STRUCTURAL PROPERTIES OF THE DYNAMICS IN FLARE FRAGMENTATION*

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Abstract. Solar flares have a fragmented structure. Dynamical systems theory, for instance in its form of dimensional analysis, can analyze such structures. It answers the question whether the underlying process is deterministic or stochastic. If the process is deterministic, it provides a measure of how complicated the process is (the fractal dimension). In order to be reliable, the analysis has to be combined with the investigation of stationarity.

We apply this method to ms-spikes, observed in the decimetric range, which are possibly a manifestation of flare fragmentation. We compare the system-theoretical properties – such as stationarity, stochasticity or deterministic behaviour – of the ms-spikes to the properties of several classes of suggested scenarios. This permits us to discuss different scenarios from a general point of view and to derive general properties of the source.

1. Introduction

Chaos theory, a branch of dynamical systems theory, provides tools to analyze general properties of systems. It can find out whether a process has a regular (i.e., a quasi-periodic), a deterministic chaotic (i.e., governed by nonlinear equations, however very sensitive to initial conditions) or an accidental (stochastic) temporal structure. Regular systems can be identified by Fourier analysis. Chaos theory is able to rule out the other two possibilities. It turns to the phase space of a system and looks at the set spanned by the trajectories, of which it determines for instance the fractal dimension. This is a measure of system-complexity, in the sense that it corresponds to the lower limit of the number of variables which are necessary to describe a system. Stochasticity corresponds to an infinite fractal dimension. The method uses only one observed variable of the system, e.g., in the form of a measured time series.

We will apply the method to three ms-spike events, recorded with the Ikarus spectrometer at ETH Zürich in 1982 (they are published in Güdel and Benz, 1990). Under the assumption that spike events are a manifestation of the flare fragmentation process, this yields fundamental properties of the fragmentation process itself.

In recent years, there have been several criticisms on a straightforward use of dimensional analysis. Cannizzo, Goodings, and Mattei (1990) find that a periodic system with noise might mimic chaotic behaviour. They, however, need a signal-to-noise ratio of about 50%. As we are far below this ratio in our spike measurements, we will not have to fear any deceptive results from this effect.

More serious is the finding of Osborne and Provenzale (1989). They investigate a variation of a random walk process, known as fractional Brownian motion. They show that this stochastic process can have a finite fractal dimension. Isliker and Kurths (1992)

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have developed a test to check whether dimensions are due to fractional Brownian motion. They argue that fractional Brownian motion belongs to the more general class of non-stationary processes. They find a fast and comprehensive tool to investigate stationarity in using an invariant measure in phase space associated with a process.

We will outline in the following the method of dimensional analysis and of testing stationarity. This will then be applied to the millisecond spike events. The conclusion discusses the consequences on the fragmentation process.

2. The Methods

2.1. Correlation dimensions

The notions of chaos theory characterize the movement of a system in phase space. Generally there will be given a time series $\{X(t_i)\}_{i=1}^{N}$. According to the time-delay method, developed by Takens (1981), a copy of phase space can be reconstructed: we build up vectors $\xi(t_i)$ in a d-dimensional space from the time series by

$$\xi(t_i) := (X(t_i), X(t_i + \Delta t), \ldots, X(t_i + (d - 1)\Delta t)) \tag{1}$$

(the time delay Δt is any multiple of the time resolution $\tau = t_{i+1} - t_i$). These vectors span the reconstructed phase space. The real state space of dimension D is related to this reconstructed space by an embedding, whenever

$$d > 2D + 1 \tag{2}$$

holds. For the movement of the system in phase space, there will generally be a limit set to which trajectories are attracted after transients have died out, the attractor. It will be an n-torus in the case of a regular process, which is the sum of n-independent periodic modes. It will fill the entire phase space for a stochastic movement. And last, for deterministic chaotic movement, it will be a highly complicated set: an invariant set of which two initially close trajectories will separate from each other exponentially fast. Generally, it is a fractal set in this latter case. Its fractal dimension is less than infinity – all these features can geometrically be characterized by an adequate notion of dimension.

