Damping and Propagation of Geodesic Acoustic Modes in Gyrokinetic Simulations

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GAM - open problems

- Observations indicate that GAM can play an important role in the L-H transition

  GAMs are observed in L- and I- mode regimes but are never observed in H-mode

- At the present is not clear if GAM velocity is constant or not in the experiments

- Most of the experiments show a radial propagation outward the tokamak device but some observations show an inward propagation

- Linear theory predicts velocities lower than that one measured in experiments/simulations (theory $v \sim 10 m/s$ experiments $v \sim 10^2$ - $10^3 m/s$)

Is this discrepancy due to nonlinear effects?

Although a general framework exists to formulate a correct theory, no further new effects have been found in order to explain the gap between theory and experiments

Conway G.D. et al. PPCF 2008

Simon P. PhD Thesis 2017

Qiu Z. et al. PoP 2015

The electromagnetic ORB5 code

- Upgrade of the PIC **ORB5** code (*Jolliet S. et al. CPC 2007*)
- Global tokamak geometry (including magnetic axis).
- Full-f Gyrokinetic Vlasov equation for multiple ion species (DK for electrons).
- Linearized Polarization equation and parallel Ampére's law.
- Electron-ion collisions (pitch angle scattering).
- Ideal MHD equilibria (CHEASE code).
- Equilibrium strong flows.
- Heat and particle sources.
- Advanced particle noise control techniques.

GAM dispersion relation (uniform profiles)

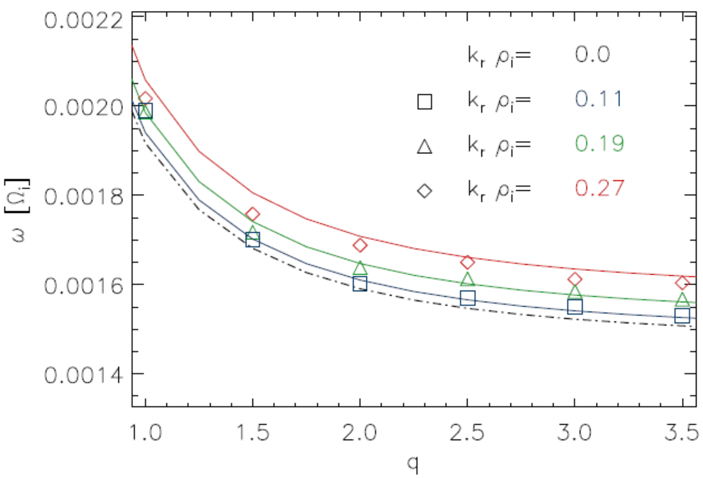
$$\omega = \omega_G(1 + 1/2\alpha_1 k_r^2 \rho_i^2) + i\gamma$$

Real part (frequency)

$$\omega_G = \frac{\sqrt{7+4\tau_e}}{2} q \left(\frac{v_{T_i}}{R_0 q}\right) \left[1 + \frac{2(23+16\tau_e+4\tau_e^2)}{q^2(7+4\tau_e)^2}\right]^{1/2}$$

Imaginary part (damping)

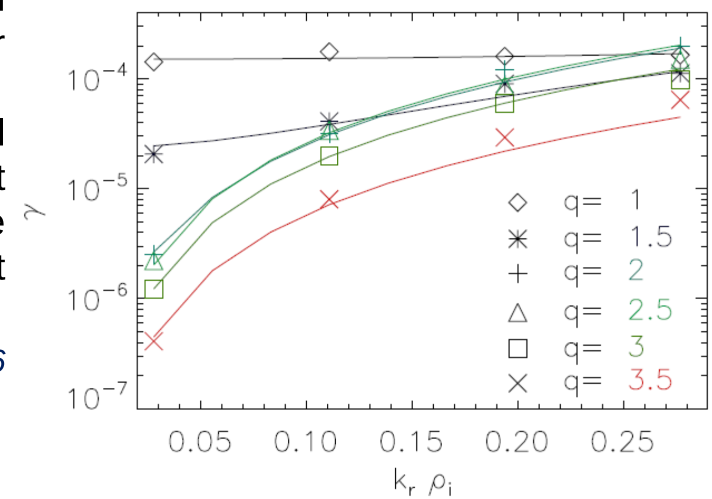
$$\gamma = -f(v_{Ti}, q, \tau_e) + k_{r0}^2 g(v_{Ti}, q, \tau_e)$$



Many expressions not consistent with each other are available for α_1

Due to the small second order effects it is difficult to choose between the different theories

