



PARTICLE ACCELERATION AND HEATING BY TURBULENT RECONNECTION

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ABSTRACT

Turbulent flows in the solar wind, large-scale current sheets, multiple current sheets, and shock waves lead to the formation of environments in which a dense network of current sheets is established and sustains “turbulent reconnection.” We constructed a 2D grid on which a number of randomly chosen grid points are acting as scatterers (i.e., magnetic clouds or current sheets). Our goal is to examine how test particles respond inside this large-scale collection of scatterers. We study the energy gain of individual particles, the evolution of their energy distribution, and their escape time distribution. We have developed a new method to estimate the transport coefficients from the dynamics of the interaction of the particles with the scatterers. Replacing the “magnetic clouds” with current sheets, we have proven that the energization processes can be more efficient depending on the strength of the effective electric fields inside the current sheets and their statistical properties. Using the estimated transport coefficients and solving the Fokker–Planck (FP) equation, we can recover the energy distribution of the particles only for the stochastic Fermi process. We have shown that the evolution of the particles inside a turbulent reconnecting volume is not a solution of the FP equation, since the interaction of the particles with the current sheets is “anomalous,” in contrast to the case of the second-order Fermi process.

Key words: acceleration of particles – diffusion – magnetic reconnection – Sun: corona – Sun: flares – turbulence

1. INTRODUCTION

Fermi (1949) introduced a fundamental stochastic process to solve the problem of particle energization (heating and/or acceleration) in space and astrophysical plasmas. His goal was to resolve the mystery of the stable energy distribution of cosmic rays (CRs; see details in Longair 2011). The core of his idea had a larger impact on nonlinear processes in general and has been the driving force behind all subsequent theories on charged particle energization. He assumed that high-energy particles with speed close to the speed of light collide with magnetic clouds that move in random directions with speed V close to the local Alfvén speed. The reflections of the charged particles at the magnetic clouds heat or accelerate the particles to substantial energies. The rate of the energy gain for the charged particles is proportional to the square of the ratio of the magnetic cloud speed to the speed of light $(V/c)^2$. A more realistic proposal was put forward initially by Kulsrud & Ferrari (1971). The magnetic clouds were replaced by a Kolmogorov spectrum of low-amplitude MHD waves, and the energization processes were called “stochastic heating and acceleration by (weak) turbulence.”

Research on reconnecting magnetic fields has undergone a dramatic evolution recently due mostly to the development of the numerical simulation techniques. Long current sheets or multiple interacting current sheets will form, on a short timescale, a turbulent environment, consisting of a collection of current sheets (Matthaeus & Lamkin 1986; Galsgaard & Nordlund 1996; Drake et al. 2006; Onofri et al. 2006; see also the recent reviews by Cargill et al. 2012; Lazarian et al. 2012). On the other hand, Alfvén waves and large-scale disturbances traveling along complex magnetic topologies will drive magnetic discontinuities by reinforcing existing current sheets or form new unstable current sheets (UCSs; see Biskamp & Welter 1989; Lazarian & Vishniac 1999; Arzner & Vlahos 2004; Dmitruk et al. 2004).

The goals of this article are to introduce three new and important elements in the current discussion of turbulent reconnection in large-scale systems: (a) the study of the characteristics of the energy gain of individual particles; (b) the use of the same framework of global and statistical analysis for two types of scatterers: (i) magnetic clouds, which are representative of stochastic energy gain, (ii) UCSs, which are representative of systematic energy gain; (c) the development of a new method to estimate the transport coefficients from the dynamics of the interaction of the particles with the scatterers.

2. FERMI-TYPE ENERGIZATION OF PARTICLES

Fermi (1949) based his estimates for the proposed acceleration mechanism on several assumptions (see Longair 2011). The particles move with relativistic velocity u , and the scatterers (“magnetic clouds”) move with mean speed V much smaller than the speed of light. The energy gain or loss of the particles interacting with the scatterers is

$$\frac{\Delta W}{W} \approx \frac{2}{c^2} (V^2 - \mathbf{V} \cdot \mathbf{u}), \quad (1)$$

where for head-on collisions $\mathbf{V} \cdot \mathbf{u} < 0$ and the particles gain energy, and for overtaking collisions $\mathbf{V} \cdot \mathbf{u} > 0$ and the particles lose energy. The rate of energy gain in Equation (1) includes both a first- and second-order term. For relativistic particles the first-order term dominates the energy gain. For non-relativistic particles both terms are second order.

