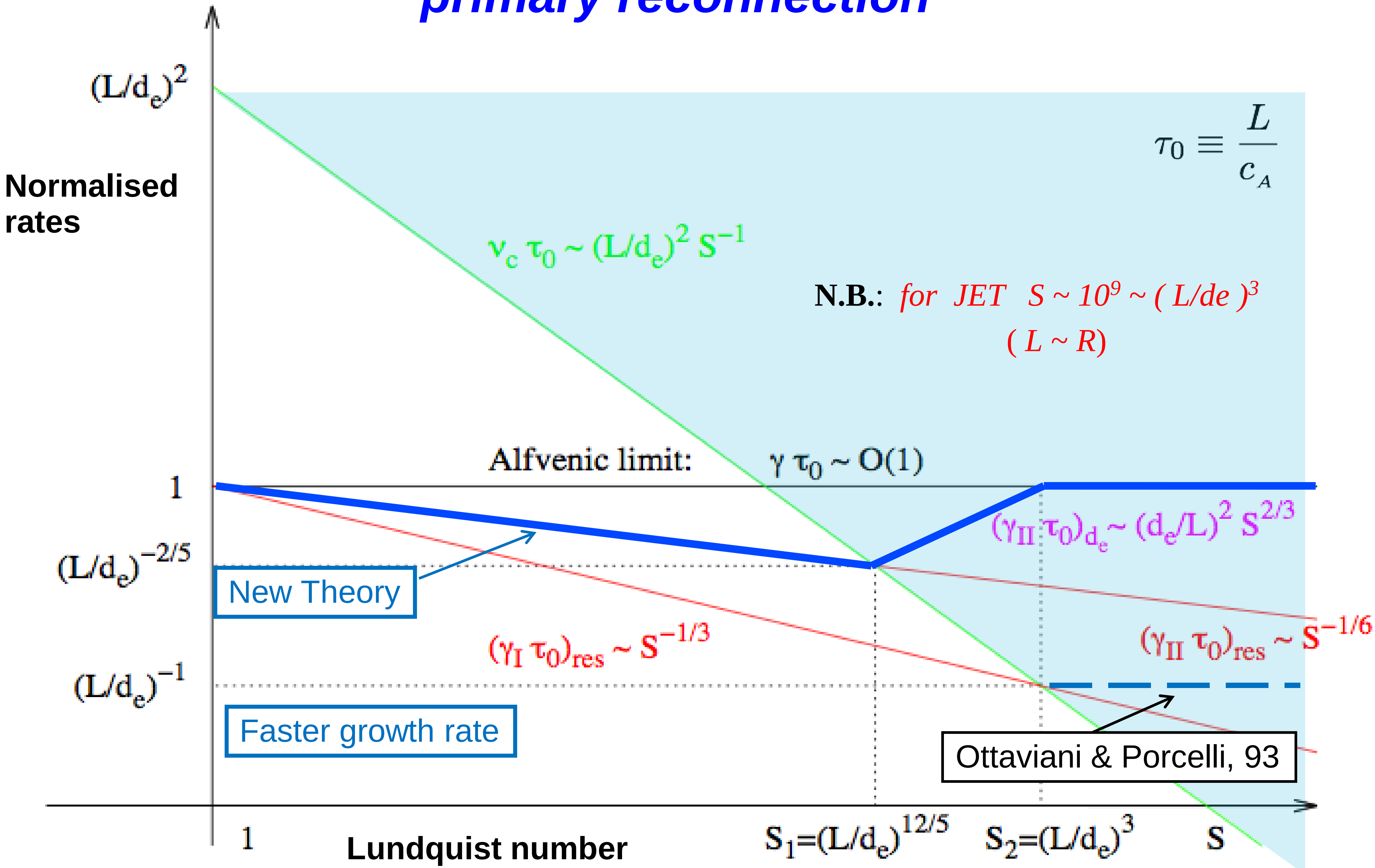
Results : secondary collisionless reconnection, possibly at near Alfvenic rate, in tokamak devices with resistive primary reconnection



