# Large-scale flows and coherent structure phenomena in flute turbulence

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The properties of zonal and streamer flows in the flute mode turbulence are investigated. The stability criteria and the frequency of these flows are determined in terms of the spectra of turbulent fluctuations. Furthermore, it is shown that zonal flows can undergo a further nonlinear evolution leading to the formation of long-lived coherent structures which consist of self-bound wave packets supporting stationary shear layers, and thus can be characterized as regions with a reduced level of anomalous transport. © *2005 American Institute of Physics*. [DOI: 10.1063/1.1883183]

## I. INTRODUCTION

Low-frequency electrostatic turbulence, driven by spatial gradients, is believed to be the dominant source of anomalous transport in magnetically confined fusion plasma. Special emphasis has been given lately on the properties of large-scale anisotropic flows generated by the drift-type turbulence, due to the critical role they play in the regulation of low-frequency drift instabilities and consequently of the levels of turbulent transport.[1,2] The spontaneous generation of large-scale flows driven by electrostatic wave turbulence has been experimentally observed in plasma discharges in various machines, e.g., in Texas Experimental Tokamak, in the reversed field pinch experiment, and in the doublet III-D tokamak (DIII-D).[3,4] Zonal (or poloidal) flows correspond to structures which spatially depend on the radial coordinate $x$ (the coordinate along the axis of plasma inhomogeneity), while radial flows or streamers are radially elongated structures which spatially depend on the poloidal coordinate $y$.[5]

In tokamak plasmas, zonal flows have the ability to limit the radial size of turbulent eddies through the shear decorrelation mechanism,[6] and hence to regulate turbulent transport. As a consequence, the high plasma confinement modes are attributed to the presence of the zonal flows. Streamers, on the other hand, are ineffective at inhibiting radial transport and, due to their long radial correlation length, may lead to enhanced or bursty levels of transport.[7] The flow formation is commonly attributed to several mechanisms such as Reynolds stress[8] or modulation[9] instabilities. The radially dependent zonal flows, as well as the poloidally dependent streamers, have relatively larger structure, and thus may dominate the turbulent transport in magnetized plasmas. Therefore, the study of the interplay between different spatiotemporal scales of turbulence is rather important. As it was shown for the case of drift wave–zonal flow turbulence, zonal flows can be spontaneously generated as a result of a resonant interaction between the flow and the modulation of the small-scale turbulence.[10,11] Later on,[12] it was shown that a coherent hydrodynamic generalization of this resonant type flow instability also exists leading to the generation of large-scale flows. The nonlinear evolution of these instabilities can lead to the formation of long-lived coherent structures in the drift-wave zonal flow system[10] which constitute paradigms for intermittency in drift wave turbulence and manifests itself by regions with a reduced level of anomalous transport.

In this work, we investigate the interactions and the associated instabilities between the large-scale flows and the background magnetic-curvature-driven flute turbulence. The magnetic-curvature-driven flute instability belongs to the class of reactive instabilities, so that no dissipation is needed for its development and growth. In the flute limit ($k_{\parallel}=0$), plasma particles do not follow the Boltzmann relation and cannot cancel the charge separation induced by the difference between the perturbed electron and ion curvature drift velocities. This leads to the development of an electric field component perpendicular to the magnetic field direction that amplifies the initial perturbation, which becomes unstable. The flute instability is also termed interchange instability, as it tends to interchange "flux tubes" of different pressure causing convective transport. Thus, it is considered to be one of the most dangerous instabilities in thermonuclear fusion devices.

The generation of large-scale flows in flute turbulence has been studied analytically in Ref. 13 and numerically in Ref. 14, where it was shown that streamers can be generated through both linear and nonlinear mechanisms, while zonal flows can be excited only nonlinearly. In contrary to the usual drift wave turbulence, the diamagnetic component of the polarization drift nonlinearity, attributed to the finite ion Larmor radius, becomes important and leads to a direct cascading of the fluctuation energy towards short scales.[15] Under some conditions, this may result in the suppression of the large-scale flows.

The interaction between disparate scales of flute turbulence can be described by employing the evolution equations for the mean flow and the wave kinetic equation for an actionlike invariant of the wave turbulence, with slowing vary-

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ing parameters due to the mean sheared flows. The solution of these equations results in the determination of a stability criterion in terms of the spectra of the flutelike electrostatic oscillations. Furthermore the formation of coherent structures, which corresponds to propagating shear layer "domain walls" between regions of different flow velocities, is depicted.

