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Drift Eigenmodes in Rotating Tokamak Plasma

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Abstract

The effects of sheared plasma rotation on the toroidal drift eigenmodes are studied for an axisymmetric, large aspect-ratio tokamak with concentric, circular magnetic surfaces. The performed analysis is based on a consistent derivation of the drift eigenmode equations, which are coupled in the poloidal mode numbers due to toroidicity. Analytical dispersion relations for both global and propagating drift waves are obtained and their approximative solutions are found. It is shown that the type of the mode is defined by the sign of the Doppler-shifted eigenfrequency that varies from one magnetic surface to another due to sheared poloidal and toroidal plasma rotation velocities. The global drift mode existence is attributed to the case of the negative Doppler-shifted eigenfrequencies, whereas the propagating drift waves are generated by the positive ones.

1. Introduction

An important issue for high-temperature plasma confinement in toroidal magnetic configurations is to understand and to determine the cause of cross-field transport. It is generally believed that under normal operating conditions, the plasma turbulence is almost always driven by microinstabilities associated with low frequency drift modes. For static equilibria, these microinstabilities driven by the expansion of free energy associated with the density and the temperature gradients of the confined plasma, have been reviewed by Tang [1] and Liewer [2].

It was shown that for the most interesting applications (plane plasma slab with strongly sheared magnetic field, tokamak) the local approximation for the description of drift waves is not valid. Thus, in order to determine the radial structure and stability of the drift modes it is necessary to solve the eigenvalue problem [3].

In absence of toroidal effects [4,5] the collisionless drift modes are always stable. However, as Taylor [6] has pointed out, in a realistic system with non-uniform magnetic field strength and shear, the waves associated with different surfaces are coupled to each other. This coupling changes the characteristics of the drift waves. In the tokamak geometry, drift modes are coupled to each other due to toroidicity. Such coupling effects can form local potential wells which reduce or eliminate the outward convection of wave energy and hence, the shear damping [7]. The ion toroidal drifts cause the shear stabilizing radial anti-well to become radial well, localizing the mode away from the regions of ion Landau damping [8]. In such a well, marginally stable quasi-modes exist for $s < 1/2$, where s is the magnetic shear parameter. These are the so-called global modes which are localized in both radial and poloidal directions and correspond to bound states in the potential well. For $s > 1/2$ the results [6,8] predict that the shear damping is further enhanced by the toroidal coupling, due to the anti-well shape of the coupling induced potential.

Experimental and theoretical investigations during the last decade have revealed the importance of plasma rotation in the confinement of tokamak plasma. Both poloidal and toroidal plasma rotations have been observed in various tokamaks [9–19]. Usually the poloidal plasma rotation is associated with the $\boldsymbol{E} \times \boldsymbol{B}$ drift, induced by a strong inhomogeneous radial electric field E_r whereas the toroidal plasma rotation is attributed to external sources.

Obviously, both poloidal and toroidal plasma rotations introduce a Doppler shift into the eigenfrequency of the drift wave that varies from one magnetic surface to another due to sheared plasma rotation velocity $\boldsymbol{v}_0(r)$. Therefore, the conventional ballooning representation is no longer appropriate, since the necessary equivalence [20] of the normal modes localized on neighboring magnetic surfaces is now violated by the sheared rotation [21].

The problem of drift eigenmodes in poloidally rotating tokamak plasma has recently drawn much attention [22–25]. Here the plasma rotation was incorporated into the toroidal coupling term of the model equation and it was shown that the instability may be stabilized by the velocity curvature, in both collision and collisionless limits. However, the introduction of the sheared plasma rotation into the final model equation looks somewhat artificial due to several approximations in the derivation of this model equation. A consistent way to study the sheared rotation effects on the drift eigenmodes should introduce the inclusion of the sheared plasma flow from the starting equations. Therefore, the effects of radially varying poloidal flow on the collisionless drift eigenmodes in tokamak plasma have been revised in Refs. [26,27]. The analysis was based on a consistently derived eigenmode equation coupled in poloidal mode numbers through toroidal effects, in the strong and weak coupling approximations respectively. In order to avoid the difficulties with the employment of the ballooning representation for sheared rotating plasmas, the treatment by Horton *et al.* [8,28,29] was used which follows the method by Taylor [6] but employs a Fourier decomposition for the eigenmodes instead of the "Fourier ballooning" representation used by Taylor and Wilson [21] and earlier by Hastie *et al.* [30]. It was shown that both global drift modes localized in the corresponding potential well and propagating drift waves

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outgoing from rational magnetic surfaces may exist in rotating tokamak plasmas depending on the plasma parameters and the poloidal rotation velocity. In particular, for sufficient high rotation velocity, global drift modes can exist even for large values of the magnetic shear $s > 1/2$, in contrast to the criterion mentioned above for non-rotating plasmas. On the contrary, the propagating drift waves move freely experiencing the magnetic shear damping.

At the present paper we study the effects of both poloidal and toroidal sheared plasma rotation on the toroidal drift eigenmodes in tokamaks. In what follows, we restrict ourselves to the collisionless or "universal" drift modes [31]. We assume the presence of both toroidal and poloidal velocity components attributed for example to the NBI lines aimed at the plasma. The two fluid model is used and the performed analysis is based on a consistent derivation of the eigenmode equations for collisionless drift oscillations in axisymmetric, large aspect-ratio tokamak with concentric, circular magnetic surfaces, where the toroidal coupling effects appear due to ion ∇B and curvature drifts. The dispersion relations for the drift-type eigenmodes are obtained using both the strong and the weak coupling approximations. The former assumes that a large number of poloidal harmonics are coupled by the equilibrium toroidal variation, whereas the latter is appropriate, when a given harmonic is coupled only with its nearest neighbors.

This paper is organized as follows: The theoretical model and the basic set of differential–difference equations which describe drift eigenmodes in rotating tokamak plasma are given in Section 2. These eigenmode equations are reduced to a one-dimensional equation by employing the strong and the weak coupling approximations in Sections 3 and 4 respectively, where solutions of both global and propagating drift eigenmodes are found. Concluding remarks follow in Section 5.

2. Basic equations

Let us consider collisionless electrostatic drift waves in a rotating toroidal plasma. The plasma is confined by an inhomogeneous magnetic field $\boldsymbol{B}$ with vanishing component along the equilibrium density gradient, and rotates in both toroidal and poloidal directions. This situation is typical for high confinement modes in most of the modern tokamaks with supplementary heating.