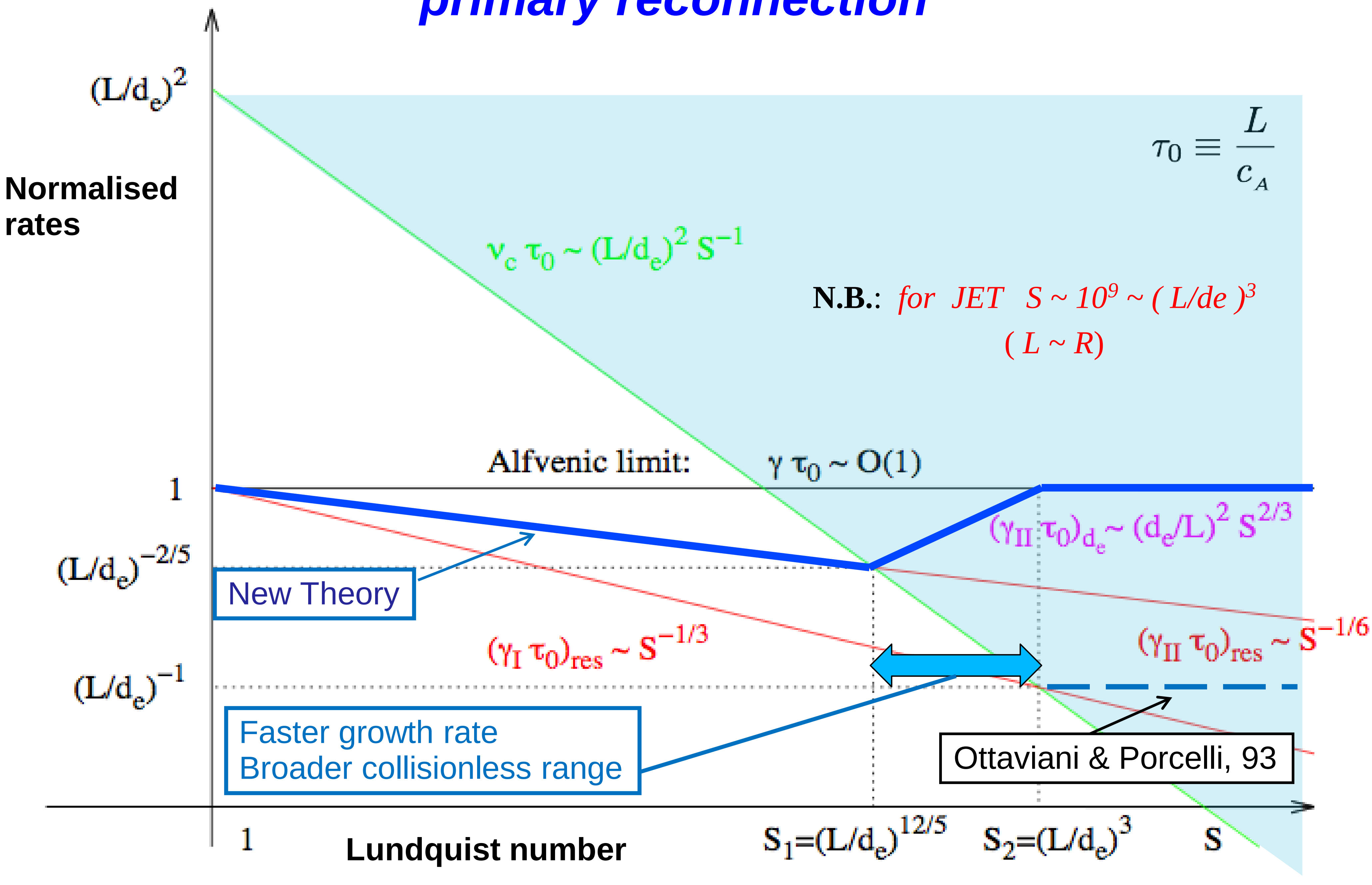
Results : secondary collisionless reconnection, possibly at near Alfvenic rate, in tokamak devices with resistive primary reconnection



