Magnetic-curvature-driven interchange modes in dusty plasmas

I. Sandberg[a] and P. K. Shukla
Institut für Theoretische Physik IV, Fakultät für Physik und Astronomie, Ruhr-Universität Bochum, D-44780 Bochum, Germany

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The magnetic-curvature-driven interchange mode instability of a weakly inhomogeneous dusty plasma is rigorously investigated. It is shown that the electric drift convection of the equilibrium dust charge density is a stabilizing factor for long wavelength interchange modes. In a fully nonlinear regime, the finite amplitude interchange modes may self-organize in the form of a dipolar vortex. The present results should be useful in the understanding of the properties of the interchange mode turbulence in nonuniform magnetized plasmas containing charged dust particles. © *2004 American Institute of Physics.* [DOI: 10.1063/1.1640621]

I. INTRODUCTION

The magnetic-curvature-driven interchange (flute) mode is a low-frequency ($\omega \ll \omega_{ci}$) unstable oscillation of a nonuniform magnetoplasma, which is driven by the combined action of the plasma density gradient and the curvature of the magnetic field line. The interchange mode instability originates from the difference between the curvature drift velocities of the electrons and ions, which in combination with a density perturbation leads to charge separation. In the flute limit (i.e., $k_{\parallel}=0$), electrons cannot cancel this charge separation, leading to the development of an electric field perpendicular to the magnetic field direction, that amplifies the initial perturbation which becomes linearly unstable. Rosenbluth *et al.*[1] showed that the diamagnetic drift effect influences the flute instability and under certain condition may lead to the plasma stabilization. The interchange instability tends to interchange "flux tubes" of different pressure causing convective transport and is considered to be one of the most dangerous instabilities in thermonuclear fusion machines and especially in the mirror machine. It is expected that any additional factor which influence the difference between the perturbed drifts of electrons and ions over the flute perturbations will considerably affect the plasma stability (see, for example, Ref. 2).

The nonlinear evolution of the convective flute instability may give rise to the formation of a random set of solitary two-dimensional dipolar vortices as shown in Ref. 3, which propagate across the external magnetic field direction. These vortex structures may enhance cross-field plasma particle transport, similar to convective cells in neutral fluids. However, recent studies which have been focused on the generation of large-scale zonal flows by the interchange mode turbulence (see, for example, Refs. 4–6), show that such flows may inhibit the radial transport in a magnetized electron–ion plasma.

During the last decade, there has been a rapidly growing interest in understanding the properties of low-frequency electrostatic oscillations and their excitations in plasmas that are contaminated by high-Z impurities or charged dust particulates. The presence of charged dust particulates gives rise to new dust-associated electrostatic and electromagnetic modes in magnetized dusty plasmas as shown in Ref. 7. The latter are composed of electrons, ions, and highly charged massive dust grains. Dust grains in plasmas are typically charged negatively due to the collection of electrons from the ambient plasma,[8,9] but under UV radiation dust can also be charged positively.[10]

For the case of a straight magnetic field line geometry, Shukla and Varma in Ref. 11 showed that the **E**×**B** convection of the equilibrium dust charge number density leads to the appearance of a dust convective cell mode [which is now known as the Shukla–Varma (SV) mode] in a nonuniform plasma, since here the divergence of the **E**×**B** particle flux is finite. In a later work,[12] the same authors studied the dusty plasma dynamics in the presence of a gravity force and reported that the dust gravity force may give rise to a Rayleigh–Taylor (RT) type dust instability, while Birk in Ref. 13 determined the onset criteria for the RT instability in a configuration where a heavy partially ionized dusty plasmas is supported by a lighter one.

Previous investigations on the interchange mode instability in a dusty magnetoplasma[14,15] adopted the concept of a real or effective "gravitational" field which drives the instability. This description may be suitable for the ion RT mode in dusty plasmas but not for the magnetic-curvature-driven interchange mode since it neglects the electron curvature effects which are of the same importance as that of ions for $T_e \sim T_i$ (see, for example, Ref. 16). We may refer, for instance, to Ref. 6, where the interchange instability is considered to be driven solely by the electron curvature drift. Hence, the previous descriptions[14,15] have to be reconsidered to account for the magnetic-curvature-driven interchange (flute) instability in a systematic manner.