The rate of energy gain for relativistic particles is estimated as $dW/dt = W/t_{\text{acc}}$, where $t_{\text{acc}} = (3\lambda c)/(4V^2)$ and λ is the mean free path the particles travel between the scatterers. Assuming that the distribution of the scatterers is uniform inside the acceleration volume and their density is n_{sc} , the mean free path will be $\lambda \approx (\sqrt[3]{n_{\text{sc}}})^{-1}$. The particles are not trapped inside the scatterers, their interaction is instantaneous, and the

temporal evolution of the mean energy is

$$\langle W(t) \rangle = W_0 e^{t/t_{\text{acc}}}. \quad (2)$$

Fermi (1949) used the Fokker–Planck (FP) equation in order to estimate the change of the energy distribution $n(W, t)$ of the accelerated particles. In order to simplify the diffusion equation, he assumed that spatial diffusion is not important and the particles diffuse only in energy space,

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial W} \left[F n - \frac{\partial [D n]}{\partial W} \right] = -\frac{n}{t_{\text{esc}}} + Q, \quad (3)$$

where t_{esc} is the escape time from an acceleration region with characteristic length L , Q is the injection rate, D is the energy diffusion coefficient

$$D(W, t) = \frac{\langle (W(t + \Delta t) - W(t))^2 \rangle_W}{2\Delta t}, \quad (4)$$

and

$$F(W, t) = \frac{\langle W(t + \Delta t) - W(t) \rangle_W}{\Delta t}, \quad (5)$$

is the energy convection coefficient representing the systematic acceleration, which, as mentioned, here takes the form $F(W, t) = W/t_{\text{acc}}$. With $\langle \dots \rangle_W$ we denote the conditional average that $W(t) = W$ (see, e.g., Ragwitz & Kantz 2001). Fermi reached his famous result by assuming that: (a) the particles reach a steady state before escaping from the acceleration volume and (b) the energy diffusion coefficient approaches zero asymptotically for the relativistic particles and the acceleration is mainly due to the systematic acceleration term (F). Based on these assumptions, the stationary solution of Equation (3) simply is $n(W) \sim W^{-k}$, where $k = 1 + t_{\text{acc}}/t_{\text{esc}}$. The index k approaches 2 (which is close to the observed value for the CR) only if $t_{\text{acc}} \approx t_{\text{esc}}$. In most recent theoretical studies of the second-order Fermi acceleration, the escape time (which is so crucial for the estimate of k) is difficult to estimate quantitatively.

We will expand the initial Fermi model in this article by replacing the scatterers by randomly distributed UCSs, which represents the environment present in turbulent reconnection in a fragmented large-scale system. In several recent articles, the 3D evolution and the fragmented UCSs have been analyzed (see Dahlin et al. 2015; Guo et al. 2015), using particle-in-cell numerical codes, and it has been found that the curvature drift competes with the electric field in the efficiency of particle acceleration inside the UCSs. It will be a natural continuation of the work presented here to study also the curvature drift case, here we focus on the acceleration by the electric fields. The particle dynamics inside the UCSs is complex since internally the UCSs are also fragmented, and the particles that interact with the fragments of the UCSs can lose and gain energy on the microscopic level of description. Yet, on average and over the entire simulation domain, the particles gain energy systematically before exiting the UCSs; see Figure 6(c) of Guo et al. (2015) and the related discussion. The energy gain is a weak function of energy in the case of electric field acceleration and proportional to the energy in the case of curvature drift. In this Letter, we estimate the macroscopic energy gain by the simple formula

$$\Delta W = |q| E_{\text{eff}} \ell_{\text{eff}}, \quad (6)$$

where $E_{\text{eff}} = |-\vec{V} \times \vec{B}|/c \approx (V/c) \delta B$ is the measure of the effective electric field of the UCSs, and δB is the fluctuating magnetic field encountered by the particle, which is of a stochastic nature, as related to the stochastic fluctuations induced by reconnection. ℓ_{eff} is the characteristic length of the interaction of the particle with the UCSs and should be proportional to E_{eff} , since small E_{eff} will be related to small-scale UCSs. The scenario of the method used here is: particles approach the scatterers with an initial energy W_0 and depart with an energy $W = W_0 + \Delta W$, where ΔW on the macroscopic level always is positive and follows the statistical properties of the fluctuations δB .