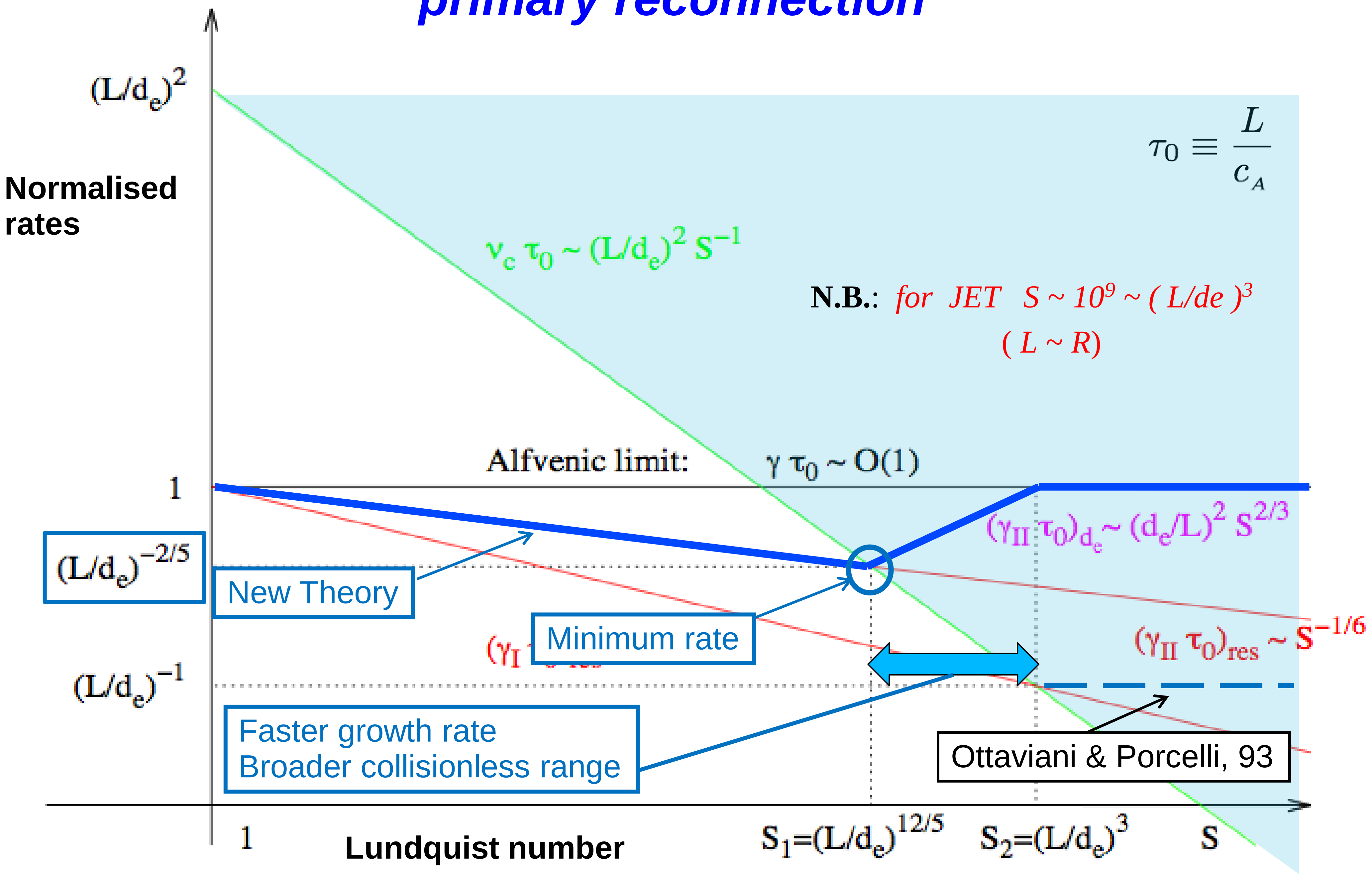
Results : secondary collisionless reconnection, possibly at near Alfvenic rate, in tokamak devices with resistive primary reconnection



Results : secondary collisionless reconnection, possibly at near Alfvenic rate, in tokamak devices with resistive primary reconnection