To the best of our knowledge, there has not been any rigorous study of the interchange mode instability in nonuniform dusty magnetoplasmas driven by the intrinsic curvature of the magnetic field configuration. Since the presence of

[a]Present address: Section of Astrophysics, Astronomy and Mechanics, Observatory, Department of Physics, Aristotle University of Thessaloniki, 54124 Thessaloniki, Greece; electronic mail: sandberg@astro.auth.gr

charged dust particles introduces new wave modes (e.g., the Shukla–Varma mode) in a nonuniform magnetoplasma, we expect that it will also influence the growth rate and threshold of the interchange mode instability. This consideration is of particular interest to space[17] as well as to magnetoinertial and magnetically confined fusion plasmas where charged dusty particulates are frequently encountered.[18] Hence, a complete knowledge of interchange mode turbulence and associated transport properties of a nonuniform dusty magnetoplasma is of practical importance to both cosmic and laboratory environments.

In the present paper, the electrostatic interchange mode instability is studied by considering rigorously the magnetic curvature-driven effects in the dynamics of both ions and electrons. Furthermore, looking for linear and nonlinear wave phenomena on a timescale much shorter than the dust plasma and dust gyrofrequencies, we ignore the dynamics of massive dust grains. However, since the presence of immobile charged dust grains gives rise to a finite value of the $\mathbf{E}\times\mathbf{B}$ particle flux in a nonuniform magnetoplasma, we expect that it will also affect the interchange mode instability. Here, $\mathbf{E}$ is the electric field perturbation and $\mathbf{B}$ represents the inhomogeneous external magnetic field.

The manuscript is organized in the following fashion. In Sec. II, we derive a system of two coupled nonlinear equations which governs the dynamics of interchange modes in a nonuniform dusty magnetoplasma. The linear instability of these modes is examined in Sec. III. Possible stationary solutions of the nonlinear equations are presented in Sec. IV. We find that both the polarity and the speed of an interchange dipolar vortex are altered when charged dust components are present in an electron–ion plasma that is nonuniform and magnetized. Section V contains conclusions and discussion.

II. MODEL EQUATIONS

We consider a weakly inhomogeneous dusty plasma of slab geometry immersed in an external curved magnetic field $\mathbf{B}=B(x)\mathbf{b}$. The magnitude $B(x)$ and the direction of the magnetic field can be written as $B(x)=B_0(1-x/R)$ and $\mathbf{b}=\hat{z}-(z/R)\hat{x}$, respectively, where R describes the effective radius of curvature of the magnetic field lines.

At equilibrium, we have $n_{i0}(x)=n_{e0}(x)+\epsilon Z_d(x)n_{d0}(x)$, where n_{l0} is the unperturbed number density of the particle species l (l equals i for ions, e for electrons, and d for dust grains), $\epsilon=1$ for negatively charged dust, $\epsilon=-1$ for positively charged dust, and $Z_d>0$ is the dust charge. The unperturbed profile of the charge density of the $j=i,e$ plasma species is assumed to be of the form $n_{j0}(x)=n_{j0}\exp(-x/L_j)$, with the characteristic inhomogeneity scalelength L_j being smaller than the curvature radius $|L_j|<R/2$. In the following, we will consider for the ions that $L_i>0$, viz. $n_i'(x)<0$. The characteristic inhomogeneity scalelength of the dust charge density $L_d=d\ln[Z_d(x)n_{d0}(x)]/dx$ comprises both the dust charge and the dust density inhomogeneities and can be given in terms of the equilibrium plasma densities and inhomogeneity scale lengths by

$$\frac{\epsilon Z_d(x)n_{d0}(x)}{L_d}=\frac{n_{i0}(x)}{L_i}-\frac{n_{e0}(x)}{L_e}. \tag{1}$$

Dust charging effects and temperature inhomogeneity will be neglected.

To describe flute-like perturbations, we use the two-fluid macroscopic equations for the electrons and ions, i.e.,

$$\partial_t N_j+\nabla\cdot(N_j\mathbf{V}_j)=0, \tag{2}$$

and

$$m_jN_j(\partial_t+\mathbf{V}_j\cdot\nabla)\mathbf{V}_j=N_jq_j(\mathbf{E}+\mathbf{V}_j\times\mathbf{B})-\nabla P_j-\nabla\cdot\Pi^j, \tag{3}$$

with the scalar pressure given by $P_j=N_jT_j$. In Eq. (3) we have assumed a collisionless magnetized plasma while the viscosity tensor Π^j contains only gyroviscosity components. In a strong magnetic field, the components of the tensor are given by[19]

$$\pi_{xx}=-\pi_{yy}=-\frac{1}{2}\frac{P_j}{\omega_{cj}}W_{xy}$$

and

$$\pi_{xy}=\pi_{yx}=\frac{1}{4}\frac{P_j}{\omega_{cj}}(W_{xx}-W_{yy}),$$

where $\omega_{cj}=q_jB/m_j$ and the rate-of-strain tensor for the case under consideration can be written in the form

$$W_{\alpha\beta}=\frac{\partial V_\alpha}{\partial x_\beta}+\frac{\partial V_\beta}{\partial x_\alpha}.$$

