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Outgoing and Global Drift Waves in Rotating Toroidal Plasma Configuration

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Abstract

The drift waves in rotating toroidal plasma are studied for an axisymmetric, large-aspect-ratio tokamak with concentric and circular magnetic surfaces. Plasma rotation is driven by the radial electrostatic field typical for the H confinement mode of plasma in tokamaks. Low-frequency electrostatic oscillations of low-beta plasma are considered in assumptions of adiabatic electrons and plasma quasineutrality. In order to describe drift oscillations in the plasma edge region, where the radial electric field and plasma rotation velocity are high, a weak coupling approximation that takes into account the toroidal coupling of normal modes centred on the neighbouring rational surfaces is considered. The derived eigenmode equation of the Weber type has two classes of solutions giving either marginally stable global drift modes or propagating drift waves which experience shear damping. The analytical dispersion equations, for both global and propagating drift waves are derived and the simple dispersion relations for some limiting cases are determined.

1. Introduction

It is well known that the L-H transition is usually marked by an abrupt reduction in the intensity of neutral atom radiation, $H_a(D_a)$, and by a substantial change in the radial electric field at the plasma edge. The experimental results indicate that the radial electric field at the plasma boundary, plays a major role in reducing turbulence and establishing the transport barrier of the H mode (see Ref. [1] and references therein). It has been found empirically that the transport barrier exists in a region in which the radial electric field achieves large absolute values and varies rapidly in space.

At present, it is generally believed that transport reduction is strongly connected with decorrelation of turbulence, which is predominantly electrostatic at the edge. Normally, the observed transport in tokamak experiments exceeds greatly that of collisional transport theory. This anomalous transport is usually attributed to turbulent fluctuations arising from various microinstabilities, e.g. drift waves or pressure gradient driven ballooning modes [2].

Microinstabilities and microturbulence of the drift-type waves have been investigated extensively both theoretically and experimentally (see reviews by Tang [3] and Liever [4]). These microinstabilities occur as low frequency collective oscillations arised from large-scale charged particle interactions. The oscillation frequency is of the order of the diamagnetic drift frequency, $\omega \approx \omega_*$, and low compared to the ion gyrofrequency ω_{ci}. The parallel to the magnetic field phase velocity lies between the ion and electron thermal velocities, $v_{Ti} \ll \omega/k_\parallel \ll v_{Te}$, where $k_\parallel$ is the component of the wave vector parallel to the magnetic field. In a tokamak however, $k_\parallel$ is a function of radius, since the direction of the magnetic field depends on the radial coordinate. Thus, the local approximation is not valid and it is necessary to solve a differential equation to determine the radial structure and stability of a drift mode [5].

In the uniform plasma slab, magnetic field shear produces an inherent damping of drift waves [6,7]. This damping arises because, in the presence of shear, a mode centred on a given magnetic surface radiates energy outwards from the surface [5]. However, as it was pointed out by Taylor [8], in a realistic system, where the field strength and shear are not uniform, waves associated with different surfaces are coupled together. This changes the propagation of drift waves and radiation is inhibited or reflected. In the tokamak geometry, a mode centred at r with $m = nq(r)$, is coupled with modes localised on surfaces at $r \pm \Delta r$ such that $m \pm 1 = nq(r \pm \Delta r)$ (m and n are poloidal and toroidal wavenumbers, respectively and $q(r)$ is the safety factor). If the coupling to adjacent modes is included, toroidal coupling effects can form local potential wells which reduce or eliminate the outward convection of wave energy and hence, the shear damping [9]. Horton *et al.* [10] have found that the ion toroidal drifts cause the shear stabilizing radial anti-well to become a radial well which localises the mode away from the regions of ion Landau damping. In such a well, marginally stable quasimodes exist for $s < 1/2$, where $s = rq'(r)/q(r)$ is the magnetic shear parameter.

Subsequent works [11–14] show that toroidicity-induced marginally stable quasimodes exist for $s > 1/2$. However, modes of this type occur only if the diamagnetic frequency $\omega_*(r)$ has a maximum at r, and they affect only a small fraction, $O(1/\sqrt{n})$, of the plasma radius around this maximum. Recently, Connor *et al.* [15] and Taylor *et al.* [16] have found another class of toroidal drift waves with very different properties. The new modes have greater shear damping (closer to that in a plane-slab) than the conventional ones and thus have a higher instability threshold. However, they occur for any plasma profile and at all radii, and they have larger radial extent over a fraction $O(\varepsilon)$ of the plasma radius (ε being the inverse aspect radius).

The structure of drift waves in a rotating toroidal plasma has been discused by Taylor and Wilson [17]. They have incorporated a Doppler frequency shift that varies from one magnetic surface to another into the basic model. This shift variation is due to a sheared plasma velocity $v_E(r)$. The standard ballooning representations are no longer effective because they are based on the fact that all the magnetic surfaces are approximately equivalent [18]. This equivalence

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is destroyed by the sheared rotation. In seeking an alternative to the ballooning representation, Taylor and Wilson [17] supposed that, although magnetic surfaces are not equivalent to each other in rotating plasma, the relationship of a surface to its neighbours is still the same for all the surfaces. This assumption led them to the conclusion that the shear damping of toroidal drift modes in a rotating plasma, is no longer reduced by toroidal effects and is similar to that in cylindric geometry.

Drift waves in rotating toroidal plasma have been investigated in Ref. [19] on the base of rigorously derived eigenmode equations, coupled in poloidal mode numbers through the toroidal effects. In order to avoid the above mentioned difficulties with employment of the ballooning representation for rotating plasmas, the treatment by Horton *et al.* [10] and Tang [3] with a method by Taylor [8] was used instead. This method employs the so-called strong coupling approximation, while assuming that a significant number Δ_l ($1 \ll \Delta_l \ll m_0$, m_0 is the poloidal mode number at the reference rational surface around which the mode is centred) of poloidal harmonics are coupled by the equilibrium toroidal variations without the supposition that the relationship of a surface to its neighbours is the same for all the surfaces. Therefore, the Fourier decomposition for the eigenmodes was used instead of the "Fourier ballooning" representation of Ref. [17]. The obtained eigenmode equation has two types of solutions depending on the value of magnetic shear parameter and on the sign of the Doppler-shifted eigenfrequency. The first type corresponds to global drift modes with a basic structure resembling a "quasimode". In the framework of strong coupling approximation, global drift modes exist for a strong magnetic shear, $s > 1/2$, when the Doppler-shifted eigenfrequency is negative. The solutions of the second type describe propagating drift waves that experience shear damping as slablike outgoing waves [5].

