Electron-cyclotron wave scattering by edge density fluctuations in ITER

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The effect of edge turbulence on the electron-cyclotron wave propagation in ITER is investigated with emphasis on wave scattering, beam broadening, and its influence on localized heating and current drive. A wave used for electron-cyclotron current drive (ECCD) must cross the edge of the plasma, where density fluctuations can be large enough to bring on wave scattering. The scattering angle due to the density fluctuations is small, but the beam propagates over a distance of several meters up to the resonance layer and even small angle scattering leads to a deviation of several centimeters at the deposition location. Since the localization of ECCD is crucial for the control of neoclassical tearing modes, this issue is of great importance to the ITER design. The wave scattering process is described on the basis of a Fokker–Planck equation, where the diffusion coefficient is calculated analytically as well as computed numerically using a ray tracing code. © *2009 American Institute of Physics*. [doi:10.1063/1.3264105]

I. INTRODUCTION

In the electron-cyclotron resonance heating (ECRH) system planned for ITER,[1] the wave beam is expected to propagate over a large distance before it reaches the resonance. In order to accomplish localized absorption and current drive, the beam is launched with parameters such that the minimum width ($\approx$2 cm) occurs close to the resonance. However, the propagation path crosses the plasma edge, where density fluctuations can be rather large and may deflect the wave. The amplitude of such fluctuations may be 10%–50% of the local density,[2] whereas the correlation length scales as $5-10\rho_s$ (ρ_s is the ion gyroradius).[3,4] For ITER parameters $\rho_s \approx 0.2$ cm at the edge, which implies length scales between 1 and 2 cm (or 0.5%–1% of the minor radius). In addition, the total region of edge turbulence may be as broad as 10% of the minor radius.

Since the beam needs to travel several meters from the edge region to the resonance, even a small deflection could lead to a considerable deviation (of several centimeters) from the location of intended power deposition. This effect has a different consequence compared with the normal refraction in inhomogeneous plasma. Although the latter can potentially be larger (in case the wave is not injected in the direction of the background density gradient), it always generates an effect in the same direction, so that a small change in the injection angle can be compensative. Turbulent eddies, however, scatter the beam randomly, leading to a diffractivelike broadening of the beam rather than a shift in its position.

Therefore, it appears that the effect of edge turbulence can be sizable enough to reduce the efficiency of electron-cyclotron (EC) beams for the major issue of neoclassical tearing mode (NTM) stabilization in ITER.[5,6] Regarding specific tokamak experiments, the problem of electromagnetic wave scattering by density fluctuations has been addressed in the past for lower-hybrid waves in ASDEX,[7] JET,[8] and FTU,[9] as well as for EC waves in JET.[10]

In this paper we present a statistical model for the wave scattering based on the Fokker–Planck (FP) equation, where the diffusion coefficient is estimated analytically as well as calculated from the numerical solution of the exact ray tracing equations for many different initial conditions. We estimate the broadening of the wave beam under ITER conditions and show that the design should consider this limitation.

II. STATISTICAL MODEL OF EC WAVE SCATTERING

Consider the propagation of an EC wave in inhomogeneous anisotropic plasma. In the short-wavelength limit, the propagation is described by the ray equations[11]

$$\frac{d\boldsymbol{r}}{dt} = \frac{\partial \omega}{\partial \boldsymbol{k}}, \quad \frac{d\boldsymbol{k}}{dt} = -\frac{\partial \omega}{\partial \boldsymbol{r}}, \tag{1}$$

where $\boldsymbol{r}$ and $\boldsymbol{k}$ are the position and the wave vector and ω is the wave frequency, which is determined by the specific dispersion relation. Here, the case of the perpendicular O-mode propagation in ITER is considered

$$\omega^2 = \omega_p^2 + c^2 k^2, \tag{2}$$

where ω_p is the plasma frequency.

