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ON DETERMINISTIC CHAOS, STATIONARITY, PERIODICITY AND INTERMITTENCY IN CORONAL BURSTS AND FLARES

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Abstract. Solar and stellar flares are highly structured in space and in time, as is indicated for example by their radio signatures: the narrowband spikes, type III, type II and IV, and pulsation events. Structured in time are also the not flare related type I events (noise storms). The nature of this observationally manifest fragmentation is still not clear. Either, it can be due to stochastic boundary or initial conditions of the respective processes, such as inhomogeneities in the coronal plasma. Or else, a deterministic non-linear process is able to cause complicated patterns of these kinds.

We investigate the nature of the fragmentation in time. The properties of processes we enquire are stationarity, periodicity, intermittency, and, with dimension estimating methods, we try to discriminate between stochastic and low-dimensional deterministic processes. Since the measured time series are rather short, the dimension estimate methods have to be used with care: we have developed an extended dimension estimate procedure consisting of five steps. Among others, it comprises again the questions of stationarity and intermittency, but also the more technical problems of temporal correlations, judging scaling and convergence, and limited number of data points (statistical limits).

We investigate 3 events of narrowband spikes, 13 type III groups, 10 type I storms, 3 type II bursts and 1 type IV event of solar origin, and 3 pulsation-like events of stellar origin. They have in common that all of them have stationary phases, periodicities are rather seldom, and intermittency is quite abundant. However, the burst types turn out to have different characteristics. None of the investigated time series reveals a low-dimensional behaviour. This implies that they originate from complex processes having dimensions (degrees of freedom) larger than about 4 to 6, which includes infinity, *i.e.* stochasticity. The lower limit of the degrees of freedom is inferred from numerical experiments with known chaotic systems, using time series of similar lengths, and it depends slightly on the burst types.

Key words: Solar and Stellar Flares — chaotic phenomena — fragmentation — radio emission — stationarity — nonlinearity.

1. Introduction

Radio emissions of flares and noise storms appear in multiple bursts at different frequencies and are usually much more structured in time than the hard X-ray emission. Is there any pattern of order hidden in the fragmentation of the radio emissions?

We investigate the time evolution of the emissions. Different tools from dynamical systems theory are used to consider general properties of the system dynamics: (i) In the reconstructed state space of a system, we perform a stationarity test. (ii) We calculate power spectra to detect the possible presence of modes. (iii) Intermittency is looked for. (iv) In order to discern between stochastic and deterministic processes, correlation dimensions are

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estimated in state space. Dimension estimate has some intricacies, however, which can even yield spurious dimensions and it must therefore be accompanied by several tests and precautions.

A related analysis has been performed for solar decimetric pulsations by Kurths et al. (1987). They analyzed the data for finite dimension and found frequent low-dimensional behaviour in them. This burst type is not analyzed in the presented work.

The data are briefly described in Section 2, Section 3 introduces the used tools, the results are presented in Section 4, and their astrophysical impact is discussed in Section 5, the conclusion.

2. The investigated data

The following types of coherent radio burst have been investigated (see Güdel & Benz 1988 for an overview): (i) 10 series of *Type I bursts* of solar noise storms; (ii) 3 events of solar *type II bursts*; (iii) 13 events of solar *type III bursts*; (iv) the temporal fine structure of 1 solar *Type IV* event; (v) 3 events of solar *narrowband spikes*; (vi) 3 events of radio emission from *stellar flares*, resembling solar decimetric pulsations.

The solar radio data have been recorded by the frequency-agile radio spectrometer IKARUS at ETH Zurich (Perrenoud 1982), between 1980 and 1982, in the frequency range 100-1000 MHz, with a time resolution of 100 ms or less.

The stellar data are flares of the dMe star AD Leo, observed in Arecibo in November 1987 by Bookbinder and Bastian (published in Bastian et al. 1990, and in Güdel et al. 1989). The time resolution is 20 ms, the frequency channels of 40 MHz bandwidth are centered at 1415 MHz.