The quasimode structure of global drift modes, obtained in the strong coupling approximation [19], indicates that the mode can be represented as a sum of degenerate radially localised normal modes. Each of these component normal modes is centred on a different rational surface and has comparable amplitude to the others. Since the spacing between rational surfaces is generally much smaller than the typical equilibrium scale lengths in realistic tokamak plasma conditions, the radial equilibrium gradients can be considered constants with a good approximation. However as it has been observed at the tokamaks TEXT [20], JFT-2M [21], TEXTOR [22], ASDEX [23], DIII-D [24] and JT-60U [25–27], the plasma poloidal rotation velocity and the radial electric field change dramatically in the plasma edge region during the L-H transition. Moreover, the profiles of rotation velocity and radial electric field are characterised by large radial gradients in this region. So, the strong coupling approximation may not be valid near the plasma edge.

In this work, we consider toroidal drift modes in rotating tokamak plasma in a weak coupling approximation, truncating the harmonic expansion at two nearest terms. As in Ref. [19], the performed analysis is based on a rigorously derived eigenmode equation for drift waves in an axisymmetric, large-aspect-ratio tokamak with concentric, circular magnetic surfaces where the toroidal coupling effects appear due to ion ∇B and curvature drifts. The eigenmode solutions obtained in the weak coupling approximation belong to two classes depending on the ratio ω'/ω_* and the sign of ω_* (ω' being the Doppler-shifted eigenfrequency). These two classes are the global drift modes and the propagating (or outgoing) drift waves. The paper is organised as follows: The basic set of equations described mode structure and dispersion properties of a drift wave in a rotating tokamak plasma, is derived in Section 2. This set is reduced to an ordinary differential equation of the Weber type by using the weak coupling approximation in Section 3. The latest equation is analysed and its simple asymptotic solutions are given for some limit cases for global and propagating drift modes in Section 4 and 5, respectively. Summary and discussions follow in Section 6.

2. Basic equations

We consider a toroidal plasma configuration and suppose that a plasma rotation occurs due to inhomogeneous electrostatic potential $\Phi(r)$. The plasma is confined by the inhomogeneous magnetic field $\boldsymbol{B}$ with vanishing component along the equilibrium density, $n_0(r)$, gradient. Restricting ourselves to the case of low $-\beta$ (the ratio of the plasma pressure to the magnetic one), we can describe the key features of collisionless electrostatic drift modes by a system of two-fluid equations for electrons and ions and the Poisson equation. The frequency of these modes is low, $\omega \ll \omega_{ci}$ and the magnetic field perturbations are negligible. The smallness of electron inertia compared with their thermal motion, $\omega \ll k_\parallel v_{Te}$, permits us to neglect the charge separation and use the quasineutrallity condition instead of the Poisson equation. In this limit the electrons are thermalised along the magnetic field lines, i.e. they follow the Boltzmann distribution. As a result, we get the electron and ion densities N_e and N_i, respectively to be:

$$N_e = N_i = n_0(r) \exp\left(\frac{e\Phi}{T_e}\right) \tag{1}$$

where $-e$ and T_e are electron charge and temperature, respectively. The drift modes can be described by the continuity equation

$$\frac{\partial}{\partial t} N_i + \nabla \cdot (N_i \boldsymbol{V}_i) = 0 \tag{2}$$

and the momentum equation

$$m_i\left(\frac{\partial}{\partial t} + \boldsymbol{V}_i \cdot \nabla\right)\boldsymbol{V}_i = e\left(-\nabla\Phi + \frac{1}{c}\boldsymbol{V}_i \times \boldsymbol{B}\right) \tag{3}$$

for cold ions. Here, m_i and $\boldsymbol{V}_i$ are the ion mass and fluid velocity, respectively. As it can been seen, we have only kept the adiabatic electron response and ignored any electron destabilizing, temperature gradient, or trapped electron effects.

In dealing with a toroidal coordinate system, we use simple toroidal orthogonal coordinates r, θ, φ, where r is the radius in the minor cross section of the torus, θ and φ are the poloidal and toroidal angles, respectively. For simplicity, we consider an axisymmetric tokamak, with large-aspect-ratio, $R/r \equiv 1/\varepsilon \gg 1$ (R being the major radius of the torus) and with circular and concentric magnetic surfaces. The equilibrium magnetic field can be expressed as

$$\boldsymbol{B} = B_{\rm p}(r)\hat{\boldsymbol{e}}_\theta + B_{\rm t}(1+\varepsilon\cos\theta)^{-1}\hat{\boldsymbol{e}}_\varphi. \quad (4)$$

Here, $B_{\rm p}$ and $B_{\rm t}$ are the poloidal and toroidal components of the equilibrium magnetic field, respectively, and the caret ($\hat{}$) denotes the corresponding unit vector.

Decomposing as usually, electrostatic potential, ion velocity and density into equilibrium (denoted by the subscript 0) and fluctuating parts, $\Phi = \Phi_0 + \phi$, $\boldsymbol{V}_{\rm i} = \boldsymbol{v_0} + \boldsymbol{v}$ and $N = n_0 + n_{\rm i}$, we obtain for the equilibrium rotation velocity, averaged over the poloidal angle θ, the following expression

$$\boldsymbol{v_0} = \left(\frac{c}{B_{\rm t}}\right)\hat{\boldsymbol{b}} \times \nabla\Phi_0. \quad (5)$$

Here, $B_{\rm t} = \text{const.}$ according to the usual tokamak approximation and $\hat{\boldsymbol{b}} = \boldsymbol{B}/B$. Since the ratio of the toroidal component of $\boldsymbol{v_0}$ to the poloidal one is of the order of ε, in what follows we neglect the effects of the toroidal plasma rotation. So, the equilibrium plasma rotation velocity (5) actually describes plasma drift in the radial electric field which depends on r only.

