Theory of Cavitons in Complex Plasmas

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(Received 6 February 2003; published 15 August 2003)

Nonlinear coupling between Langmuir waves with finite amplitude dispersive dust acoustic perturbations is considered. It is shown that the interaction is governed by a pair of coupled nonlinear differential equations. Numerical results reveal the formation of Langmuir envelope solitons composed of the dust density depression created by the ponderomotive force of bell-shaped Langmuir wave envelops. The associated ambipolar potential is positive. The present nonlinear theory should be able to account for the trapping of large amplitude Langmuir waves in finite amplitude dust density holes. This scenario may appear in Saturn's dense rings, and the Cassini spacecraft should be able to observe fully nonlinear cavitons, as presented herein. Furthermore, we propose that new electron-beam plasma experiments should be conducted to verify our theoretical prediction.

DOI: 10.1103/PhysRevLett.91.075005 PACS numbers: 52.27.Lw, 52.35.Mw, 52.35.Sb

More than a decade ago, Rao, Shukla, and Yu [1] predicted for the first time the existence of linear and nonlinear dust acoustic waves (DAWs) in an unmagnetized dusty plasma. In the DAWs, the restoring force comes from the pressures of inertialess electrons and ions, while the mass of charged dust grains provides the inertia in order to maintain the waves. The nonlinear dust acoustic waves have subdust acoustic speed and localized negative potential, contrary to the usual ion-acoustic solitary waves [2,3] in an electron-ion plasma which have compressive potential and density variations. A number of laboratory experiments [4–7] have verified the prediction of Rao *et al.* [1]. For typical laboratory conditions, the dust acoustic wave frequencies (phase speeds) range between 5 to 25 Hz (0.5 to 10 cm/s), while the wavelength lies between 0.1 to 0.4 cm. Hence, the visual images of the DAWs are possible. The possible impact of nonlinear effects on the dust acoustic waves has also been presented [8]. The status of linear and nonlinear dust acoustic waves has been summarized in review articles [9,10] and books [11–13].

It is well known [14] that finite amplitude Langmuir waves in a plasma can be excited by electron beams. Large amplitude Langmuir waves interacting with ion-acoustic waves produce an envelope of Langmuir waves which are trapped in a subsonic density cavity that is created by the ponderomotive force of the Langmuir waves. Nonlinear structures composed of trapped Langmuir waves in a density cavity are known as cavitons or Langmuir envelope solitons [15–18], and these structures have been observed in many laboratory plasma experiments [19–21] without dust. A preliminary investigation of *small amplitude cavitons* in a dusty plasma has been presented by Shukla and Mamun [12], based on a model of the cubic nonlinear Schrödinger equation for the modulated Langmuir wave envelops. Because of their infinitely small amplitudes, the cavitons in Ref. [12] are unlikely to be observed in space and laboratory environments. Therefore, there is a need to develop a finite amplitude theory for Langmuir envelope solitons in dusty plasmas.

In this Letter, we present for the first time a fully nonlinear theory for cavitons in a complex (dusty) plasma whose constituents are electrons, ions, and negatively charged massive dust grains. At equilibrium, we have $n_{i0} = n_{e0} + Z_d n_{d0}$, where n_{j0} is the unperturbed number density of the particle species j (j equals i for ions, e for electrons, and d for dust grains) and Z_d is the number of electrons residing on the dust grain surface. Dust grains in laboratory plasmas are typically charged negatively due to collection of electrons from the ambient plasma [22,23], but under UV radiation dust can also be charged positively [24]. We suppose that the presence of electron beams in a dusty plasma generates large amplitude Langmuir waves whose frequency is $\omega = (\omega_{pe}^2 + 3k^2V_{Te}^2)^{1/2}$, where $\omega_{pe} = (4\pi n_e e^2/m_e)^{1/2}$ is the electron plasma frequency, n_e is number density of the electrons, e is the magnitude of the electron charge, m_e is the electron mass, k is the wave number, $V_{Te} = (T_e/m_e)^{1/2}$ is the electron thermal speed, and T_e is the electron temperature. Large amplitude Langmuir waves interacting nonlinearly with finite amplitude dust acoustic perturbations generate a Langmuir wave electric field envelope whose electric field E evolves slowly in comparison with the electron plasma wave period according to a nonlinear Schrödinger equation (see, e.g., Chapter 7.4 in [12])

$$2i\omega_p\left(\frac{\partial}{\partial t} + v_g\frac{\partial}{\partial x}\right)E + 3V_{Te}^2\frac{\partial^2 E}{\partial x^2} + \omega_p^2\left(1 - \frac{n_e}{n_{e0}}\right)E = 0, \tag{1}$$

where $\omega_p = (4\pi n_{e0}e^2/m_e)^{1/2}$ is the unperturbed electron plasma frequency and $v_g = 3kV_{Te}^2/\omega_p$ is the group velocity of the Langmuir waves. We note that (1) is derived by combining the electron continuity and momentum equations as well as by using Poisson's equation with