However, in the limit of inertialess electrons the left-hand side in Eq. (3) for $j=e$ is zero, and $\Pi^e=0$. In what follows, we consider a flute type ($\partial_z\ll\nabla_\perp$) electrostatic perturbation (i.e., $\mathbf{E}=-\nabla_\perp\Phi$) of a low-$\beta$ plasma with the frequency ω much smaller than the ion gyrofrequency, i.e., $\omega\ll\omega_{ci}$. Here, β is the ratio between the plasma kinetic energy and the magnetic energy. By separating the velocity and density in Eqs. (2) and (3), into their equilibrium and perturbed parts, viz., $\mathbf{V}_j=\mathbf{v}_{j0}+\mathbf{v}_j$, $N_j=n_{j0}+n_j$, we express the perpendicular component of the electron fluid velocity as $\mathbf{V}_{\perp e}=\mathbf{V}_{De}+\mathbf{v}_E$, where the diamagnetic and electric field drifts are, respectively,

$$\mathbf{V}_{Dj}=\frac{\mathbf{b}\times\nabla P_j}{q_jN_jB}=\mathbf{v}_{D0j}+\mathbf{v}_{Dj},$$

and

$$\mathbf{v}_E=\frac{\mathbf{b}\times\nabla\Phi}{B}.$$

The perpendicular component of the ion fluid velocity is determined using an iterative method based on the smallness of the frequency of the perturbation with respect to the ion gyrofrequency, and is of the form

$$\mathbf{V}_{\perp i}=\mathbf{V}_{Di}+\mathbf{v}_E+\mathbf{v}_{pi}+\mathbf{v}_{\pi i}, \tag{4}$$

where the ion polarization drift is

$$\mathbf{v}_{pi}=\frac{m_i\mathbf{b}}{eB}\times\left(\frac{\partial}{\partial t}+\mathbf{V}_i\cdot\nabla\right)\mathbf{V}_i,$$

and the ion drift velocity due to the stress tensor reads

$$\mathbf{v}_{\pi i}=\frac{\mathbf{b}\times\nabla\cdot\Pi^i}{q_j N_j B}.$$

Considering quasineutral perturbations ($n=n_e=n_i$), we obtain from $\nabla\cdot[n(\mathbf{v}_i-\mathbf{v}_e)]=0$ the following equation:

$$\mathbf{v}_g\cdot\nabla n+n_{i0}\nabla_\perp\cdot(\mathbf{v}_{pi}+\mathbf{v}_{\pi i})+\epsilon\nabla\cdot(n_{d0}Z_d\mathbf{v}_E)=0. \quad (5)$$

The first term in Eq. (5) describes the difference between the curvature drifts of ions and electrons, and is given by

$$\mathbf{v}_g\cdot\nabla n=n_{i0}\nabla\cdot\mathbf{v}_{Di}-n_{e0}\nabla\cdot\mathbf{v}_{De}=\frac{T_i+T_e}{e}\left(\nabla\times\frac{\mathbf{b}}{B}\right)\cdot\nabla n. \quad (6)$$

This is the responsible term for the development of the interchange instability and in various treatments, e.g., Ref. 15, it is modelled by a gravitational drift term caused by a fictional ‘‘gravity’’ force which is inserted in the fluid equation of motion for the ions. However, since the curvature drift of electrons can be of the same order as that of ions for $T_e\sim T_i$, we have included both of these effects in our consideration.

In the second term of Eq. (5), we can derive an analytic expression for the perpendicular component of the ion fluid velocity provided that the velocity components $\mathbf{v}_{pi}$, $\mathbf{v}_{\pi i}$ can be regarded as small corrections with respect to $\mathbf{v}_E$ and $\mathbf{V}_{Di}$. This is true for slowly varying perturbations (compared to ω_{ci}) with characteristic wavelength in the perpendicular direction much shorter than the length scale of the background quantities (WKB approximation) and much larger than the ion Larmor radius (fluid approximation). Hence, we evaluate the convective nonlinear terms and the velocities $\mathbf{v}_{pi}$ and $\mathbf{v}_{\pi i}$ to the leading order by setting $\mathbf{V}_{\perp i}\rightarrow\mathbf{V}_{Di}+\mathbf{v}_E$, $B\rightarrow B_0$ and $\mathbf{b}\rightarrow\hat{z}$. In the limit of long wavelength perturbations, viz., $\rho_i^2\nabla_\perp^2\ll 1$, with $\rho_i^2\nabla_\perp\ll R\omega/\omega_{ci}$ the second term in Eq. (5) can be written in the form