The rest of the paper is organized as follows; in the following section, we briefly present the model of equations that describe the flute modes. The evolution of large-scale flows driven by the flute instability is considered in Sec. III, and in Sec. IV we present the properties of the zonal and streamer flows. Finally, the summary and conclusions follow in Sec. V.

## II. BASIC EQUATIONS

Flute (or interchange) modes are low-frequency ($\omega \ll \omega_{ci}$) electrostatic oscillations of a nonuniform magnetoplasma which are elongated along the magnetic field and may become unstable due to the combined effects of the density inhomogeneity and the curvature of the magnetic field lines, which always exist in the magnetic confinement devices. To describe flute modes we use the two-fluid equations,[16] for a weakly inhomogeneous magnetized plasma with characteristic inhomogeneity scale length $L_n$ along the radial axis $x$. For the slab geometry $\mathbf{r}\equiv(x,y)$, we model the curved magnetic field by $B(x)=B_0(1-x/R)$ and $\mathbf{b}=\hat{z}-(z/R)\hat{x}$, where $B(x)$ is the magnitude, $\mathbf{b}$ is the unit vector, and $R(>L_n)$ is the curvature radius of the curved magnetic field lines.

Assuming flute-type ($k_\parallel=0$), quasineutral, low-frequency, electrostatic oscillations, it is found that the magnetic-curvature-driven flute modes are described by the following set of dimensionless coupled equations for the perturbed electrostatic potential $\Phi$ and density $n$:[17]

$$(\partial_t - v_{ni}\partial_y)\nabla_\perp^2\Phi + v_g\partial_y n = \tau_i \mathrm{div}\{\boldsymbol{\nabla}_\perp\Phi, n\} + \{\nabla_\perp^2\Phi,\Phi\}, \tag{1}$$

$$(\partial_t + v_{ge}\partial_y)n + (v_{ne}-v_{ge})\partial_y\Phi = \{n,\Phi\}, \tag{2}$$

where $\{f,g\}=\hat{z}\times\boldsymbol{\nabla}f\cdot\boldsymbol{\nabla}g$ denotes the Poisson bracket. The first equation originates from the quasineutrality condition $\boldsymbol{\nabla}_\perp\cdot[n(\vec{v}_i-\vec{v}_e)]=0$ and the second one results from the electron continuity equation. Here, we have neglected the effect of temperature gradient and the collisional viscosity of the stress tensor. The system of equations (1) and (2) generalizes previous descriptions of the magnetic-curvature-driven flute instability[13] as it includes rigorously the diamagnetic drift, $v_{nj}=T_i/(eB_0L_n)$, the magnetic curvature drift, $v_{gj}=2T_j/(eRB_0)$, of both electron and ion fluids ($j=i,e$), and similar to Refs. 18 and 19, the finite ion Larmor radius effect which is described here by the term proportional to $\tau_i$ ($=T_i/T_e$) in the right-hand side (rhs) of Eq. (1). For details on the derivation of these equations, we refer to Refs. 13 and 17. In Eqs. (1) and (2) the electrostatic potential has been normalized by $T_e/e$, the time by the ion cyclotron frequency $\omega_{ci}$, the length scales by the ion Larmor radius $\rho=c_s/\omega_{ci}$ defined at the electron temperature (here $c_s^2=T_e/m_i$), and the density by the unperturbed density $n_0$. Furthermore, $v_g=v_{ge}+v_{gi}$. Linearizing Eqs. (1) and (2), one obtains the linear dispersion relation of the magnetic-curvature-driven flute modes given by

$$\omega_k = -\frac{k_y(v_{ni}-v_{ge})}{2}\left(1 \pm \epsilon\sqrt{1-\frac{k_{cr}^2}{k_\perp^2}}\right). \tag{3}$$

Here, $\epsilon\equiv(v_{ni}+v_{ge})/(v_{ni}-v_{ge})>0$ and $k_{cr}^2\equiv 4v_g(v_{ne}-v_{ge})/(v_{ni}+v_{ge})^2$ determines the critical for the development of the linear instability perpendicular wave number as the modes of finite poloidal wave number with $k_\perp \leqslant k_{cr}$ are linearly unstable.