Smolyakov et al. WIP 2016



- Flat profiles T, n, q
- adiabatic electrons
- circular magnetic surface

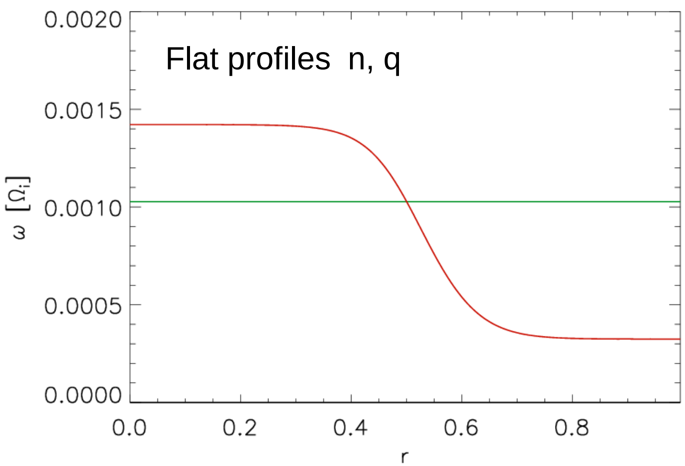
Sugama H. and Watanabe T.-H. JPP 2006

Jolliet S. et al. CPC 2007

Palermo F. et al. PoP 2017

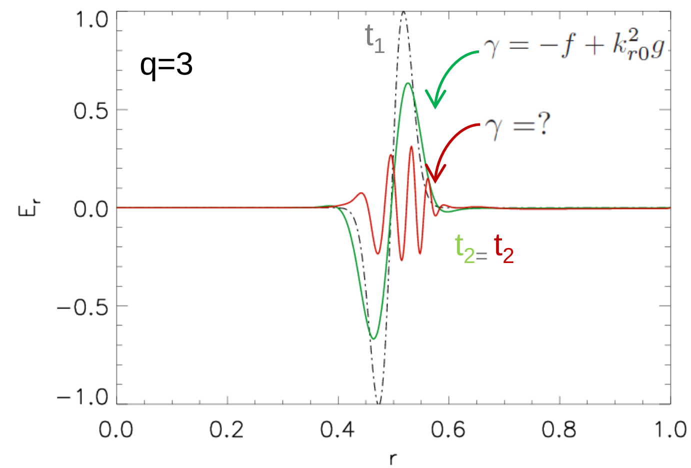
GAMs in nonuniform temperature profile

In the presence of a non-uniform temperature profile GAM constitutes a continuum spectrum



$$\omega_G = \omega_G(v_{Ti}, q, \tau_e)$$

$$v_{Ti} \propto \sqrt{T_i(r)}$$



By phase mixing, the electric field oscillates at each radial position with $\delta E_r(r,t) = A_1 e^{-i\omega_G(r)t}$

Phase mixing does not imply a dissipation of energy $\quad Q \propto |\delta E_r|^2$

Local approximation for the continuum spectrum $\quad \omega_G(r) = \alpha + \beta r$

$$\beta = 0.5\omega_G \kappa_T \qquad \kappa_T = -1/T(\partial T/\partial r)|_{r_0} \qquad r_0 = 0.5$$

Energy is increasingly shifted towards high k_r numbers in time with a rate

$$\delta\tilde{E}_r = 2A_1 e^{-i\alpha t} \lim_{c\to\infty} \frac{\sin[(\beta t + k_r)c]}{(\beta t + k_r)} \longrightarrow k_r = (k_{r0} + \beta t)$$

F. Zonca et al. EPL 2008

The PL- damping mechanism

In the presence of a continuum spectrum the energy of radial waves is shifted in time with $k_r=(k_{r0}+\beta t)$ in the region in which Landau damping is more and more efficient

Phase mixing

$$k_r = (k_{r0} + \beta t)$$

Landau damping

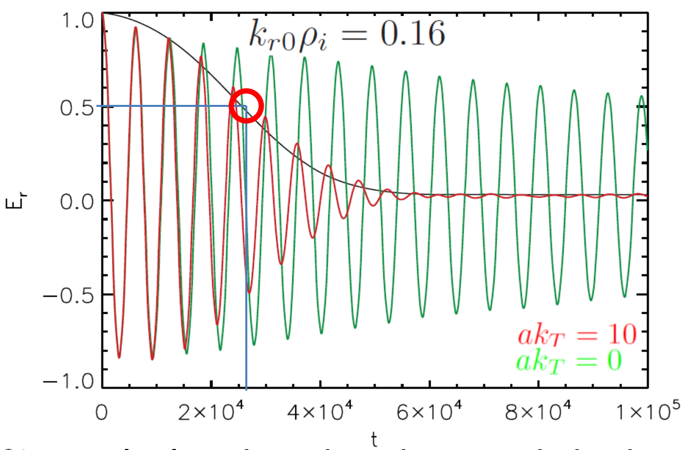
$$\gamma = -f(v_{T_i}, q, \tau_e) + k_{r0}^2 \; g(v_{T_i}, q, \tau_e)$$

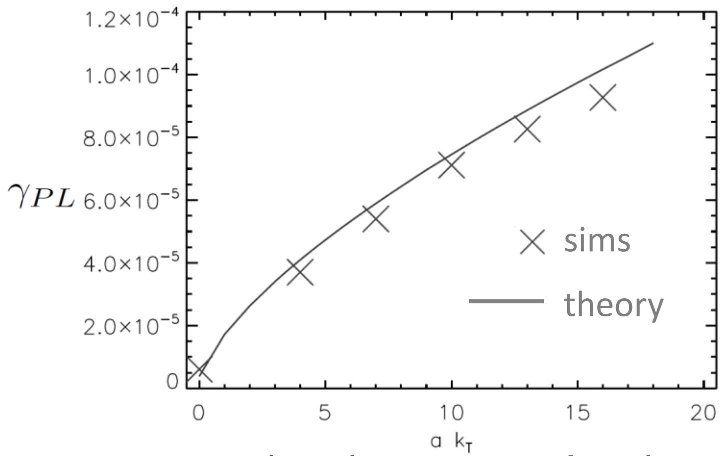
PL-mechanism

$$\gamma_{PL}(t, k_T) = -f(v_{T_i}, q, \tau_e) + (k_{r0} + \beta t)^2 g(v_{T_i}, q, \tau_e)$$

$$\gamma_{PL}(t, k_T) = -\frac{1}{E_r}\frac{\partial E_r}{\partial t}$$

Time evolution of GAM in non-/uniform T profiles





γ_{PL} calculated at the characteristic time t_{PL} in which the GAM electric field is half of its initial value