In the next order of perturbation theory, we obtain equations for the perturbed values. The relative size of the perturbed to the equilibrium quantities is of the order of $\varepsilon = r/R \simeq \omega/\omega_{\rm ci}$. Then, in the lowest order of ε, we get from Eq. (1) and (2)

$$\left(\frac{\partial}{\partial t} + \boldsymbol{v_0}\cdot\nabla\right)\frac{e\phi}{T_{\rm e}} + \nabla\cdot\boldsymbol{v} + \boldsymbol{v}\cdot\nabla\ln n_0 = 0. \quad (6)$$

The ion motion is assumed to be three-dimensional, i.e. an adequate model includes the ion dynamics parallel and perpendicular to the magnetic field. So, we can express the perturbed ion velocity in the form

$$\boldsymbol{v} = \boldsymbol{v}_\perp + \hat{\boldsymbol{b}}v_\parallel \quad (7)$$

with $\boldsymbol{v}_\perp = \hat{\boldsymbol{b}} \times (\boldsymbol{v} \times \hat{\boldsymbol{b}})$ and $v_\parallel = \hat{\boldsymbol{b}}\cdot\boldsymbol{v}$. Taking the dot and cross products of Eq. (3) with the unit vector $\hat{\boldsymbol{b}}$, we obtain the following equation for the parallel component

$$\frac{\partial v_\parallel}{\partial t} + \hat{\boldsymbol{b}}\cdot(\boldsymbol{v_0}\cdot\nabla)\boldsymbol{v} + \hat{\boldsymbol{b}}\cdot(\boldsymbol{v}\cdot\nabla)\boldsymbol{v_0} = -\frac{e}{m_{\rm i}}\hat{\boldsymbol{b}}\cdot\nabla\phi \quad (8)$$

and for the transverse component

$$\boldsymbol{v}_\perp = \boldsymbol{v_E} + \frac{\hat{\boldsymbol{b}}}{\omega_{\rm ci}} \times \left[\frac{\partial \boldsymbol{v_E}}{\partial t} + (\boldsymbol{v_0}\cdot\nabla)\boldsymbol{v_E} + (\boldsymbol{v_E}\cdot\nabla)\boldsymbol{v_0}\right] \quad (9)$$

of the perturbed velocity. Here, $\boldsymbol{v_E} = (c/B)\hat{\boldsymbol{b}} \times \nabla\phi$ is the $\boldsymbol{E}\times\boldsymbol{B}$ drift velocity. It should be noted, that deriving Eq. (9) the usual drift expansion was used.

Inserting Eq. (9) into Eqs (6) and (8) and assuming that the fluctuating quantities depend on time and toroidal angle as $\exp(-{\rm i}\omega t - {\rm i}n\varphi)$ (n being the toroidal wavenumber), we obtain the following coupled equations for the perturbations of parallel velocity $v_\parallel$ and electrostatic potential ϕ,

$$\left(\omega + \frac{{\rm i}v_0}{r}\frac{\partial}{\partial\theta}\right)v_\parallel + \frac{{\rm i}v_0}{R}\left(\cos\theta\frac{\partial}{\partial\theta} - \sin\theta\right)v_\parallel = -\frac{{\rm i}e}{m_{\rm i}qR}\left(\frac{\partial}{\partial\theta} - {\rm i}nq\right)\phi \quad (10)$$

and

$$\left(\omega + \frac{{\rm i}v_0}{r}\frac{\partial}{\partial\theta}\right)\left[\phi - \frac{c_{\rm s}^2}{\omega_{\rm ci}^2}\left(\frac{\partial^2\phi}{\partial r^2} + \frac{1}{r}\frac{\partial\phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\phi}{\partial\theta^2} + \frac{{\rm d}\ln n_0}{{\rm d}r}\frac{\partial\phi}{\partial r}\right)\right]$$
$$- \frac{{\rm i}c_s^2}{\omega_{\rm ci}r}\left[\frac{v_0}{r^2} - \frac{1}{r}\frac{{\rm d}v_0}{{\rm d}r} - \frac{{\rm d}^2v_0}{{\rm d}r^2} + \frac{{\rm d}\ln n_0}{{\rm d}r}\left(\omega_{\rm ci} - \frac{v_0}{r} - \frac{{\rm d}v_0}{{\rm d}r}\right)\right]\frac{\partial\phi}{\partial r}$$
$$- \frac{2{\rm i}c_{\rm s}^2}{\omega_{\rm ci}R}\left[\sin\theta\frac{\partial\phi}{\partial r} + \left(\frac{1}{r} + \frac{1}{2}\frac{{\rm d}\ln n_0}{{\rm d}r} - \frac{\omega_{\rm ci}v_0}{2c_{\rm s}^2}\right)\cos\theta\frac{\partial\phi}{\partial\theta}\right]$$
$$+ \frac{{\rm i}T_{\rm e}}{eqR}\left(\frac{\partial}{\partial\theta} - {\rm i}nq\right)v_\parallel = 0. \quad (11)$$

Here, $v_0(r) = (c/B_{\rm t}){\rm d}\Phi_0/{\rm d}r$ is the equilibrium poloidal rotation velocity, $q(r) = rB_{\rm t}/RB_{\rm p}(r)$ is the safety factor and $c_{\rm s}^2 = T_{\rm e}/m_{\rm i}$ is the ion sound velocity.