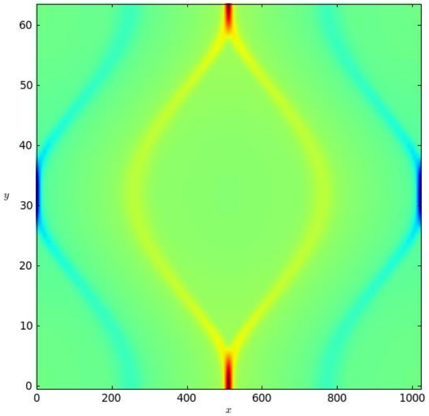


Results : secondary collisionless reconnection, possibly at near Alfvenic rate, in tokamak devices with resistive primary reconnection



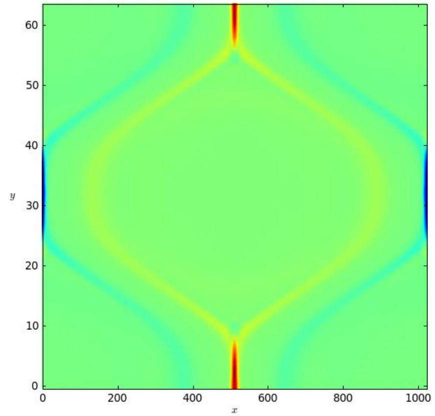
Numerical simulations (ongoing work)