In the course of its propagation through the plasma edge, the ray may encounter a series of density fluctuations, also known as "blobs"[12,13] [see Fig. 1(a)]. Assuming that the wave scattering is random, where the dimensions and amplitudes of the blobs follow statistics relevant to edge turbulence, the angle by which the wave is deflected follows a statistical rather than a deterministic law. Therefore, it is necessary to introduce a probability distribution function $F(\alpha, l)$ for the description of the propagation in the turbulent envi-

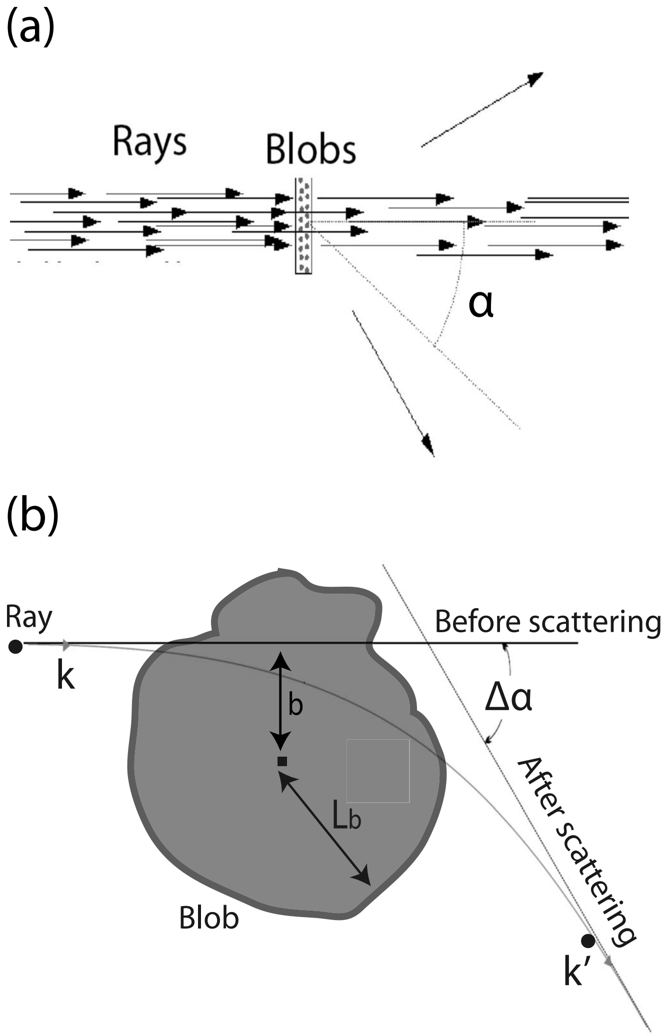

FIG. 1. Schematic representation of (a) a bunch of rays interacting with a turbulent environment of density fluctuations (blobs), (b) one ray scattered by one blob.

ronment (α is the total scattering angle and l is the propagation length inside the turbulent region). We should note that in general, α and l are not independent quantities; however under the assumption of small-angle scattering ($\Delta\alpha \ll \alpha$) this dependence can be neglected.

In this framework, the scattering process may be described by a FP equation[14]

$$\frac{\partial F(\alpha,l)}{\partial l} = -V(\alpha)\frac{\partial F(\alpha,l)}{\partial \alpha} + D(\alpha)\frac{\partial^2 F(\alpha,l)}{\partial \alpha^2}, \quad (3)$$

where $V(\alpha)=\langle\Delta\alpha\rangle/\Delta l$ and $D(\alpha)=\langle(\Delta\alpha)^2\rangle/(2\Delta l)$ are, respectively, the "friction" and the "diffusion" coefficients for the variations $\Delta\alpha$ due to the presence of a typical blob of length Δl [Fig. 1(b)]. By choosing Δl as the characteristic length scale of fluctuating structures in edge turbulence, assuming that the turbulent layer contains blobs that the ray successively encounters and is scattered off, the evolution of α may be calculated in terms of the FP equation, provided that $F(\alpha,l)$ is slowly varying in l with respect to Δl (the specific condition is $\Delta l\,\partial[\ln F(\alpha,l)]/\partial l \ll 1$).