## III. COUPLED DYNAMICS OF FLUTE MODE TURBULENCE AND LARGE-SCALE FLOWS

For the description of the dynamics of large-scale plasma flows that vary on a longer time scale compared to the small-scale fluctuations, a multiple scale expansion is usually employed assuming that there is a sufficient spectral gap separating the large- and the small-scale motions. In what follows, $[\tilde{\Phi}(\mathbf{r},t),\tilde{n}(\mathbf{r},t)]$ denote the small-scale fluctuations and $[\overline{n}(\mathbf{r},t),\overline{\Phi}(\mathbf{r},t)]$ the large-scale ones. By averaging Eqs. (1) and (2), we get

$$(\partial_t - v_{ni}\partial_y)\nabla_\perp^2\overline{\Phi} + v_g\partial_y\overline{n} = -\overline{R^\Phi} - \overline{R^n}, \tag{4}$$

$$(\partial_i + v_{ge}\partial_y)\overline{n} + (v_{ne}-v_{ge})\partial_y\overline{\Phi} = \overline{\{\tilde{n},\tilde{\Phi}\}}, \tag{5}$$

where $R^\Phi=\{\tilde{\Phi},\nabla^2\tilde{\Phi}\}$ is the standard Reynolds force due to the polarization drift nonlinearity and $R^n=\tau_i\{\overline{n},\nabla_\perp^2\tilde{\Phi}\}+\tau_i\{\nabla_\perp\overline{n},\nabla_\perp\tilde{\Phi}\}$ is the diamagnetic Reynolds force due to the fluctuating ion pressure and it is a finite ion Larmor radius effect. The equations above describe the formation of large-scale structures by the flute turbulence. This is ensured by the inverse cascade properties of the polarization drift nonlinearity.[20] However, the diamagnetic component of the polarization drift nonlinearity leads to direct energy cascade towards short scales.[15] Hence, the description of the formation of large-scale flows during the temporal evolution of flute turbulence is more complicated compared to the electrostatic drift wave turbulence.

Equations (1) and (2) conserve the following energy integral:

$$I_1 = \int\left\{\overline{n}^2 + \tilde{n}^2 - \frac{v_{ne}-v_{ge}}{v_g}[(\boldsymbol{\nabla}\overline{\Phi})^2 + (\boldsymbol{\nabla}\tilde{\Phi})^2]\right\}dx\,dy = \mathrm{const}, \tag{6}$$

which shows that the modulations of flows and turbulence are coupled and cannot be addressed in isolation. The propagation of the flute modes in weakly inhomogeneous media can be described by employing the wave kinetic equation for the wave-action density $N_k(r,t)$ in the $\mathbf{r}$-$\mathbf{k}$ space. The source of the slow spatial and temporal variations are the large-scale flows induced by the velocity and the density perturbations. The wave kinetic equation for the generalized wave action allows us to determine the modulations of $N_k(r,t)$ due to the mean flow. In the flute mode turbulence, we deal with two-field ($\Phi$ and $n$) perturbations and thus, we have to determine a proper combination of these fields in order to form the action invariant. The method of constructing the adiabatic invariant has been previously discussed in Ref. 21. By introducing the variable $\Psi_k = n_k + \alpha_k \Phi_k$ and following the standard method, the parameter $\alpha_k$ is determined to be $\alpha_k = k_\perp^2[(v_{ni} + v_{ge})/2v_g](1 - i\sqrt{k_{cr}^2/k_\perp^2 - 1})$ for the case under consideration. As a consequence, the generalized action density is found to be

$$N_k \equiv |\Psi_k|^2 = k_\perp^4 \left(\frac{v_{ni} + v_{ge}}{v_g}\right)^2 \left|\frac{k_{cr}^2}{k_\perp^2} - 1\right| |\Phi_k|^2. \quad (7)$$

The WKB-type wave kinetic equation which describes the evolution of the generalized action invariant $N_k(\mathbf{r}, t)$ in the flute mode turbulence due to the interaction between the mean flow and the small fluctuations is given by[13]

$$\frac{\partial N_k}{\partial t} + \frac{\partial N_k}{\partial_{\mathbf{r}}} \frac{\partial \omega_k^{NL}}{\partial_{\mathbf{k}}} - \frac{\partial \omega_k^{NL}}{\partial \mathbf{r}} \frac{\partial N_k}{\partial \mathbf{k}} = \gamma_k N_k - \Delta\omega_k N_k^2. \quad (8)$$