3. A FERMI LATTICE GAS MODEL FOR TURBULENT RECONNECTION

We constructed a 2D grid ($N \times N$), with linear size L . Each grid point is set as either *active* or *inactive*, i.e., scatterer or not. Only a small fraction R (1%–15%) of the grid points are active. The mean free path of the particles moving inside the grid with minimum distance $\ell = L/(N - 1)$ is $\lambda_{\text{sc}} = \ell/R$. When a particle encounters an active grid point it is renewing its energy state depending on the physical characteristic of the scatterer (magnetic cloud or UCS).

At time $t = 0$ all particles are located at random positions on the grid. The injected distribution $n(W, t = 0)$ is Maxwellian with temperature T . The initial direction of motion of every particle is selected randomly. The particles' individual time t_i is also adjusted between scatterings as $t_{i+1} = t_i + \Delta t$, $\Delta t = l_i/u_i$, with u_i the particle velocity and l_i the distance the particle travels between scatterings. The particles move in a random direction after interaction with the scatterers, being always confined to follow the grid lines. It is to note that the consequent large angle scattering takes place in position space, and not in velocity space, the large angle scattering is unrelated with the particle energy, and its role is to implement a spatial random walk process on a grid that basically is influencing only the timing of the energization process. We mainly consider electrons and will just briefly comment on the energization of ions.

Random “scattering” by magnetic clouds—We start our analysis using the standard stochastic Fermi accelerator, Equation (1), in order to validate our method for the estimate of the transport coefficients and the solution of the Fokker–Planck equation, since this accelerator has been already discussed in the literature using many different approaches. The parameters used in this article are related to the plasma parameters in the low solar corona. We choose the strength of the magnetic field to be $B = 100$ G, the density of the plasma $n_0 = 10^9 \text{ cm}^{-3}$, and the ambient temperature around 10 eV. The Alfvén speed is $V_A \approx 7 \times 10^8 \text{ cm s}^{-1}$, so V_A is comparable with the thermal speed of the electrons. The energy increment is $(\Delta W/W) \sim (V_A/c)^2 \approx 5 \times 10^{-4}$, and the length of the simulation box is 10^{10} cm . We consider an open grid, so particles escape from the accelerator when they reach any boundary of the grid, at $t_i = t_{\text{esc},i}$. We assume in this setup that only $R = 10\%$ of the 601×601 grid points are active.

The temporal evolution of the mean kinetic energy of the particles and the kinetic energy evolution of typical particles are shown in Figure 1(a). The motion of the particles is typical for a stochastic system with random-walk-like gain and loss of energy before exiting the simulation box. The mean energy