To model a large aspect-ratio tokamak with circular, concentric magnetic surfaces, we use the toroidal orthogonal coordinate system r, φ, θ, where r is the radius in the minor cross section of the torus, and θ, φ are the poloidal and toroidal angles respectively with the inverse aspect ratio as a characteristic small parameter, $\varepsilon(r) \equiv r/R \ll 1$ (R being the major radius of the torus). In this approximation, the equilibrium magnetic field takes the form

$$\boldsymbol{B} = B_\theta(r)\hat{\theta} + B_\varphi(1 + \varepsilon\cos\theta)^{-1}\,\hat{\varphi}$$

where, $B_\theta(r)$ is the poloidal component, $B_\varphi = \text{const}$ in the toroidal one, and the caret ($\hat{}$) denotes the corresponding unit vector.

Furthermore, we assume that the radially varying equilibrium rotation velocity for the ion fluid has sheared poloidal and toroidal components, $\boldsymbol{v}_0(r) = \boldsymbol{v}_{0\theta}(r) + \boldsymbol{v}_{0\varphi}(r)$ that can be attributed to various sources of rotation such as to Neutral Beam Injection. In our previous considerations [26,27], the poloidal plasma rotation was assumed to be the result of an arbitrary external steady state potential and the possibility of toroidal rotation was ignored. To describe the key features of collisionless electrostatic drift waves in a low-β plasma (the ratio of plasma pressure to the magnetic field one) we use the two fluid plasma model.

The analysis of linearised electron fluid equations shows that as long as electrons are freely move along the magnetic field lines and cancel space charge, the Boltzmann relation is fulfilled. There are however some effects (e.g., electron–ion collisions, Landau damping, electron inertia or inductance) that can limit electrons mobility. In that case, a phase shift δ between the perturbed density n_e and the potential Φ can be incorporated in order to represent the various electron dissipative mechanism effects [32]

$$\frac{n_e}{n_0} = \frac{e\Phi}{T_e}(1 - \mathrm{i}\delta)$$

where n_0 is the equilibrium plasma density. Using the quasineutrality condition $n_e = n_i$, we reduce the set of the linearised electron and ion fluid equations to the ion momentum and ion continuity equations. The ion velocity and electrostatic potential can be divided into equilibrium and fluctuating parts, assuming that the relative size of the spatial and temporal scales of the fluctuating parts to the equilibrium ones is of the order of $\varepsilon \sim \omega/\omega_{ci} \ll 1$, where ω is the oscillation frequency and ω_{ci} is the ion gyro-frequency. It is convenient to subdivide the ion fluid velocity into parts perpendicular and parallel to the magnetic field, $\boldsymbol{v} = \boldsymbol{v}_\perp + \boldsymbol{b}v_\parallel$, with $\boldsymbol{v}_\perp = \boldsymbol{b} \times (\boldsymbol{v} \times \boldsymbol{b})$, $v_\parallel = \boldsymbol{b} \cdot \boldsymbol{v}$ and $\boldsymbol{b} = \boldsymbol{B}/B$. In the low-frequency limit $\omega/\omega_{ci} \ll 1$, the ion polarization drift can be consider as a correction to the electric drift $\boldsymbol{v}_E$. This is the usual drift ordering and allows us to express the perpendicular component of the ion velocity by a perturbation expansion,

$$\boldsymbol{v}_\perp \simeq \boldsymbol{v}_E + \frac{\boldsymbol{b}}{\omega_{ci}} \times \left[\frac{\partial \boldsymbol{v}_E}{\partial t} + (\boldsymbol{v}_0 \cdot \nabla)\boldsymbol{v}_E + (\boldsymbol{v}_E \cdot \nabla)\boldsymbol{v}_0\right] \tag{1}$$

where

$$\boldsymbol{v}_E = \frac{c}{B}\boldsymbol{b} \times \nabla\Phi.$$

Similar, we obtain for the parallel component of the ion fluid velocity the following equation,

$$\frac{\partial v_\parallel}{\partial t} + \boldsymbol{b} \cdot (\boldsymbol{v}_0 \cdot \nabla)\boldsymbol{v} + \boldsymbol{b} \cdot (\boldsymbol{v} \cdot \nabla)\boldsymbol{v}_0 = -\frac{e}{m_i}\boldsymbol{b} \cdot \nabla\Phi \tag{2}$$

where m_i is the ion mass. Inserting Eq. (1) into the linearized ion continuity equation and Eq. (2), we obtain a system of two equations for the perturbed electrostatic potential Φ and the parallel velocity $v_\parallel$. Since the symmetry of the magnetic configuration is not broken in the toroidal direction, we express the fluctuating quantities into Fourier component form as

$$\Phi(r, \theta, \varphi, t) = \Phi(r, \theta)\exp(-\mathrm{i}\omega t + \mathrm{i}n\varphi),$$

$$v_\parallel(r, \theta, \varphi, t) = v_\parallel(r, \theta)\exp(-\mathrm{i}\omega t + \mathrm{i}n\varphi)$$

where n is the toroidal mode number. Then, the equations for the functions $\Phi(r,\theta)$ and $v_\parallel(r,\theta)$ take the following form

$$\frac{\mathrm{i}T_e}{eqR}\left(\frac{\partial}{\partial\theta}+\mathrm{i}nq\right)v_\parallel=\hat{\omega}\left(\frac{\partial^2\Phi}{\partial r^2}+\frac{1}{r}\frac{\partial\Phi}{\partial r}+\frac{1}{r^2}\frac{\partial^2\Phi}{\partial\theta^2}\right)$$
$$+\frac{\mathrm{i}v'_{0\varphi}}{qR}\left(\frac{\partial}{\partial\theta}+\mathrm{i}nq\right)\frac{\partial\Phi}{\partial r}+\frac{\mathrm{i}}{r}\left(\frac{v_{0\theta}}{r^2}-\frac{v'_{0\theta}}{r}-v''_{0\theta}-\frac{s-1}{qR}v'_{0\varphi}\right.$$
$$\left.+\frac{\varepsilon(r)}{q}v'_{0\varphi}\right)\frac{\partial\Phi}{\partial\theta}+\frac{\mathrm{d}\ln n_o}{\mathrm{d}r}\left[\hat{\omega}\frac{\partial\Phi}{\partial r}+\frac{\mathrm{i}}{r}\left(1+\varepsilon(r)\cos\theta\right.\right.$$
$$\left.\left.-\frac{v_{0\theta}}{r}-v'_{0\theta}+\frac{\varepsilon(r)}{q}v'_{0\varphi}\right)\frac{\partial\Phi}{\partial\theta}\right]+\mathrm{i}\frac{2}{R}\left(\sin\theta\frac{\partial\Phi}{\partial r}+\frac{\cos\theta}{r}\frac{\partial\Phi}{\partial\theta}\right)$$
$$-(1-\mathrm{i}\delta)\left[\hat{\omega}+\varepsilon(r)\cos\theta\left(\mathrm{i}\frac{v_{0\theta}}{r}\frac{\partial\Phi}{\partial\theta}+\frac{nv_{0\varphi}}{R}\Phi\right)\right] \quad (3)$$