The nonlinear frequency is defined through $\omega_k^{NL} = \omega_k + \mathbf{k} \cdot \mathbf{V}_0$, where the nonlinear shift is due to the presence of the large-scale flows and it is given by $\mathbf{V}_0 = \mathbf{V}_\Phi + \mathbf{V}_n$, where

$$\mathbf{V}_\Phi = -\frac{1}{2}(\nabla \overline{\Phi} \times z), \quad \mathbf{V}_n = -\frac{\tau_i}{4}(\nabla \overline{n} \times z). \quad (9)$$

The nonlinear frequency shift $\Delta\omega_k$ in the rhs of Eq. (8) represents the part of the nonlinear interactions among the flute modes which balance the linear growth rate.

Considering small deviations of the spectrum function from the equilibrium, we may write the adiabatic action invariant as a sum of $N_k = N_k^0 + \widetilde{N}_k$, where $N_k^0 = \langle N_k \rangle$ describes the equilibrium part of the turbulent spectrum and $\widetilde{N}_k$ is the perturbed part. For the equilibrium part, we may consider a balance between the terms in the right-hand side of Eq. (8), which corresponds to the case of stationary turbulence and gives $N_0^k \simeq 2\gamma_k/\Delta\omega_k$. Using the standard quasilinear theory, the quasilinear equation for $N_k^0$ has the following form:

$$\frac{\partial N_k^0}{\partial t} - \left\langle \frac{\partial}{\partial \mathrm{r}}(\mathbf{k} \cdot \mathbf{V}_0) \frac{\partial \widetilde{N}_k}{\partial \mathbf{k}} \right\rangle = 0. \quad (10)$$

The perturbed density of the "quasiparticles" $\widetilde{N}_k$ can be calculated by the linearized wave kinetic equation, for a uniform equilibrium $\partial N_k^0/\partial \mathbf{r} = 0$, and becomes

$$\frac{\partial \widetilde{N}_k}{\partial t} + \frac{\partial}{\partial \mathbf{k}}(\omega_k + \mathbf{k} \cdot \mathbf{V}_0) \frac{\partial \overline{N_k}}{\partial_{\mathbf{r}}} - \frac{\partial}{\partial \mathbf{r}}(\omega_k + \mathbf{k} \cdot \mathbf{V}_0) \frac{\partial N_k^0}{\partial \mathbf{k}} = -\gamma_k \widetilde{N}_k. \quad (11)$$

In the local approximation, i.e., $\partial\omega_k/\partial \mathbf{r} = 0$, Eq. (11) can be solved by assuming that the large-scale variation of the action density is of the form $N_k \sim \exp[i\mathbf{qr} - i\Omega t]$. This yields the resonant part of the distribution

$$\widetilde{N}_k^{res} = \frac{\partial}{\partial \mathbf{r}}(\mathbf{k} \cdot \mathbf{V}_0) \frac{\partial N_k^0}{\partial \mathbf{k}} R(\Omega, q, \delta\omega_k). \quad (12)$$

Here $R$ is the response function defined by $R(\Omega, \mathbf{q}, \delta\omega_k) = i/(\Omega - \mathbf{q} \cdot \mathbf{V_g} + i\delta\omega_k)$, $\delta\omega_k$ is the total decorrelation frequency which includes the linear growth rate and a nonlinear shift and $\mathbf{V_g}$ is the group velocity defined by $\mathbf{V_g} = \partial\omega_k/\partial \mathbf{k}$. In a weakly nonlinear regime it is $R(\Omega, q, \delta\omega_k) \to \pi\delta(\Omega - \mathbf{q} \cdot \mathbf{V_g})$, while for a wide fluctuating spectrum $R(\Omega, \mathbf{q}, \delta\omega_k) \to 1/\delta\omega_k$. The broad spectrum of large-scale structures regulates the flute turbulence by the process of random shearing which is now understood to be a key mechanism that governs the self-regulative and saturation mechanism of the flute mode turbulence.[17]