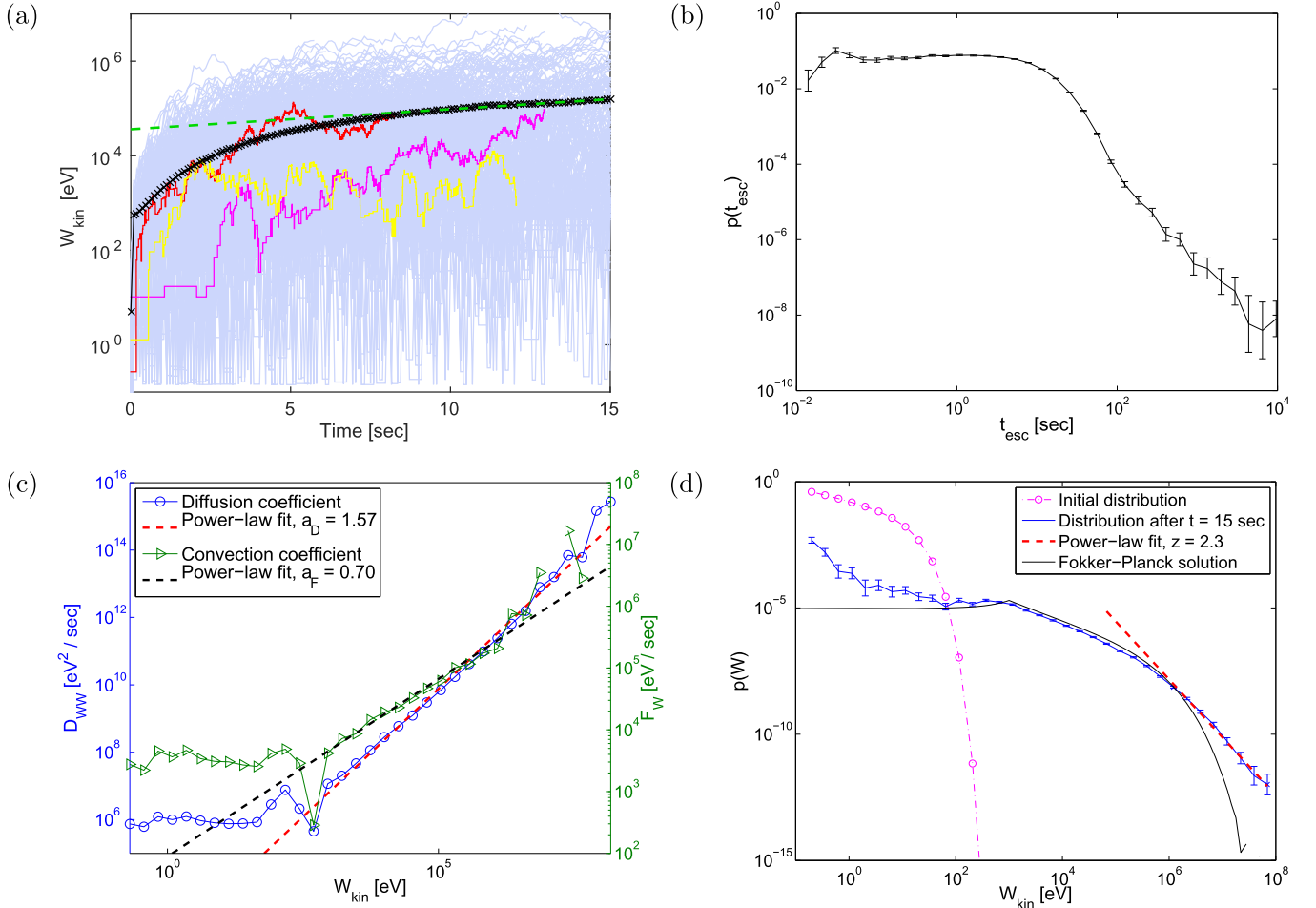


Figure 1. (a) Mean energy increase as a function of time and the energy evolution of typical particles. (b) The escape time distribution of particles. (c) The energy diffusion and convection coefficients as functions of the kinetic energy. (d) Energy distribution at $t = 0$ and $t = 15$ s for particles remaining inside the box, together with the solution of the FP equation at final time.

increases exponentially (after a brief initial period of a few seconds), as is expected from the analysis presented by Fermi (see Equation (2)). The mean free path is given as $\lambda_{\text{sc}} = \ell/R \approx 1.67 \times 10^8$ cm, and, using the analytical expression derived by Fermi, we find $t_{\text{acc,th}} = (3\lambda_{\text{sc}}c)/(4V_A^2) \approx 8$ s. We can also estimate the acceleration time from our simulation (see Figure 1(a)), by fitting the asymptotic exponential form to the mean kinetic energy, as predicted by Equation (2), which yields $t_{\text{acc,num}} \approx 10$ s, a value close to the analytically determined one. Figure 1(b) presents the escape time, which is different for each particle, and we use the median value (≈ 8 s) as an estimate of a characteristic escape time. In Figure 1(d), we show the energy distribution function of the particles remaining inside the box after 15 s. The distribution is a synthesis of a hot plasma and a power-law tail, which is extended to 100 MeV, with slope $k \approx 2.3$. If we use the estimates of t_{acc} and t_{esc} reported, we can estimate the index of the power-law tail $k = 1 + t_{\text{acc}}/t_{\text{esc}} \approx 2.3$. Therefore, the slope of the accelerated particles agrees with the estimates provided by the theory of the stochastic Fermi process.