current



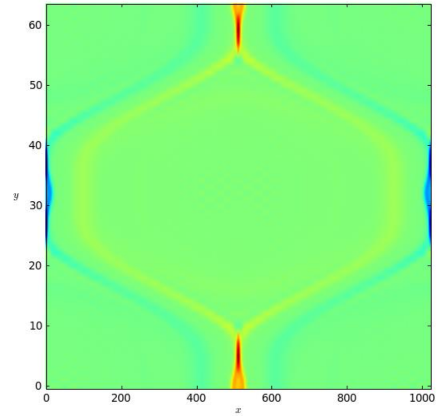
t=301

current



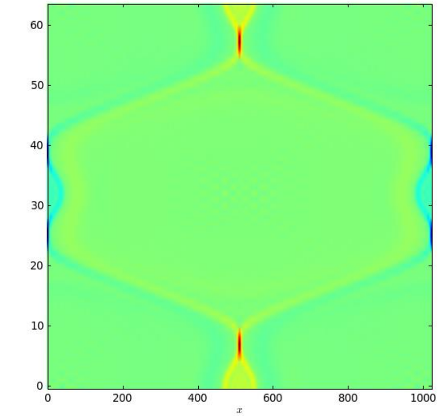
t=321

current



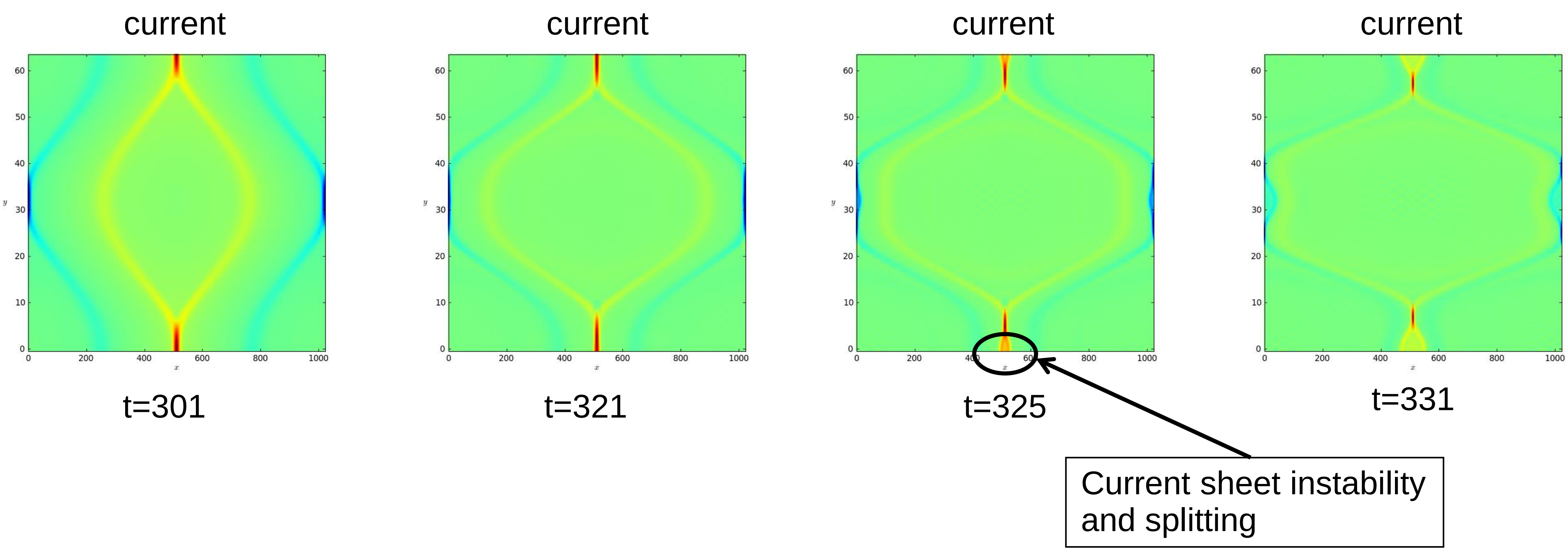
t=325

current