The process of the random shearing of the flute turbulence can be expressed in terms of the diffusion of the stationary spectra in the $\mathbf{k}$ space. Indeed, substituting Eq. (12) into Eq. (10), the following diffusionlike equation for $N_k^0$ is obtained:

$$\frac{\partial N_k^0}{\partial t} - \frac{\partial}{\partial \mathbf{k}} D_{\mathbf{k}} \frac{\partial N_k^0}{\partial \mathbf{k}} = 0. \quad (13)$$

For the case of shearing by a zonal flow $(q_x \gg q_y)$, the diffusivity in the radial wave number is $D_{k_x} = (q_x^4 k_y^2/4)|\overline{\Phi'}|^2 R(\Omega, q_x, \delta\omega_k)$. Similarly, for the case of shearing by a streamer flow $(q_y \gg q_x)$ the diffusivity in the poloidal wave number is $D_{k_y} = (q_y^4 k_x^2/4)|\overline{\Phi'}|^2 R(\Omega, q_y, \delta\omega_k)$ as in Ref. 13. Here, $\overline{\Phi'} \equiv \overline{\Phi} + \tau_i \overline{n}/2$. In what follows, we focus on the dynamics and the properties of the large-scale flows in the presence of flute mode turbulence.

## IV. LONG TERM DYNAMICS OF LARGE-SCALE FLOWS

Calculating the averaged Reynold stress forces in Eqs. (4) and (5), we obtain the equations describing the evolution of the mean flows. First, we consider zonal flows with $q(q_x, q_y) = q(q_x, 0)$. The resulting equations become decoupled and can be written, similar to Ref. 13, as

$$\frac{\partial \overline{\Phi}_{q_x}}{\partial t} = \int k_x k_y \left(1 - \frac{v_{ni}}{2v_g} k_\perp^2\right) |\widetilde{\Phi}_k|^2 \, d^2k, \quad (14)$$

$$\frac{\partial \overline{n}_{q_x}}{\partial t} = 0. \quad (15)$$

The second term in the right-hand side of Eq. (14) is attributed to the ion diamagnetic drift and to the finite ion Larmor radius. As one may see, the ion diamagnetic effects may lead to the suppression of the zonal flow generation. Adding the equations above and using Eqs. (7) and (9), we get a relation which connects the zonal flow velocity with the spectra of the short-scale fluctuations,

$$\frac{\partial V_{0y}}{\partial t} = \frac{1}{2} \frac{\partial}{\partial x} \int k_x k_y \zeta(k_\perp) |\Psi_k|^2 d^2k. \quad (16)$$

Here $\zeta(k_\perp)$ is defined by

$$\zeta(k_\perp) = \frac{1}{2k_\perp^2} \frac{v_g v_{ni}}{(v_{ni} + v_{ge})^2} \left(\frac{2}{k_\perp^2} \frac{v_g}{v_{ni}} - 1\right) \left|\frac{k_{cr}^2}{k_\perp^2} - 1\right|^{-1}. \quad (17)$$

Closure conditions for the $N_k$ modulations in terms of the mean flow are provided by Eq. (8).

In the presence of a turbulent induced zonal flow ($\partial/\partial y = 0$ and $V_{0x}=0$), the resonant response of the spectrum modulation is given by

$$\overline{N}_k^{res} = \frac{\partial}{\partial x}(k_y V_{0y})\frac{\partial N_k}{\partial k_x}R(\Omega, q_x, \delta\omega_k). \quad (18)$$

Inserting Eq. (18) into Eq. (16) and assuming the large-scale variations to be of the form $V_{0y} \sim \exp[iq_x x]$, we obtain

$$\frac{\partial V_{0y}}{\partial t} = q_x^2 D_{xx} V_{0y}.$$

This equation determines the stability of the zonal flow since $\gamma_{zf} = q_x^2 D_{xx}$. The coefficient $D_{xx}$ is given by

$$D_{xx} = -\frac{1}{2}\int k_x k_y^2 \frac{\partial N_k}{\partial k_x}\zeta(k_\perp)R(\Omega, q_x, \delta\omega_k)d^2k. \quad (19)$$

As it turns out the zonal flow gets unstable when $D_{xx} > 0$. This instability can be interpreted as a result of the resonant interaction between zonal flow and the small-scale modulations of the turbulence. In the fluid dynamics the analogous mechanism of spontaneous excitation of large-scale structures from the small-scale turbulence is known as a negative eddy viscosity. The sign of $D_{xx}$ depends strongly on the sign of the product $k_x(\partial N_k^0/\partial k_x)(2v_g/v_{ni} - k_\perp^2)$ along the distribution in the **k** space.