and

$$\hat{\omega}v_\parallel=-\mathrm{i}\frac{e}{T_eqR}\left[\left(\frac{\partial}{\partial\theta}+\mathrm{i}nq\right)-\left(\frac{v_{0\theta}}{r}+v'_{0\theta}+\frac{v_{0\varphi}q}{r}\cos\theta\right.\right.$$
$$\left.\left.+\frac{v'_{0\varphi}q}{\varepsilon(r)}(1+\varepsilon(r)\cos\theta)\right)\frac{\partial}{\partial\theta}-v_{0\varphi}q\sin\theta\frac{\partial}{\partial r}\right]\Phi \quad (4)$$

with

$$\hat{\omega}\equiv\omega+\mathrm{i}\frac{v_{0\theta}}{r}\frac{\partial}{\partial\theta}-\frac{nv_{0\varphi}}{R}.$$

We note here that we have normalized the space and the time variables with respect to the ion Larmor radius ($\rho=C_s/\omega_{ci}$) defined at the electron temperature T_e, and to the ion angular gyrofrequency ω_{ci}, respectively. As a consequence the velocities are normalized with respect to the ion sound velocity $C_s^2=T_e/m_i$. Furthermore, $q(r)=rB_\varphi/RB_\theta(r)$ is the safety factor, $s(r)=rq'(r)/q(r)$ is the magnetic shear parameter, and the prime denotes the derivative with respect to the normalized radial coordinate r. It should be noted that we only keep the leading order terms in the left-hand side of Eq. (4).

The dependence of Eqs. (3) and (4) on the poloidal angle θ implies that the azimuthal modes are coupled to each other. This coupling is due to toroidicity and arises from the ∇B drift. As it was already pointed out in Section 1, the standard ballooning transformation is not appropriate for the rotating plasma, since the necessary *equivalence* between the rational magnetic surfaces is destroyed by the sheared rotation [21]. Therefore we make use of the method employed by Horton *et al.* [8], instead, seeking for solutions of Eqs. (3) and (4) which are localized about some rational magnetic surface with radius $r=r_0$ defined by $m_0+nq(r_0)=0$. Since the perturbed potential and parallel velocity must be periodic functions with respect to the poloidal angle θ, we represent these functions in the form of the Fourier expansion

$$\Phi(r,\theta)=\exp(\mathrm{i}m_0\theta)\sum_l\Phi_l(r)\exp(\mathrm{i}l\theta),$$
$$v_\parallel(r,\theta)=\exp(\mathrm{i}m_0\theta)\sum_l v_{\parallel l}(r)\exp(\mathrm{i}l\theta) \quad (5)$$

where the summation is over the index of the lth neighboring to m_0 poloidal mode. In the limit of a large aspect ratio we have, from the definition of m_0, that $|\Phi_{\pm1}(r)/\Phi_0(r)|\simeq\varepsilon\ll1$. The drift eigenmode localized on the rational surface $r=r_0$ is coupled with the eigenmodes of the neighboring surfaces $r_0\pm\Delta r$ such that $m_0\pm1+nq(r_0\pm\Delta r)=0$. Thus, the excitation of m_0 mode corresponds to a group excitation of modes with poloidal numbers $m_0\pm l$. For the drift waves of principal interest we have that $m_0\gg1$ and the spread of azimuthal modes number Δ_l can be regarded as small compared with m_0. Here, Δ_l is a measure of the dominant harmonics in the sums (5).

Substituting Eqs. (5) into Eqs. (3) and (4), we expand the radial dependent equilibrium parameters into the Taylor series in the vicinity of the reference magnetic surface $r=r_0$. Due to the smallness of the last term of Eq. (3), we may neglect the velocity harmonic coupling and eliminate the $v_\parallel$ term from Eqs. (3) and (4) reducing our problem to a set of Δ_l coupled equations for the potential poloidal harmonics. Thus, by keeping only the main terms and introducing a new dimensionless variable, $x=r-r_0$ we obtain the following differential equation for the coupled harmonics of the potential,

$$\frac{\partial^2\Phi_l}{\partial x^2}+[A_1+A_2(l-k_\theta sx)]\frac{\partial\Phi_l}{\partial x}+[A_3+A_4(l-k_\theta sx)$$
$$+A_5(l-k_\theta sx)^2]\Phi_l+[A_6+A_7(l-k_\theta sx)](\Phi_{l+1}+\Phi_{l-1})$$
$$+[A_8+A_9(l-k_\theta sx)]\frac{\partial}{\partial x}(\Phi_{l+1}-\Phi_{l-1})=0 \quad (6)$$

with the following coefficients

$$A_1=r_0^{-1}-r_n^{-1},\quad A_2=-\frac{\xi_\varphi}{r_0q_0R}\frac{V_\varphi}{\omega'},$$
$$A_3=\frac{k_\theta}{\omega' r_n}\left[1-\frac{\omega'}{\omega_*}-\frac{V_\theta}{r_0}(1+\xi_\theta)+\frac{V_\varphi}{q_0R}\xi_\varphi\right]+\mathrm{i}\delta,$$
$$A_4=\frac{k_\theta}{\omega'}A_2,\qquad A_5=(\omega' q_0R)^{-2},$$
$$A_6=\varepsilon_0\frac{k_\theta}{2\omega'}\left[r_n{}^{-1}-2r_0^{-1}+\mathcal{V}(1-\mathrm{i}\delta)\right],$$
$$A_7=-k_\theta(1+\xi_\varphi)\frac{q_0V_\varphi}{2}A_5,\quad A_8=-(\omega'R)^{-1},$$
$$A_9=-\frac{q_0V_\varphi}{2}A_5 \quad (7)$$