$$\nabla_\perp\cdot(\mathbf{v}_{pi}+\mathbf{v}_{\pi i})=\nabla_\perp\left\{\frac{1}{\omega_{ci}}\left(\frac{\partial}{\partial t}+\mathbf{v}_E\cdot\nabla\right)[\hat{z}\times(\mathbf{v}_E+\mathbf{V}_{D\,i})]\right\}$$
$$=-\frac{1}{\omega_{ci}B_0}\nabla_\perp\cdot\left\{\frac{\partial\nabla_\perp\Phi}{\partial t}+\frac{T_i}{eB_0 n_{i0}}\{N_i,\nabla_\perp\Phi\}+\frac{1}{B_0}\{\Phi,\nabla_\perp\Phi\}\right\}, \quad (7)$$

where $\{f,g\}=\hat{z}\times\nabla f\cdot\nabla g$ denotes the Poisson bracket (or the Jacobian). As one can see the convective diamagnetic contributions are exactly cancelled by the stress tensor contribution. This well known result is justified from the fact that diamagnetic drift is not a particle drift and thus cannot transfer any information by convection (see, for example, Ref. 16).

The last term in Eq. (5) appears due to the presence of charged dust particles in a nonuniform magnetoplasma, and it describes the electric field convection of the equilibrium charge density of the dust. It can be written in the form

$$\epsilon\nabla\cdot(Z_d n_{d0}\mathbf{v}_E)=\epsilon\mathbf{v}_E\cdot\nabla(Z_d n_{d0})+\epsilon Z_d n_{d0}\left(\nabla\times\frac{\mathbf{b}}{B}\right)\cdot\nabla\Phi, \quad (8)$$

which includes both the dust charge density inhomogeneity and the magnetic curvature effects.

Using the relation $\nabla\times\mathbf{b}/B=-2/(B_0R)\hat{y}$, we substitute (6) and (7) into Eq. (5) to obtain

$$\frac{1}{\omega_{ci}}\left(\frac{\partial}{\partial t}-\frac{T_i}{eB_0L_i}\frac{\partial}{\partial y}\right)\nabla_\perp^2\Phi+\frac{2}{R}\frac{T}{e}\frac{\partial n}{\partial y}+\epsilon Z_d n_{d0}\left(\frac{2}{R}-\frac{1}{L_d}\right)\frac{\partial\Phi}{\partial y}$$
$$=\frac{T_i}{eB_0 n_{i0}\omega_{ci}}\operatorname{div}\{\nabla_\perp\Phi,n\}+\frac{1}{B_0\omega_{ci}}\{\nabla_\perp^2\Phi,\Phi\}. \quad (9)$$

This equation combined with the electron continuity equation for an inhomogeneous plasma immersed in a curved magnetic field

$$\left(\frac{\partial}{\partial t}+\frac{2T_e}{B_0Re}\frac{\partial}{\partial y}\right)n+\frac{n_{e0}}{B_0}\left(\frac{1}{L_e}-\frac{2}{R}\right)\frac{\partial\Phi}{\partial y}=\frac{1}{B_0}\{n,\Phi\} \quad (10)$$

forms a closed system of equations. Here, T is the sum of the electron and ion temperatures. The system of Eqs. (9) and (10) generalizes for dusty plasmas the description of the interchange mode instability[3,4] including rigorously the magnetic field curvature effect. Through the next sections, we normalize the electrostatic potential by T_i/e, the length by the ion Larmor radius $\rho_i=v_{Ti}/\omega_{ci}$, the time by ω_{ci}, and the densities (temperatures) with respect to the ion density n_{i0} (temperature T_i). However, we keep, for simplicity, the same notations as before.