The drift modes are known to have long wavelengths along the magnetic field lines, but short wavelengths in the radial direction. Let us consider solutions of Eqs (10) and (11) for the mode localised on the rational surface $r = r_0$, defined by $m_0 - nq(r_0) = 0$, where m_0 is the poloidal mode number at the reference rational surface around which the mode is centred. It should be noted that $m_0 \gg 1$ for the drift modes of principal interest. The excitation of m_0 mode results, due to toroidal mode coupling, in a chain excitation of the modes with poloidal numbers $(m_0 \pm l)$, which are localised on the neighbouring rational surfaces. Supposing that the relationship of a magnetic surface to its neighbours is different for different surfaces due to strong inhomogeneity of equilibrium radial electric field and poloidal rotation velocity, we search solutions of Eq. (10) and (11) in form of the Fourier decomposition

$$\phi(r,\theta) = \exp({\rm i}m_0\theta)\sum_l \phi_l(r)\exp({\rm i}l\theta),$$
$$v_\parallel(r,\theta) = \exp({\rm i}m_0\theta)\sum_l v_l(r)\exp({\rm i}l\theta). \quad (12)$$

Substituting Eq. (12) into Eq. (10) and (11), we expand the coefficients that are functions of r into Taylor series in the vicinity of the reference rational surface, $r = r_0$. In order to keep the main terms only, we neglect corrections much smaller than $k_\theta^2\rho_s^2$ (where $k_\theta = m_0/r_0$ is the local poloidal wavenumber and $\rho_s = c_{\rm s}/\omega_{\rm ci}$ is the ion Larmour radius defined at the electron temperature) and the terms with the second derivative of the equilibrium density $n_0(r)$, since they are smaller in the conventional low-β tokamak ordering. As a result, we reduce Eqs (10) and (11) to the form

$$\omega' v_l - k_\theta V_0\frac{r_0}{2R}(v_{l+1} + v_{l-1}) + \frac{V_0}{2R}(v_{l+1} - v_{l-1}) = \frac{e}{m_{\rm i}q_0R}[l - k_\theta s(r - r_0)]\phi_l \quad (13)$$

and

$$\omega'\phi_l + \rho_s^2\left[k_\theta\left(\frac{V_0}{r_0^2} - \frac{V_0'}{r_0} - V_0''\right)\phi_l - \omega'\left(\frac{\partial^2\phi_l}{\partial r^2} + \frac{1}{r_0}\frac{\partial\phi_l}{\partial r} - k_\theta^2\phi_l\right)\right] - \frac{\rho_s^2}{r_n} \times \left[k_\theta\omega_{\rm ci}\phi_l + k_\theta\omega_{\rm ci}\frac{r_0}{2R}(\phi_{l+1} + \phi_{l-1}) - \omega'\frac{\partial\phi_l}{\partial r} - k_\theta\left(\frac{V_0}{r_0} + V_0'\right)\phi_l\right] + \frac{\rho_s c_{\rm s}}{R}\left[\frac{\partial}{\partial r}(\phi_{l+1} - \phi_{l-1}) + k_\theta(\phi_{l+1} + \phi_{l-1})\right] - k_\theta V_0\frac{r_0}{2R}(\phi_{l+1} + \phi_{l-1}) - \frac{T_{\rm e}}{eq_0R}[l - k_\theta s(r - r_0)]\, v_l = 0 \tag{14}$$

respectively. Here, $V_0 = v_0(r_0)$ is the local rotation velocity, $V_0' = ({\rm d}V_0/{\rm d}r)_{r=r_0}$, $V_0'' = ({\rm d}^2V_0/{\rm d}^2r)_{r=r_0}$, $q_0 = q(r_0)$, $\omega' = \omega - k_\theta V_0$ is the Doppler-shifted eigenfrequency, $r_n^{-1} = -({\rm d}\ln n_0/{\rm d}r)_{r=r_0}$ is the inverse scale of density inhomogeneity and $s = r_0q_0'/q_0$ is the magnetic shear with $q_0' = ({\rm d}q_0/{\rm d}r)_{r=r_0}$.

In what follows, we exclude from the consideration the limit case $\omega' = 0$ which corresponds to the ship waves [28]. Then, substituting Eq. (13) into Eq. (14) and neglecting the terms of the order of ε^2, we obtain

$$\rho_s^2\left[\frac{\partial^2\phi_l}{\partial r^2} + \left(\frac{1}{r_0} - \frac{1}{r_n}\right)\frac{\partial\phi_l}{\partial r}\right] + \left(\frac{c_{\rm s}}{q_0R\omega'}\right)^2[l - k_\theta s(r - r_0)]^2\phi_l - \left[1 + k_\theta^2\rho_s^2 + \frac{k_\theta\rho_s^2}{\omega'}\left(\frac{V_0}{r_0^2} - \frac{V_0'}{r_0} - V_0''\right) - \frac{k_\theta\rho_s^2}{\omega' r_n}\left(\omega_{\rm ci} - \frac{V_0}{r_0} - V_0'\right)\right]\phi_l + \frac{r_0}{2\omega' R} \times \left(k_\theta V_0 - \frac{k_\theta\rho_s c_{\rm s}}{r_0} + \frac{k_\theta\rho_s^2\omega_{\rm ci}}{r_n}\right)(\phi_{l+1} + \phi_{l-1}) - \frac{\rho_s c_{\rm s}}{\omega' R}\frac{\partial}{\partial r}(\phi_{l+1} - \phi_{l-1}) = 0. \tag{15}$$

Introducing a new potential function $U_l(r)$, through

$$\phi_l(r) = U_l(r)\exp\left[-\frac{r - r_0}{2}\left(\frac{1}{r_0} - \frac{1}{r_n}\right)\right] \tag{16}$$

and the dimensionless variable $x = (r - r_0)/\rho_s$, we reduce Eq. (15) to the form

$$\frac{\partial^2 U_l}{\partial x^2} - DU_l + \sigma^2\left(x - \frac{l}{k_\theta\rho_s s}\right)^2 U_l = \frac{c_{\rm s}k_\theta\rho_s}{\omega' R}\left(1 - \frac{r_0}{2r_n} - \frac{r_0V_0}{2c_{\rm s}\rho_s}\right)(U_{l+1} + U_{l-1}) + \frac{c_{\rm s}}{\omega' R}\frac{\partial}{\partial x}(U_{l+1} - U_{l-1}) \tag{17}$$

Here,

$$D = 1 + k_\theta^2\rho_s^2 - \frac{c_{\rm s}k_\theta\rho_s}{\omega' r_n}\left[1 - \frac{\rho_s}{r_0}\frac{V_0}{c_{\rm s}}(1 + \xi)\right] + \frac{\rho_s^2}{r_0^2}\frac{k_\theta V_0}{\omega'}(1 - \xi - \xi\xi') \tag{18}$$

and

$$\sigma = \left|\frac{c_{\rm s}k_\theta\rho_s s}{\omega' q_0 R}\right| \tag{19}$$

with $\xi = r_0V_0'/V_0$ and $\xi' = r_0V_0''/V_0'$.