Assuming that the wave scattering process is symmetric ($\langle\Delta\alpha\rangle=0$), so that $V(\alpha)=0$, and that the diffusion coefficient does not depend explicitly on the scattering angle [$D(\alpha)=D_\alpha$], the FP equation reduces to

$$\frac{\partial F(\alpha,l)}{\partial l} = D_\alpha \frac{\partial^2 F(\alpha,l)}{\partial \alpha^2}. \quad (4)$$

For an initial condition of the type $F(\alpha,l=0)=\delta(\alpha)$, the solution is a Gaussian function of α with amplitude $1/\sqrt{4\pi D_\alpha l}$ and half-width $\sqrt{2D_\alpha l}$,

$$F(\alpha,l) = \frac{1}{\sqrt{4\pi D_\alpha l}} \exp[-\alpha^2/(4D_\alpha l)]. \quad (5)$$

In the calculation of $\langle(\Delta\alpha)^2\rangle$ the averaging is either over a sample of typical blobs or, equivalently, over rays injected with different initial conditions for one typical blob. Regarding the latter, $\langle(\Delta\alpha)^2\rangle$ may be expressed in terms of the transition probability $\psi(\alpha,\Delta\alpha)$ that a ray scattered over an angle α undergoes a change $\Delta\alpha$ over a step-length Δl equal to a blob's linear size

$$\langle(\Delta\alpha)^2\rangle = \int_{-\infty}^{\infty} (\Delta\alpha)^2 \psi(\alpha,\Delta\alpha)\, d(\Delta\alpha). \quad (6)$$

The transition probability function $\psi(\alpha,\Delta\alpha)$ is essentially determined by the statistics of the blobs. From another point of view, which is connected to the above mentioned, $\langle(\Delta\alpha)^2\rangle$ may be determined in terms of the distance b from the blob's center

$$\langle(\Delta\alpha)^2\rangle = \frac{1}{L_b}\int_{-L_b/2}^{L_b/2} (\Delta\alpha)^2(b)\, db, \quad (7)$$

where L_b is the length scale of the blobs [see Fig. 1(b)].

From the ray tracing equations one may estimate the variation in the scattering angle due to one blob, and then $\langle(\Delta\alpha)^2\rangle$ by averaging over many different blobs. We use a coordinate system with the x-axis in the direction of the major radius, the z-axis in the vertical direction, and the y-axis in the direction of the toroidal magnetic field. The main approximations are (a) one-dimensional (1D) poloidal injection (only $k_{x0} \neq 0$) and (b) 1D vertical scattering (only $\Delta k_z \neq 0$). These approximations are adopted in order to simplify the ray equations and reduce the dimensionality of the diffusive process, while keeping the physical picture the same. In the case of two-dimensional/three-dimensional propagation and scattering there is one scattering angle per transverse direction, which leads to a more complicated FP equation.

In standard ITER scenarios, the electron density close to the plasma edge is low and the dispersion relation (2) can be approximated as

$$\omega \approx ck\left(1 + \frac{1}{2}\frac{\omega_p^2}{c^2k^2}\right), \quad (8)$$

which yields for the partial derivatives of ω involved in the ray tracing equations

$$\frac{\partial\omega}{\partial k_x} \approx c\left(1-\frac{1}{2}\frac{\omega_p^2}{c^2k^2}\right), \quad \frac{\partial\omega}{\partial z} \approx \frac{1}{2}\frac{\omega_p^2}{ck}\frac{1}{n_e}\frac{\partial n_e}{\partial z}. \tag{9}$$

In the case of small-angle scattering, the variation in the perpendicular wave number can be found by integrating along the "unperturbed" rays

$$\Delta k_z = \int \frac{\partial k_z}{\partial x}dx \approx \int \frac{\partial\omega/\partial z}{\partial\omega/\partial k_x}dx. \tag{10}$$