PL damping rate as a function of the ak_T gradient

Palermo F. et al. EPL 2016
Biancalani A., Palermo F. et al. PoP 2016

Time of Damping

$$\boxed{t_{PL}}$$

$$\text{GAM} \xrightarrow{\text{energy}} \text{background}$$

$$\gamma_{PL}(t, k_T) = -\frac{1}{E_r}\frac{\partial E_r}{\partial t}$$

Characteristic time at which the GAM E_r is half of its initial value

Palermo F. et al. EPL 2016

Time of Drive

$$\boxed{t_{RD} = 1/\gamma_{RD}}$$

$$\text{background} \xrightarrow{\text{energy}} \text{GAM}$$

$$\gamma_{RD}^2 = \frac{\boxed{\alpha_i}}{4}(k_\theta \rho_i k_r \rho_i \Omega_i)^2 \boxed{\langle\langle|\phi_0|^2\rangle\rangle \left|\frac{e}{T_e}A_0\right|^2} \sim |k_\theta| \left|\frac{\delta n}{n}\right|^2$$

$$\alpha_i = \alpha_i(\tau_e, L_T, \eta_i)$$

$$\eta_i = \frac{d\ln T_i}{d\ln n} = \frac{L_n}{L_T}$$

F. Zonca et al. EPL 2008

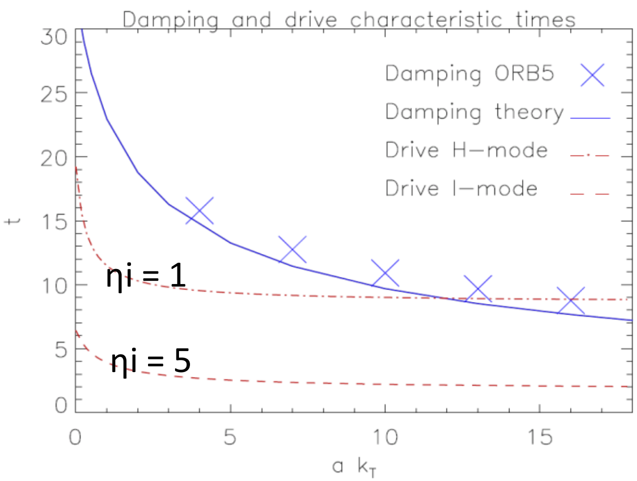
$\boxed{t_{RD}} > \boxed{t_{PL}}$ The PL damping rate exceeds the energy transfer rate from the turbulence to the GAM → Damping of GAM

$\boxed{t_{RD}} < \boxed{t_{PL}}$ Energy flux towards GAMs is larger than the one that leaves GAMs → Amplitude of GAM increases

Time competition in several regimes

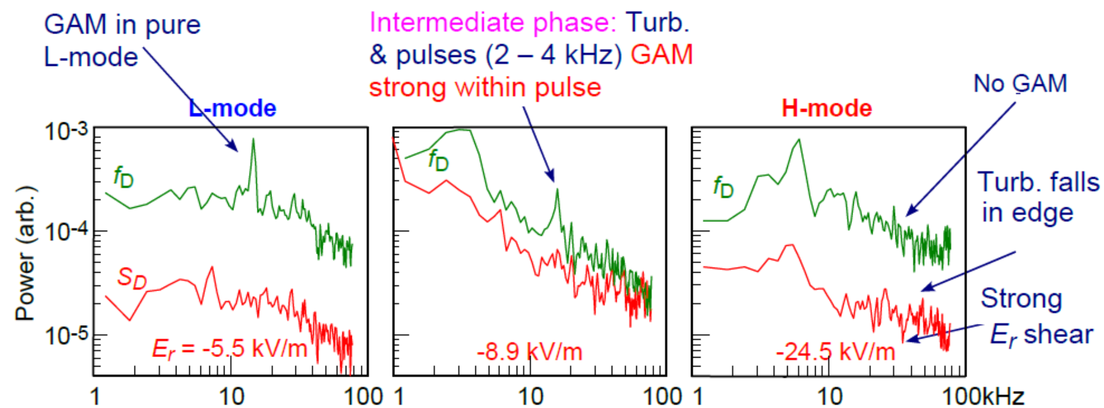
ASDEX Upgrade

AUG #24750



The times are given in sound unit $t_s = R_0/v_{thi}$ with $t_s\Omega_i = 1741.6$.

Palermo F. et al. EPL 2016



$$\boxed{t_{RD}} < \boxed{t_{PL}} \qquad \boxed{t_{RD}} \approx \boxed{t_{PL}} \qquad \boxed{t_{RD}} > \boxed{t_{PL}}$$

Conway et al. PRL 2011

The PL mechanism is consistent with the presence of GAMs in the different confinement regimes