III. INTERCHANGE INSTABILITY IN DUSTY PLASMAS

Assuming small perturbations of the form $(n,\Phi)\sim\exp(-i\omega t+i\mathbf{k}_\perp\cdot\mathbf{r})$, we linearize the system of Eqs. (9) and (10) by discarding the nonlinear terms in the right-hand side. Fourier transforming the resultant equations and combining them, we obtain a quadratic dispersion relation which has the following normalized (by ω_{ci}^{-1}) solution:

$$\omega=\tfrac{1}{2}[\omega_{\mathrm{SV}d}+\omega_{*i}+\omega_{De}\pm\sqrt{(\omega_{\mathrm{SV}d}+\omega_{*i}-\omega_{De})^2-4\omega_{\mathrm{SV}e}\omega_D}]. \quad (11)$$

Here we have introduced the notations

$$\omega_{*i}=-\frac{k_y}{L_i}<0,\quad \omega_{Dj}=\frac{2k_yT_j}{R}>0,$$
$$\omega_{\mathrm{SV}l}=\frac{k_y\epsilon Z_l n_{l0}}{k_\perp^2}\left(\frac{1}{L_l}-\frac{2}{R}\right),$$

for the normalized ion diamagnetic drift frequency, the curvature drift frequency and a generalization of the Shukla–Varma (SV) frequency (with $\epsilon Z_e=1$) in the presence of a curved magnetic field, respectively. In the absence of curvature effects (viz., $R\rightarrow\infty$), Eq. (11) gives a stable mode with frequency given by the sum of the ion diamagnetic and SV frequencies, i.e., $\omega=\omega_{\mathrm{SV}d}+\omega_{*i}$. Equation (11) recovers the well-known criterion for the interchange instability of an electron–ion plasma when dust is absent ($\omega_{SVd}=0$, $\omega_{SVe}\neq 0$) and when the ion diamagnetic drift dominates over the electron curvature drift. However, in the presence of charged dust grains and a curved magnetic field, Eq. (11) describes an unstable dust interchange mode only when the electron density varies as the ion one, viz., $L_e>0$. In what follows, we will consider that $L_e>0$, noting that the inhomogeneity scalelength of the charge density of the dust L_d can acquire both signs according to Eq. (1).

Provided that the instability criterion holds, $2\sqrt{\omega_{SVe}\omega_D}\geqslant|\omega_{SVd}+\omega_{*i}-\omega_{De}|$, the frequency of the dust interchange mode will be given by half of the sum of the ion diamagnetic drift, the electron curvature drift and the SV frequency.

The source of the interchange mode instability is the difference in the curvature drifts of ions and electrons, which in combination with a perturbation leads to a charge separation. In the flute limit, electrons cannot cancel this charge separation, leading to the development of an electric field perpendicular to the plane of the density gradient and magnetic field.

However, the electric field ($\mathbf{E}\times\mathbf{B}$) convection of the equilibrium dust charge density [cf. Eq. (8)], modifies essentially the instability threshold, the frequency and the growth rate of the interchange instability.

A rigorous analysis of the dispersion relation shows that necessary condition for the development of the interchange instability in dusty plasmas is

$$f_d>-\frac{4Tn_{e0}}{R}\left(\frac{1}{L_e}-\frac{2}{R}\right)\left(\frac{1}{L_i}+\frac{2T_e}{R}\right)^{-1}, \tag{12}$$

where the parameter $|f_d|<1$ is defined through

$$f_d=\epsilon Z_d n_{d0}\left(\frac{1}{L_d}-\frac{2}{R}\right)\simeq\frac{\epsilon Z_d n_{d0}}{L_d}.$$

In absence of dust grains we have $f_d=0$, otherwise we can have both positive and negative values for f_d depending on the density profiles of the ions, electrons and the dust charge density.

Provided that the inequality (12) holds, the fastest growing dust flute mode occurs for the perpendicular wave number

$$k_{\perp\max}^4=f_d^2\left(\frac{1}{L_i}+\frac{2T_e}{R}\right)^{-2}, \tag{13}$$

and has (maximum) growth rate

$$\gamma_{\max}^2=\sin^2\theta\left[\frac{2T(1-\epsilon Z_d n_{d0})}{R}\left(\frac{1}{L_e}-\frac{2}{R}\right)-\frac{|f_d|-f_d}{2}\left(\frac{1}{L_i}+\frac{2T_e}{R}\right)\right]. \tag{14}$$

Here, θ denotes the propagation angle with respect to the x-axis of the plasma inhomogeneity. It is also interesting to note here that the real part of the frequency of the most unstable interchange mode is $\omega_{\max}=\omega_{De}(k_{\perp\max})$ in dusty plasmas with $f_d>0$ and $\omega_{\max}=\omega_{*i}(k_{\perp\max})$ in dusty plasmas with $f_d<0$.