Embeddings have the property that they conserve quantities such as dimensions. From that it is possible to calculate fractal dimensions in order to determine the character of a system in the reconstructed phase space. We use the correlation dimension proposed by Grassberger and Procaccia (1983a, b). The correlation integral is defined as

$$C_d^{(2)}(\varepsilon) := \lim_{N \to \infty} \frac{2}{N(N-1)} \sum_{i<j}^{N} \Theta(\varepsilon - |\xi_i - \xi_j|), \tag{3}$$

with the Heaviside function $\Theta(.)$ and any vector norm '$|.|$'. The correlation dimension $D^{(2)}$ is defined via the scaling property of this quantity:

$$C_d^{(2)}(\varepsilon) \sim \varepsilon^{D^{(2)}}, \quad \text{for } \varepsilon \to 0 . \tag{4}$$

Details on technical questions around the evaluation of this correlation dimension can be taken from Atmanspacher, Scheingraber, and Voges (1988). A finite correlation dimension in the absence of any peaks in the power spectrum is indicative of a low-dimensional deterministic chaotic process. In fact, the method will not be able to distinguish between *high*-dimensional chaos and stochasticity, because there will be only a finite number of measured data points. They contain just a limited amount of information about the process. With too few points, the dimension will be infinity, the dimension of noise.

Isliker (1992) showed that the deciding quantity is not the number of points but the number of 'least structures of interest', i.e., of spikes. A single peak (spike) in the time series should be covered by about 10 to 20 points. Then, as a rule, 500 points are sufficient to find dimensions $D^{(2)}$ up to about 5 or 6, corresponding to about 50 spikes. The commonly used $10^{D^{(2)}}$ as a lower limit for the necessary number of points (Brandstater and Swinney, 1987; Ruelle, 1990) is too high and ambiguous, unless the number of least structures is specified.

It is important to have an estimate of the error of the correlation dimension. To determine the correlation dimension itself, one would take the logarithms of Equation (4). The correlation dimension is equal to the slope of $\log C_d^{(2)}(\varepsilon)$ against $\log \varepsilon$. It is widely in use to take the error of $D^{(2)}$ to be the error of a least-square fit of a straight line into this logarithmic relation. This error estimate has the disadvantage to be not an intrinsic error, its values are unreasonably small. We follow the proposition of Ellner (1983): the expected form $\varepsilon^{D(2)}$ of the correlation integral, $C_d^{(2)}$, is interpreted as a probability distribution in the space of distances $|\xi_i - \xi_j|$. The average value of this distribution and its mean error are functions of the correlation dimension $D^{(2)}$. Inverting these relations, we get for the 5% significance error Δ of the correlation dimension:

$$\Delta = \frac{D^{(2)} 1.96}{\sqrt{n}} \, \frac{\sqrt{1 + 2 \ln r_0^\alpha r_0^\alpha - r_0^{2\alpha}}}{1 + \ln r_0^\alpha r_0^\alpha - r_0^\alpha} \,, \tag{5}$$

with n chosen about $N/2$, half of the number of points, and $r_0 = r_1/r_2$, where $r_1 \le \varepsilon \le r_2$ is the linear scaling region of $\log C_d^{(2)}(\varepsilon)$ as a function of $\log \varepsilon$.

2.2. A Test of Stationarity

Stationarity is the property that all statistical quantities of a process are independent of absolute time; they are at most a function of relative times.

The fractional Brownian motion of Osborne and Provenzale (1989) is an example of a non-stationary, stochastic process. It is a self-affine process, and self-affinity is a scaling behaviour; for that reason it shows a finite correlation dimension. Therefore, to prevent misinterpretation, a dimensional analysis must include a test of stationarity. If one is able to assert the stationarity of a process, a large class of processes is excluded, above all the class of stochastic processes with finite correlation dimension which is known at present, the fractional Brownian motion.

To investigate stationarity, a time series is usually divided into several parts and statistical properties of each part are compared. A detailed picture of a stationary process is given by one of its invariant measures, ρ. It is operationally defined in the d-dimensional state space as the time average of Dirac δ-distributions along a trajectory $\mathbf{x}(t)$,

$$\rho := \lim_{T \to \infty} \frac{1}{T} \int_0^T \delta \mathbf{x}(t) \, \mathrm{d}t \tag{6}$$