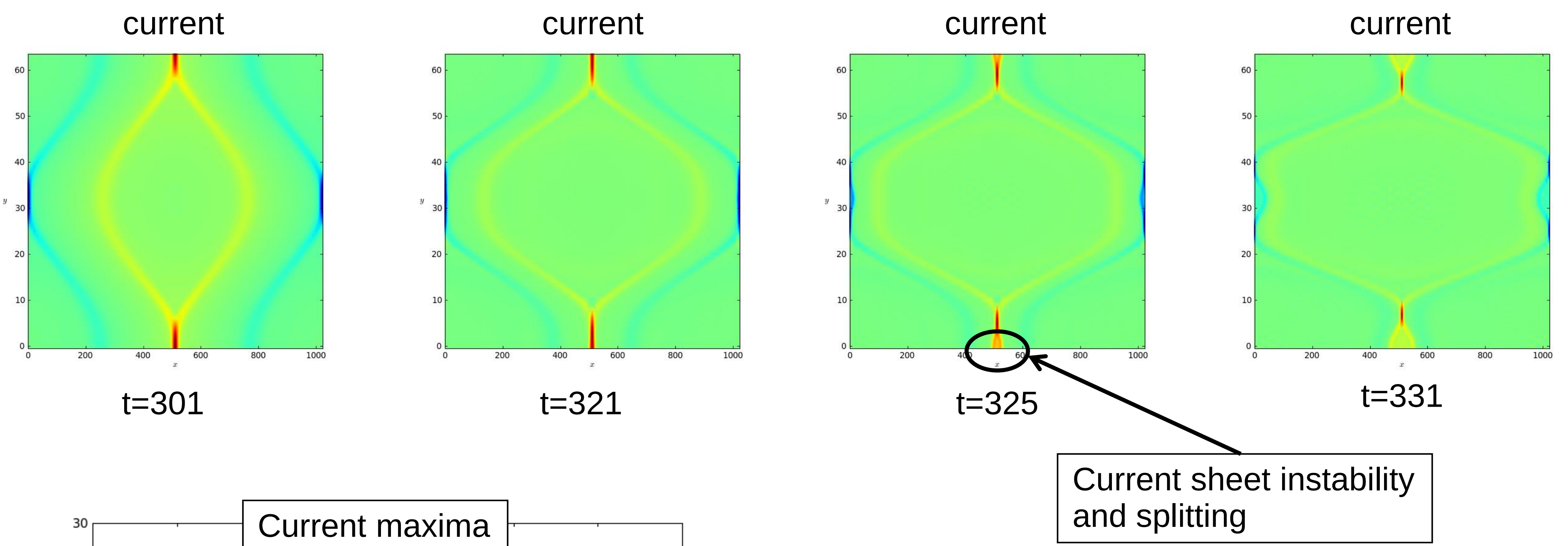


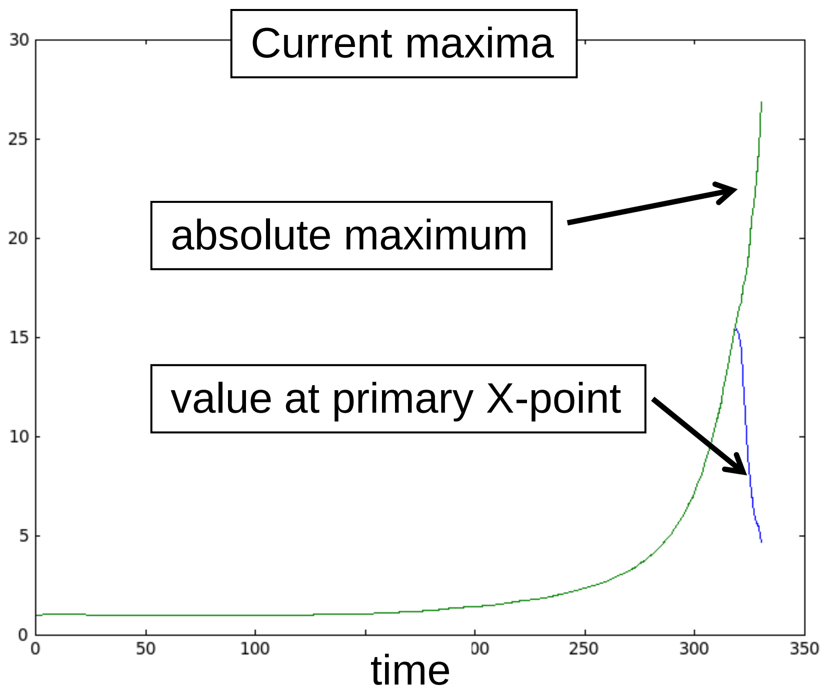
t=331

Numerical simulations (ongoing work)

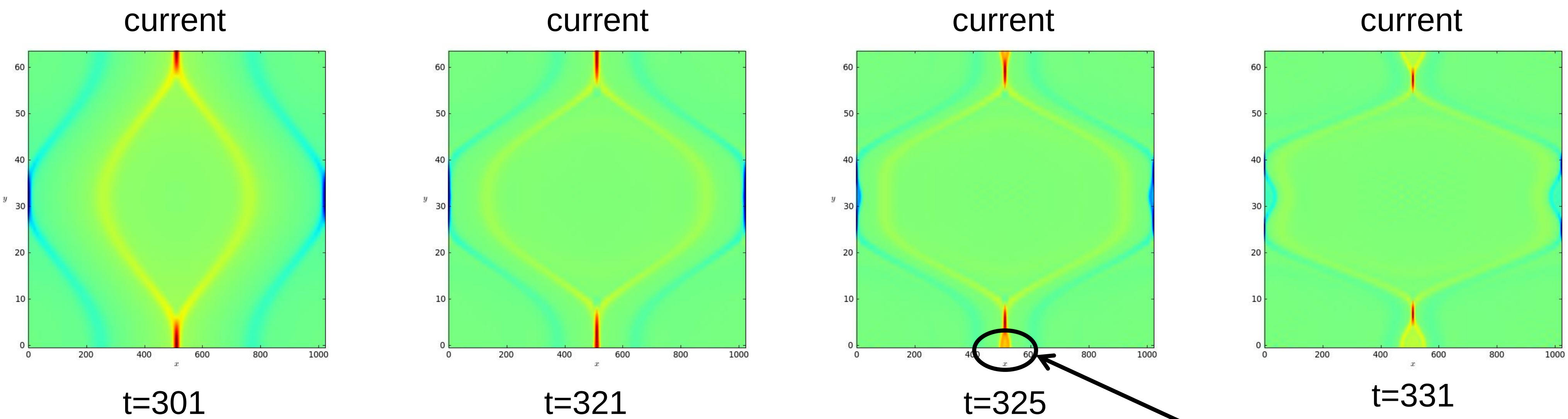


Numerical simulations (ongoing work)