The spectra of unstable convective flute modes in a dusty magnetoplasma are now given by $k_{\perp -}<k_\perp<k_{\perp +}$, where

$$k_{\perp\pm}^2=k_{\perp 0}^2\left(1\pm\sqrt{1-\frac{k_{\perp\max}^4}{k_{\perp 0}^4}}\right), \tag{15}$$

and

$$k_{\perp 0}^2=\left[\frac{4Tn_{e0}}{R}\left(\frac{1}{L_e}-\frac{2}{R}\right)+f_d\left(\frac{1}{L_i}+\frac{2T_e}{R}\right)\right]\left(\frac{1}{L_i}+\frac{2T_e}{R}\right)^{-2}. \tag{16}$$

Contrary to the electron–ion plasma case where $k_{\perp -}=0$, the presence of (negative or positive) charged dust particles leads to the suppression of interchange modes with large wavelength, $k_\perp<k_{\perp -}$. The width of the spectra, defined here by $\Delta k_\perp^2\equiv k_{\perp +}^2-k_{\perp -}^2$ is given by

$$\Delta k_\perp^2=\frac{8Tn_{e0}}{R}\left(\frac{1}{L_e}-\frac{2}{R}\right)\left(\frac{1}{L_i}+\frac{2T_e}{R}\right)^{-2}\times\sqrt{1+\frac{f_d}{2}\frac{R}{Tn_{e0}}\left(\frac{1}{L_e}-\frac{2}{R}\right)\left(\frac{1}{L_i}+\frac{2T_e}{R}\right)^{-1}}, \tag{17}$$

and it is expected to be wider in the presence of positive dust grains since it is proportional to the electron density $n_{e0}=1-\epsilon Z_d n_{d0}$.

In Fig. 1, we plot the normalized growth rate (upper panel) and the real frequency (lower panel) of the interchange mode vs the normalized $k_\perp$ for different sign of the dust charge density but for equal inhomogeneity scale lengths. We see that the presence of charged dust suppresses completely the long wavelength modes with $k_\perp\leqslant k_{\perp -}$, leading to the appearance of a finite critical value $k_{\perp\max}$ which corresponds to the wave number of the most unstable flute mode. When the charge inhomogeneity scale lengths L_l's of all the plasma species are equal, the growth rate and the width of the spectra of unstable interchange modes is higher in presence of positive dust grains ($\epsilon=-1$) compared to that in presence of negative dust grains ($\epsilon=1$).

In the upper panel of Fig. 2, we present the normalized growth rate vs the normalized $k_\perp$ for different values of negative dust charge density by keeping the same inhomogeneity scale length for all the species. However, in the lower panel of the same figure we plot the normalized growth rate by keeping fixed the ion inhomogeneity scale length and choosing various combination for the dust charge density and the rest inhomogeneity scale sizes which satisfy Eq. (1). As it was expected from Eqs. (13)–(16), the growth rate and the spectra of unstable interchange modes depend significantly on the density profiles of the plasma species and the dust charge.

IV. INTERCHANGE DIPOLAR VORTICES

The nonlinear terms in the left-hand side of Eqs. (9) and (10) are expected to convert the dust-interchange perturbations into an array of localized vortices which flows across the magnetic field in a similar way as in an electron–ion plasma.[3] In what follows, we consider without loss of generality that $R\gg|L_j|$. Seeking for a steady state solution of the