In Figure 1(c), the diffusion and convection coefficients at $t = 15$ s, as functions of the energy, are presented. The estimate of the coefficients is based on Equations (4) and (5), with Δt small, whereto we monitor the energy of the particles at a number of regularly spaced monitoring times $t_k^{(M)}$, $k = 0, 1, \dots$,

K , with K typically chosen as 200, and we use $t = t_{K-1}^{(M)}$, $\Delta t = t_K^{(M)} - t_{K-1}^{(M)}$ in the estimates. Also, in order to account for the conditional averaging in Equations (4) and (5), we divide the energies $W(t_{K-1}^{(M)})_i$ of the particles into a number of logarithmically equi-spaced bins and perform the requested averages separately for the particles in each bin. As Figure 1(c) shows, both transport coefficients exhibit a power-law shape, with indexes $a_D = 1.57$ and $a_F = 0.70$, for energies above 1 keV, $F(W) = AW^{0.70}$, $D(W) = BW^{1.57}$. These estimates clearly depart from the assumptions made initially by Fermi.

In order to verify the estimates of the transport coefficients, we insert them in the form of the fit into the FP equation (Equation (3)) and solve the FP equation numerically (including the escape term, and with $Q = 0$). For the integration of the FP equation on the semi-infinite energy interval $[0, \infty)$, we use the pseudospectral method, based on the expansion in terms of rational Chebyshev polynomials in energy space, combined with the implicit backward Euler method for the time stepping (see, e.g., Boyd 2001). The resulting energy distribution at final time is also shown in Figure 1(d), and it turns out to coincide very well with the distribution from the particle simulation in the intermediate energy range that corresponds to the heating of the population, the power-law tail though cannot be reproduced by the FP solution. The differences below energies of about 10 eV are of

