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Are solar flares random processes?

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Abstract. We show that the current high time-resolution observations are compatible with considering the flare process stochastic. To establish this proposition we analytically investigate a particular class of stochastic processes. These *marked and filtered point processes* (a generalization of shot noise) describe the radio-emission occurring during a flare in a realistic way, namely as a trigger signal causing observable pulses, where the trigger and the plasma response are treated as separated processes. The existing empirical inquiries, with which we confront our model, are based on radio observations (type III and narrow-band spikes events), and they refer to the shape of the power-spectrum, the possible existence of a low-dimensional attractor (correlation dimension estimate), and the statistics of the time intervals between subsequent peaks. It turns out that the stochastic model can describe the properties as they are found in these empirical investigations. Particularly, what has been found in the observations and termed 'periodic' or 'almost periodic' can as well be the signature of temporal correlations in a stochastic model. Such correlations arise for instance from the pulse shape, but also from a blocking time, if such a one is inherent to the peak-detection algorithm.

Key words: Sun: flares – chaos – methods: statistical – Sun: radio radiation

1. Introduction

The question we address in this paper is whether stochastic models for the solar flare process are compatible with today's observational results with high temporal resolution. The existing empirical investigations are controversial, mainly concerning the point whether flares essentially are a periodic process or not. After all, the topical question still is whether the nature of the flare process is stochastic or deterministic, and, within the latter, whether it is periodic (also multiply periodic) or chaotic (i.e. deterministic, with sensitive dependence on initial conditions, such that the system appears random-like). Agreement exists on the assertion that flares are fragmented: Hard X-ray and radio emissions (in the form of type III, microwave, and possibly narrowband-spikes events) are the electromagnetic emissions which are most directly linked to the energy release process in flares, and they reveal very rich spatial as well as temporal fine structures (see e.g. the review of Benz and Aschwanden 1991).

Several investigations exist which are dedicated to the understanding of the temporal properties of flares (the flare dynamics). Since radio measurements are less contaminated by noise than present-day hard X-ray observations, and since they show to be at least as rich in details as the hard X-rays emissions, most inquiries on temporal fine structures deal with observations of type III and narrowband spike events. They consequently are the subject of our investigations.

Mangeney and Pick (1989), continued by Zhao et al. (1991), searched for periodic behaviour in type III events by estimating the power-spectrum (using Fourier transformation). Aschwanden et al. (1994) have investigated the statistics of the spread of the times between subsequent peaks in type III events. All these authors conclude that the flare process is basically periodic. Contrary to these investigations, Isliker and Benz (1994) did not find any hint to periodic behaviour, analyzing type III as well as narrowband spikes events. They had estimated correlation dimensions (with the *extended dimension estimate algorithm*) and power-spectra to search for possible periodicities or low-dimensional chaos. Neither of the two was detected, which infers that the flare process is high-dimensional, or, as can be concluded from the typical duration of the events, the trajectory of the system variables moves in state space on a set which is characterized by a dimension of at least about 4 or 5. This includes infinity, i.e. the stochastic case, and it excludes periodic behaviour with just one or a few active modes.

We introduce and analyze a particular stochastic flare model. The aim is to see whether such a model can explain the described observational results. If it can, then a counter-example is found against the conclusion of the above authors who claim that the observed time series must be due to a periodic process (to show that the chosen model is the true one would be a different problem). The analyzed model belongs to the class of the *marked and filtered point processes*, a generalization of *shot noise* (it is also termed *real-valued point process*). It is an adequate way of describing the radio emissions during flares, assuming basically

Send offprint requests to: H. Isliker (Patras) that a trigger signal causes radio pulses. The nature of the trigger and of the pulse process (including the pulse shape) are independent free parameters, though observations give hints how they roughly should be chosen. We just note that the recently proposed model of Vlahos (1994) and Vlahos and Raoult (1995) is of this type, as well as the model presented in Aschwanden et al. (1994), whose suggestion for an empirically reasonable pulse shape we take over.

After introducing the model (Sect. 2), we analytically deduce the properties of interest, namely power spectrum (Sect. 3), and statistics of the times between subsequent peaks (Sect. 4); correlation dimension estimate needs no further treatment, it will be discussed in the conclusion (Sect. 5). In each section, the respective empirical investigations are discussed and compared, and the influence particularly of the shortness of the data is treated. The considerations on the times between subsequent peaks (Sect. 4) include also an analytical treatment of the peak-identification problem in measured time series, and results are also derived for the case of a periodic trigger process.

2. A stochastic flare model

The flare model we investigate belongs to the very general class of stochastic processes called *marked and filtered point processes* (see e.g. Cox & Miller 1965, p. 366ff., where they are termed *real-valued point processes*; Papoulis 1991, p. 361f., where they are termed *generalized shot noise*; the terminology we use is taken from Gardner 1986, p. 118ff.). They apply to the situations where pulses of possibly random shape occur at possibly random times: The onset time t_i of the ith pulse is taken as the outcome of a random process T_i (capital letters denote random variables, and small letters concrete realizations; a set $\{T_i\}$ of random points on the time axis is termed a *point process*, see e.g. Papoulis 1991, p. 297). The pulse-shape is described by a *deterministic* function $f(t; \boldsymbol{a})$, which depends on time t and on an ensemble of parameters $a^{(1)}, ..., a^{(L)} =: \boldsymbol{a}$, written in vector-form, for conciseness. To account for *randomness* of these pulse shapes, the parameter $\boldsymbol{a}_i$, which determines the shape of the ith pulse, is considered as the outcome of a vector-valued random variable $\boldsymbol{A}_i$, the parameter process. The ith pulse is then described as $f(t - T_i, \boldsymbol{A}_i)$, and the observed signal $X(t)$ is the superposition of the individual pulses:

$$X(t) = \sum_{i=1}^{N_p} f(t - T_i, \boldsymbol{A}_i) \ . \tag{1}$$

The total number of pulses N_p is finite in a finite time interval (say Λ). If the pulses occur uncorrelated in time, an average number of pulses per unit time n_1 can be defined: $n_1 := N_p/\Lambda$, the pulse rate. The process in Eq. (1) is a generalization of the so-called *shot noise* model, where the pulse shape is constant, and the observed time series is $X(t) = \sum_{i=1}^{N_p} f(t - T_i)$.

Important examples for point processes $\{T_i\}$ are the so-called *Poisson point processes*. In their case, the points $\{T_i\}$ are completely random, i.e. uncorrelated and even independent.