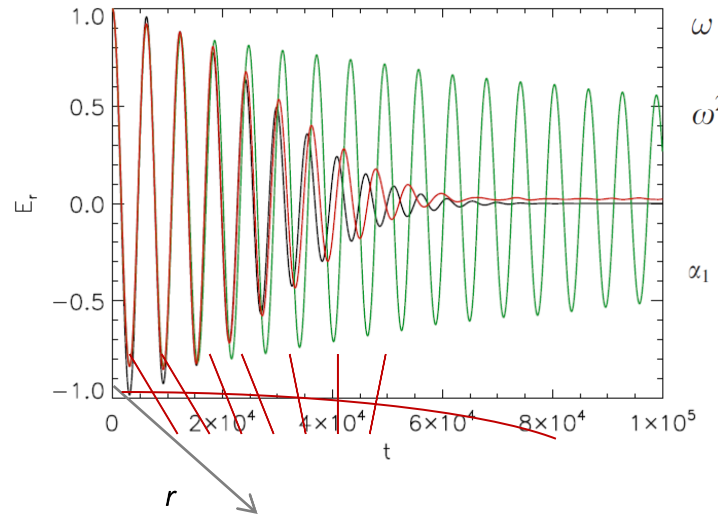
I-mode and L-mode: at each temperature gradient $\boxed{t_{RD}} < \boxed{t_{PL}}$ *(GAM existence)*

H-mode: there is a threshold in k_T, above which $\boxed{t_{RD}} > \boxed{t_{PL}}$ *(GAM suppression)*

The competition between the two times opens interesting scenarios close the H-mode transition, such as the intermittent behaviour of the GAM characteristic of the prey/predator dynamics

Time evolution of frequency and radial propagation

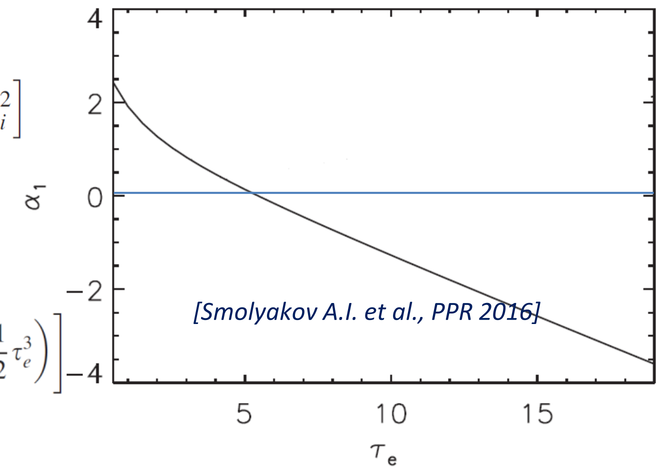
Time evolution of GAM in non/-uniform T profiles

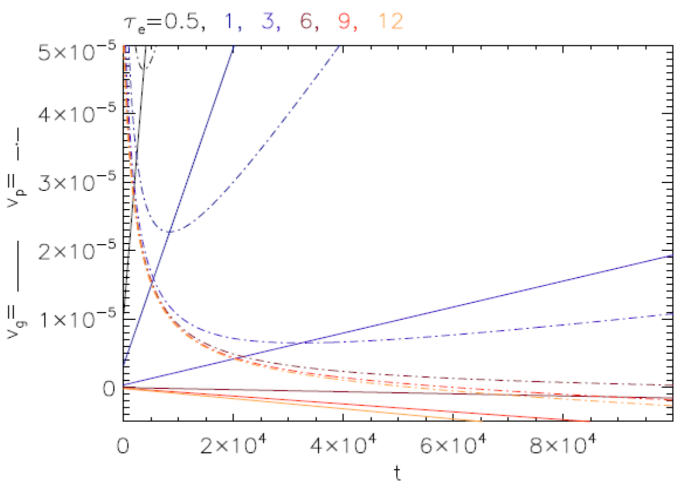


$$\omega = \omega(r,t)?$$

$$\omega^2(r,t) = \omega_G^2(r)\left[1 + \alpha_1 (k_{r0} + \beta t)^2 \rho_i^2\right]$$

$$\alpha_1 = \frac{1}{2}\left[\frac{3}{4} - \left(\frac{7}{4} + \tau_e\right)^{-1}\left(\frac{13}{4} + 3\tau_e + \tau_e^2\right) + \left(\frac{7}{4} + \tau_e\right)^{-2}\left(\frac{747}{32} + \frac{481}{32}\tau_e + \frac{35}{8}\tau_e^2 + \frac{1}{2}\tau_e^3\right)\right]$$





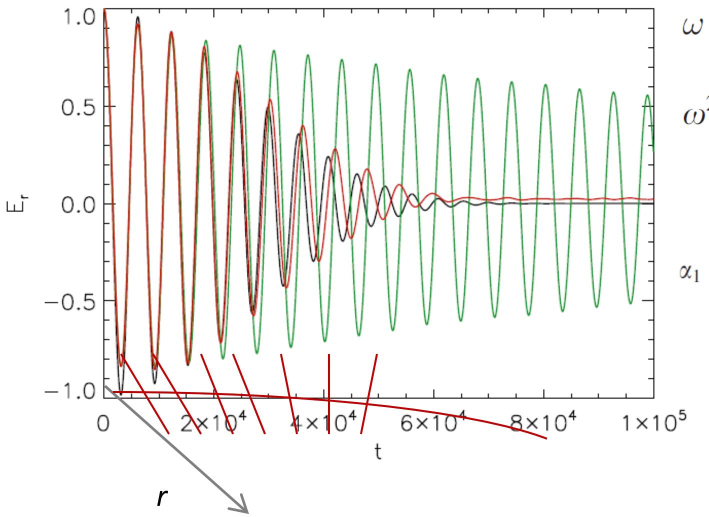
$$v_g = \frac{\partial \omega}{\partial k_r} = \alpha_1 \omega_G (k_{r0} + \beta t)\rho_i^2 \longrightarrow a_c = \frac{\partial v_g}{\partial t} = \frac{\partial}{\partial t}\frac{\partial \omega}{\partial k_r} = \alpha_1 \omega_G \beta \rho_i^2$$

$$v_p = \omega_G \frac{(1 + 1/2\alpha_1 (k_{r0} + \beta t)^2 \rho_i^2)}{(k_{r0} + \beta t)} \longrightarrow \frac{\partial v_p}{\partial t} = \frac{1}{2}\omega_G \alpha_1 \beta \rho_i^2 - \frac{\omega_G \beta}{(k_{r0} + \beta t)^2}$$