Assuming a density fluctuation of the form $\delta n_e = \delta n_{e0}\exp[-4(x^2+z^2)/L_b^2]$, Eqs. (9) and (10) give the following expression for the variation $\Delta\alpha=\Delta k_z/k_x$:

$$\Delta\alpha = -\sqrt{\pi}\frac{\omega_p^2}{\omega^2}\frac{\delta n_{e0}}{n_e}\frac{2z}{L_b}\exp\left(-\frac{4z^2}{L_b^2}\right), \tag{11}$$

which by utilizing $\omega_p^2=e^2n_e/(\varepsilon_0 m_e)$ and averaging over many different blobs becomes

$$\langle(\Delta\alpha)^2\rangle = \frac{4\pi e^4}{\varepsilon_0^2 m_e^2\omega^4}\frac{\delta n_{e0}^2}{L_b^3}\int z^2\exp\left(-\frac{8z^2}{L_b^2}\right)dz. \tag{12}$$

The result of the above calculation yields for the diffusion coefficient D_α $(\Delta l=L_b)$,

$$D_\alpha = \frac{\langle(\Delta\alpha)^2\rangle}{2\Delta l} = \frac{\sqrt{2\pi^3}}{32}\frac{e^4}{\varepsilon_0^2 m_e^2\omega^4}\frac{\delta n_{e0}^2}{L_b}. \tag{13}$$

III. RESULTS FOR ITER

Using this diffusion coefficient, we can calculate the solution in Eq. (5) for parameters relevant to ITER. We assume the values $\delta n_{e0}=0.1\times 3\times 10^{19}$ m^{-3} (corresponding to a density fluctuation level of 10% of the mean background density at the plasma edge), $L_b=0.01$ m, and $\omega=170$ GHz for the wave frequency, for which the diffusion coefficient is $D_\alpha=0.001\,75$ rad^2/m. In Fig. 2(a) we show the exit distribution function, as calculated from Eq. (5) for the specific values of the parameters, for turbulent region lengths $l=n_bL_b$, where n_b is the ratio of the length of the turbulent layer to the length scale of one typical blob. As expected, the distribution spreads to larger angles as the scattering process goes on.

The diffusion coefficient may also be calculated numerically with a ray tracing code by averaging the variation in the scattering angle over a (statistically) large number of ray orbits. The numerical result may be directly compared with the analytic one presented above. We illustrate this procedure in Fig. 2(b), where the propagation path in the absence of blobs is compared with the trajectory of a ray scattered by one blob, and the change in the scattering angle $\Delta\alpha$ is calculated for the same parameters as above (the part of the electron density corresponding to the perturbation is a Gaussian function of the distance from the blob center). The result for $\Delta\alpha$, as expected, varies with initial conditions due to the different collision parameter for each ray [see Eq. (7)]. For the specific initial ray corresponding to Fig. 2(b) the scattering angle is $\Delta\alpha=8.6\times 10^{-4}$; however a more detailed calculation shows that for initial conditions crossing the density gradient, $\Delta\alpha$ is within $[10^{-5},\ 10^{-3}]$. The numerical