3. Concepts and methods

Stationarity in the strong sense is the property that all statistical moments are independent of time. In other words, a stationary process does not change in time if only its statistical properties are considered. To detect stationarity we use a *test of stationarity* proposed by Isliker and Kurths (1993). It is based on the invariant measure associated with a motion in state space (Eckmann & Ruelle 1985). The test yields a list of parts of a time series which are stationary.

Eventual quasi-periodicity (i.e. the property that one or several harmonic modes govern the behaviour) of a process is revealed in the power spectrum, which we calculate by a standard Fast Fourier Transform (FFT).

The time series are inspected for intermittency, which is the phenomenon that a process is interrupted with one or many quiet phases of arbitrary lengths, or with phases of smaller amplitudes (intensities). It is often visually obvious, and it can be quantified e.g. by calculating variances in a window gliding through a time series (Isliker & Benz 1994).

Measuring the radio flux of an event, the entire phase space, whose coordinates are the system variables, can be reconstructed (Takens 1981): From a given time series $\{X(t_i)\}_{i=1}^{N}$, vectors $\boldsymbol{\xi}(t_i)$ are built up in a d-dimensional space as

$$\boldsymbol{\xi}(t_i) := \Big[\ X(t_i),\ X(t_i+\Delta t), ..., X(t_i+(d-1)\Delta t)\ \Big]. \tag{1}$$

The time delay Δt is a multiple of the time resolution $\tau = t_{i+1} - t_i$.

State space motion is characteristically different for different kinds of systems. On one hand, a limit set of a process' motion may exist, the so-called attractor, the motion being consequently *stationary.* On the other hand, there may be no stationary limit set. Then the system is either in a transient phase, or it is an inherently non-stationary process.

If a stationary attractor exists for a process which is not quasi-periodic, and if its dimension is smaller than the dimension d of the embedding space, then the underlying process is deterministic, e.g. deterministic chaotic. The latter are deterministic, however non-linear processes which, in their case, leads to sensitive dependence on initial conditions. The dimension of the attractor corresponds to the degrees of freedom of the respective process, and it therefore can be used to discern between stochastic and deterministic chaotic processes (Liapunov exponents and Kolmogorof entropies are other tools to achieve this distinction, see Eckmann & Ruelle 1985. They are not considered here.).

We estimate the correlation dimension $D^{(2)}$ with the Grassberger Procaccia (GP) algorithm (Grassberger & Procaccia 1983a, 1983b) and the Maximum Likelihood formalism (ML) (Takens 1984; Ellner 1988).

There are serious sources of misinterpretation of the results of dimension estimates. We list them together with the precautionary measures we applied (for details see Isliker & Benz 1994):

- Numerical problems:
  - What is the minimum number of data points necessary ? — The length of a time series is best measured by n_S, the number of peaks or structures, defined as

    $$n_S := \frac{N\tau}{t_{corr}}, \tag{2}$$

    where t_{corr} is the auto-correlation time, and τ the time resolution. If $n_s \gtrsim 50$, dimensions up to about 3.5 are reliably estimated, and up to 5 they are detected (Isliker 1992, Isliker & Benz 1994; see also Brandstater & Swinney 1987, Ruelle 1990 and Eckmann & Ruelle 1992 for alternative formulae).

  - — What is the maximum noise level allowed ? — Numerical experiments show that the ratio noise/signal should be below about 10% (Isliker, 1992).
- — Non-stationary *stochastic* processes, such as fractional Brownian motion (see e.g. Osborne & Provenzale 1989), can yield a finite correlation dimensions. — Consequently, only data which pass the stationarity test must be analyzed.
- — Temporal correlations can lead to finite dimensions, even in stochastic processes (Theiler 1986, 1991). — A slight modification of the GP and the ML algorithm, which forbids temporarily correlated vectors, inhibits this effect (Theiler 1986, 1991).
- — Intermittency can mimic a finite correlation dimension, no matter whether a process is stochastic or deterministic (Isliker & Benz 1994). — To detect this effect, an 'omitting test' must be performed: the GP algorithm should be resistant to omitting vectors from an eventually quiet regime (l.c.).
- — Some shapes of power spectra, combined with shortness of time series, force a finite correlation dimension, even for stochastic processes. — A 'surrogate data' test must be done: conserving the power spectrum, a time series must be artificially randomized. If some out of different realizations of surrogate data have the same dimension as the original data, then the dimension is almost surely due to the mentioned effect (the test is first described in Kurths & Herzel 1987; see also Provenzale et al. 1992).
- — Problems with interpretation: are the algorithms really converging ? — We use a test proposed by Isliker (1992) which checks scaling of the correlation integral, consistency of the GP method with the ML method, and convergence (independence of the embedding dimension).