Palermo F. et al. PoP 2017

Time evolution of frequency and radial propagation

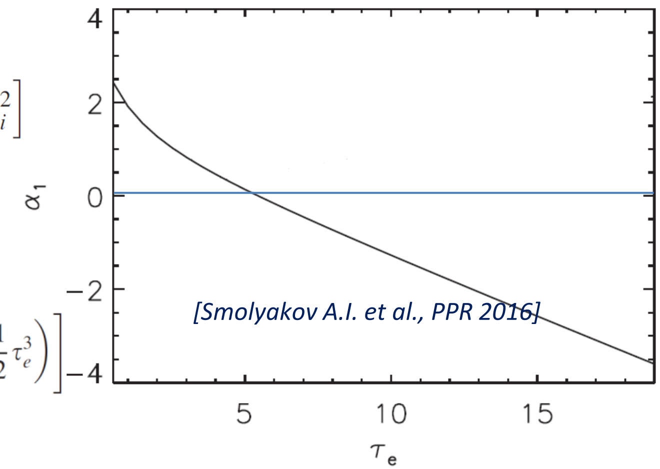
Time evolution of GAM in non/-uniform T profiles

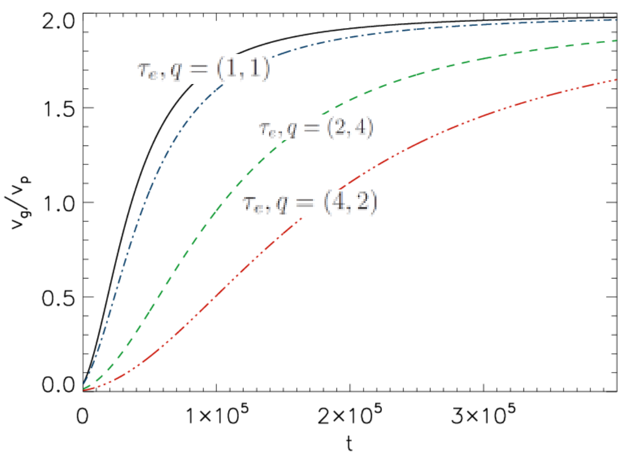


$$\omega = \omega(r,t)?$$

$$\omega^2(r,t) = \omega_G^2(r)\left[1 + \alpha_1 (k_{r0} + \beta t)^2 \rho_i^2\right]$$

$$\alpha_1 = \frac{1}{2}\left[\frac{3}{4} - \left(\frac{7}{4}+\tau_e\right)^{-1}\left(\frac{13}{4} + 3\tau_e + \tau_e^2\right) + \left(\frac{7}{4}+\tau_e\right)^{-2}\left(\frac{747}{32} + \frac{481}{32}\tau_e + \frac{35}{8}\tau_e^2 + \frac{1}{2}\tau_e^3\right)\right]$$





$$v_g = \frac{\partial \omega}{\partial k_r} = \alpha_1 \omega_G (k_{r0} + \beta t)\rho_i^2 \quad \longrightarrow \quad a_c = \frac{\partial v_g}{\partial t} = \frac{\partial}{\partial t}\frac{\partial \omega}{\partial k_r} = \alpha_1 \omega_G \beta \rho_i^2$$

$$v_p = \omega_G \frac{(1 + 1/2\alpha_1 (k_{r0} + \beta t)^2 \rho_i^2)}{(k_{r0} + \beta t)} \quad \longrightarrow \quad \frac{\partial v_p}{\partial t} = \frac{1}{2}\omega_G \alpha_1 \beta \rho_i^2 - \frac{\omega_G \beta}{(k_{r0} + \beta t)^2}$$

Initial times

$$\frac{v_g}{v_p} \approx \alpha_1 (k_{r0} + \beta t)^2 \rho_i^2$$

Large times

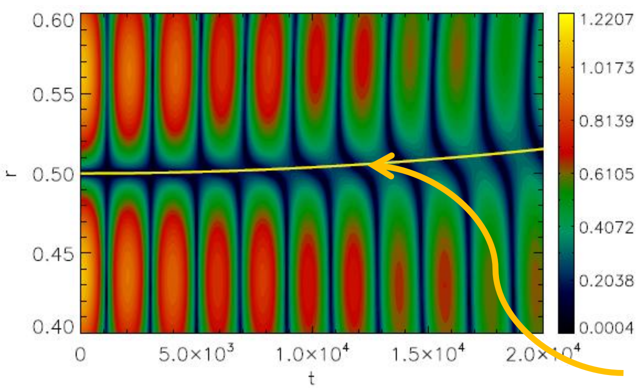
$$\frac{v_g}{v_p} \approx constant$$

Palermo F. et al. PoP 2017

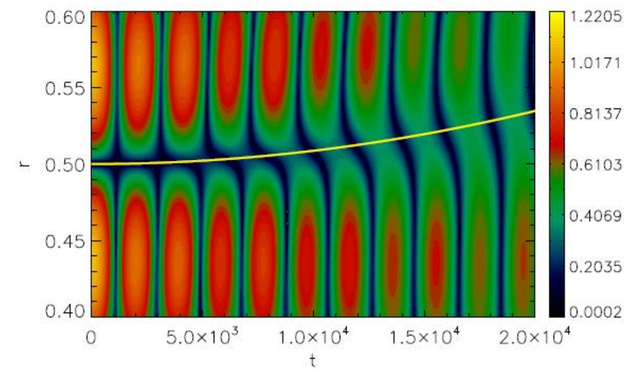
Comparison Theory-Simulations

ASDEX Upgrade

Regime in which the *PL-damping* is not very strong $t = 1/\gamma_{PL} \gg \Delta r / v_g$