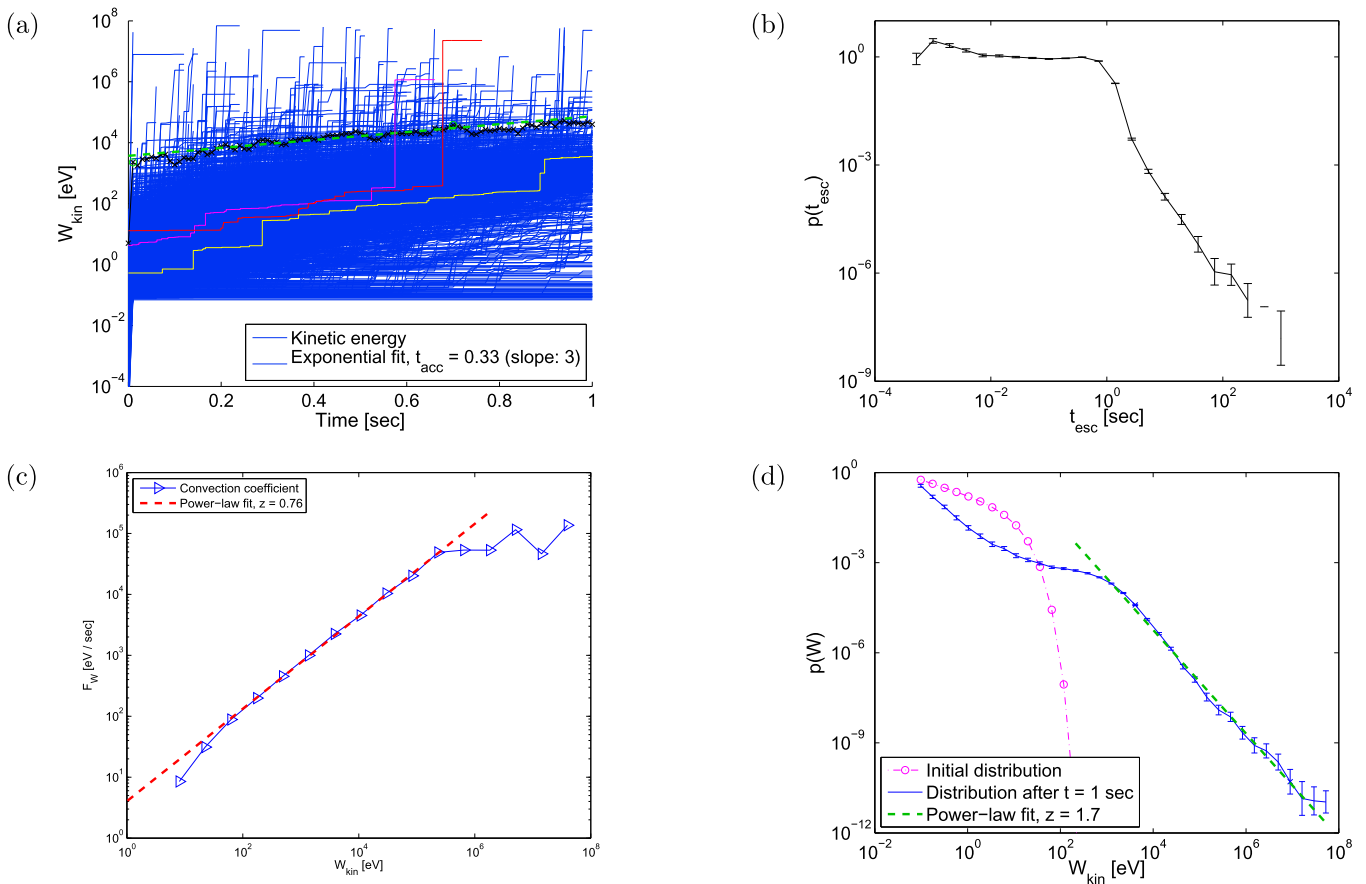


Figure 2. (a) The mean energy increase as a function of time and the energy evolution of some typical particles are shown. (b) The escape time distribution of particles. (c) The energy convection coefficient as a function of the kinetic energy at $t = 1$ s. (d) Initial and asymptotic (here, $t = 1$ s) kinetic energy distribution.

less importance and can most likely be attributed to the fact that for simplicity we just assumed the transport coefficients to be constant at low energies.

Varying the density of the scatterers in a parametric study in the range $0.01 < R < 0.2$ and keeping the characteristic length of the acceleration volume constant, we find that the main characteristics of the distribution remain the same but the heating and the slope of the accelerated particles vary.

The ions in the asymptotic stage do not appear to have significant differences from the evolution of the electrons. We can then conclude that stochastic Fermi processes can heat and accelerate both ions and electrons in the solar corona, yet on different timescales.

A model for turbulent reconnection—We now use the lattice gas model to estimate the heating and acceleration of particles inside a large-scale turbulent reconnection environment, where a fragmented distribution of UCSs is present. The setup is $R = 0.1$, $N = 601$, $V = V_A$, and the simulation box has length 10^8 cm and is open. The energy change of a particle that encounters a UCS is now given by Equation (6), and we assume that δB takes random values following a power-law distribution with index $5/3$ (Kolmogorov spectrum), and $\delta B \in [10^{-5} \text{ G}, 100 \text{ G}]$. We also assume the effective length ℓ_{eff} to be a linear function of E_{eff} , $\ell_{\text{eff}} = aE_{\text{eff}} + b$, and by restricting the size of ℓ_{eff} to $\ell_{\text{eff}} \in [10^3 \text{ cm}, 10^5 \text{ cm}]$, we determine the constants a , b . Combining all the above we find that the effective electric field lies approximately in $E_{\text{eff}} \in [10^{-7} E_D, E_D]$, where E_D is the Dreicer field, $E_D \approx 1.6 \cdot 10^{-7} \text{ statV/cm}$.