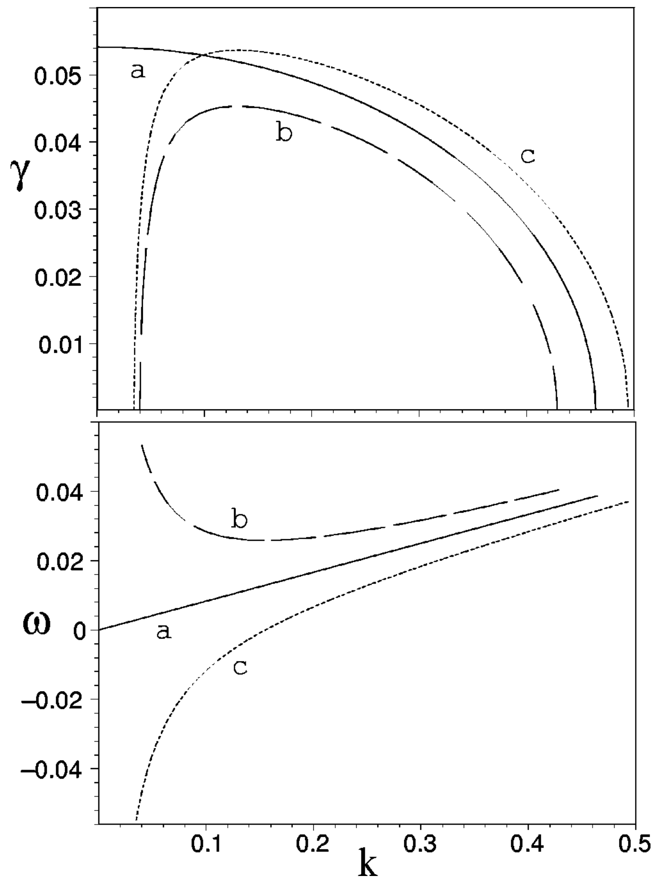

FIG. 1. The normalized growth rate (upper panel) and normalized frequency (lower panel) of the flute unstable mode vs the normalized perpendicular wave number for different dust charge densities: (a) $\epsilon Z_d n_{d0}=0$ (solid line), (b) $\epsilon Z_d n_{d0}=0.3$, negative charged dust (dashed line), and (c) $\epsilon Z_d n_{d0}=-0.3$, positive charged dust (dotted line). The other parameters are kept fixed: $T_e=10$, $R=100$, $L_l=30$, and $\theta=\pi/2$. All normalizations are explained in the text.

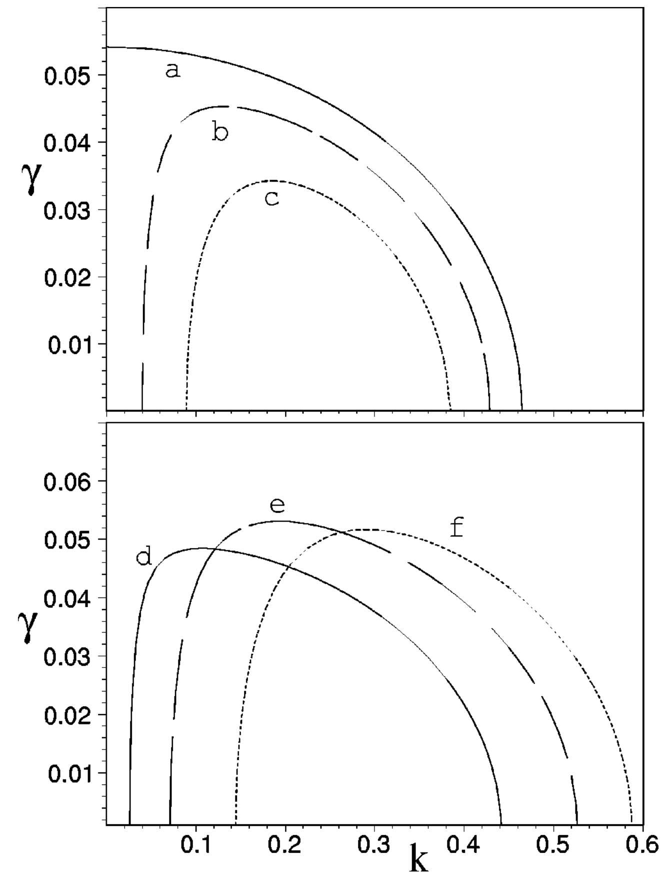

FIG. 2. The normalized growth rate of the flute unstable mode vs the normalized perpendicular wave number. (i) For different dust charge densities and fixed $L_d=30$ (upper panel); (a) $\epsilon Z_d n_{d0}=0$ (solid line), (b) $\epsilon Z_d n_{d0}=0.3$ (dashed line), and (c) $\epsilon Z_d n_{d0}=0.6$ (dotted line). (ii) For different charge inhomogeneity scale lengths (lower panel); (d) $\epsilon Z_d n_{d0}=0.2$, $L_d=L_e=30$ (solid line), (e) $\epsilon Z_d n_{d0}=0.2$, $L_d=-42$, $L_e=21$ (dashed line), (f) $\epsilon Z_d n_{d0}=-0.2$, $L_d=8.4$, $L_e=21$ (dotted line). The other parameters are kept fixed: $T_e=10$, $R=100$, $L_i=30$, and $\theta=\pi/2$. All normalizations are explained in the text.