where $\varepsilon_0=r_0/R$, $q_0=q(r_0)$, $k_\theta=m_0/r_0$ is the normalized local poloidal number, $r_n=-(\mathrm{d}\ln n_0/\mathrm{d}r)^{-1}_{r=r_0}$ is the characteristic scale of inhomogeneity, $\omega'=\omega-\boldsymbol{k}\cdot\boldsymbol{v_0}(r_0)=\omega-k_\varphi V_\varphi-k_\theta V_\theta$ is the Doppler-shifted eigenfrequency with $k_\varphi=n/R$, $V_{\theta,\varphi}=v_{0\theta,\varphi}(r_0)$ are the normalized local poloidal/toroidal rotation velocities with corresponding shears $\xi_{\theta,\varphi}=r_0v'_{\theta,\varphi}(r_0)/V_{\theta,\varphi}$, and $\mathcal{V}=V_\theta+\varepsilon_0q_0^{-1}V_\varphi$. It should be noted, that in the expressions of the coefficients (7) we have neglected high order corrections with respect to ε^2 assuming that the Doppler-shifted eigenfrequency ω' has the same order of magnitude as the diamagnetic drift frequency,

$$\omega_* \equiv \frac{k_\theta r_n^{-1}}{1+k_\theta^2}.$$

The drift eigenmode described by the set of Eqs. (6) is broad in its radial dependence being composed of a superposition of normal modes. The plasma rotation is not necessary for the existence of such eigenmodes, but it can affect substantially their type and structure. In particular as it was shown in Ref. [26], poloidal plasma rotation changes the criterion of existence of global drift modes from $s < 1/2$ for $\omega' > 0$ to $s > 1/2$ for $\omega' < 0$. For further analytical progress with the set of equations (6), we have to replace the discrete set of functions $\Phi_l(x)$ by a continuous function of two variables $\Phi(x, l)$, or to truncate the set of equations. This can be done by applying the so called strong and weak coupling approximations. The corresponding results are presented in the following two sections respectively.

3. Strong coupling approximation

Assuming that a large number $\Delta_l \gg 1$ of poloidal harmonics are coupled due to equilibrium toroidal variations, we can consider Eq. (6) as an infinite set of coupled equations for the harmonics Φ_l with $l = 0, \pm 1, \pm 2, \ldots$. Then, following Horton *et al.* [8], in the limit $m_0 \gg \Delta_l \gg 1$ that is called the strong limit, we replace the discrete set of functions $\Phi_l(x)$ by a continuous function $\Phi(x, l)$ of two variables using the ansatz

$$\Phi_{l\pm 1}(x) \to \Phi(x, l) \pm \frac{\partial \Phi(x, l)}{\partial l} + \frac{1}{2}\frac{\partial^2 \Phi(x, l)}{\partial l^2}. \tag{8}$$

Substituting Eq. (8) into Eq. (6) we obtain a partial differential equation with respect to x and l, i.e., a two-dimensional (2D) drift mode equation with radial variable x and "poloidal" variable l. Since it is a formidable task to find a general solution of the 2D eigenmode equation, one should try to make some simplifying assumptions to reduce essentially the problem to a one-dimensional (1D) one. The assumption that the basic structure of a drift mode resembles a quasi-mode (see review by Tang [1] and references therein), i.e., a mode which can be represented as a sum of normal modes with components centered on different rational surfaces, allows the reduction of the 2D equation to a 1D one by choosing a special combination of two variables x and l. Here, we consider the special case of solutions $\Phi(x, l) = \Phi(y)$ with $y = x - l/k_\theta s$, which permit us to reduce the obtained 2D eigenmode equation to the following ordinary second order differential equation,

$$\begin{aligned}(a_0 + a_1 y)\Phi''(y) + (b_0 + b_1 y)\Phi'(y)\\ + (c_o + c_1 y + c_2 y^2)\Phi(y) = 0\end{aligned} \tag{9}$$

with coefficients given by

$$\begin{aligned}&a_0 = 1 + A_6(k_\theta s)^{-2} + 2A_8(k_\theta s)^{-1}, \qquad a_1 = 2A_9 - A_7(k_\theta s)^{-1},\\ &b_0 = A_1, \quad b_1 = -A_2 k_\theta s, \qquad c_0 = A_3 + 2A_6,\\ &c_1 = -(A_4 + 2A_7)k_\theta s, \qquad c_2 = A_5(k_\theta s)^2.\end{aligned} \tag{10}$$

Supposing $\omega' \sim \omega_*$, subsonic rotation velocities $V_{\theta,\varphi} < 1$, and using the plasma parameter ordering written below (in dimensional form)

$$\rho \le r_n \ll r_0 \quad \text{and} \quad r_n^2/\rho R \ll 1, \tag{11}$$

one can see that the term $a_1 y$ in Eq. (10) is much smaller than a_0 even for the upper limit values of the variable y, namely $y \sim r_0$. This ordering seems to be realistic for the edge region of modern tokamaks.

For example, in the distance of a few cm within the last closed magnetic surface of the DIII-D tokamak [11] $r_n \simeq 3\,\text{cm}$, $\rho \simeq 2\,\text{cm}$ and $r_0 \simeq 20\,\text{cm}$ while $R \simeq 1.75\,\text{m}$ (the dimensional parameters).

This implies that we can divide Eq. (9) by $(a_0 + a_1 y)$ and expand over the small parameter $a_1 y/a_0 \ll 1$. After some algebraic manipulations, we reduce Eq. (9) to a second order ordinary differential equation

$$\Phi''(y) + P(y)\Phi'(y) + Q(y)\Phi(y) = 0 \tag{12}$$

with the polynomials $P(y)$ and $Q(y)$ given as combinations of the coefficients (10),

$$\begin{aligned}P(y) &\simeq \frac{1}{a_0}[b_0 + (b_1 - a_1 b_0/a_0)y],\\ Q(y) &\simeq \frac{1}{a_0}\left[c_0 + (c_1 - a_1 c_0/a_0)y + c_2 y^2\right].\end{aligned}$$