Equation (17) is an extension of Eq. (7) by Horton *et al.*[10] for the case of poloidally rotating plasma. It is the basic mode equation which describes the mode structure and dispersion properties of a drift wave in a rotating tokamak plasma.

3. Weak Coupling Approximation

Equation (17) identifies an infinite system of equations for harmonics U_l with $l = 0, \pm 1, \pm 2, \ldots$. In the weak coupling approximation, we truncate this system keeping only the harmonic $l = 0$, which corresponds to the reference rational surface around which the mode is centred and the two nearest harmonics $l = \pm 1$. Such approximation is justified, if the Doppler-shifted eigenfrequency ω' is of the order of the local drift frequency

$$\omega_* = \frac{\rho_s}{r_n}\frac{k_\theta c_{\rm s}}{1 + (k_\theta\rho_s)^2} \tag{20}$$

since in this case the ratio of the terms of the right-hand side of Eq. (17) to the left-hand side ones is of the order of ε.

Thus, the system of equations (17) is reduced to the equation for the reference harmonic $l = 0$

$$\frac{\partial^2 U_0}{\partial x^2} - DU_0 + \sigma^2x^2U_0 = \frac{c_{\rm s}k_\theta\rho_s}{\omega' R}\left(1 - \frac{r_0}{2r_n} - \frac{r_0V_0}{2c_{\rm s}\rho_s}\right) \times (U_{+1} + U_{-1}) + \frac{c_{\rm s}}{\omega' R}\frac{\partial}{\partial x}(U_{+1} - U_{-1}) \tag{21}$$

and to the following equations for the nearest harmonics $l = \pm 1$

$$\frac{\partial^2 U_{\pm 1}}{\partial x^2} - DU_{\pm 1} + \sigma^2\left(x \mp \frac{1}{k_\theta\rho_s s}\right)^2 U_{\pm 1} = \frac{c_{\rm s}k_\theta\rho_s}{\omega' R}\left(1 - \frac{r_0}{2r_n} - \frac{r_0V_0}{2c_{\rm s}\rho_s}\right)U_0 \mp \frac{c_{\rm s}}{\omega' R}\frac{\partial U_0}{\partial x}. \tag{22}$$

Since the right-hand side of Eq. (22) contains the smallness parameter $\varepsilon \ll 1$, we can solve this equation by the successive approximation method. Changing the variable to

$$y = \sqrt{2\sigma}\left(x \mp \frac{1}{k_\theta\rho_s s}\right) \tag{23}$$

we reduce the equation for zero-order functions $U_{\pm 1}^{(0)}$ to a standard form of the Weber equation

$$\frac{\partial^2 U_{\pm 1}^{(0)}}{\partial y^2} - \left(\alpha - \frac{1}{4}y^2\right)U_{\pm 1}^{(0)} = 0 \tag{24}$$

with $\alpha = \frac{D}{2\sigma}$.

The Weber equation (24) has the following even and odd solutions [29]

$$\left(U_{\pm 1}^{(0)}\right)_{\text{even}} = \exp(-\frac{\mathrm{i}}{4}y^2)\Big[1+\left(a+\frac{1}{2}\mathrm{i}\right)\frac{y^2}{2!}+\left(a+\frac{1}{2}\mathrm{i}\right)\times\left(a+\frac{5}{2}\mathrm{i}\right)\frac{y^4}{4!}+...\Big],$$
$$\left(U_{\pm 1}^{(0)}\right)_{\text{odd}} = \exp(-\frac{\mathrm{i}}{4}y^2)\Big[1+\left(a+\frac{3}{2}\mathrm{i}\right)\frac{y^3}{3!}+\left(a+\frac{3}{2}\mathrm{i}\right)\times\left(a+\frac{7}{2}\mathrm{i}\right)\frac{y^5}{5!}+...\Big], \tag{25}$$

convergent for all values of y. Differentiating only the most rapidly changing term $\exp(-\frac{\mathrm{i}}{4}y^2)$, we obtain an estimation for the second derivative

$$\frac{\mathrm{d}^2 U_{\pm 1}^{(0)}}{\mathrm{d}y^2} \simeq \frac{-y^2}{4} U_{\pm 1}^{(0)}. \tag{26}$$

Substituting this estimation instead of $\partial^2 U_{\pm 1}/\partial x^2$ into Eq. (22), we find

$$DU_{\pm 1} \simeq -\frac{c_{\mathrm{s}} k_\theta \rho_s}{\omega' R}\left(1-\frac{r_0}{2r_n}-\frac{r_0 V_0}{2c_{\mathrm{s}}\rho_s}\right)U_0 \pm \frac{c_{\mathrm{s}}}{\omega' R}\frac{\partial U_0}{\partial x}. \tag{27}$$

Multiplying Eq. (21) by D and inserting the approximated expressions (27) for the functions $U_{\pm 1}$, we obtain the following equation for the function U_0

$$\frac{\partial^2 U_0}{\partial x^2} + \left(\Lambda - \zeta x^2\right)U_0 = 0 \tag{28}$$

with

$$\Lambda = \frac{D^2 - 2\left(\frac{c_{\mathrm{s}} k_\theta \rho_s}{\omega' R}\right)^2\left(1-\frac{r_0}{2r_n}-\frac{r_0 V_0}{2c_{\mathrm{s}}\rho_s}\right)^2}{2\left(\frac{c_{\mathrm{s}}}{\omega' R}\right)^2 - D} \tag{29}$$

and

$$\zeta = \frac{\sigma^2 D}{2\left(\frac{c_{\mathrm{s}}}{\omega' R}\right)^2 - D}. \tag{30}$$

The eigenfunctions of Eq. (28) describe the radial structure of the drift eigenmode, localised at the reference rational surface and the eigenvalues of this equation define the dispersion properties of excited drift eigenmodes. The solutions of Eq. (28) belong to two classes depending on the sign of the parameter ζ.