2.1. The context of flares

The translation of the model Eq. (1) into the context of flares is as follows: at some time t_i, an energy-release event is triggered (most likely localized reconnection). Among others, particles are accelerated, leading in turn to a plasma instability, which possibly is converted into electromagnetic waves (a type III or narrowband spikes event). The latter are finally radiated off the corona and detected by some terrestrial instrument. The plasma response will depend on local parameters, say $a_i^{(1)}, ..., a_i^{(L)} \equiv \boldsymbol{a}_i$ (e.g. local magnetic field, local particle densities, local temperature), which yield the burst time-profile of the form $f(t-t_i, \boldsymbol{a}_i)$: the $\boldsymbol{a}_i$-dependent pulse shape $f(t, \boldsymbol{a}_i)$ is shifted in time to t_i. To take causality into account, $f(t, \boldsymbol{a}_i) = 0$ for $t < 0$ must hold.

The assumptions which allow to write a radio measurement in the form of Eq. (1) do not decide whether the energy release process in flares is stochastic or deterministic (periodic or chaotic). This decision is hidden in the choice of the trigger process T_i. The same holds for the parameter-process $\boldsymbol{A}_i$, which can be adjusted to different assumptions on the flare-scenario: one possibility is that the local parameters appear as completely random elements in the process (corresponding for instance to some assumptions on the inhomogeneity of the flaring plasma), another possibility is that the parameters are constant throughout (so that the model reduces to shot noise).

2.2. The analytical treatment of marked and filtered point processes

To derive properties of the time series $X(t)$, it is convenient to separate clearly the deterministic from the stochastic part in Eq. (1). Thereto, we introduce the *counting process* $\Delta\Xi(t, \boldsymbol{a})$, the random variable denoting the number of pulses in $[t, t + \Delta t]$, associated with a value of the parameter-vector $\boldsymbol{A}$ in $\left[a^{(1)}, a^{(1)} + \Delta a^{(1)}\right] \times \ldots \times \left[a^{(L)}, a^{(L)} + \Delta a^{(L)}\right]$. The counting process $\Delta\Xi(t, \boldsymbol{a})$ takes the values $0, 1, 2, \ldots$. This allows to write Eq. (1) in the equivalent form

$$X(t) = \sum_{\boldsymbol{a}} \sum_{t'=-\infty}^{\infty} f(t - t', \boldsymbol{a}) \, \Delta\Xi(t', \boldsymbol{a}) \ , \tag{2}$$

i.e. as temporal convolutions of a deterministic function f with random processes $\Delta\Xi(t', \boldsymbol{a})$, summed over the parameter $\boldsymbol{a}$. The sums range now over *all times* on one hand, and over all possible parameter values $\boldsymbol{a}$, which we assume here to be a discrete set. The generalization to continuous time and parameters is straightforward. Since Eq. (2) is linear, the statistical properties of $X(t)$ can be expressed in terms of the ones of the $\Delta\Xi(t', \boldsymbol{a})$. The latter up to the second moments are presented in the following.

Properties of the counting processes $\Delta\Xi(t, \boldsymbol{a})$

To arrive at a concise and lucid presentation we introduce the counting process

$$\sum_{\boldsymbol{a}} \Delta\Xi(t, \boldsymbol{a}) =: \Delta\Xi(t) \ , \tag{3}$$

which obviously is the total number of pulses generated in the time interval $[t, t+\Delta t]$. The benefit of introducing this process is that — under quite general conditions — the properties of the $\Delta\Xi(t, \boldsymbol{a})$ in Eq. (2) can be reduced to the ones of $\Delta\Xi(t)$.

Throughout the following we assume

(a) stationarity (independence of absolute time),
(b) and Δt to be small, so that we can expand the quantities of interest and keep only the terms of first order in Δt.

These assumptions allow the expectation and variance of $\Delta\Xi(t)$ to be written as

$$\mathrm{E}\left[\Delta\Xi(t)\right] = \mu_n \, \Delta t \, , \tag{4}$$

$$\mathrm{Var}\left[\Delta\Xi(t)\right] = \sigma_n^2 \, \Delta t \, , \tag{5}$$

and the covariance as

$$\begin{aligned} &\mathrm{Cov}\left[\Delta\Xi(t'), \, \Delta\Xi(t'')\right] \\ &:\equiv \mathrm{E}\Big[\big(\Delta\Xi(t') - \mathrm{E}\left(\Delta\Xi(t')\right)\big)\big(\Delta\Xi(t'') - \mathrm{E}\left(\Delta\Xi(t'')\right)\big)\Big] \\ &=: \gamma_n^{(0)}(t''-t')\Delta t'\Delta t'' + \delta_{t'}(t'')\Delta t'' \; \sigma_n^2 \Delta t' \, , \end{aligned} \tag{6}$$

introducing the covariance density $\gamma_n^{(0)}(t''-t')$ — for more clarity we have written the covariance in a form such that the lag 0 appears separated, defining $\gamma_n^{(0)}(0) = 0$, and using that the covariance for $t' = t''$ is the variance. The absence of correlations can now be referred to as the case $\gamma_n^{(0)}(h) = 0$. Note that μ_n, σ_n^2, and $\gamma_n^{(0)}(t''-t')$ are densities, they are the respective quantities per unit time. μ_n for instance is the pulse rate. The definition of $\delta_{t'}(t'')$ in Eq. (6) is so that $\delta_{t'}(t'')\Delta t'' = 1$ if $t'' = t'$, and zero else. This facilitates greatly translating the given formulae to the continuous time case ($\sum \Delta t \to \int dt$).

Let $w(\boldsymbol{a})\,\Delta\boldsymbol{a}$ denote the probability distribution of $\boldsymbol{A}_i$ (independent of i, due to stationarity). With the further assumption that

(c) the amplitude process $\boldsymbol{A}_i$ is independent of the trigger process (and of the counting process $\Delta\Xi(t)$)

we have

$$\mathrm{E}\left[\Delta\Xi(t, \boldsymbol{a})\right] = w(\boldsymbol{a})\,\Delta\boldsymbol{a} \;\; \mathrm{E}\left[\Delta\Xi(t)\right] \, , \tag{7}$$

since $\Delta\Xi(t)$ as well as $\Delta\Xi(t, \boldsymbol{a})$ are counting processes.

The variance and covariance of $\Delta\Xi(t, \boldsymbol{a})$ can in general not be reduced to the properties of $\Delta\Xi(t)$. However, we can make further assumptions which are still general enough for our purposes, and which allow to express the wanted properties in terms of $\Delta\Xi(t)$. These not very restrictive assumptions are that for small time intervals Δt (according to assumption (b), so that Eqs. (4), (5), and (6) hold)

(d) either pulses are rare events (i.e. it is much more probable that there is no pulse at all), or, equivalently, the occurring of several pulses at a time is highly unlikely (at best a higher order effect);
(e) mutual independence of the processes $\Delta\Xi(t, \boldsymbol{a}')$ and $\Delta\Xi(t, \boldsymbol{a}'')$ if the parameters $\boldsymbol{a}'$ and $\boldsymbol{a}''$ are different (so that $\mathrm{Cov}\left[\,\Delta\Xi(t', \boldsymbol{a}')\,,\;\Delta\Xi(t', \boldsymbol{a}'')\,\right] = 0$ for $\boldsymbol{a}'' \neq \boldsymbol{a}'$).