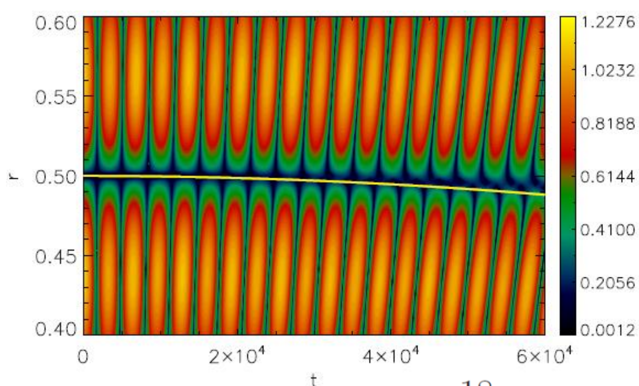


$ak_T = 0.5$ $\tau_e = 1$

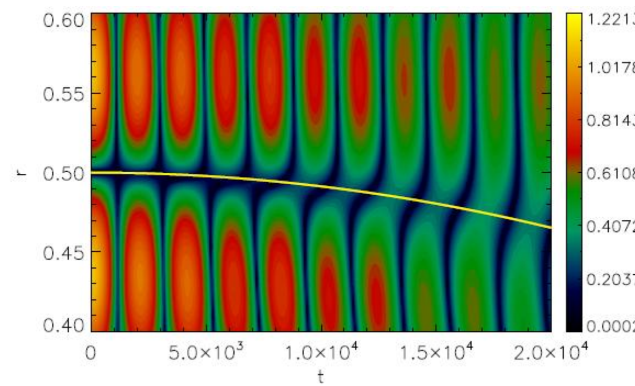


Overplotted trajectory by theoretical acceleration

$ak_T = 1$ $\tau_e = 1$



$ak_T = 1$ $\tau_e = 18$

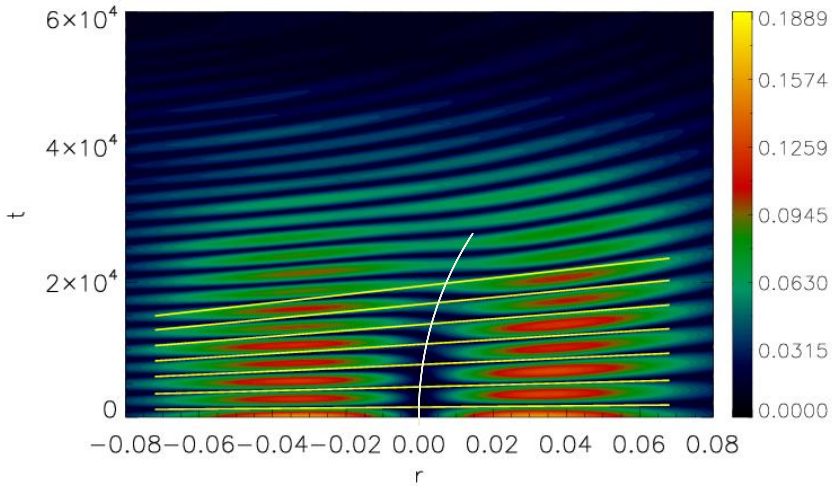


$ak_T = -1$ $\tau_e = 1$

Module of the Electric field amplitude in the (t, r) plane

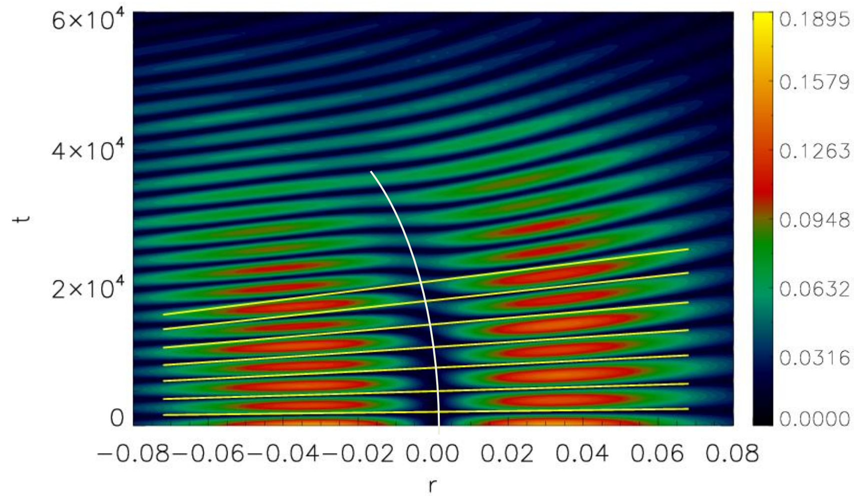
Evolution of the central node of the electric field signal corresponding to time evolution of the potential peak