Introducing the function $\eta(y)$ by

$$\Phi(y) = \eta(y)\exp\left[-\frac{1}{2}\int^y P(y')\,dy'\right]$$

we eliminate the first derivative term. Then, for the new variable $\psi = y + (c_1 - c_0 a_1/a_0)/2c_2$, Eq. (12) takes the form

$$\frac{d^2\eta}{d\psi^2} + (\Lambda - \zeta\psi^2)\eta(\psi) = 0 \tag{13}$$

with the parameters

$$\Lambda = \frac{k_\theta}{a_0 r_n \omega'} \times \left[1 - \frac{\omega'}{\omega_*} + \varepsilon_0 + \varepsilon_0 r_n \mathcal{V}(1 - i\delta) - \Delta + i\frac{\delta}{k_\theta} r_n \omega'\right] \tag{14}$$

and

$$\zeta = -\frac{1}{a_0}\left(\frac{k_\theta s}{\omega' R q_0}\right)^2 \tag{15}$$

where

$$a_0 = 1 + \frac{\varepsilon_0}{2k_\theta s^2 \omega'}\left[2(2s-1){r_0}^{-1} + {r_n}^{-1} + \mathcal{V}(1-\mathrm{i}\delta)\right] \quad (16)$$

and

$$\Delta = \frac{V_\theta}{r_0}(1+\xi_\theta) - \frac{V_\varphi}{q_0 R}\xi_\varphi. \quad (17)$$

In the above expressions we have retained only the first order corrections with respect to ε. Equation (13) is an extension of Eq. (23) in Ref. [26], which includes the effects of toroidal rotation. It has the form of the Schrödinger equation, but the parameters (14) and (15) contain small imaginary parts. Nevertheless, we shall use this analogy for the clearness of further interpretations. Equation (13) has two types of solutions depending on the sign of the parameter ζ. For $\zeta > 0$ this equation describes localized states in a potential well (global eigenmodes), whereas the case $\zeta < 0$ corresponds to a potential hump which results to unbound states (propagating eigenmodes). These two types of solutions are analyzed in the next two subsections.

3.1. *Global modes*

A global mode has a structure of a non-propagating quasi-mode [33] that can be considered as a sum of degenerated normal modes localized on different magnetic rational surfaces. It has a substantial radial extent and includes a large number of rational surfaces due to toroidal coupling. This mode corresponds to the bound state in a potential well which is marginally stable. The dissipative effects, for the energy levels of such a well, lead to a broadening of the spectral lines, without changing the dispersion properties. Therefore, in the analysis of the global modes, we can omit the phase shift δ which was introduced to represent the electron dissipative mechanism effects. So, through this subsection, we imply that $\Lambda = \mathrm{Re}[\Lambda]$ and $\zeta = \mathrm{Re}[\zeta]$ for Eq. (13). In the case of $\zeta > 0$, Eq. (13) has the form of a Schrödinger equation with a parabolic potential well. It is well known that the eigenfunctions of this equation are given through $\eta_N \propto \exp(-\sqrt{\zeta}\psi^2)H_N(\zeta^{1/4}\psi)$, where H_N is the Hermite polynomial of the N^{th} order and the corresponding eigenvalues are defined by the dispersion equation

$$\Lambda/\sqrt{\zeta} = 2N+1, \quad N = 0, 1, 2\ldots \quad (18)$$

The necessary condition for the global mode existence in the strong coupling approximation is $\zeta > 0$, i.e., $a_0 < 0$ as follows from Eq. (15). This inequality is satisfied for the positive Doppler-shifted eigenfrequencies $\omega' > 0$, when

$$\mathcal{V} + {r_n}^{-1} + 2(2s-1){r_0}^{-1} < 0.$$

We have to consider the probability of existence of global drift modes in this case as very low, for the considered plasma parameter ordering (11) at least in the limit of subsonic plasma rotation velocities. For the case of negative Doppler-shifted frequencies, $\omega' < 0$, the condition of global drift mode existence $a_0 < 0$ is rewritten as

$$\frac{|\omega'|}{\omega_*} < \frac{\varepsilon_0 r_n}{2s^2}\left(1 + k_\theta^{-2}\right)\left[\mathcal{V} + {r_n}^{-1} + 2(2s-1){r_0}^{-1}\right].$$

This inequality is satisfied easily for the ordering (11), and one can see that the role of plasma rotation in the formation of the global drift mode is much more crucial than that of magnetic shear. Thus, for the considered plasma parameter ordering, the global drift eigenmode existence may be attributed to the negative Doppler-shifted frequencies (i.e., $\omega < \boldsymbol{k}\cdot\boldsymbol{v}_0$). In this case inserting Eqs. (14) and (15) for Λ and ζ respectively into Eq. (18), we obtain the following dispersion equation for the global mode,

$$\frac{|\omega'|}{\omega_*} + 1 + \varepsilon_0 r_n \mathcal{V} - \Delta = (2N+1)\frac{r_n s}{q_0 R} \times \sqrt{\frac{\varepsilon_0}{2|\omega'|k_\theta s^2}\left[\mathcal{V} + {r_n}^{-1} + 2(2s-1){r_0}^{-1}\right] - 1}. \quad (19)$$

The latter takes the form of a cubic algebraic equation for the value $\Omega = |\omega'|/\omega_*$,

$$\Omega^3 + 2(1+\alpha)\Omega^2 + (1 + 2\alpha + L)\Omega - \kappa L = 0 \quad (20)$$

where the following parameters,

$$\alpha = \varepsilon_0 r_n \mathcal{V} - \Delta, \quad (20a)$$

$$L = (2N+1)^2\,(r_n s/q_0 R)^2, \quad (20b)$$

$$\kappa = \frac{\varepsilon_0 r_n}{2s^2}\left(1 + k_\theta^{-2}\right)\left[2(2s-1)r_0^{-1} + r_n^{-1} + \mathcal{V}\right] \quad (20c)$$

are introduced. It is a formidable task to find the general solutions of Eq. (20), since the coefficients L and κ vary in wide ranges depending on the values of N and k_θ. However, one can easily obtain an approximative solution for $\Omega \ll 1$. In this limit, the first two terms of Eq. (20) are negligible and we find

$$\Omega \simeq \kappa L/(1 + 2\alpha + L). \quad (21)$$

The requirement of smallness of Ω is ensured either for arbitrary L in the limit $\kappa \ll 1$ or for substantially small values of k_θ, with small numbers N, when $L \ll 1$. It should be noted that the solution (21) is valid even for the values $\Omega \sim 1$ in the limit $L \gg 1$.