The assumptions imply that generally

$$\mathrm{Var}\left[\Delta\Xi\right] = \mathrm{E}\left[\Delta\Xi\right] \, , \tag{8}$$

where $\Delta\Xi$ stands for $\Delta\Xi(t)$ or $\Delta\Xi(t, \boldsymbol{a})$ (with $\boldsymbol{a}$ arbitrary), and furthermore

$$\begin{aligned} &\mathrm{Cov}\left[\,\Delta\Xi(t', \boldsymbol{a}')\,,\;\Delta\Xi(t'', \boldsymbol{a}'')\,\right] \\ &\quad = w(\boldsymbol{a}')\Delta\boldsymbol{a}' \; w(\boldsymbol{a}'')\Delta\boldsymbol{a}'' \; \mathrm{Cov}\left[\,\Delta\Xi(t')\,,\;\Delta\Xi(t'')\,\right] \, . \end{aligned} \tag{9}$$

Inserting Eq. (6) into the previous expression yields that

$$\begin{aligned} &\mathrm{Cov}\left[\,\Delta\Xi(t', \boldsymbol{a}')\,,\;\Delta\Xi(t'', \boldsymbol{a}'')\,\right] \\ &\quad = w(\boldsymbol{a}')\Delta\boldsymbol{a}' \; w(\boldsymbol{a}'')\Delta\boldsymbol{a}'' \; \gamma_n^{(0)}(t''-t')\Delta t'\Delta t'' \\ &\qquad + \delta_{t'}(t'')\Delta t'' \; \delta_{\boldsymbol{a}'}(\boldsymbol{a}'')\Delta\boldsymbol{a}'' \; w(\boldsymbol{a}')\Delta\boldsymbol{a}' \; \sigma_n^2 \Delta t' \, , \end{aligned} \tag{10}$$

where again, analogously to $\delta_{t'}(t'')\Delta t''$, we define $\delta_{\boldsymbol{a}'}(\boldsymbol{a}'') \times \Delta\boldsymbol{a}''$ to be 1 if $\boldsymbol{a}'' = \boldsymbol{a}'$, and 0 else, in order to facilitate the transition to continuous parameters.

Eqs. (7), (8), and (10) are the second order properties of $\Delta\Xi(t, \boldsymbol{a})$. Via Eq. (2), they allow to derive the respective properties of $X(t)$. This will be shown for the covariance in the next section.

3. Power spectra

For analytical calculations it is convenient to use the definition of the power spectrum as the Fourier transform of the autocovariance:

$$\begin{aligned} P(\omega) &= \big(\mathscr{F}\mathrm{Cov}(h)\big)(\omega) \\ &= \int dh \, e^{i\omega h} \, \mathrm{Cov}\Big[X(t), X(t+h)\Big] \, , \end{aligned} \tag{11}$$

where $\mathscr{F}$ denotes Fourier transformation (a definition of covariance is given on the occasion of Eq. 6). This definition is equivalent under quite general conditions to the definition as the square Fourier amplitude per unit time ($\sim (1/T) \cdot \left|\big(\mathscr{F}X\big)(\omega)\right|^2$), according to the Wiener-Khinchin theorem.

To derive the covariance of the process $X(t)$, Eq. (2) has to be inserted into the definition of covariance. After some algebraic calculations, the covariance of the processes $\Delta\Xi(t, \boldsymbol{a})$ can be recovered. Inserting Eq. (10) for the latter yields the general formula

$$\begin{aligned} &\mathrm{Cov}\Big[X(t), X(t+h)\Big] \\ &= \sum_{\boldsymbol{a}'}\sum_{\boldsymbol{a}''}\sum_{t'=-\infty}^{\infty}\sum_{t''=-\infty}^{\infty} f(t', \boldsymbol{a}')\, f(t''+h, \boldsymbol{a}'') \\ &\qquad \times w(\boldsymbol{a}')\Delta\boldsymbol{a}' \; w(\boldsymbol{a}'')\Delta\boldsymbol{a}'' \; \gamma_n^{(0)}(t'-t'') \, \Delta t'\Delta t'' \\ &\quad + \mu_n \sum_{\boldsymbol{a}'}\sum_{t'=-\infty}^{\infty} f(t', \boldsymbol{a}')\, f(t'+h, \boldsymbol{a}') \; w(\boldsymbol{a}')\Delta\boldsymbol{a}' \, \Delta t' \, . \end{aligned} \tag{12}$$

We recall that behind this formula are hidden the assumptions (a) to (e) in Sect. 2.2, which are sensible in the context of flares, as noted, and therewith they are hardly restrictive.

In case where the point process $\Delta\Xi(t)$ is uncorrelated in time (i.e. $\gamma_n^{(0)}(t'-t'')=0$), Eq. (12) obviously reduces to the second term, and the power spectrum (Eq. 11) may be written as

$$\begin{aligned} P(\omega) &= \mu_n \sum_{\boldsymbol{a}'} (\mathscr{F} f)(\omega, \boldsymbol{a}') \cdot (\mathscr{F} f)^*(\omega, \boldsymbol{a}') \cdot w(\boldsymbol{a}')\Delta \boldsymbol{a}' \\ &= \mu_n \sum_{\boldsymbol{a}'} \left|(\mathscr{F} f)(\omega, \boldsymbol{a}')\right|^2 w(\boldsymbol{a}')\Delta \boldsymbol{a}' , \end{aligned} \qquad (13)$$

as can be derived e.g. with the convolution theorem (the asterisk denotes complex conjugation, μ_n is defined in Eq. 4). After all, the power spectrum is determined by the shape of the pulses and its parameter dependence, the (uncorrelated) point process has only a minor influence on it. To discuss Eq. (13), it is worthwhile treating the case of shot noise first, i.e. the case where the pulse shape is independent of the parameters $\boldsymbol{a}$, and thereafter to turn to the more general case.