4. Results and discussion

(More details of the results than presented in the following are given in Isliker & Benz 1994.) All burst types have been found in stationary states for some time. How long these states last depends on the individual burst types. In Table I $\bar{n}_S(stat.)$, the average length of stationary sections in units of the respective auto-correlation time, is listed. The type I events, narrowband spikes and stellar flares have the longest stationary states. It is therefore adequate to search for physical models which are able to bring forth stationary solutions, as e.g. electron cyclotron masers, or quasi-steady reconnection, two models which are in discussion for the latter two burst types.

The stationary sections of type II, type III and type IV bursts are so short that the transient states at the beginning and at the end have to be taken into account. This means that the start phase and the decay phase of

TABLE I

Statistics of the results: In some ms-spikes, type III and pulsation events we have analyzed different frequencies at equal times. Therefore, they have two entries in the table. The entries marked by an asterisk are just the lowest of the accessible frequencies taken into account, whereas the unmarked entry is summing up the different frequencies as if they were different events. The entries are: $\bar{n}_S \equiv < \frac{t(stat.)}{t_{corr}} >$, the average length of a stationary sections in units of the auto-correlation time, $D^{(2)}_{min}$, a rough estimate of the lower limit for the dimensions of the respective processes.

	$\bar{n}_S \equiv < \frac{t(stat.)}{t_{corr}} >$		$D^{(2)}_{min}$
type I	94	± 71	5
type II	28	± 12	(4)
type III *)	47	± 29	3 – 4
type III	46	± 26	3 – 4
type IV *)	38	± 0	(4)
type IV	35	± 5	(4)
ms-spikes *)	333	± 495	5
ms-spikes	1041	± 1380	5
stellar	76	± 42	?

the operating processes are a relevant part of any available measurement, and they can hardly be neglected in the models.

The narrowband spikes and type I events have a tendency to be interrupted in their long stationary phases. Type III's are less homogeneous, intermittency is present not just in the form of stopped emission, but also in the form of reduced amplitudes. Type II and type IV events are mixed, some are homogeneous, some are not. The stellar flares are not intermittent. Concerning models, we note that there are examples of non-linear stochastic processes (Provenzale et al. 1992), as well as of deterministic equations which show features of intermittency.

Periodicities are rare, except in the pulsation-like stellar flares. Their periods are not very prominent. Because of this and because the power spectrum is not at all uniform, these modes do not explain the entire dynamics of a process. It remains unclear whether there is a hidden chaotic or a stochastic process with a dominant mode.

The GP algorithm was not observed to converge, we can infer therefore

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that there is no low-dimensional chaos. The processes are high dimensional, with the dimensions so high that the length of the time series does not allow to detect them. Numerical experiments with known chaotic systems (Isliker 1992; Isliker & Benz 1994) give a very rough estimate of the lower limit for the respective dimensions. This value $D^{(2)}_{min}$ is given in Table I. It should be interpreted in the sense that the correlation dimension of for instance a type I event must be expected to be larger than 5. The type II's and type IV's limits are in brackets, since the statistics is too poor for a definite answer. The noise in the stellar data prohibits to give a lower limit.

Since a lower limit on the dimension allows dimensions to be infinite, we are left with the dichotomy that

— either the dimension is infinite and the systems are stochastic (a sum of many uncorrelated processes, or a process governed by stochastic boundary or initial conditions),

— or the dimension is finite and the systems are high-dimensional deterministic chaotic ($D^{(2)} \gtrsim 4 - 5$).

This latter distinction could be done with dimension estimate methods only if the processes revealed themselves in time series which were substantially longer than the ones considered in this article. It seems, however, that it is the very nature of the respective processes to be restricted to the typical durations reported.