3.2. *Propagating drift waves*

In the case of $\zeta < 0$, the potential of the Schrödinger equation (13) becomes a hump and this equation describes drift waves which leave the magnetic surface, where they were generated, and propagate radially. Since a propagating wave can be substantially affected by dissipative processes, we retain the imaginary corrections due to the phase shift δ and hence, Eq. (13) is a complex one. Introducing the new variable $z = \sqrt{\mathrm{i}|\zeta|^{1/2}}\psi$, we transform Eq. (13) into a Weber-type equation [34],

$$\frac{\partial^2\eta}{\partial z^2} + \left(\frac{\Lambda}{\mathrm{i}\sqrt{|\zeta|}} - z^2\right)\eta = 0 \quad (22)$$

which has eigenfunctions of the form $\eta_\nu \propto \exp(-z^2)H_\nu(z)$, where H_ν is the complex Hermite function with the index ν defined by the equation $2\nu + 1 = \Lambda/\mathrm{i}\sqrt{|\zeta|}$. If $\nu = N$, with $N = 0, 1, 2\ldots$, the Hermite function H_ν coincides with the Hermite polynomial H_N. The requirement that the solution of the Weber equation has to be limited in the whole z interval including the infinite points, leads to the condition that the parameter ν has to be a real integer [34]. So, the eigenvalues of Eq. (22) are defined by the dispersion equation

$$\Lambda/\mathrm{i}\sqrt{|\zeta|} = 2N + 1. \tag{23}$$

It should be noted that the binding of the solution in the whole z interval corresponds to the outgoing boundary condition as was introduced by Perlstein and Berk [3]. The necessary condition $\mathrm{Re}[\zeta] < 0$, for propagating drift wave existence is reduced to $\mathrm{Re}[a_0] > 0$. This is easily satisfied for positive Doppler-shifted eigenfrequencies, $\omega' > 0$, but practically not for $\omega' < 0$, due to the reason that global modes are impossible for $\omega' > 0$, (see previous subsection). Thus, we may attribute the propagating drift waves to the case of positive Doppler-shifted eigenfrequencies $\omega' > 0\,(\omega > \boldsymbol{k}\cdot\boldsymbol{v})$. The dispersion equation for these waves is obtained by substituting Eqs. (14) and (15) for Λ and ζ respectively, into Eq. (23),

$$\frac{\omega'}{\omega_*} - 1 - \varepsilon_0 - \varepsilon_0 r_n \mathcal{V}(1 - \mathrm{i}\delta) + \Delta - \mathrm{i}\frac{\delta r_n \omega'}{k_\theta}$$
$$= -\mathrm{i}(2N+1)\frac{r_n s}{q_0 R}\sqrt{1 + \frac{\varepsilon_0}{2\omega' k_\theta s^2}[\mathcal{V} + r_n{}^{-1} + 2(2s-1)r_0{}^{-1}]}. \tag{24}$$

Here, the eigenfrequency ω' has complex value, with $\mathrm{Re}[\omega'] = \omega'_0$ giving the characteristic frequency and $\mathrm{Im}[\omega'] = \gamma$ representing the growth ($\gamma > 0$) or the decrement rate ($\gamma < 0$) of the propagating wave. Using the ansatz $\omega' = \omega'_0 + \mathrm{i}\gamma$, with $|\gamma| < \omega'_0$, we separate the dispersion equation (24) into real and imaginary parts. From the real part we determine the frequency of the propagating drift mode

$$\frac{\omega'_0}{\omega_*} \simeq 1 + \alpha, \tag{25}$$

and from the imaginary part of Eq. (24), we find the growth/damping rate of this wave

$$\frac{\gamma}{\omega_*} \simeq \frac{\delta(1+\alpha)}{1+k_\theta^2} - (2N+1)\frac{s r_n}{q_0 R}\sqrt{1 + \frac{\omega_*}{\omega'_0}\kappa}. \tag{26}$$

The situation when electrostatic potential lags behind the density $\delta > 0$ corresponds to the destabilization of the drift wave. The resulting microinstability is usually referred to as "universal drift instability" [31] and it is described by the first term in the right hand side of Eq. (26). The stability of the mode is now influenced by the sheared plasma rotation.

Indeed, as can be seen from Eqs. (17) and (20a), for subsonic rotation velocity and strong velocity shear (i.e. $\xi_\theta \simeq r_0 L_E^{-1}$), the parameter α can be approximated by $\alpha \simeq -V_\theta \rho / L_E$, where the "shear length" L_E is given by $L_E^{-1} = |\partial \ln(v_{0\theta}/r)/\partial r|_{r=r_0}$. If the ratio $V_\theta r_n / L_E$ which characterizes the steepness of the flow shear, is held fixed and larger than one, then $\alpha < -1$ and the universal drift instability will essentially be suppressed by the poloidal shearing. Similar result can be obtained from the estimation of the eddy decorrelation rate of turbulence due to poloidal flow shear. As it was predicted recently, from a two-point non-linear analysis [35], the reduction of the radial correlation length below its ambient turbulence value and the consequent fluctuation suppression occur when the shearing rate $\omega_s \simeq \left|L_r \partial(v_{0\theta}/r)/\partial r\right|_{r=r_0}$ (L_r is the radial correlation length of the ambient turbulence) exceeds the decorrelation rate of the ambient turbulence $\Delta\omega_T$. Assuming that $\Delta\omega_T$ scales like the diamagnetic drift frequency $\Delta\omega_T \sim \omega_*$ (this assumption is supported by fluctuation measurements [36], as well as by gyrokinetic and gyrofluid simulations [37]), the condition of flow-shear-induced suppression of turbulence is given by

$$\frac{\omega_s}{\Delta\omega_T} \sim \frac{L_r}{r_0}\frac{r_n}{L_E} V_\theta > 1. \tag{27}$$

Theories of the toroidal drift instability turbulence indicate for the scaling of the radial correlation length, $L_r \simeq \rho^{1-\beta} r_m^\beta$ (r_m is the minor radius of the tokamak), that β ranges $1/2 < \beta \leq 1$. Hence, one can estimate the radial correlation length L_r by a length typically of the order of minor radius. Hence, the fluctuation reduction ($\omega_s > \Delta\omega_T$) takes place if $V_\theta r_n / L_e > 1$. This coincides with the condition of stabilization of drift instability by flow shear as derived above. Using the approximation $L_E \simeq \rho$ for the "shear length" [35], we conclude that the condition (27) can easily be satisfied.