3.1. Shot noise

If the pulse shape $f(t, \boldsymbol{a})$ is independent of the parameter $\boldsymbol{a}$ (shot noise), and if the point process is uncorrelated, then, according to Eq. (13), the power spectrum is directly given by the squared absolute value of the Fourier transform of the pulse shape. Consequently, the power spectrum can have very different forms, depending solely on the particular pulse shape. In Fig. 1, the power spectra for three different pulse shapes are shown, a Gaussian ($f(t)=e^{-(2t)^2}$), a cosine ($f(t)=\cos(\pi t)$, for $-0.5 \le t \le 0.5$), and a single-sided exponential ($f(t)=e^{-2t}$, for $t \ge 0$). The location of these pulses on the time axis has no influence on the power spectrum.

We note that the general slope of the power spectra in Fig. 1 varies strongly with the different burst shapes.

3.2. Generalized shot noise

More generally, we consider a point process which is uncorrelated in time, and which is marked and filtered (generalized shot noise). The pulse shape $f(t, \boldsymbol{a})$ depends now explicitly on the parameter $\boldsymbol{a}$, and the $\boldsymbol{a}$-summation (integration) in Eq. (13) has to be performed. To evaluate the formula one has to choose a concrete pulse shape and a concrete point process:

The example of a pulse shape we use is the one proposed by Aschwanden et al. (1994). These authors consider the burst time profile to be basically sine-shaped, $a \cdot \widetilde{\sin}\left(\pi \frac{t'}{D(a)}\right)$, with the amplitude a, and the burst duration $D(a)$ a function of the amplitude ($\widetilde{\sin}(t)$ is understood as half a period of the sine, the rest set to zero: $\widetilde{\sin}(t) := \sin(t)$ for $0 \le t \le \pi$, and 0 else). This profile is assumed to be instantaneously damped, so that the final burst shape is the convolution

$$f(t,a) = \int_{-\infty}^{t} a\,\widetilde{\sin}\left(\pi \frac{t'}{D(a)}\right) \frac{\exp\left(-(t-t')/\tau\right)}{\tau}\, dt' , \qquad (14)$$

with the damping constant τ. The involved parameter values and parameter distributions are all taken from empirical estimates for type III events at 310MHz (references in Aschwanden et al. 1994): $\tau = 0.44$ sec, the amplitude distribution $w(a)\,\Delta a = a^{-1.8}\,\Delta a$, with the amplitudes restricted to the range $a_{min} = 5\,\mathrm{SFU} \le a \le 2000\,\mathrm{SFU} = a_{max}$. The duration depends on the amplitude and is given by $D(a) = 0.1421 \cdot \ln(a)$ (so that $D_{min} = 0.23$ sec $\le D(a) \le 1.08$ sec $= D_{max}$).

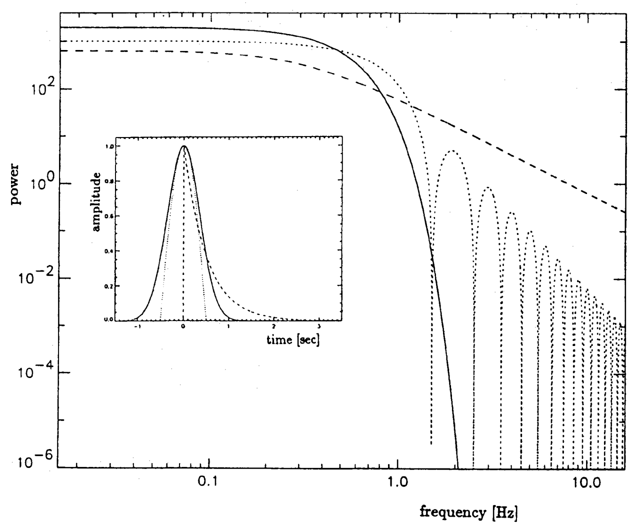

Fig. 1. Power spectra of shot noise for three different pulse shapes (depicted in the inlet): Gaussian (solid), half a period of a cosine (dotted), and a single-sided exponential (dashed), all three of similar length (about 1 sec). The pulse rate n_1 is assumed as 1.

The point process is assumed to be a Poisson process (see Sect. 2), which implies that it is uncorrelated in time. Poisson point processes can explicitly be defined by the following two properties (see e.g. Papoulis 1991, p. 59):

(i) the probability for a certain number of points to fall into a time interval Δt has Poisson distribution,

$$\mathrm{prob}\left[k \text{ points in } \Delta t\right] = e^{-n_1 \Delta t}\, \frac{(n_1 \Delta t)^k}{k!} , \qquad (15)$$

where the constant n_1 is the average number of points per unit time interval.

(ii) The events "$[k'$ points in $\Delta t']$" and "$[k''$ points in $\Delta t'']$" are independent if the two intervals $\Delta t'$ and $\Delta t''$ do not overlap.

The parameter n_1 is set to the empirically estimated value $n_1 = 1/1.47$ sec (with the help of Eq. (20) below, which is not quite correct, as shown in Remark 2 of Sect. 4, but this has no deciding influence on the power spectrum).

Inserting Eq. (14) into Eq. (13), a lengthy calculation yields a sum of several exponential integrals and a few oscillating terms which are due to the cutoffs in the amplitude range. The formula is difficult to survey unless presented graphically: In Fig. 2a, $P(\omega)$ is plotted against $\omega/2\pi$, the frequency. This spectrum is qualitatively similar to the shot noise spectra shown in Fig. 1: a (steep) power-law fall-off at high frequencies (modulated by the mentioned oscillations), and a turn-over to a constant value for smaller frequencies (which accounts for the integrability of

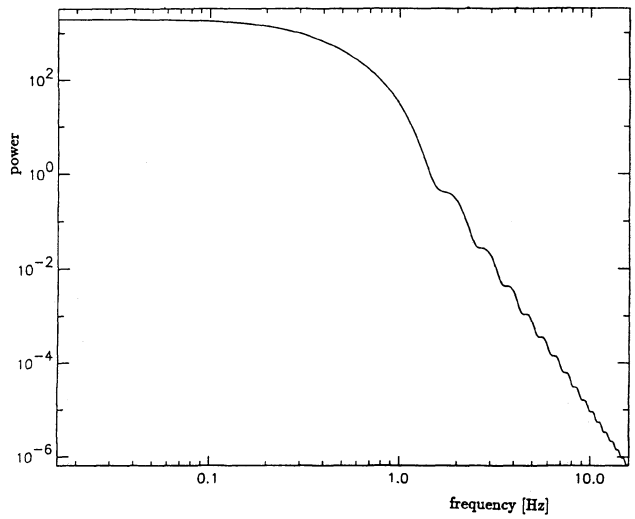

Fig. 2a. The analytically derived power spectrum $P(\omega)$ of the marked and filtered Poisson point process described in Sect. 3.2.