above system of nonlinear equations which describes a perturbation travelling across the magnetic field, we consider solutions of the form $\Phi(x,y,t)=\Phi(x,y')$ and $n(x,y,t)=n(x,y')$. Here, $y'=y-Ut$ and U (=constant) is the vortex translational speed. Furthermore, employing a polar coordinate system with $[r=\sqrt{x^2+y'^2},\ \theta=\tan^{-1}(y'/x)]$, and working in an analogy with Ref. 3, we divide the $x-y'$ plane into an inner ($r\leqslant r_0$) and an outer region ($r\geqslant r_0$). Assuming a linear relation between n and Φ we obtain $n=(n_{e0}/U'L_e)(1-2L_e/R)\Phi$, where $U'=U-2T_e/R$, from the normalized form of Eq. (9), which allows us to rewrite the vorticity Eq. (8) in a compact form. We then seek for a solution Φ which is continuous at $r=r_0$ up to second order derivative and vanish at infinity. Following standard procedures,[20–22] we then obtain the dipolar vortex solution

$$\Phi_I(r,\theta)=-\frac{s^2}{p^2}\frac{J_1(pr)}{J_1(pr_0)}v r_0\cos\theta+\left(1+\frac{s^2}{p^2}\right)v r\cos\theta \quad \text{for} \quad r\leqslant r_0,$$

$$\Phi_O(r,\theta)=\frac{K_1(sr)}{K_1(sr_0)}v r_0\cos\theta \quad \text{for} \quad r\geqslant r_0 \tag{18}$$

for the inner and outer regions, respectively. Here, $J_m(pr)$ and $K_m(sr)$ are the Bessel and Hankel functions of the mth order. The vortex solution is finite and smooth through the (x,y') plane, and describes a pair of vortices with opposite polarity propagating with speed U along the $\hat{y}$ direction. Using Eq. (1), we find that the condition for the existence of the dipolar vortex solution can be written in the form

$$s^2=\left[\frac{2T}{RU'}\left(\frac{1}{L_i}-f_d\right)-f_d\right]\left(U+\frac{1}{L_i}\right)^{-1}>0, \tag{19}$$

where the rest of the parameters are determined by

$$v=\left(U+\frac{1}{L_i}\right)\left[1+\frac{1}{U'}\left(\frac{1}{L_i}-f_d\right)\right]^{-1}$$

$$\text{and}\ \frac{J_2(pr_0)}{J_1(pr_0)}=-\frac{p}{s}\frac{K_2(sr_0)}{K_1(sr_0)}. \tag{20}$$

An analysis of Eq. (19) reveals that U is restricted by the following intervals:

$$U < -\frac{1}{L_i} \quad \text{or}$$
$$\min\left\{\frac{2T_e}{R}, \frac{2T}{RL_i f_d} - \frac{2}{R}\right\} < U < \max\left\{\frac{2T_e}{R}, \frac{2T}{RL_i f_d} - \frac{2}{R}\right\} \tag{21}$$

in dusty plasmas with $f_d \geqslant 0$ and within

$$U > \frac{2T_e}{R}$$
$$\text{or} \quad \min\left\{-\frac{1}{L_i}, -\frac{2T}{RL_i|f_d|} - \frac{2}{R}\right\} < U < \max\left\{-\frac{1}{L_i}, -\frac{2T}{RL_i|f_d|} - \frac{2}{R}\right\} \tag{22}$$

in dusty plasmas with $f_d < 0$.

An inspection of the above inequalities shows that the single inequalities in the left-hand side of Eqs. (21) and (22) are outside the interval of the phase velocities of the linear modes propagating in the y direction. In contrary, the double inequalities in the right-hand side of Eqs. (21) and (22) are in the neighborhood of the fastest growing interchange mode and always inside the phase velocities interval of the linear modes. Thus, we have to discard the velocity intervals in the right-hand side of Eqs. (21) and (22), since any stationary vortex structure propagating in these intervals will be oscillating and contradict to the solution given by Eq. (18).

In conclusion, the interchange mode dipolar vortex in dusty plasmas propagates in the opposite direction to the phase velocity of the most unstable interchange mode with velocity bounded by $U < -1/L_i$ in dusty plasmas with $f_d \geqslant 0$ and $U > 2T_e/R$ in dusty plasmas with $f_d < 0$.

Finally, we note that Eqs. (8) and (9) may also admit a vortex chain solution.[23]

V. CONCLUSIONS

The interchange flute mode instability of a nonuniform dusty magnetoplasma has been investigated by considering rigorously the curvature driven effects in the dynamics of both the electrons and ions. It has been found that the electric drift convection of the equilibrium inhomogeneous dust charge density stabilizes the long wavelength interchange modes, leading to the appearance of a maximally growing dust flute mode of finite wavelength. The instability threshold and the spectrum of the interchange modes depend on the dust charge and on the density profiles of the plasma species. The nonlinear evolution of the convective dust-flute instability may lead to the formation of a dipolar vortex whose properties are different from those in an electron–ion plasma without dust. The present results should help to understand the properties of low-frequency interchange mode turbulence and associated coherent structures in cosmic and laboratory dusty plasmas containing nonuniform magnetic fields and equilibrium density inhomogeneities.

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