Comparison Theory-Simulations



$\tau_e = 5$ $\qquad ak_T = 7$

$$v_g v_p > 0$$



$\tau_e = 10$ $\qquad ak_T = 7$

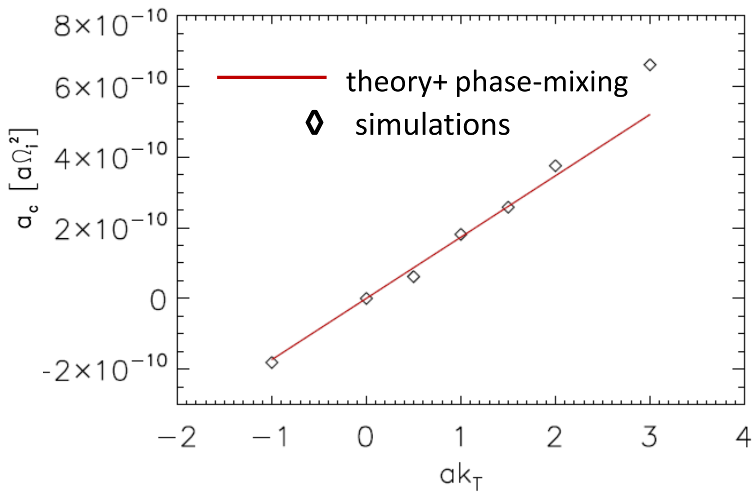
$$v_g v_p < 0$$

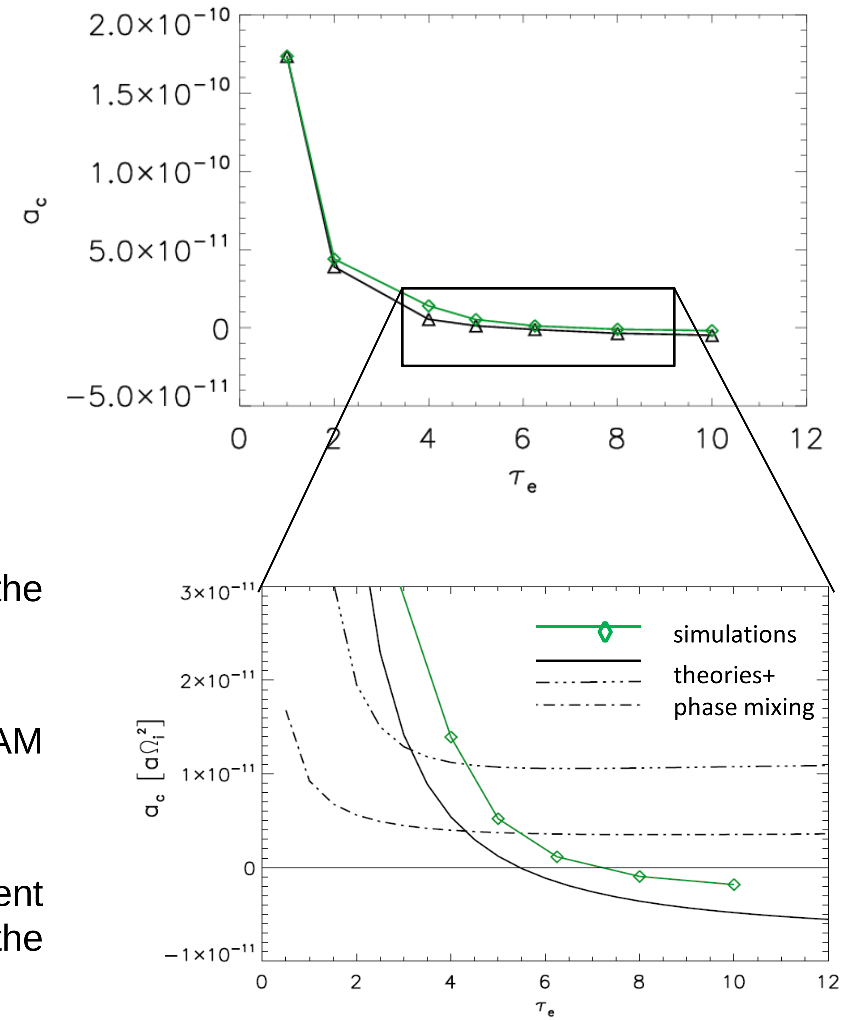
$$v_g = \frac{\partial \omega}{\partial k_r} = \alpha_1 \omega_G (k_{r0} + \beta t) \rho_i^2$$

$$v_p = \omega_G \frac{(1 + 1/2\alpha_1 (k_{r0} + \beta t)^2 \rho_i^2)}{(k_{r0} + \beta t)}$$

- ❑ The relative sign of group and phase velocity is at the basis of a kind of Doppler shift
- ❑ Experimental measurements of GAM frequency can be influenced by this effect

Radial acceleration of GAMs





- GAM acceleration linearly increases with the temperature gradient
- The α_1 changes sign and consequently GAM acceleration reverses around $\tau_e \sim 6$
- Simulations allow to discriminate between different theoretical predictions available in literature for the coefficient α_1 of the dispersive term

Group and phase velocity in realistic equilibria

$$B_0 = 2T$$
$$\Omega_i = 1.92 \cdot 10^8 rad/s$$
$$q = 3$$

$$\rho_i = 9.8 \cdot 10^{-4} m$$
$$\epsilon = a/R = 0.5/1.65 = 0.3$$
$$\lambda_{GAM} = 5.6cm$$

Values of parameters close to the ASDEX Upgrade shot #20878.