4. Global drift mode solutions

In the case

$$\zeta > 0 \tag{31}$$

using the new variable $y = \zeta^{\frac{1}{4}}x$, we transform Eq. (28) to a form of the Hermite equation

$$\frac{\partial^2 U_0}{\partial y^2} + \left(\frac{\Lambda}{\sqrt{\zeta}} - y^2\right)U_0 = 0. \tag{32}$$

The eigenfunctions of Eq. (32) are given through the Hermite polynomials $\exp(-y^2/2)H_N(y)$ and the corresponding eigenvalues are defined by the dispersion equation

$$\frac{\Lambda}{\sqrt{\zeta}} = 2N+1, \qquad N = 0, 1, 2... \tag{33}$$

It can be easily seen from Eq. (33), that positive eigenvalues exist only when

$$\Lambda > 0. \tag{34}$$

This class of solutions describes non-propagating modes localised inside a "potential well", i.e. the global modes. In order to simplify the apparent form of the dispersion equation (33), we use the following assumptions

$$\frac{r_0}{2r_n} \gg 1, \quad \frac{\rho_s^2}{r_0^2} \ll 1, \quad V_0 < c_{\mathrm{s}}, \quad \frac{k_\theta V_0}{\omega'} \ll \frac{r_0^2}{\rho_s^2} \tag{35}$$

which may be considered as typical for the edge plasma region. Then, the terms of Eq. (18), which contain the first and second derivatives of poloidal rotation velocity (velocity shear) are negligibly small. In other words, the poloidal rotation velocity shear does not effect essentially the dispersion properties of global drift modes in this approximation and under these assumptions.

Substituting the apparent expressions (29), (30), (18) and (19) into Eq. (33) and taking the above inequalities into account, we obtain the analytical dispersion equation for the global drift modes in rotating tokamak plasma

$$\left(\frac{R}{r_n}\right)^2\left(\frac{\omega'}{\omega_*}-1\right)^2 - \frac{1}{2}\left(\frac{r_0}{r_n}+\frac{r_0 V_0}{c_{\mathrm{s}}\rho_s}\right)^2 = (2N+1)\left\{\frac{2s^2}{q_0^2}\frac{1+k_\theta^2\rho_s^2}{k_\theta^2\rho_s^2}\left(1-\frac{\omega_*}{\omega'}\right)\times\left[1-\frac{1}{2}\frac{k_\theta^2\rho_s^2}{1+k_\theta^2\rho_s^2}\left(\frac{R}{r_n}\right)^2\left(\frac{\omega'}{\omega_*}\right)^2\left(1-\frac{\omega_*}{\omega'}\right)\right]\right\}^{1/2}. \tag{36}$$

4.1. *Positive Doppler-shifted eigenfrequency, $\omega' > 0$*

In this case the conditions (31) and (34) are satisfied simultaneously, when

$$1+\frac{r_0}{\sqrt{2}R}\left(1+\frac{r_n V_0}{\rho_s c_{\mathrm{s}}}\right) < \frac{\omega'}{\omega_*} < \frac{1}{2}+\frac{1}{2}\left[1+8\frac{1+k_\theta^2\rho_s^2}{k_\theta^2\rho_s^2}\left(\frac{r_n}{R}\right)^2\right]^{1/2}. \tag{37}$$

If, for example

$$\frac{r_n V_0}{\rho_s c_{\mathrm{s}}} \ll 1 \quad \text{and} \quad 8\frac{1+k_\theta^2\rho_s^2}{k_\theta^2\rho_s^2}\left(\frac{r_n}{R}\right)^2 \ll 1 \tag{38}$$

the conditions (37) and (38) are reduced to the inequalities

$$8 \ll \left(\frac{R}{r_n}\right)^2\left(k_\theta\rho_s\right)^2 < 2^{3/2}\frac{R}{r_0} \tag{39}$$

which lead to the limit

$$\frac{\omega'}{\omega_*} - 1 \ll 1. \tag{40}$$

Such modes can exist at very low rotation velocities

$$\frac{V_0}{c_{\mathrm{s}}} \ll \frac{\rho_s}{r_n} \ll 1 \tag{41}$$

and in this case the dispersion equation (36) takes the form

$$\left(\frac{R}{r_n}\right)^2\left(\frac{\omega'}{\omega_*}-1\right)^2-\frac{1}{2}\left(\frac{r_0}{r_n}\right)^2= (2N+1)\left\{\frac{2s^2}{q_0^2}\frac{1}{k_\theta^2\rho_s^2}\left(1-\frac{\omega_*}{\omega'}\right)\times\left[1-\frac{1}{2}k_\theta^2\rho_s^2\left(\frac{R}{r_n}\right)^2\left(\frac{\omega'}{\omega_*}\right)^2\left(1-\frac{\omega_*}{\omega'}\right)\right]\right\}^{1/2}. \tag{42}$$

From the inequalities (37) and (38), it follows that

$$\frac{r_0}{\sqrt{2}R}<\frac{\omega'}{\omega_*}-1<\frac{2}{k_\theta^2\rho_s^2}\left(\frac{r_n}{R}\right)^2 \quad \text{and} \quad (k_\theta\rho_s)_{\min}\gg 2^{3/2}\frac{r_n}{R}. \tag{43}$$

Using these estimations, we can express the condition of smallness of the righ-hand side of Eq. (42) compared to the second term of its left hand side as

$$2N+1<\frac{8q_0}{s}\frac{r_0}{r_n}\frac{r_0}{R}. \tag{44}$$

Then Eq. (42) can be simplified to the following dispersion relation

$$\frac{\omega'}{\omega_*}\simeq 1+\frac{r_0}{\sqrt{2}R}\times\left[1+(2N+1)\frac{s\sqrt{2}}{q_0}\left(\frac{r_n}{r_0}\right)^2\left(\frac{1}{\sqrt{2}k_\theta^2\rho_s^2}\frac{r_0}{R}-\frac{1}{4}\frac{r_0^2}{r_n^2}\right)^{1/2}\right]. \tag{45}$$