In the flute turbulence, where it is usually $k_x(\partial N_k^0/\partial k_x) < 0$, the zonal flow may become unstable due to the contribution of the modes with $k_\perp^2 < 2v_g/v_{ni}$, according to the integral (19). A part of these modes is expected to be linearly unstable since it may have relatively small wave number, i.e., $k_\perp < k_{cr}$. However, when $v_{gj}/v_{nj} < \sqrt{3\tau_i^2 + 2\tau_i} - 2\tau_i$ (for $\tau_i < 2$), it turns out that $k_{cr}^2 > 2v_g/v_{ni}$ and subsequently the modes that are responsible for the (nonlinear) instability of the zonal flow may have rather significant contribution to the value of the integral (19). From the above, it follows that when $k_x(\partial N_k^0/\partial k_x) > 0$, it is more likely that the zonal flow is stable. It is worthwhile to point out here that this conclusion is qualitatively similar to that of the zonal flow stability in the drift wave turbulence.

For perturbations with $\Omega \ll q_x V_{gx}$, we can take into account the nonresonant response $\overline{N}_k^{(1)}$ of the turbulent spectra over the perturbations of the induced zonal flow. In this case the solution of the linearized wave kinetic equation (8) yields

$$\tilde{N}_k^{(1)} = k_y V_{0y}\left(\frac{\partial \omega_k}{\partial k_x}\right)^{-1}\frac{\partial N_k^0}{\partial k_x}.$$

Substituting the later expression into Eq. (16), we obtain the oscillation frequency of the zonal flow, $\Omega_{zf} \simeq -u_x q_x$, where $u_x$ is defined by

$$u_x = \frac{1}{2}\int k_x k_y^2 \left(\frac{\partial \omega_k}{\partial k_x}\right)^{-1}\frac{\partial N_k^0}{\partial k_x}\zeta(k_\perp)d^2k. \quad (20)$$

However, as the amplitude of the zonal flow grows, nonlinear effects become significant. Using the derived expression of $\tilde{N}_k^{(1)}$, we determine iteratively from Eq. (11), the next order nonlinear response $\tilde{N}_k^{(2)}$ for the nonresonant interactions,

$$\tilde{N}_k^{(2)} = \frac{1}{2}(k_y V_{0y})^2\left(\frac{\partial \omega_k}{\partial k_x}\right)^{-1}\frac{\partial}{\partial k_x}\left[\left(\frac{\partial \omega_k}{\partial k_x}\right)^{-1}\frac{\partial N_k^0}{\partial k_x}\right].$$

Including the total response, $\tilde{N}_k = \tilde{N}_k^{res} + \tilde{N}_k^{(1)} + \tilde{N}_k^{(2)}$ into Eq. (16), we obtain, similar to the case of the drift wave-zonal flow turbulence,[10] a nonlinear equation which describes the evolution of the zonal flow:

$$u_x \frac{\partial^2}{\partial x^2}V_{0y} + b_x \frac{\partial^2}{\partial x^2}V_{0y}^2 - D_{xx}\frac{\partial^3}{\partial x^3}V_{0y} = \frac{\partial}{\partial t}\frac{\partial}{\partial x}V_{0y}. \quad (21)$$

The coefficient $b_x$ in the nonlinear term is given by

$$b_x = \frac{1}{4}\int k_x k_y^3 \zeta(k_\perp)\left(\frac{\partial \omega_k}{\partial k_x}\right)^{-1}\frac{\partial}{\partial k_x}\left[\left(\frac{\partial \omega_k}{\partial k_x}\right)^{-1}\frac{\partial N_k^0}{\partial k_x}\right]d^2k. \quad (22)$$

It is interesting to see here that Eq. (21) admits localized solutions. Indeed, for the family of stationary solutions of the type $V_{0y}(x - u_{0x}t)$, Eq. (21) can be solved by integrating twice,

$$(u_{0x} + u_x)V_{0y} + b_x V_{0y}^2 = D_{xx}\frac{\partial}{\partial x}V_{0y} + C. \quad (23)$$