We initiate the simulation with a Maxwellian distribution with temperature 10 eV. Figure 2(a) shows the mean energy and the energy of some typical particles as a function of time, up to final time or until they escape from the simulation box. The rate at which the particles on the average gain energy is exponential, so $\ln \langle W \rangle \approx t/t_{\text{acc}}$, and we estimate the asymptotic value of the acceleration time to be ≈ 0.3 s.

The acceleration is systematic and the particles feel a rapid increase of their energy any time they cross a UCS with variable strength of the effective electric field (see the similar behavior observed in Dahlin et al. 2015; Guo et al. 2015). The energy distribution reaches an asymptotic state (see Figure 2(d)) in a fraction of a second. It is obvious that particles are very efficiently accelerated inside the turbulent reconnecting volume and form a power-law tail with index ≈ 1.7 .

Figure 2(b) presents the escape time, which is different for each particle, and we use the median value (≈ 0.5 s) as an estimate of a characteristic escape time. If we use the estimates of t_{acc} and t_{esc} reported, we can estimate the index of the power-law tail $k = 1 + t_{\text{acc}}/t_{\text{esc}} \approx 1.6$, which is close to the slope of the distribution of the accelerated particles in the simulation.

In Figure 2(c), the convection coefficient F at $t = 1$ s is presented as function of the energy, and it exhibits a power-law shape, with index $a_F = 0.76$ for energies above 100 eV, an index close to the one found above in Fermi's original scenario. For the diffusion coefficient, the estimate D based on Equation (4) yields a power law, applying through the finite time correction of Ragwitz & Kantz (2001),

$D_{\text{true}} = D - 0.5\Delta t F^2$, we find that $D_{\text{true}} \approx 0$; the energization process is purely convective in nature, the non-zero D is an artifact resulting from the finite time contribution of F to D (we just note that in the Fermi case the finite time correction was negligible).

Using F and D_{true} in the numerical solution of the FP equation, we find only heating, on timescales though of the order of tens of seconds, much larger than the time of 1 s considered here. This result is in accordance with and a generalization of the result in Guo et al. (2014, 2015), who also find only heating when analytically solving the FP equation (for $D=0$ and $F \sim W$ in their case). On the other hand, the asymptotic distribution can be calculated from Equation (3) (assuming $\partial n/\partial t = 0$) as $n \sim W^{-0.76}$. The reason for the discrepancy between the FP solution and the asymptotic solution must be attributed to the fact that the asymptotic solution, determined as a stationary solution, cannot be reached with the initial condition being a Maxwellian (in analogy to the case in Guo et al. 2014 with $F \sim W$).

Concerning the difference between the FP solution and the lattice model, we find that the sample of energy differences $W_i(t + \Delta t) - W_i(t)$ in Equation (5) (with i the particle index), on which the estimate of F is based, actually follows a power-law distribution, and, as a consequence, the particles occasionally perform very large jumps in energy space (Levy flights), as illustrated in Figure 2(a), in contrast to the second-order Fermi process (see Figure 1(a)). The fact that the energy increments have a power-law distribution with the specific index has several consequences. (1) The estimate of F as a mean value theoretically is finite, yet it is very noisy. (2) Both the mean (or the median, as used here) are not representative of a scale-free power-law distribution. (3) The variance of the distribution of energy increments tends to infinity. After all, in the case at hand, the applicability of the classical random walk theory (classical Langevin and FP equation) breaks down, as it is manifested in the inability of the FP equation to reproduce the test particles' energy distribution, and in the practical difficulties of the expressions for F and D in Equations (5) and (4) to yield meaningful transport coefficients. Thus, modeling tools like the fractional FP equation become appropriate here. Similar cases of Levy flights have been observed by Arzner & Vlahos (2004) and Bian & Browning (2008), without further analyzing the consequences for the transport coefficients and the FP equation.

We also have explored the role of collisions, and they are important for impulsive energization longer than the collision time of the system, though they play a crucial role only for the bulk of the energized plasma and just slightly modify the slope of the tail.