4.2. *Negative Doppler-shifted eigenfrequency, $\omega'<0$*

In this case, conditions (31) and (34) are satisfied simultaneously, when

$$0<\frac{|\omega'|}{\omega_*}<-\frac{1}{2}+\frac{1}{2}\left[1+8\frac{1+k_\theta^2\rho_s^2}{k_\theta^2\rho_s^2}\left(\frac{r_n}{R}\right)^2\right]^{1/2} \tag{46}$$

and the dispersion equation (47) takes the form

$$\left(\frac{R}{r_n}\right)^2\left(\frac{|\omega'|}{\omega_*}+1\right)^2-\frac{1}{2}\left(\frac{r_0}{r_n}+\frac{r_0V_0}{c_s\rho_s}\right)^2= (2N+1)\frac{\sqrt{2}s}{q_0}\left[\frac{1+k_\theta^2\rho_s^2}{k_\theta^2\rho_s^2}\frac{\omega_*}{|\omega'|}\left(\frac{|\omega'|}{\omega_*}+1\right)-\frac{1}{2}\left(\frac{R}{r_n}\right)^2\left(\frac{|\omega'|}{\omega_*}+1\right)^2\right]^{1/2}. \tag{47}$$

If we consider that

$$8\frac{1+k_\theta^2\rho_s^2}{k_\theta^2\rho_s^2}\left(\frac{r_n}{R}\right)^2<<1 \tag{48}$$

then we have, according to Eq. (46)

$$\frac{|\omega'|}{\omega_*}<2\frac{1+k_\theta^2\rho_s^2}{k_\theta^2\rho_s^2}\left(\frac{r_n}{R}\right)^2<<1 \tag{49}$$

and the approximate solution of Eq. (47) is given by

$$\frac{|\omega'|}{\omega_*}\simeq 2\frac{1+k_\theta^2\rho_s^2}{k_\theta^2\rho_s^2}\times\left\{\left(\frac{R}{r_n}\right)^2+\frac{1}{(2N+1)^2}\frac{q_0^2}{s^2}\left[\left(\frac{R}{r_n}\right)^2-\frac{1}{2}\left(\frac{r_0}{r_n}+\frac{r_0V_0}{c_s\rho_s}\right)^2\right]^2\right\}^{-1}. \tag{50}$$

One can easy check that the solution (50) satisfies the inequalities (49). On the other hand if

$$8\frac{1+k_\theta^2\rho_s^2}{k_\theta^2\rho_s^2}\left(\frac{r_n}{R}\right)^2>>1 \tag{51}$$

the inequalities (46) are transformed to

$$0<\frac{|\omega'|}{\omega_*}<\frac{r_n}{R}\left(2\frac{1+k_\theta^2\rho_s^2}{k_\theta^2\rho_s^2}\right)^{1/2}-\frac{1}{2} \tag{52}$$

and Eq. (47) can be simplified to the following dispersion relation

$$\left(\frac{|\omega'|}{\omega_*}\right)^2\simeq(2N+1)\frac{\sqrt{2}s}{q_0}\left(\frac{r_n}{R}\right)^2\times\left[\left(\frac{1+k_\theta^2\rho_s^2}{k_\theta^2\rho_s^2}\right)^{\frac{1}{2}}+\frac{(2N+1)^2}{16}\frac{s^2}{q_0^2}\left(\frac{k_\theta^2\rho_s^2}{1+k_\theta^2\rho^2}\right)^{\frac{1}{2}}-\frac{2N+1}{2^{\frac{3}{2}}}\frac{s_0}{q_0}\right] \tag{53}$$

in the supposition $|\omega'|\gg\omega_*$.

5. Propagating mode solutions

In the case

$$\zeta<0 \tag{54}$$

we transform Eq. (28) to a form of the Weber equation

$$\frac{\partial^2U_0}{\partial y^2}+\left(\frac{\Lambda}{i\sqrt{|\zeta|}}-y^2\right)U_0=0 \tag{55}$$

with $y=\sqrt{i|\zeta|^{\frac{1}{2}}}x$.
Lebedev [30] shows that the solutions of Eq. (55) are bound in the whole interval of y, only if $N=0,1,2,..$, where N is defined by the dispersion equation

$$\frac{\Lambda}{i\sqrt{|\zeta|}}=2N+1. \tag{56}$$

These solutions have the form

$$U_{0N}(y)=M_NH_N(y)\exp(-y^2/2) \tag{57}$$

where M_N is an arbitrary constant and $H_\nu(z)$ is the Hermite function that is an entire function of the complex variable z and parameter ν. For the case $\nu=N$ the Hermite function H_N coincides with the Hermite polynomial of the N^{th} order. The boundness of the solution (57) on the whole y interval corresponds to the outgoing boundary condition introduced by Perlstein and Berk [5].

5.1. *Positive Doppler-shifted eigenfrequency, $\omega' > 0$*

In this case, inequality (54) is satisfied if either

$$\frac{\omega'}{\omega_*} > \frac{1}{2} + \frac{1}{2}\left[1 + 8\frac{1 + k_\theta^2\rho_s^2}{k_\theta^2\rho_s^2}\left(\frac{r_n}{R}\right)^2\right]^{1/2} \tag{58}$$

or

$$0 < \frac{\omega'}{\omega_*} < 1. \tag{59}$$

The apparent form of the dispersion equation (56) is

$$\left(\frac{R}{r_n}\right)^2\left(\frac{\omega'}{\omega_*} - 1\right)^2 - \frac{1}{2}\left(\frac{r_0}{r_n} + \frac{r_0 V_0}{c_s\rho_s}\right)^2 = -\mathrm{i}(2N+1)\frac{s}{q_0} \times \left[\left(\frac{R}{r_n}\right)^2\left(\frac{\omega'}{\omega_*} - 1\right)^2 - 2\frac{1 + k_\theta^2\rho_s^2}{k_\theta^2\rho_s^2}\frac{\omega_*}{\omega'}\left(\frac{\omega'}{\omega_*} - 1\right)\right]^{1/2} \tag{60}$$

where the assumptions (35) have been taken into account.

For the region (58) a simple dispersion can be obtained in the limit

$$2\frac{1 + k_\theta^2\rho_s^2}{k_\theta^2\rho_s^2}\left(\frac{r_n}{R}\right)^2 \ll \frac{\omega'}{\omega_*} - 1. \tag{61}$$

The dispersion relation in this case has the form

$$\frac{\omega'}{\omega_*} \simeq 1 + \frac{r_n}{R}\left[\frac{1}{2}\left(\frac{r_0}{r_n} + \frac{r_0 V_0}{c_s\rho_s}\right)^2 - \left(N + \frac{1}{2}\right)^2\frac{s^2}{q_0^2}\right]^{1/2} - \mathrm{i}\left(N + \frac{1}{2}\right)\frac{s}{q_0}\frac{r_n}{R}. \tag{62}$$

Here Im $\omega' < 0$, that corresponds to the damping of the drift wave.