For weak magnetic shear the characteristic length of the mode structure may become smaller than the shear length. In this case the velocity shear does not distort the mode structure any longer [38] and the universal instability may be stabilized by the magnetic shear. This stabilization effect is described by the second term in the right hand-side of Eq. (26). It is worthwhile to note that these results are obtained for the ordering described in Eqs. (11) and thus are related to edge turbulence. In the center of a tokamak, the shear rate of the poloidal plasma rotation is usually much less than the linear mode growth rate [39] and hence does not play any substantial role on the drift wave stability.

4. Weak coupling approximation

The weak coupling approximation implies a truncation of the harmonic expansion (5) and hence, of the set of Eq. (6). However, we will simplify Eq. (6) before proceeding further. First of all, we note that the coefficients in the terms containing harmonics $\Phi_{l\pm1}$ are of the order of ε compared to the coefficients in the terms with Φ_l. One can also see this in Eq. (3) where the terms with $\cos\theta$ and $\sin\theta$, which are responsible for the appearance of the harmonics $\Phi_{l\pm1}$, contain the small parameter ε. Moreover, by assuming that in the transition from a given rational surface to its nearest neighboring surface, the change of the value $(l - k_\theta s x)$ is of the order of unity, we see that the ratios A_7/A_6 and A_9/A_8 are of the order of $\varepsilon_0 \sim \omega'$. So, we can neglect the terms containing A_7 and A_9 in Eq. (6), since they are of the order of ε^2. Furthermore by introducing a new potential function by

$$\Phi(x) = U_l(x)\exp\left\{-\frac{1}{2}\int^x [A_1 + (l - k_\theta s x')A_2]\,dx'\right\}$$

we eliminate the first derivative terms in Eq. (6). In the following, we truncate the harmonic expansion at $l \pm 1$, restricting ourselves in the equations for the fundamental harmonic $l = 0$,

$$U_0'' + \left[\left(A_3 - \frac{A_1^2}{4}\right) - A_4 k_\theta s x + A_5(k_\theta s x)^2\right]U_0 = -A_6(U_1 + U_{-1}) - A_8\left(U_1' - U_{-1}'\right) + \tfrac{1}{2}A_1 A_8(U_1 - U_{-1}), \quad (28)$$

and for the two nearest harmonics $l \pm 1$,

$$U_{\pm1}'' + \left[\left(A_3 - \frac{A_1^2}{4} + A_5 \pm A_4\right) - (A_4 \pm 2A_5)k_\theta s x + A_5(k_\theta s x)^2\right]U_{\pm1} = -(A_6 \pm \tfrac{1}{2}A_1A_8)U_0 \pm A_8 U_0', \quad (29)$$

where we have neglected small terms considering that the right-hand side of Eq. (28) is a small correction of the order of ε to the left hand side. In the case of weak coupling with the nearest harmonics only, we can assume following Horton *et al.* [8], that the variation of the variable x is limited by the distance between the neighboring rational surfaces $\Delta x \sim (k_\theta s)^{-1}$, and thus, the second derivative term $U_{\pm1}''$ may be estimated as $U_{\pm1}'' \simeq (k_r)^2 U_{\pm1}$, where $k_r \sim r_0^{-1} \ll k_\theta$. In addition to this we neglect the coefficients A_4 and A_5 as small compared with A_3 and A_1. These approximations allow us to write Eq. (29) in the form

$$\left(A_3 - \frac{A_1^2}{4}\right)U_{\pm1} = -\left(A_6 \pm \tfrac{1}{2}A_1A_8\right)U_0 \pm A_8 U_0'.$$

In what follows, we substitute the latter equation into Eq. (28), we introduce a new function $\eta_w(x)$ by $U_0(x) = \eta_w(x)\exp\left(A_1A_8^2 d_0^{-1}x\right)$ where $d_0 = A_3 - A_1^2/4\ +2A_8^2$, and we change to a new variable $\psi_w = x -\ A_4(2A_5 k_\theta s x)^{-1}$. As a result, Eq. (28) takes the form of Eq. (13) with parameters $\Lambda = \Lambda_w$ and $\zeta = \zeta_w$, given by

$$\Lambda_w \simeq [(A_3^2 - A_1^2/4)^2 - 2A_6^2]d_0^{-1} \quad (30)$$

and

$$\zeta_w \simeq -\left(A_3 - A_1^2/4\right)A_5(k_\theta s)^2 d_0^{-1}. \quad (31)$$

Here and in what follows, we use the subscript "w" to mark the variables and the parameters for the weak coupling approximation in order to distinguish them from the corresponding ones in the strong coupling approximation in Section 3.

4.1. *Global modes*

As discussed in Section 3, Eq. (13) has the solutions of the global type for $\zeta > 0$. Substituting the apparent expressions for the coefficients (7) into Eq. (31) and neglecting, as previously, the imaginary corrections due to the phase shift δ, the inequality $\zeta_w > 0$ takes the form

$$\frac{1}{d_1} < \frac{\omega'}{\omega_*} < \frac{1 + (1 + 4d_1d_2)^{1/2}}{2d_1} \quad (32)$$

for the positive Doppler-shifted frequencies, $\omega' > 0$, and

$$0 < \frac{|\omega'|}{\omega_*} < \frac{(1 + 4d_1d_2)^{1/2} - 1}{2d_1} \quad (33)$$

for the negative Doppler-shifted frequencies, $\omega' < 0$, where

$$d_1 = 1 + \frac{1}{4r_n^2}\left(1 + k_\theta^2\right)^{-1} \quad \text{and} \quad d_2 = \frac{2r_n^2}{R^2}\frac{1 + k_\theta^2}{k_\theta^2}.$$