4. SUMMARY AND DISCUSSION

Turbulent reconnection is a new type of accelerator that can be modeled with the use of tools borrowed from Fermi-type accelerators, namely, by replacing the “magnetic clouds” with a new type of “scatterers,” the UCSs. This generalization can handle large-scale astrophysical systems composed from local accelerators like current sheets appearing randomly in reconnecting turbulence. We developed a 2D lattice gas model where a number of active points act as “scatterers” in order to model the new accelerator. Our main contribution in this article is the

estimate of the transport coefficients from the particle dynamics and their use in solving the FP equation. Our main results from this study are as follows. (a) Stochastic Fermi accelerators can reproduce a well-known energy distribution in laboratory and astrophysical plasmas, where heating of the bulk and acceleration of the runaway tail co-exist. The density of the scatterers plays a crucial role in controlling the heating and the acceleration of particles. (b) The transport coefficients show a general power-law scaling with energy. (c) The replacement of the scatterers with UCSs has several effects on the energization of the particles. (i) The acceleration time is an order of magnitude faster than in the stochastic Fermi process. (ii) Estimating the transport coefficients from the dynamic particle orbits, we have shown that the final energy distribution cannot be a solution of the FP equation since the orbits of the energetic particles in energy space depart radically from Brownian motion, showing characteristics of Levy flights. (iii) The asymptotic distribution of the accelerated particles is similar to the ones obtained in different simulations (see Arzner & Vlahos 2004; Dmitruk et al. 2004; Drake et al. 2006, 2013; Onofri et al. 2006; Dahlin et al. 2015), where turbulent reconnection is established.

We can conclude that the stochastic Fermi acceleration and turbulent reconnection processes can play a crucial role in many astrophysical plasmas and their role depends strongly on their physical properties, such as the nature of the scatterers (e.g., large-amplitude Alfvén waves or UCSs), their spatio-temporal statistical properties (e.g., their spatial density), and the time evolution of the driver of the explosions.

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REFERENCES

- Arzner, K., & Vlahos, L. 2004, *ApJL*, 605, L69
 Bian, N. H., & Browning, P. K. 2008, *ApJL*, 687, L111
 Biskamp, D., & Welter, H. 1989, *PhFIB*, 1, 1964
 Boyd, J. P. 2001, *Chebyshev and Fourier Spectral Methods* (2nd ed.; Mineola, NY: Dover)
 Cargill, P., Vlahos, L., Baumann, G., Drake, J., & Nordlund, Å. 2012, *SSRv*, 173, 223
 Dahlin, J. T., Drake, J. F., & Swisdak, M. 2015, *PhPI*, 22, 100704
 Dmitruk, P., Matthaeus, W. H., & Seenu, N. 2004, *ApJ*, 617, 667
 Drake, J. F., Swisdak, M., Che, H., & Shay, M. A. 2006, *Natur*, 443, 553
 Drake, J. F., Swisdak, M., & Fermo, R. 2013, *ApJL*, 763, L5
 Fermi, E. 1949, *PhRv*, 75, 1169
 Galsgaard, K., & Nordlund, Å. 1996, *JGR*, 101, 13445
 Guo, F., Li, H., Daughton, W., & Liu, Y.-H. 2014, *PhRvL*, 113, 155005
 Guo, F., Liu, Y.-H., Daughton, W., & Li, H. 2015, *ApJ*, 806, 167
 Kulsrud, R. M., & Ferrari, A. 1971, *Ap&SS*, 12, 302
 Lazarian, A., & Vishniac, E. T. 1999, *ApJ*, 517, 700
 Lazarian, A., Vlahos, L., Kowal, G., et al. 2012, *SSRv*, 173, 557
 Longair, M. S. 2011, *High Energy Astrophysics* (3rd ed.; New York: Cambridge Univ. Press)
 Matthaeus, W. H., & Lamkin, S. L. 1986, *PhFI*, 29, 2513
 Onofri, M., Isliker, H., & Vlahos, L. 2006, *PhRvL*, 96, 151102
 Ragwitz, M., & Kantz, H. 2001, *PhRvL*, 87, 254501