5. Conclusion

We first draw conclusions on the radio bursts occurring in the impulsive phase of flares, the type III and the narrowband spike events. They are directly connected to the primary energy release. Groups of type III bursts and narrowband spikes last similar times, typically one minute. Single type III bursts have a much longer duration than the narrowband spike bursts. It is quite natural then that the type III groups appear more transient, whereas the short narrowband-spikes have long phases of stationary bursting. The narrowband spikes would have a better chance to reveal a possible low dimension. They do not, so the relevant process must be complex, i.e. with a dimension higher than about 5. This calls for high-dimensional deterministic or stochastic models (which means infinite-dimensional deterministic, here).

Later in the flare and probably further away from the acceleration site occur the type II and IV events. Type II bursts are emitted in association with a shock wave emerging from the flare. The fine structures at a fixed frequency (suggesting an approximately constant height in the corona) has in one case a simple structure, namely a weak period. Such an oscillation may be produced by a wave structure in the upstream medium modulating the emission at the shock front.

The type I events are not flare related. They appear in the present analysis similar to the narrowband spikes, with long stationary sections. The analysis infers that the type I storms are complex, with dimensions greater than about 5. They have a stronger tendency to be intermittent than the narrowband spikes. The results exclude a self-contained source model that operates as a non-linear system of a few free variables. The results, on the contrary, support a scenario for type I storms in which the variability is caused by a stochastic input or a high-dimensional mechanism. Possibilities include inhomogeneous plasma flowing into a reconnection region or activity at a shock front modulated by the upstream medium.

The stellar flares, finally, appear pulsation-like, with long stationary phases and quite strong ordering, as is indicated by the frequent presence of quasi-periodicity (i.e. one or several modes are dominant in the power spectrum). We cannot consider the absence of low-dimensions to be a final result, since the quality of the data is not adequate. Better data is needed, with a lower noise level, or an efficient, but conservative noise reduction method in phase space.

We note that this analysis does not disprove deterministic chaos in the flare radio emission. It merely shows that if deterministic chaos is present in the analyzed data, then it is not low-dimensional. The results of this work give constraints to models for all types of investigated bursts. Stationarity, intermittency and dimensionality are new characteristics of radio bursts which should be taken into account in their interpretation.

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References

Bastian, T.S., Bookbinder, J., Dulk, G.A., Davis, M., 1990, ApJ 353, 265
Brandstater, A., Swinney, H.L., 1987, Phys. Rev. A 35, 2207
Eckmann, J.-P., Ruelle, D., 1985, Review of Modern Physics 57, 617
Eckmann, J.-P., Ruelle, D., 1992, Physica D 56, 185
Ellner, S., 1988, Phys. Lett. A 133, 128
Grassberger, P., Procaccia, I., 1983a, Physica 9D, 189
Grassberger, P., Procaccia, I., 1983b, Phys. Rev. Lett. 50, 346
Güdel, M., Benz, A.O., 1988, A&AS 75, 243
Güdel, M., Benz, A.O., Bastian, T.S., et al., 1989, A&A 220, L5
Isliker, H., 1992, Phys. Lett. A 169, 313
Isliker, H., Benz, A.O., 1994, A&A, in press
Isliker, H., Kurths, J., 1993, Intern. J. of Bifurc. and Chaos, in press
Kurths, J., Benz, A.O., Aschwanden, M.J., 1991, Astron. Astrophys. 248, 270

Kurths, J. & Herzel, H., 1987, Physica D 25, 165
Osborne, A.R., Provenzale, A., 1989, Physica D 35, 357
Perrenoud, M.R., 1982, Solar Phys. 81, 197
Provenzale, A., Smith, L.A., Vio, R., Murante, G., 1992, Physica D 58, 31
Ruelle, D., 1990, Proc. R. Soc. Lond. A 427, 241
Takens, F., 1981, in: Dynamical Systems and Turbulence, Lecture Notes in Mathematics 898. Springer, Berlin, p. 366
Takens, F., 1984, in: Dynamical systems and turbulence, Lecture Notes in Mathematics 1125. Springer, Berlin, p. 99
Theiler, J., 1986, Phys. Rev. A 34, 2427
Theiler, J., 1991, Phys. Lett. A 155, 480