Substituting the apparent expressions for Λ_w and ζ_w, (30) and (31) respectively, into (18), we obtain the dispersion equation for the global drift modes in the weak coupling approximation,

$$\left[\frac{\omega'}{\omega_*}d_1 - 1 + \Delta\right]^2 - \tfrac{1}{2}\varepsilon_0^2(1 + r_n\mathcal{V})^2 = (2N+1)\frac{s r_n}{q_0 R}\sqrt{\left(\frac{\omega'}{\omega_*}d_1 - 1\right)\left(\frac{\omega_*}{\omega'}d_2 - \frac{\omega'}{\omega_*}d_1 + 1\right)} \quad (34)$$

where Δ has been defined at Eq. (17). One can see that the interval (32) of the positive Doppler-shifted global drift eigenmodes is very narrow. Further analysis show that for the considered ordering (11) the positive solutions of Eq. (34) contradict to the condition of positiveness of the left hand side of the dispersion equation (34). Therefore, in the considered limits positive Doppler-shifted global drift eigenmodes do not exist. For the case of negative Doppler-shifted eigenfrequencies, $\omega' < 0$ of the range (33), the dispersion relation (34) has approximative solutions

$$\frac{|\omega'|}{\omega_*} \simeq d_2\left[1 + (2N+1)^{-2}\frac{q_0^2R^2}{r_n^2 s^2}\right]^{-1}. \quad (35)$$

The spectrum of these eigenfrequencies ranges from zero-order line

$$\frac{|\omega_0'|}{\omega_*} \simeq \frac{2r_n^4 s^2}{q_0^2 R^4}(1 + k_\theta^{-2}) \quad \text{up to the accumulation point}$$

$$\frac{|\omega_\infty'|}{\omega_*} \simeq \frac{2r_n^2}{R^2}(1 + k_\theta^{-2}).$$

Thus, in the weak coupling approximation and for the considered plasma parameter ordering, the existence of the global drift eigenmode can be attributed to the negative Doppler-shifted frequencies $\omega' < 0$ (i.e., $\omega < \boldsymbol{k}\cdot\boldsymbol{v}_0$).

4.2. *Propagating Drift Waves*

The existence of propagating waves in the case of weak coupling is attributed to the condition $\zeta_w < 0$. Employing Eqs. (7) and (31), we find that this condition is satisfied in the intervals

$$0 < (\omega'/\omega_*)d_1 < 1 \tag{36}$$

and

$$(\omega'/\omega_*)d_1 > 1 + d_1 d_2 \tag{37}$$

for the positive Doppler-shifted eigenfrequencies, $\omega' > 0$, and in the interval

$$\frac{|\omega'|}{\omega_*} > d_2 \tag{38}$$

for the negative Doppler-shifted eigenfrequencies, $\omega' < 0$. The general dispersion equation for the propagating waves has the form of Eq. (23) for $\Lambda = \Lambda_w$ and $\zeta = \zeta_w$,

$$\left[1 - \Delta - \frac{\omega'}{\omega_*}\left(d_1 - \frac{\mathrm{i}\delta}{1 + k_\theta^2}\right)\right]^2 - \frac{\varepsilon_0^2}{2}[1 + r_n\mathcal{V}(1 - \mathrm{i}\delta)]^2$$
$$= \mathrm{i}(2N+1)\frac{sr_n}{q_0 R}\,\mathrm{sgn}\left[d_2 + \frac{\omega'}{\omega_*} - \frac{\omega'^2}{\omega_*^2}d_1\right]$$
$$\times \sqrt{\frac{\omega_*}{\omega'}\left(1 - \frac{\omega'}{\omega_*}d_1\right)\left(d_2 + \frac{\omega'}{\omega_*} - \frac{\omega'^2}{\omega_*^2}d_1\right)}. \tag{39}$$

From the real part of (39) we obtain two approximative solutions for the frequency ranges of (36) and (37),

$$\frac{\omega_0'}{\omega_*} \simeq \left[1 - \Delta \pm \frac{\sqrt{2}}{2}\varepsilon_0(1 + r_n\mathcal{V})\right]d_1^{-1} \tag{40}$$

where Δ has been defined by (17). The solution with the upper sign contradicts to the right-hand side of inequality of (36) since $\Delta \ll \varepsilon_0$ for the considered ordering (11) and for subsonic plasma rotation velocities. Thus, the dispersion relation for the propagation drift waves with frequencies in the range (36) corresponds to the case of lower sign of (40). Similarly, one can see that dispersion relation for the range (37) corresponds to the case of the upper sign of (40). From the imaginary part of Eq. (39), we find

$$\frac{\gamma}{\omega_0'} \simeq \frac{\delta(1 - \Delta)}{1 + k_\theta^2 + (1/4r_n^2)} - \left(N + \tfrac{1}{2}\right)\frac{sr_n}{q_0 R} \tag{41}$$

for both signs of (40), which is similar to Eq. (26) obtained in the case of the strong coupling approximation. Again, the "universal drift instability" described by the first term in the right hand side of Eq. (39) may essentially be suppressed by the strong velocity shear ($\xi_\theta \to r_0/L_E$) for subsonic plasma rotation velocities as it was discussed in Section 3.2. On the other hand if the condition (27) is not satisfied, then, the propagating drift wave will experience the magnetic shear damping described by the second term. Finally, for the negative Doppler-shifted frequencies of the range (38), we see that the dispersion relation (40) does not have any solution for the accepted ordering.

5. Conclusions

In conclusion, we have studied the effects of sheared plasma rotation on the structure of toroidal drift eigenmodes, for low-β plasma in a tokamak with concentric, circular magnetic surfaces and large aspect-ratio. The two fluid model is used and the performed analysis is based on a consistent derivation of the eigenmode equations for collisionless drift oscillations in terms of the usual drift ordering, where the toroidal coupling effects appear due to ion ∇B and curvature drifts.

Both strong and weak coupling approximations are implied in order to take into account the toroidal coupling of local normal modes centered on neighboring rational magnetic surfaces. In the strong coupling approximation it is assumed that a large number of poloidal harmonics is coupled due to toroidicity, whereas in the weak coupling is considered that the given harmonic is coupled only with its nearest neighbors. It is shown that two types of eigenmodes exist for both approximations. The first one corresponds to global drift modes that have a structure of a quasi-mode localized in both radial and poloidal directions with a small wave number component along the confining magnetic field. This mode corresponds to the bounds states in a potential well which is marginally stable. The other type of eigenmodes corresponds to the propagating drift wave, which leaves the magnetic surface, where it was generated, propagates radially and experiences the shear damping.

The results obtained in both strong and weak coupling approximations are similar qualitatively. The Doppler-shifted eigenfrequency varies from one magnetic surface to another due to sheared poloidal and toroidal plasma rotation velocities, whereas its sign depends on the values and directions of the velocities. For the considered plasma parameter ordering, the existence of the global drift mode is attributed to the case of negative Doppler-shifted eigenfrequencies, whereas the propagating waves are possible for positive Doppler-shifted eigenfrequencies.

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