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fixed ions and retaining the arbitrary large electron number density perturbation n_{e1} associated with the dust acoustic waves in the presence of the Langmuir wave ponderomotive force. For our purposes, we have

$$n_e = n_{e0} \exp(\varphi - W^2), \tag{2}$$

where $\varphi = e\phi/T_e$, $W^2 = |E|^2/16\pi n_{e0} T_e$, and ϕ is the electrostatic potential of the DAWs whose phase speed is much smaller than the electron and ion thermal speeds. The ion number density perturbation associated with the DAWs is

$$n_i = n_{i0} \exp[-\tau(\varphi + \mu_i W^2)], \tag{3}$$

where $\tau = T_e/T_i$, T_i is the ion temperature, $\mu_i = m_e/m_i$, and m_i is the ion mass. We note that the W terms in Eqs. (2) and (3) come from the averaging of the nonlinear term $m_j \mathbf{v}_{hj} \cdot \nabla \mathbf{v}_{hj}$ over the Langmuir wave period $2\pi/\omega_{pe}$, where $\mathbf{v}_{hj} \approx q_j E/m_j \omega_{pe}$ is the quiver velocity of the particle species j in the Langmuir wave electric field, $q_e = -e$, $q_i = e$, and $q_d = -Z_d e$. The dust dynamics is governed by the dust continuity and momentum equations

$$\frac{\partial n_d}{\partial t} + \frac{\partial (n_d u_d)}{\partial x} = 0, \tag{4}$$

and

$$m_d n_d \left(\frac{\partial u_d}{\partial t} + u_d \frac{\partial u_d}{\partial x} \right) = Z_d n_d T_e \frac{\partial (\varphi - \mu_d W^2)}{\partial x}, \tag{5}$$

where $\mu_d = Z_d m_e/m_d$, n_d is the dust number density, and u_d is the x component of the dust fluid velocity. The phase speed of the DAWs is assumed to be much larger than the dust thermal speed. The equations are closed with Poisson's equation

$$\lambda_{De}^2 \frac{\partial^2 \varphi}{\partial x^2} = \left(\frac{n_e}{n_{e0}} - \frac{n_i}{n_{e0}} + \frac{Z_d n_d}{n_{e0}} \right), \tag{6}$$

where $\lambda_{De} = (T_e/4\pi n_{e0} e^2)^{1/2}$ is the electron Debye radius. Thus, our nonlinear theory of cavitons accounts for arbitrary large amplitude density variations (and associated space charge potential ϕ) that are associated with fully nonlinear dispersive DAWs, contrary to the small amplitude cavitons based on the cubic nonlinear Schrödinger equation [12], which discards nonlinearities and the departure from the quasineutrality in the plasma slow motion.

We are interested in quasisteady state solutions of Eqs. (1)–(6). Accordingly, we insert $E(x, t) = W(\xi) \exp\{i[X(x) + T(t)]\}$, $n_d(x, t) = n_d(\xi, t)$, $u_d(x, t) = u_d(\xi, t)$, and $\varphi(x) = \varphi(\xi)$, where $\xi = x - Vt$, and V is the constant speed of the soliton and $W(x)$, $X(x)$, $T(x)$ are assumed to be real, into Eqs. (1)–(6) we finally obtain the coupled set of nonlinear equations

$$3 \frac{\partial^2 W}{\partial \xi^2} - (\lambda - 1) W - W \exp(\varphi - W^2) = 0, \tag{7}$$

and

$$\frac{\partial^2 \varphi}{\partial \xi^2} - \exp(\varphi - W^2) + \alpha \exp[-\tau(\varphi + \mu_i W^2)] + (1 - \alpha) \frac{M}{\sqrt{M^2 + 2(\varphi - \mu_d W^2)}} = 0, \tag{8}$$

where ξ is normalized by λ_{De}, $\lambda = 2\omega_p^{-1}(dT/dt) - 3k^2 \lambda_{De}^2 (1 - V^2/v_g^2)$ represents a nonlinear frequency shift, $\alpha = n_{i0}/n_{e0}$, and $M = V/C_D$ is the Mach number involving the dust acoustic speed $C_D = (Z_d T_e/m_d)^{1/2}$. Since in complex plasmas we typically have $\mu_i \ll 1$ and $\mu_d \ll 1$, the contributions of ion and dust ponderomotive forces in Eq. (8) can be safely dropped. Thus, the electron ponderomotive force is transmitted to ions and dust via the ambipolar potential.