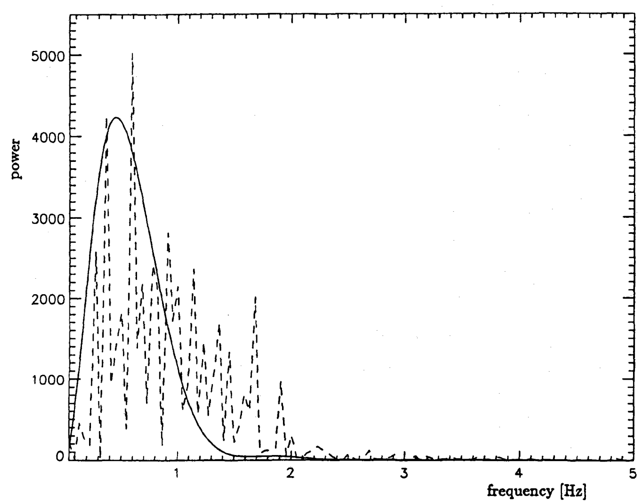

Fig. 2b. The power spectrum of Fig. 2a in a different representation: $\omega^2 P(\omega)$ is plotted, i.e. the power spectrum of the differentiated time series (solid line). The dashed line is the numerically evaluated power-spectrum for a simulation of a time series with 15 pulses (after differentiation).

the power spectrum, i.e. the second order stationarity of the process).

3.3. Comparison with empirical data

The power spectra of the shot noises (Fig. 1) and of the marked and filtered point process (Fig. 2a) are in qualitative accordance with the results of Isliker and Benz (1994): they report power spectra of different shapes, without finding any pronounced peaks in them, for type III bursts as well as for narrowband spikes events.

In order to compare the above marked and filtered point process to the power spectrum estimates of Mangeney and Pick (1989) and Zhao et al. (1991), we first have to note that they have analyzed not the data themselves, but the derivative of the time series. To differentiate a time series $X(t)$ means to multiply the corresponding Fourier transform $(\mathscr{F}X)(\omega)$ by $-i\omega$, and therewith the power spectrum $Q(\omega)$ of the derivative can be expressed as $Q(\omega) = \omega^2 P(\omega)$ (where $P(\omega) \equiv (1/T) \cdot |(\mathscr{F}X)(\omega)|^2$, the power spectrum of the original time series). In Fig. 2b, $\omega^2 P(\omega)$ vs. $\omega/2\pi$ is plotted for the power spectrum $P(\omega)$ shown in Fig. 2a. This plot can directly be compared to the results of Mangeney and Pick (1989) and Zhao et al. (1991). Obviously, our Fig. 2b is in qualitative accordance with the Fig. 1 in Mangeney and Pick (1989; the only plot of a power spectrum they give, and which they denote to be typical): There is a range of enhanced power for small frequencies, i.e. in the range roughly from 0.1Hz to 1Hz, peaked somewhere in this range. Of course, their figure is not so smooth, since they have estimated the spectrum from a finite amount of data, so the statistical error is considerable:

Using standard techniques, a power spectrum estimate for white noise has an error of about 100%, no matter how long the original time series is (see e.g. Scargle 1982). To illustrate this behaviour, we have made a numerical experiment: we have generated a realization of a time series of our marked and filtered point process with 15 pulses (which is a typical value, according to Aschwanden et al. 1994), with the parameter values chosen as above, and we have estimated the power spectrum of the resulting and differentiated time series by a Fast Fourier Transform. The result is visualized as the dashed line in Fig. 2b: the smooth curve has decayed into a spiky one. From the generation of the process it is clear that the high peaks must not be mistaken for the signature of underlying periods. (That the power of the simulation is like shifted to higher frequencies seems also to be a statistical effect: for longer time series, the curve approximates more and more the theoretical one, as we verified numerically.)

The location of the local maximum of the theoretically expected power spectrum for the differentiated time series (solid line in Fig. 2b) at about 0.45 Hz is of course depending on the details of how the model is chosen. It fits well into the distribution of "periods" as reported in Mangeney and Pick (1989) and Zhao et al. (1991). (That a local maximum does appear at all reflects only the turnover from a small slope at small frequencies to a large slope at high frequencies in the original power spectrum, Fig. 2a.)

Our results do of course not prove that the marked and filtered shot noise model with pulse shape Eq. (14), a Poisson point process as a trigger, and the parameters chosen as mentioned is the correct flare model. It is merely shown that the results of Mangeney and Pick (1989) and Zhao et al. (1991) can as well be interpreted in the frame of stochastic processes. Isolated peaks in the power spectrum can be due to statistical fluctuations, where the actual power-spectrum is continuous, and therewith the corresponding process is not periodic.

4. Time intervals between subsequent peaks

What is the statistics of the intervals between subsequent peaks for marked and filtered point processes (defined by Eq. 1) ? The details of the pulse shape are not taken into account in this analysis, and even, by considering only expectation and variance of the intervals, the information content of a time series is reduced to two numbers.

Empirical investigations suffer from difficulties to identify the peaks in a time series: Peaks have to be strong enough (not hidden in the noise), and they have to be separated enough. Else, they are not detected. We make an attempt to model these empirical insufficiencies analytically in our inquiry, assuming that after a peak has been observed a certain amount of time has to elapse before a new peak can be identified.

4.1. The general theory: renewal processes

Let $t_i^{(max)}$ denote the time when the ith local maximum (peak) in a time series occurs. The corresponding random variables $T_i^{(max)}$ generating these times form a point process. They are to be distinguished from the times T_i in Eq. (1), the point process which generates the *onset time* of the ith pulse (the trigger process). Our interest is in the time intervals ϑ_i between subsequent peaks, $\vartheta_i := t_i^{(max)} - t_{i-1}^{(max)}$. The corresponding sequence of processes Θ_i generating these intervals is termed a *renewal process* (see e.g. Papoulis 1991, p. 340ff.). It can be considered as the increment process of a random walk in time for which negative increments are forbidden.

Guided by the general form of marked and filtered point processes (Eq. 1), we assume that an observed time interval ϑ_i between subsequent peaks consists in three parts, $\vartheta_i = b_i + w_i + r_i$, i.e. in terms of random variables we consider the renewal process

$$\Theta_i = B_i + W_i + R_i \ . \tag{16}$$

The constituents are (as sketched in Fig. 3):

a) a time interval B_i, during which no bursts can be detected, since the burst identification algorithm needs a certain minimal separation in between bursts to recognize them as different events;
b) a time interval W_i, which denotes the time elapsing between the instant when the detection algorithm is able to detect bursts again and the onset of the next instability. These W_i are related to the point process (see Sect. 2);
c) a time interval R_i, describing the delay between the start of an instability and the occurrence of the respective maximum emission. R_i describes the inertia of the plasma response, and it is related to the particular pulse shape $f(t, \boldsymbol{a})$ in Eq. (1).