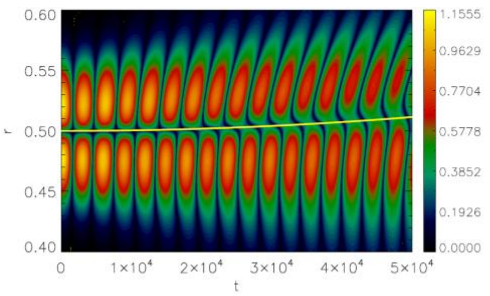
$R/L_T = 6.5$

$t = 40000\Omega_i^{-1}$
$t = 16.8R/c_s$

$$v_{g0} = 4.28 \cdot 10^{-7} a\Omega_i$$
$$v_{p0} = 1.83 \cdot 10^{-5} a\Omega_i$$
$$v_g = 41m/s$$

$$a_c = 7.8 \cdot 10^{-12} a\Omega_i^2$$

$$v_g \approx v_p \approx 4.0 \cdot 10^{-6} a\Omega_i$$
$$v_g = 400m/s$$



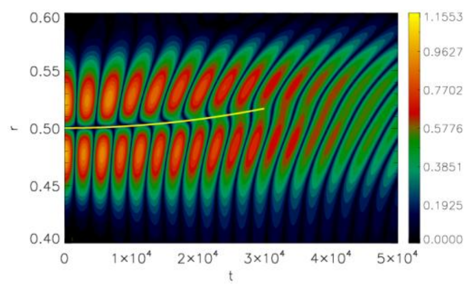
$R/L_T = 22.75$

$t = 16.8R/c_s$

$$v_{g0} = 4.28 \cdot 10^{-7} a\Omega_i$$
$$v_{p0} = 1.83 \cdot 10^{-5} a\Omega_i$$
$$v_g = 41m/s$$

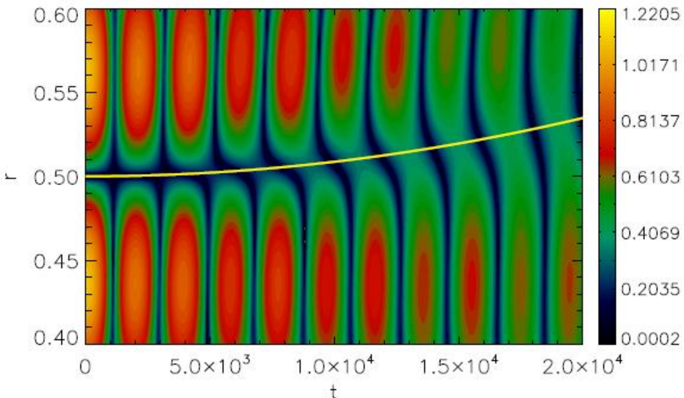
$$a_c = 2.71 \cdot 10^{-11} a\Omega_i^2$$

$$v_g \approx 1134m/s$$
$$v_p \approx 610m/s$$

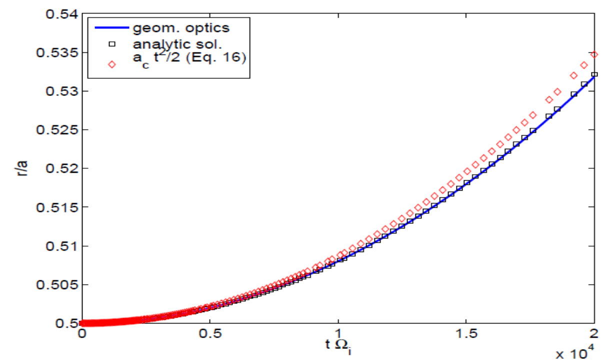


Geometrical optics

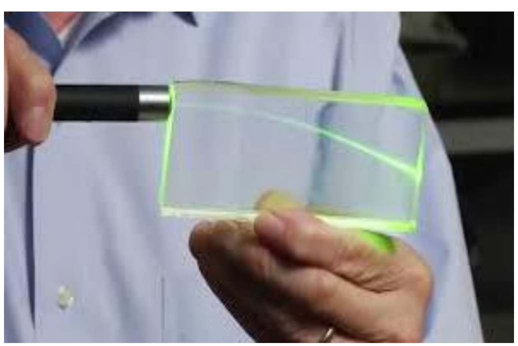
The radial propagation of GAMs, can be treated in the framework of geometrical optics as the space (time) scales involved in the oscillations are shorter (faster) than those related to the equilibrium quantities



$ak_T = 1$ $\qquad$ $\tau_e = 1$



Time evolution of the radial position corresponding to the maximum potential perturbation



Equations of geometrical optics

$$\frac{dr}{dt} = \frac{\partial\omega}{\partial k_r} \qquad \frac{dk_r}{dt} = -\frac{\partial\omega}{\partial r}$$

Characteristics of the wave-kinetic equation that gives a correct description of GAM evolution

$$\frac{\partial E_r}{\partial t} + \nabla_{k_r}\omega\nabla_r E_r - \nabla_r\omega\nabla_{k_r} E_r = -\gamma_{PL} E_r$$

Conclusions

A new picture of GAMs has been given in this work:

- ❑ New damping mechanism (PL-mechanism) has been identified
  This is consistent with the presence/absence of GAMs in the different confinement regimes

- ❑ Time evolution of GAM frequency has been shown for the first time

- ❑ Acceleration in the radial propagation of GAM has been predicted
  This effect reduces the gap between the velocities observed in the experiments/simulations and those predicted by linear and quasi-linear theories