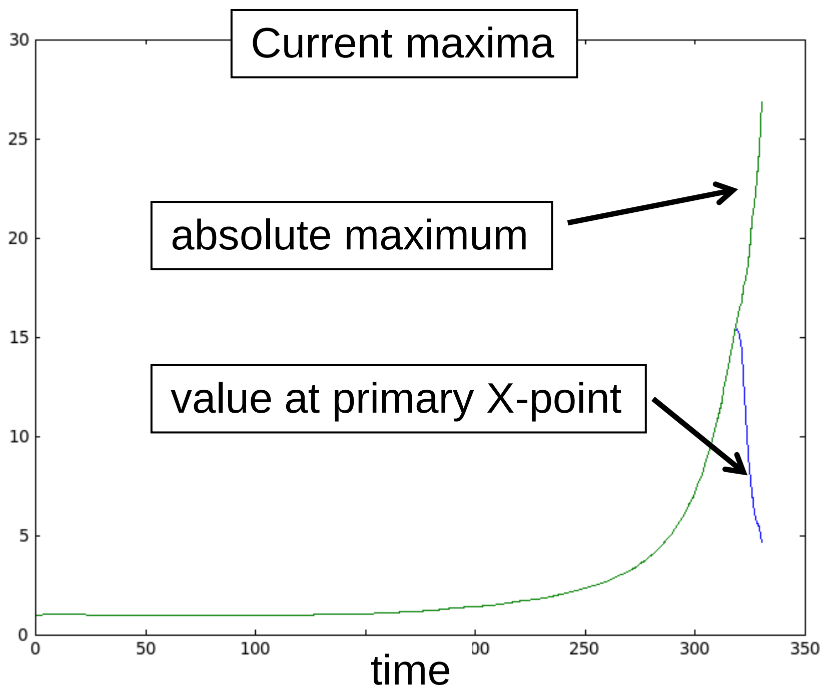




Numerical simulations (ongoing work)



Current sheet instability and splitting



Pseudo-spectral simulations

- Box aspect ratio A=2
- Lundquist number $S=10^3$

Still work to do…

Summary and conclusions

- *A current sheet generated by a primary* instability with $\Delta' = \infty$, such as the *resistive internal kink mode*, becomes *unstable at an early stage in its nonlinear development because of sub-Alfvenic modes*.

- In the case of *sawtooth phenomenon in a purely resistive framework* the current sheet becomes unstable to a *secondary mode with growth rate* $\gamma\tau_A \sim S^{-1/6}$, apparently in agreement with the numerical results of [Yu et al., *Nucl. Fusion* **54**, 072005 (2014)].

- When finite electron inertia is included in the analysis, for $(L/d_e)^{12/5} < S < (L/d_e)^3$, regime relevant to most medium size tokamak devices, *the secondary instability develops in the inertia-driven collisionless regime with a growth rate* $\gamma_{II}\tau_A \sim (d_e/a)^2 \sim (d_e/L)^2 S^{2/3}$ *which can become near-Alfvenic.*

- The overall reconnection rate is at least of order $\gamma\,\tau_A \sim (d_e/L)^{2/5}$, always larger than the internal kink growth rate.

Paper: D. Del Sarto, M. Ottaviani "*Secondary fast instability in the sawtooth crash*", Physics of Plasmas **24**, 012102 (2017)

THANK YOU FOR YOUR ATTENTION

On the plasmoid scaling

- *A «plasmoid» scaling of superfast (faster than Alfvenic) reconnection rate of current sheets has been proposed in the literature (Loureiro et al., 2005, and subsequent studies)*

$$\gamma_{plasmoid}\,\tau_A \sim S^{1/4}$$

- This scaling requires two conditions (Tajima and Shibata, *Space Astrophysics*, 2002)

  1. An intense current sheet such that $B_{CS} \sim B_0$ and $J_{CS} / J_0 \sim L/\delta_{sheet} >> 1$

     AND

  2. A sheet size such that $\boldsymbol{\delta_{sheet}/L \sim S^{-1/2}}$

- As we have seen, this is not justified in the sawtooth case

- Current sheets such as $\boldsymbol{\delta_{sheet}/L \sim S^{-1/3}}$ and $J_{CS} / J_0 \sim L/\delta_{sheet}$ produce Alfvenic rate reconnection such that

$$\gamma\,\tau_A \sim 1$$

which is a situation of likely interest in astrophysics where sheets can be produced by Alfvenic motion (Pucci and Velli, 2014)