Considering now the boundary conditions $V_{0y} \to V_{1y}$, $V'_{0y} = 0$ for $x \to -\infty$ and $V_{0y} \to V_{2y}$, $V'_{0y} = 0$ for $x \to \infty$, we determine the integration constant $C = (u_{0x} + u_x)V_{1y} + b_x V_{1y}^2$ while $V_{2y} = -V_{1y} - (u_{0x} + u_x)/b_x$. Note that these boundary conditions correspond to the solitary wave solution with different asymptotic values. This solution is known as "switching" wave or "kink" soliton, in contrast to the solution with same asymptotic value which is called "bell" soliton.[10] The simplest solution of Eq. (23) is of the kink type and is given by

$$V_{0y} = \tfrac{1}{2}\{V_{1y} + V_{2y} + (V_{1y} - V_{2y})\tanh[x\, b(V_{1y} - V_{2y})/2D_{xx}]\}. \quad (24)$$

This solution describes the transient region between two different values of the flow. So, the cooperative effects of the wave motion, steeping, and instability give the possibility to the formation of stationary or moving kink solitons. The values of the parameters which determine the characteristic lengths of these structures are determined by the value of the group velocity and by the spectral density of the background fluctuations. The above simple analysis demonstrates the self-organization properties of the flute modes–zonal flow coupled system.

The corresponding evolution equations for the streamer flows $q(q_x, q_y) = q(0, q_y)$ become

$$(\partial_t - v_{ni}\partial_y)\partial_y^2 \overline{\Phi}_{qy} + v_g \partial_y \overline{n}_{qy} = -\partial_y^2 \int k_x k_y \zeta(k_\perp)|\Psi_k|^2 d^2k, \quad (25)$$

$$(\partial_t + v_{ge}\partial_y)\overline{n}_{qy} + (v_{ne} - v_{ge})\partial_y \overline{\Phi}_{qy} = 0. \quad (26)$$

The resonance response of the action invariant spectra over streamer flows is given by the right-hand side of Eq. (18) by applying a mutual permutation between $x$ and $y$. Assuming the large-scale variations to be of the form

$[\overline{n}(\mathbf{r},t),\overline{\Phi}(\mathbf{r},t)], \sim \exp[iq_y y - i\Omega_{st} t]$, we obtain the following relation which describes the stability and the oscillation frequencies of the streamer flow:

$$\Omega_{st} = \frac{iq_y^2 D_{yy}}{2} + \frac{q_y}{2}\left[(v_{ni} - v_{ge}) \pm \sqrt{(v_{ni} + v_{ge})^2\left(1 - \frac{k_{cr}^2}{q_y^2}\right) - q_y^2 D_{yy}^2 - 2iq_y D_{yy} v_g}\right]. \tag{27}$$

The coefficient $D_{yy}$ is given by the right-hand sides of Eq. (19) by applying a mutual permutation between $x$ and $y$. The real part of this relation corresponds to the frequency of the streamer flow $q_y$ and is in accordance to the dispersion relation of the linear flute modes. However, there are three additional terms attributed to the resonant interaction which can modify the stability of the streamer. We notice that, in absence of the drifts, the terms proportional to $D_{yy}$ in Eq. (27) give a similar result to that concerning the zonal flow stability.

The third term which appears due to the combined effect of the resonant interaction and the magnetic curvature drift can modify significantly the stability of the streamer especially when $\sqrt{v_g/q_y D_{yy}} \gg 1$. For simplicity, if we restrict ourselves here in the case of linear marginal stability for the flute streamer, we obtain a growth rate of the streamer given by

$$\gamma_{str} = \frac{q_y^{3/2}}{2}\sqrt{v_g D_{yy}},$$

which is attributed to the resonant interaction.

## V. SUMMARY AND DISCUSSION

The properties of the large-scale flows, developed and interacting in an electrostatic turbulent environment of the flute type, were investigated and determined by using a kinetic wave equation coupled with averaged fluid equations which describe the flute turbulence. The resonant interaction between the variations of the mean flow and the turbulent spectra may lead to the stabilization of the large-scale flows. The nonlinear evolution of the large flows can lead to the formation of stationary coherent structures in the transition layer between surfaces of different flow velocities, modifying significantly the transport properties of the turbulent plasma.

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