For the expectation of the time interval Θ_i between subsequent peaks we have directly from Eq. (16)

$$\begin{aligned} \mathrm{E}\,[\Theta_i] \equiv& \mathrm{E}\,[B_i + W_i + R_i] \\ =& \mathrm{E}\,[B_i] + \mathrm{E}\,[W_i] + \mathrm{E}\,[R_i] \ , \end{aligned} \tag{17}$$

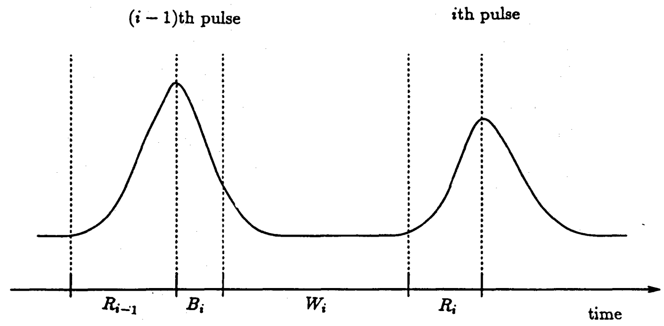

Fig. 3. Schematic drawing of a part of a time series to visualize Eq. (16): an observed time interval between two subsequent peaks consists of a blocked time B_i, the time W_i until the next onset of an instability, and the time R_i elapsed from the start of the pulse to the peak.

and, in case the three processes B_i, W_i, and R_i are *independent* or at least *uncorrelated*, the variance equals

$$\begin{aligned} \sigma^2_{\Theta_i} \equiv& \sigma^2_{(B_i+W_i+R_i)} \\ =& \sigma^2_{B_i} + \sigma^2_{W_i} + \sigma^2_{R_i} \end{aligned} \tag{18}$$

(in the presence of correlations, the respective covariances would have to be added to Eq. (18)). We assume the three processes B_i, W_i, and R_i to be *stationary*, so that the above expectation and variance are independent of the temporal index i. Eqs. (17) and (18) show that the properties of the combined process are simply given by the respective quantities of the constituent processes.

4.2. Specification

We have to specify the three processes B_i, W_i, and R_i in Eq. (16) in order to arrive at concrete results which can be compared to empirical investigations:

I *The blocked time interval B_i:* We model the insufficiency of empirical peak-detections by the simplifying assumption that after the recording of a peak a fixed time β has to elapse before a new peak can be detected. The blocked time process has probability density, expectation and variance

$$p_{B_i}(t) = \delta(t-\beta), \quad \mathrm{E}\,[B_i] = \beta, \quad \sigma^2_{B_i} = 0 \ , \tag{19}$$

with the only free parameter β. More sophisticated models are of course possible without causing any mathematical complication, since in the present context we need only to specify the expectation and the variance.

II *The time W_i until the next instability starts:* We consider the two opposite cases where the onset times of the instabilities (the trigger) are completely random, and where they are periodic:

IIa *Completely random onset times (a completely random trigger):* We assume the onset times of the instabilities to form a Poisson point process (Eq. 15). Therewith, the time between two subsequent starts of instabilities is exponentially distributed (see e.g. Papoulis 1991, p. 355).

Our random variable W_i is the waiting time from an arbitrary instant (the previous peak time plus the blocked time B_i) to the next occurring of an instability, in statistical language: the first return time. In case of Poisson processes, W_i is exponentially distributed, too (see e.g. Cox and Miller 1965, p. 356). The probability density, expectation and variance are

$$p_{W_i}(t) = n_1\, e^{-n_1 t} \,,$$
$$\mathrm{E}\,[W_i] = \frac{1}{n_1},\ \sigma^2_{W_i} = \left(\frac{1}{n_1}\right)^2 \,, \tag{20}$$

with n_1 the same constant as in Eq. (15), the expected number of pulses per unit time interval.

IIb *Periodic onset times:* Alternatively, we consider a periodic accelerator (trigger), where every P seconds a new pulse starts. Let $\widetilde{W}_i$ denote the interval from the end of the blocked time to the onset of the next pulse. A period P is the sum of the time R_{i-1} from the onset of the $(i-1)$th pulse to its peak , the blocked time B_i after the $(i-1)$th pulse, and the time $\widetilde{W}_i$ until the next pulse (number i) starts: $P = R_{i-1} + B_i + \widetilde{W}_i$ (see the sketch in Fig. 3). Whence $\widetilde{W}_i = P - B_i - R_{i-1}$, and consequently (using Eq. (19), and Eq. (22) below)

$$\begin{aligned}\mathrm{E}\left[\widetilde{W}_i\right] &= P - \mathrm{E}\,[B_i] - \mathrm{E}\left[R_{i-1}\right] \,,\\ \sigma^2_{\widetilde{W}_i} &= \sigma^2_{B_i} + \sigma^2_{R_{i-1}} \,.\end{aligned} \tag{21}$$

These considerations remain true as long as the blocked time is small enough compared to the period, more precisely as long as $B_i \le P - R_{i-1}$ holds, which we assume to be the case, here.

III *The time R_i from the onset to the peak of a pulse (the plasma response):* Assuming the pulse shape of Eq. (14), we find numerically that the peaks occur approximately $\rho_\tau \cdot D(a)$ sec after the start of the pulse. The parameter ρ_τ is restricted to the range $0.5 \le \rho_\tau \le 1.0$, depending on the damping constant τ. With $D(a) = 0.1421 \cdot \ln(a)$ and the amplitude a exponentially distributed (see text after Eq. 14), we have that R_i is exponentially distributed over a finite interval

$$p_{R_i}(t) = C_R\, e^{-k_R t} \quad \text{for } r_{min} \le t \le r_{max} \,,$$
$$\mathrm{E}\,[R_i] = \frac{1}{k_R} - \frac{\left(e^{-k_R r_{max}} r_{max} - e^{-k_R r_{min}} r_{min}\right)}{\left(e^{-k_R r_{min}} - e^{-k_R r_{max}}\right)} \,, \tag{22}$$
$$\sigma^2_{R_i} = \left(\frac{1}{k_R}\right)^2 - \frac{e^{-k_R (r_{max}+r_{min})}\left(r_{max} - r_{min}\right)^2}{\left(e^{-k_R r_{min}} - e^{-k_R r_{max}}\right)^2} \,.$$

C_R is a normalization constant, and the cutoffs r_{min} and r_{max} are given as $\rho_\tau D_{min}$ and $\rho_\tau D_{max}$, respectively, the scaled minimum and maximum burst duration.
The typical value τ = 0.44 sec (see text after Eq. 14) yields $\rho_\tau \approx 0.72$, and $k_R \approx 7.82$ for the exponent.