The system of Eqs. (7) and (8), without the $\mu_i W^2$ and $\mu_d W^2$ terms in Eq. (8), admits the first integral in the form of a Hamiltonian

$$\begin{aligned} H(W, \varphi, \lambda, M) = {} & 3 \left(\frac{\partial W}{\partial \xi} \right)^2 - \frac{1}{2} \left(\frac{\partial \varphi}{\partial \xi} \right)^2 - (\lambda - 1) W^2 + \exp(\varphi - W^2) - 1 + \frac{\alpha}{\tau} [\exp(-\tau\varphi) - 1] \\ & + (\alpha - 1) M (\sqrt{M^2 + 2\varphi} - M) = 0, \end{aligned} \tag{9}$$

where in the unperturbed state ($|\xi| = \infty$) we have used the boundary conditions $W = 0$, $\varphi = 0$, $\partial W/\partial \xi = 0$, and $\partial \varphi/\partial \xi = 0$. Because we are interested in symmetric solutions defined by $W(\xi) = W(-\xi)$ and $\varphi(\xi) = \varphi(-\xi)$, the appropriate boundary conditions at $\xi = 0$ are $W = W_0$, $\varphi = \varphi_0$, $\partial W/\partial \xi = 0$, and $\partial \varphi/\partial \xi = 0$. Hence, from Eq. (11) we have $\exp(\varphi_0 - W_0^2) - 1 - (\lambda - 1) W_0^2 + (\alpha/\tau)[\exp(-\tau\varphi_0) - 1] + (\alpha - 1) M (\sqrt{M^2 + 2\varphi_0} - M) = 0$, which shows how the maximum values of W_0 and φ_0 are related with M and λ for given values of τ and α. It should be stressed that the static dust case has to be treated separately, where the last term in the left-hand side of Eq. (9) has to be replaced by $(\alpha - 1)\varphi$. Accordingly, the last term in the left-hand side of the simplified Hamiltonian above will be replaced by $(\alpha - 1)\varphi_0$. A practical application of the Hamiltonian is to check the correctness of any numerical scheme used to solve Eqs. (7) and (8).

In the absence of the Langmuir waves, the nonlinear DAWs are governed by the energy integral

$$\frac{1}{2}\left(\frac{\partial\varphi}{\partial\xi}\right)^2 + \Psi(\varphi, M) = 0, \tag{10}$$

where the Sagdeev potential is

$$\Psi(\varphi, M) = 1 - \exp(\varphi) + \frac{\alpha}{\tau}[1 - \exp(-\tau\varphi)] + (1-\alpha)M(\sqrt{M^2 + 2\varphi} - M). \tag{11}$$

Equation (10), which is obtained from Eq. (9) in the limit of vanishing Langmuir wave electric fields, determines the profile of nonenvelope (bare) dust acoustic solitary waves. The latter exist provided that $\Psi(\varphi)$ is negative between zero and $\pm\varphi_0$. Multivalued solutions of $\Psi(0)$ are ensured provided that $\partial^2\Psi/\partial^2\varphi = 0$ while, at $\varphi = \varphi_0(-\varphi_0)$, we must have $\partial\Psi/\partial\varphi > 0(<0)$. The condition $\Psi(\varphi_0, M) = 0$ gives a relation between φ_0 and M for given values of α and τ. It turns out that dust acoustic solitons have subdust acoustic speed, negative potential, and dust density hump.