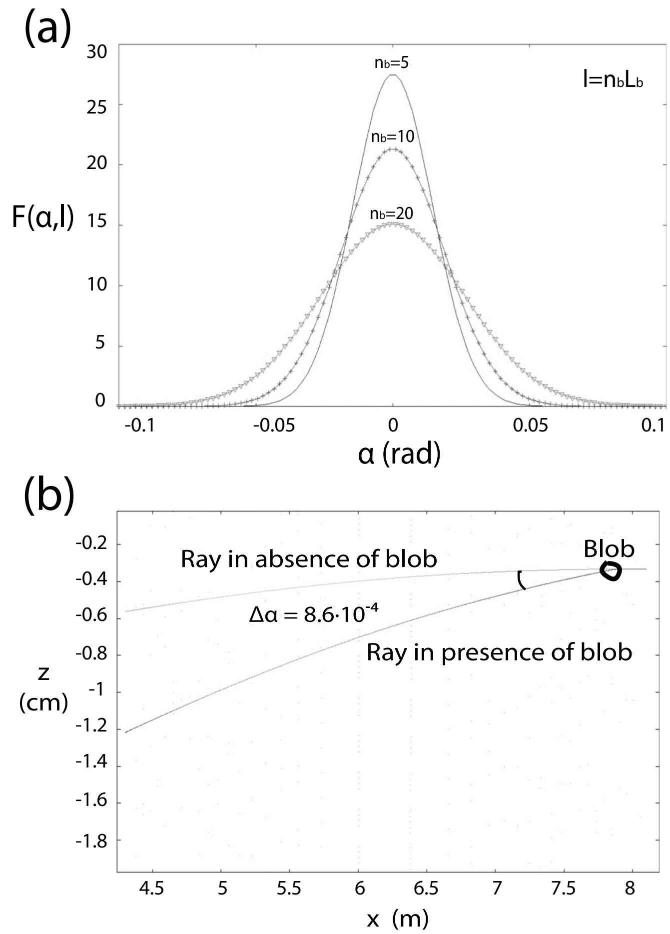

FIG. 2. (a) Distribution function of the scattering angles at $l=n_bL_b$ as calculated from Eq. (5) for ITER parameters and $\delta n_{e0}=0.1\times 3\times 10^{19}$ m^{-3}, $L_b=0.01$ m. (b) Ray trajectory in the presence of one blob [for the same parameters as in (a)] compared with the same trajectory in the absence of blobs.

calculation of the diffusion coefficient as an ensemble average of 100 rays scattered by the blob gives $D_\alpha=0.001\,27$ rad^2/m, which is very close to the analytic result.

The results presented so far imply important consequences for the evolution of collimated EC beams under edge turbulence. For the ECRH experiments in ITER different scenarios are foreseen with the wave beam propagating between 2 and 4 m from the launcher to the resonant layer and the minimum width varying between 2 (for 2 m) and 4 cm (for 4 m). Using the estimates above for $\Delta\alpha$, a turbulent layer of length $l=10$ cm in the path of $L=2$ m would give an additional broadening up to $\Delta w\approx(l/L_b)L\Delta\alpha=1.7$ cm and in the path of 4 m an additional broadening up to 3.4 cm, i.e., the width of the beam may be doubled due to the scattering. Taking into account that the marginal magnetic island width in ITER is estimated around 2 cm,[15] it seems that the widening in the EC deposition may be considerable enough to have an annihilating effect to the NTM stabilization technique based on localized electron-cyclotron current drive (ECCD) within the island's O-point.

IV. CONCLUSIONS

We have shown that the effect of edge turbulence on the EC wave propagation in ITER can be sizable and might strongly affect the use of EC beams for NTM stabilization. The topic of electromagnetic beam spreading in ITER due to strong edge turbulence is important not just for ECCD, but also for all microwave-based plasma diagnostics such as interferometry and reflectometry.

The wave scattering process has been described on the basis of a FP equation, where the diffusion coefficient was estimated analytically as well as computed with a ray tracing code. The range of scattering angles depends on the shape and amplitude of the perturbation, mostly through the diffusion coefficient, and on the width of the turbulent layer. For ITER, the results imply an effect of additional broadening, even up to 100%, for collimated EC beams in the presence of edge turbulence.

We have concentrated on the edge of the plasma since the relative density fluctuations are known to be largest there, and also because the beam has to propagate over a large distance from there to the absorption layer. Fluctuations of the density further can be expected to have a somewhat smaller impact, but they might not necessarily be completely negligible. In the plasma core the relative amplitude of the density fluctuations is an order of magnitude smaller, but at the same time the plasma frequency is larger. A first estimate shows a reduction in the effect, and since the distance from the core to the region of absorption is smaller the edge turbulence generates the largest effect. Nevertheless, one should most probably consider the core as well.

The main restrictions in the model presented here are that (a) the simplified geometry does not allow for several geometric effects which could reduce the broadening of the beam or its effect to the power deposition (which is a function of the launching angle). (b) In case the eddy structures of the density are smaller than the wavelength, the use of geometric optics is, strictly speaking, not allowed. In such cases, one should resort to methods that consistently describe diffraction (e.g., Refs. 16–18). However, a first estimate on the basis of geometric optics in simplified geometry is desirable due to its limited complexity.

In general, the scope of this presentation is to focus on the importance of the effect itself rather than to provide a detailed computation taking into account all parameters involved. A full-wave calculation in realistic geometry with a plasma equilibrium in the presence of edge turbulence is the subject of current work.

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