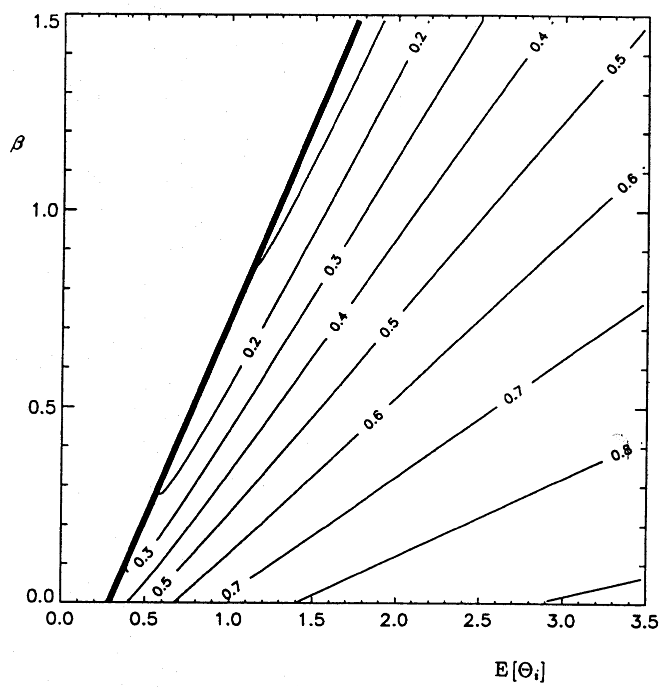

Fig. 4. Contour plot of $\frac{\sigma_{\Theta_i}}{\mathrm{E}[\Theta_i]}$ as a function of the expectation $\mathrm{E}\,[\Theta_i]$ and the blocked time β. The process is the stochastic model with a *random (Poisson distributed) trigger signal*, as described in the text.

4.3. Comparison with empirical data

Inserting Eqs. (19) to (22) into Eqs. (17) and (18) we can analytically calculate the expectation and variance for the times Θ_i between subsequent peaks for the described model.

Random acceleration

In case where the onset times of the instabilities are random (a random accelerator or trigger; Case (IIa), Eq. 20), the essential parameters are: the number n_1 of pulses per unit time interval characterizing the point process, the duration of the blocked time β, and the pulse shape (including the statistics of the respective parameters). Fig. 4 is a contour plot of the relative spread of the times between peaks $\frac{\sigma_{\Theta_i}}{\mathrm{E}[\Theta_i]}$ as a function of $\mathrm{E}\,[\Theta_i]$ and β. The parameter n_1 remains hidden in the presentation — which makes sense since $\mathrm{E}\,[\Theta_i]$ is directly measurable, whereas there is no direct access to n_1. The pulse parameters are kept constant to the typical values reported above (Eq. (22); τ = 0.44 sec, $\rho_\tau \approx 0.72$, and $k_R \approx 7.82$).

Aschwanden et al. (1994) have empirically estimated the mentioned statistics from a large amount of type III events. They report that their peak identification algorithm has a blocked time β around 0.5 sec (due to a smoothing procedure which smears out shorter structures, and due to the set-up of the applied algorithm to detect local maxima). They find that the average time between subsequent peaks $\mathrm{E}\,[\Theta_i]$ is in the range from 1 to 2 sec, and for $\frac{\sigma_{\Theta_i}}{\mathrm{E}[\Theta_i]}$, which they call 'degree of periodicity' or 'mean periodicity', they find $\frac{\sigma_{\Theta_i}}{\mathrm{E}[\Theta_i]} = 0.37 \pm 0.12$. Fig. 4 shows that this value is well compatible with a stochastic flare model:

Given that β and $E[\Theta_i]$ are in the stated ranges, the quantity $\frac{\sigma_{\Theta_i}}{E[\Theta_i]}$ varies from about 0.2 to 0.7.

Remark 1: The value of $\frac{\sigma_{\Theta_i}}{E[\Theta_i]}$ changes with parameters: increasing ρ_τ (see Eq. 22) from its minimum 0.5 to its maximum 1.0 causes a change of about 20%, and changing the lower amplitude cut-off a_{min} (see text after Eq. 14) by a factor of 2 causes a change of about 10%. The other parameters have substantially less influence on the results. Analyzing pulse shapes different from Eq. (14) would not yield principally different results, since only the expectation $E[R_i]$ and variance $\sigma^2_{R_i}$ enter the present calculations.

Remark 2: Aschwanden et al. (1994) supplement their investigations with a numerical study: they generate time series of the marked and filtered point process with pulse shape Eq. (14) and a Poisson point process. The pulse parameters are chosen as described in the text after Eq. (14). They intend to model time series with $E[\Theta_i] \approx 1.47$ sec, since this value has turned out to be the empirically determined average. However, they prescribe $n_1 = 1/1.47$ sec, so that $E[W_i] = 1.47$ sec (Eq. 20) instead of $E[\Theta_i]$, and therewith they generate time series with larger intervals between peaks than intended, namely $E[\Theta_i] \approx 2.27$ sec according to our formula. They find $\frac{\sigma_{\Theta_i}}{E[\Theta_i]} = 0.73$, in accordance with our analytical derivation which yields that $\frac{\sigma_{\Theta_i}}{E[\Theta_i]} = 0.65$ has to be expected (see also Fig. 4).

The simulations of Aschwanden et al. (1994) agree with our analytical calculations. They do not disprove stochastic models to explain features found in the data, however, since the value of $\frac{\sigma_{\Theta_i}}{E[\Theta_i]}$ strongly varies with changing parameters, as Fig. 4 illustrates.

Periodic acceleration

What values of $\frac{\sigma_{\Theta_i}}{E[\Theta_i]}$ can be expected in case of a periodic accelerator (a periodic trigger; Case (IIb), Eq. 21) ? Note that the process Θ_i is still stochastic in this case, only one of the constituent processes is replaced by a deterministic one. This causes stronger temporal correlations to be present as they are in case of a random acceleration process.

Using Eq. (21) instead of Eq. (20) we see that $E[\Theta_i] = P$ and $\sigma^2_{\Theta_i} = \sigma^2_{R_{i-1}} + \sigma^2_{R_i} = 2\sigma^2_{R_i}$. The parameter β of the blocked time does not appear (at least if we assume that $B_i \leq P - R_{i-1}$ holds, as mentioned above). The result is visualized in Fig. 5 as the solid line, where $\frac{\sigma_{\Theta_i}}{E[\Theta_i]}$ is plotted against $E[\Theta_i]$, the period P. The interesting range for $E[\Theta_i]$ is again in between 1 sec and 2 sec, where $\frac{\sigma_{\Theta_i}}{E[\Theta_i]}$ takes values roughly between 0.1 and 0.2.