In the numerical solution [25] of Eqs. (7) and (8), the second derivatives were approximated by a second-order centered difference scheme, and the values of W and φ were set to zero at the boundaries of the computational domain. The resulting nonlinear system of equations was solved with Newton's method. We used 2000 sampling points to resolve the solution. The results are displayed in Figs. 1–3 for dusty plasmas containing micronsized dust grains with $Z_d = 10^3$ and $m_d/m_i = 10^{12}$. The ion to electron mass ratio is typically 1836 and more depending upon ionized dusty gases. Figure 1 shows the profiles of a caviton for $M = 0.7$, $\lambda = 0.06$, and $\alpha = 2$, and in Fig. 2 the Mach number was changed to $M = 10$. Figures 1 and 2 show that the envelope Langmuir solitons in complex plasmas are composed of a bell-shaped Langmuir electric field and a dust density hole in association with a positive localized space charge electric potential. For higher Mach numbers, the influence of the potential on the dust dynamics becomes smaller, as can be seen from the dust density perturbations in Figs. 1 and 2. Thus, large and small amplitude cavitons move with subdust and superdust acoustic speeds, respectively. By performing several numerical experiments, we studied the influence of the electron to ion temperature ratio τ on the caviton. We found that the depth of the caviton decreases as τ increases, if the other parameters are kept constant.

In order to see the difference between the cavitons and the bare dust acoustic solitary waves [12], we integrated Eq. (10) numerically. The results are displayed in Fig. 3. The numerical results show that the dust acoustic solitary waves have subdust acoustic speed and they are associated with negative space charge potentials and the dust density hump. As can be seen from Fig. 3, the bare soliton also develops on a much smaller length scale than the envelope cavitons in Figs. 1 and 2. Evidently, the properties of cavitons are very different from the bare dust acoustic solitary waves.

In summary, we have presented a theory for finite amplitude cavitons in complex (dusty) plasmas, taking into the nonlinear coupling between finite amplitude

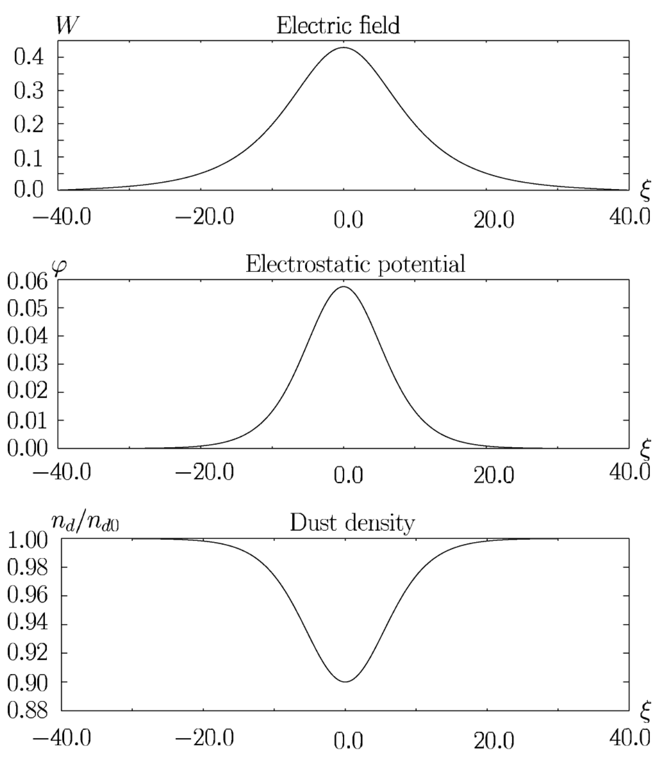

FIG. 1. Small Mach number Langmuir envelope solitons (cavitons) for $\lambda = 0.06$, $\tau = 2$, $\alpha = 2$, and $M = 0.7$.

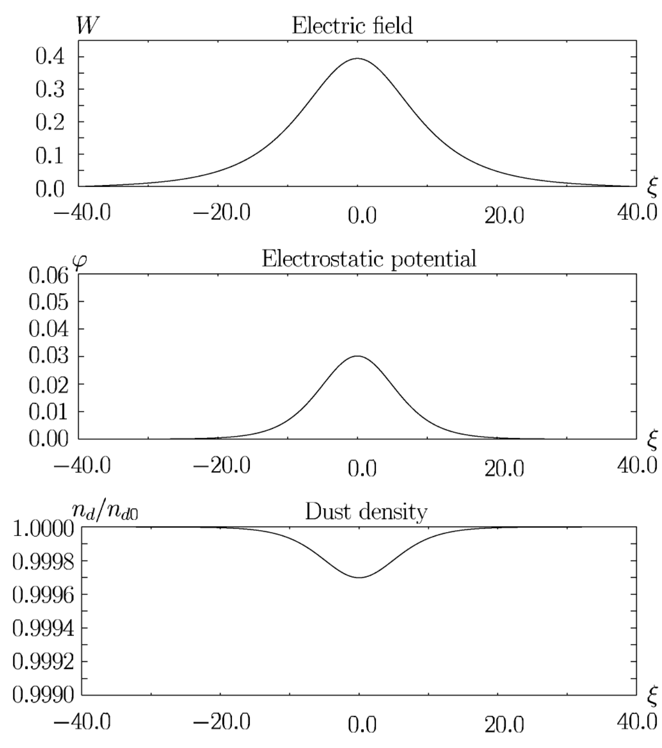

FIG. 2. High Mach number Langmuir envelope solitons (cavitons) for $\lambda = 0.06$, $\tau = 2$, $\alpha = 2$, and $M = 10$.