For the region (59) we can also simplify Eq. (60) to the following dispersion relation

$$\frac{\omega'}{\omega_*} \simeq 1 - \frac{r_n}{R}\left[\frac{1}{2}\left(\frac{r_0}{r_n} + \frac{r_0 V_0}{2c_s\rho_s}\right)^2 - \left(N + \frac{1}{2}\right)^2\frac{s^2}{q_0^2}\right]^{1/2} - \mathrm{i}\left(N + \frac{1}{2}\right)\frac{s}{q_0}\frac{r_n}{R} \tag{63}$$

for the limit case

$$2\frac{1 + k_\theta^2\rho_s^2}{k_\theta^2\rho_s^2}\left(\frac{r_n}{R}\right)^2 \ll \frac{\omega'}{\omega_*} < 1. \tag{64}$$

The dispersion relation (63) also describes weakly damping drift waves.

5.2. *Negative Doppler-shifted eigenfrequency, $\omega' < 0$*

In this case inequality (54) gives

$$\frac{|\omega'|}{\omega_*} > -\frac{1}{2} + \frac{1}{2}\left[1 + 8\frac{1 + k_\theta^2\rho_s^2}{k_\theta^2\rho_s^2}\left(\frac{r_n}{R}\right)^2\right]^{1/2} \tag{65}$$

and the dispersion equation (56) takes the form

$$\left(\frac{R}{r_n}\right)^2\left(\frac{|\omega'|}{\omega_*} + 1\right)^2 - \frac{1}{2}\left(\frac{r_0}{r_n} + \frac{r_0 V_0}{c_s\rho_s}\right)^2 = -\mathrm{i}(2N+1)\frac{s}{q_0} \times \left[\left(\frac{R}{r_n}\right)^2\left(\frac{|\omega'|}{\omega_*} + 1\right)^2 - 2\frac{1 + k_\theta^2\rho_s^2}{k_\theta^2\rho_s^2}\frac{\omega_*}{|\omega'|}\left(\frac{|\omega'|}{\omega_*} + 1\right)\right]^{1/2}. \tag{66}$$

In the limit case

$$2\frac{1 + k_\theta^2\rho_s^2}{k_\theta^2\rho_s^2}\left(\frac{r_n}{R}\right)^2 \ll \frac{|\omega'|}{\omega_*} < 1 \tag{67}$$

the solution of Eq. (56) can be approached by

$$\frac{|\omega'|}{\omega_*} \simeq \frac{r_n}{R}\left[\frac{1}{2}\left(\frac{r_0}{r_n} + \frac{r_0 V_0}{c_s\rho_s}\right)^2 - \left(N + \frac{1}{2}\right)^2\frac{s^2}{q_0^2}\right]^{1/2} - 1 - \mathrm{i}\left(N + \frac{1}{2}\right)\frac{s}{q_0}\frac{r_n}{R}. \tag{68}$$

Such solution may exist only if

$$\frac{r_n}{R}\frac{r_0}{\rho_s}\frac{V_0}{c_s} \gg 1. \tag{69}$$

The inequalities (67) and (69) are satisfied simultaneously for very high values of m_0. In the limit

$$|\omega'| \gg \omega_* \tag{70}$$

the dispersion equation (66) leads to the relation

$$\frac{|\omega'|}{\omega_*} \simeq \frac{r_n}{R}\left[\frac{1}{2}\left(\frac{r_0}{r_n} + \frac{r_0 V_0}{c_s\rho_s}\right)^2 - \left(N + \frac{1}{2}\right)^2\frac{s^2}{q_0^2}\right]^{1/2} - \mathrm{i}\left(N + \frac{1}{2}\right)\frac{s}{q_0}\frac{r_n}{R} \tag{71}$$

with the inequality (69) taken into account.

It should be noted here that all dispersion relations obtained above correspond to damping drift waves, i.e. their shear stabilization. The propagating mode damping due to magnetic field shear, is found to be the same for all modes independently of the eigenfrequency sign and value and the assumptions that we consider in each limit case. Thus, it seems that the linear damping of the propagating drift modes depends on the magnetic shear and inhomogeneity scale, but not on the velocity shear.

6. Conclusions

In summary, the effects of poloidal plasma rotation on a drift eigenmodes in a tokamak plasma have been studied. We assume that the plasma rotation is driven by the radial electrostatic field typical for the H mode of plasma confinement in tokamaks. The low-frequency potential plasma oscillations are investigated using assumptions of adiabatic electrons and plasma quasineutrality. We use a simple model of low-β tokamak plasma with concentric, circular magnetic surfaces and large-aspect-ratio. In order to truncate an infinite set of equations obtained by the above assumptions, a weak coupling approximation is used. We believe that this approximation is more suitable to describe drift oscillations in a plasma edge region, where the radial electric field and the plasma rotation velocity (and their radial gradients) reach high values.

Making use of the weak coupling approximation, we reduce the basic set of equations to an ordinary differential equation of the Weber type. The equation has two classes of solutions depending on the sign of the Doppler-shifted frequency and the value of its ratio to the diamagnetic drift frequency. These two classes are the global drift wave modes and propagating (or outgoing) drift waves.

The global drift mode has a structure of a quasimode localised in radial direction with a small wavenumber along the confining magnetic field. It includes a number of rational magnetic surfaces due to toroidal coupling of the modes localised on the neighbouring magnetic surfaces. This mode corresponds to the bound state in a potential well which is marginally stable.

The propagating drift waves correspond to unbound states and leave the magnetic surface on which they are excited. These waves are characterised by a damping which depends on the magnetic field shear and plasma density inhomogeneity scale, but not on the poloidal rotation velocity shear.

We obtain the analytical dispersion equations for both global and propagating drift waves and define the regions of plasma parameters, where each class of solutions exists. Simple dispersion relations are found for some limiting cases.

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