Remark 3: The results vary of course with the exact pulse shape, or in terms of renewal processes, they vary with the definition of the process R_i (Eq. 22). To give an impression of this variability, two more curves of $\frac{\sigma_{\Theta_i}}{E[\Theta_i]}$ are shown in Fig. 5, derived for different parameter values: they differ by the value of ρ_τ, which describes the peak of a pulse as a linear function of its duration (see (III) in Sect. 4.2), and implies different average delays $E[R_i]$ between the onset of pulses and their peaks. Additionally to the case treated by Aschwanden et al. (1994), where $\rho_\tau = 0.72$ ($E[R_i] \approx 0.29$ sec, solid curve), we also show the

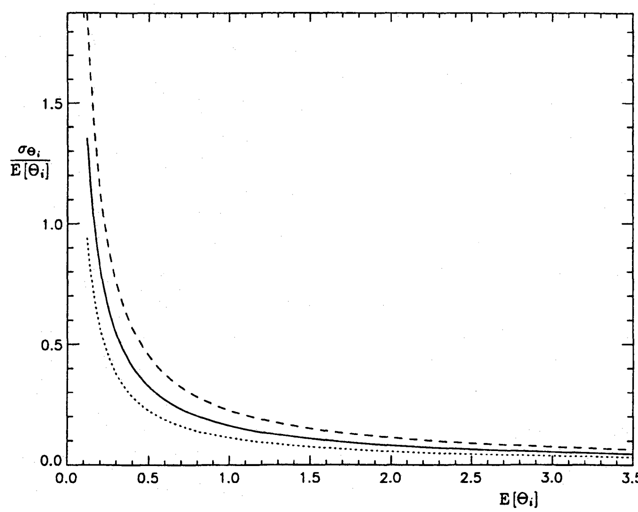

Fig. 5. $\frac{\sigma_{\Theta_i}}{E[\Theta_i]}$ as a function of the expectation $E[\Theta_i]$ for the stochastic model with a *periodic trigger* of period P ($E[\Theta_i] = P$, see text). The solid line is for the case $\rho_\tau = 0.72$, the dotted curve for $\rho_\tau = 0.5$, and the dashed curve for $\rho_\tau = 1.0$ (see text).

case $\rho_\tau = 0.5$ ($E[R_i] \approx 0.20$ sec, dotted curve), and the case $\rho_\tau = 1.0$ ($E[R_i] \approx 0.40$ sec, dashed curve). The other pulse parameters in Eq. (22), the range r_{min}, r_{max} and the exponent k_R, have much less influence on the value of $\frac{\sigma_{\Theta_i}}{E[\Theta_i]}$.

Remark 4: Aschwanden et al. (1994) have made numerical simulations of this case, too, and they find that $\frac{\sigma_{\Theta_i}}{E[\Theta_i]} = 0.2$, if the detection efficiency is high enough. This is in accordance with our Fig. 5, and it confirms again that our analytical description is reasonable.

What the values of $\frac{\sigma_{\Theta_i}}{E[\Theta_i]}$ regards, the cases of periodic acceleration and of a random trigger are not distinctly different. This parameter therefore cannot be considered as being able to distinguish the two cases. Particularly, the value empirically found by Aschwanden et al. (1994) can well be explained by a random trigger.

5. Conclusion

The discussed stochastic flare model can explain present-day's observational results concerning power-spectra, correlation dimensions, and statistics of the times between subsequent radio peaks:

Isliker and Benz (1994) find no hint for periodic behaviour and for finite dimensions in power-spectrum and correlation-dimension estimates for type III and narrowband spikes events. Their results characterize a complex and maybe even a stochastic process, and they are in accordance with the properties of the presented stochastic flare model (we verified numerically the (theoretically obvious) fact that our model yields no finite correlation dimensions).

Mangeney and Pick (1989) and Zhao et al. (1991) have analyzed differentiated time series of type III events and report to have detected many isolated peaks in the corresponding power spectra, which they consider to be indicative of periodic processes. We make the conjecture that these observed power spectra can be interpreted in terms of the introduced stochastic flare model. This has been proved in one case, where it has been shown that the power spectrum of our model (for differentiated time series) looks qualitatively similar to an empirically estimated one, the one displayed in Mangeney and Pick (1989). We generalize this result since the authors refer to the displayed example as a typical one, and therewith we claim that the isolated peaks they observe in the power spectra of differentiated time series can as well be the signature of a stochastic process, which is temporally correlated and whose time series consists in a sequence of localized and peak-shaped structures. The power spectrum resulting from such a process (a broadened peak if the time series is differentiated) decays into isolated peaks in empirical estimates since the available time series are short, and therewith the fluctuations in the estimated power spectra are considerable.

The empirical results of Aschwanden et al. (1994) can also well be reproduced by the discussed stochastic flare model. Temporal correlations in the time series cause the relative spread $\frac{\sigma_{\Theta_i}}{\mathrm{E}[\Theta_i]}$ of the times between subsequent peaks to take values in a broad range, depending on the details of the process (mainly its correlations). Further variation is added to this parameter by the uncertainties in peak detection. The sum of these variations is so considerable that the parameter $\frac{\sigma_{\Theta_i}}{\mathrm{E}[\Theta_i]}$ is not able to discriminate between periodic and deterministic behaviour.

After all, the reason why we arrive at a conclusion which is different from the one of Mangeney and Pick (1989), Zhao et al. (1991), and Aschwanden et al. (1994) is that these authors have mostly considered uncorrelated stochastic processes (white noise), with the only exception of one example in Aschwanden et al. (1994), where the sensitive dependence on parameters has not been realized, however. We, on the other hand, have inquired a large class of temporally correlated stochastic processes, and found them to be very rich in possible properties. Stressing temporal correlations is not a mathematical trick, since they are an inherent property of the observed time series, and they consequently have to appear in every realistic flare model.

A little note: we have mainly discussed the correlations induced by the pulse shape. An extension to a correlated trigger process is possible (some of the given formulae contain the respective additional terms) and would certainly make sense, depending on the assumed flare scenario. The properties of such processes must be expected to be even more complex, in the sense of being even harder to distinguish from deterministic processes than the discussed models.

We emphasize that our inquiry does not prove that the model in its introduced form, particularly with the used pulse shape and the respective parameter values, is the correct flare model. The properties vary too strongly with the parameters in order that such a conclusion were possible. We have shown, however, that the interpretations of the empirical results are not as unique as some authors seem to believe. Above all, the interpretation in terms of a (correlated) stochastic process is still possible, and therewith a strictly periodic process is rather unlikely.

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