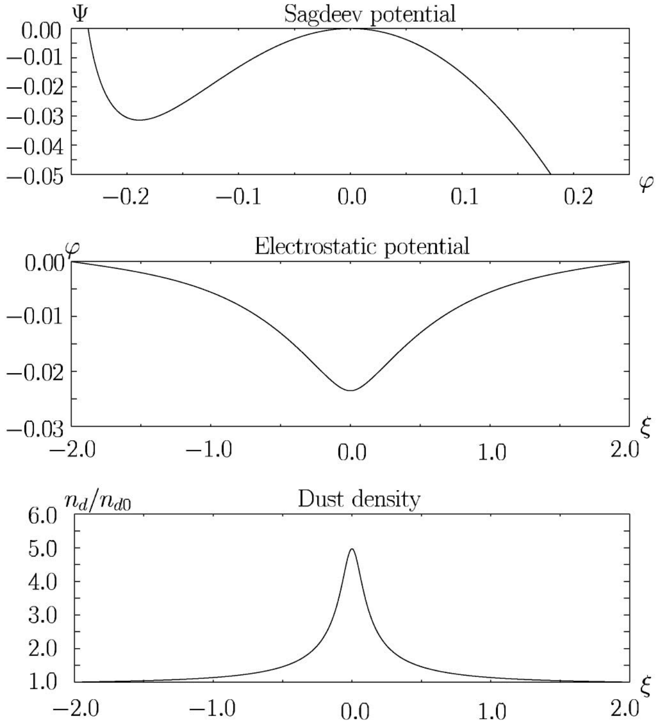

FIG. 3. Bare dust acoustic soliton ($W = 0$) for $\tau = 2$, $\alpha = 2$, and $M = 0.7$.

Langmuir and dust acoustic waves. In a stationary frame of reference, the governing equations for coupled Langmuir and DAWs have been reduced to a pair of coupled nonlinear differential equations, which admit a new class of Hamiltonian. Using appropriate boundary conditions for localized perturbations, we have numerically analyzed the coupled differential equations by demanding that the Hamiltonian is conserved. Numerical results reveal the existence of cavitons composed of a positive space charge electric potential and dust density holes in which localized Langmuir wave envelopes are trapped. Large amplitude cavitons move slower than the small amplitude cavitons. Positive space charge electric potentials associated with cavitons have a strong gradient, which can accelerate negatively charged dust grains in complex plasmas. We stress that the previously reported small amplitude Langmuir envelope soliton theory [12], which is based on a cubic Schrödinger equation, cannot account for large amplitude cavitons that will necessarily appear in space and laboratory dusty plasmas. In conclusion, we suggest that new beam-plasma experimental and computer simulation studies should be conducted for verifying our theoretical predictions of dust cavitons. We also hope that forthcoming data from the Cassini mission will also reveal signatures of magnetic field aligned large amplitude localized Langmuir electric fields as well as an associated positive potential and finite amplitude dust density hole in Saturn's rings where $n_{e0} \sim 50\ \mathrm{cm}^{-3}$, $n_{i0} \sim 950\ \mathrm{cm}^{-3}$, $n_{d0} \sim 1\ \mathrm{cm}^{-3}$, $Z_d \sim 900$, the dust radius $r_d \sim 1$–$5\ \mu\mathrm{m}$, and $T_e \sim T_i \approx 100$ eV. In such a Saturn plasma environment, we expect an ambipolar potential of 10 V and a relative dust density depletion of 10% if $E \sim 0.3$ mV/m and $M = 0.96$, which are within the observable range.

One of the authors (P. K. Shukla) thanks Robert Bingham for invaluable discussions. This work was partially supported by the European Commission through Contracts No. HPRN-CT-2000-00140 and No. HPRN-CT-2001-00314, as well as by the DFG through the Sonderforschungsbereich 591.

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