(Eckmann and Ruelle, 1985). It is a probability density, measuring how frequently the different parts of state space are visited. If the system is assumed to be ergodic, then space averages, with ρ as weight, indeed equal time averages:

$$\int_{\text{state space}} f(\mathbf{x}) \rho (\mathrm{d}\mathbf{x}) = \lim_{T \to \infty} \frac{1}{T} \int_0^T f(\mathbf{x}(t)) \, \mathrm{d}t \,, \tag{7}$$

for any function $f(\mathbf{x})$ in this state space. Simplifying, we write the density ρ as $\rho(\mathbf{x}) \, \mathrm{d}\mathbf{x}$ instead of $\rho \, (\mathrm{d}\mathbf{x})$. As we do have only one coordinate of state space accessible (say x_1), the projection $\rho(x_1) \, \mathrm{d}x_1$ of $\rho(\mathbf{x}) \, \mathrm{d}\mathbf{x}$ onto this coordinate x_1 is considered:

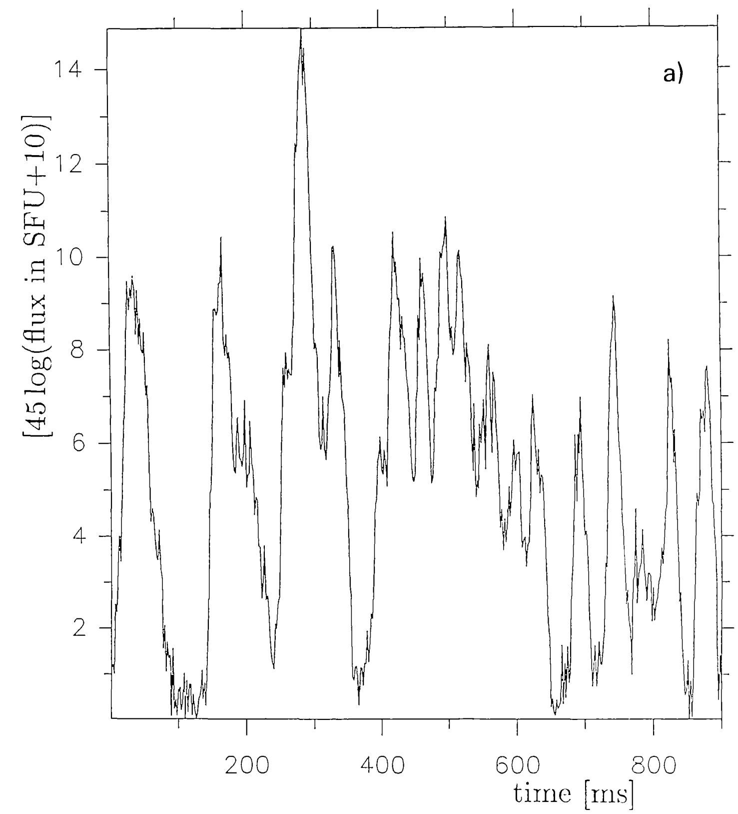
$$\rho(x_1) \, \mathrm{d}x_1 := \int \rho(\mathbf{x}) \, \mathrm{d}x_2 \, \mathrm{d}x_3 \ldots \mathrm{d}x_n \,. \tag{8}$$

We calculate this measure empirically by dividing the x_1-axis (which is the flux in our data, the measured $\{X(t_i)\}_{i=1}^{N}$) into intervals, counting the measured points falling into these intervals, and normalizing. This is actually a normalized histogram.

Stationarity for a certain range of time is tested by calculating the invariant measure for the entire time range, and the one based just on the first half of the same time range. The two probability distributions are compared then by means of a χ^2-test. This is illustrated in Figure 1. In Figure 1(a), the section 10:06:35.5–10:06:36.4 out of the spike event on 17 July, 1982 is plotted, and in Figure 1(b) the histogram of $\rho(x_1) \, \mathrm{d}x_1$ is shown (solid line), together with the histogram basing only on the first half of this same time section. The significance of the χ^2-test is fairly high (95%). Figure 2 presents the same for the section 10:38:46.0–13:38:50.0 of the event on 4 June, 1982. In Figure 2(a) is the time profile, and in Figure 2(b) are the histograms. The coincidence is very bad, the two distributions are significantly different at the 95% level. The event changes substantially during this time section, it is not stationary, and it is definitely not appropriate for a dimensional analysis.

3. Results

We investigated three ms-spike events. The dates, times, frequencies and time resolutions are given in Table I. The data are taken in a temporal, high-resolution measurement combined with spectral resolution. For a discussion of the time profiles of these



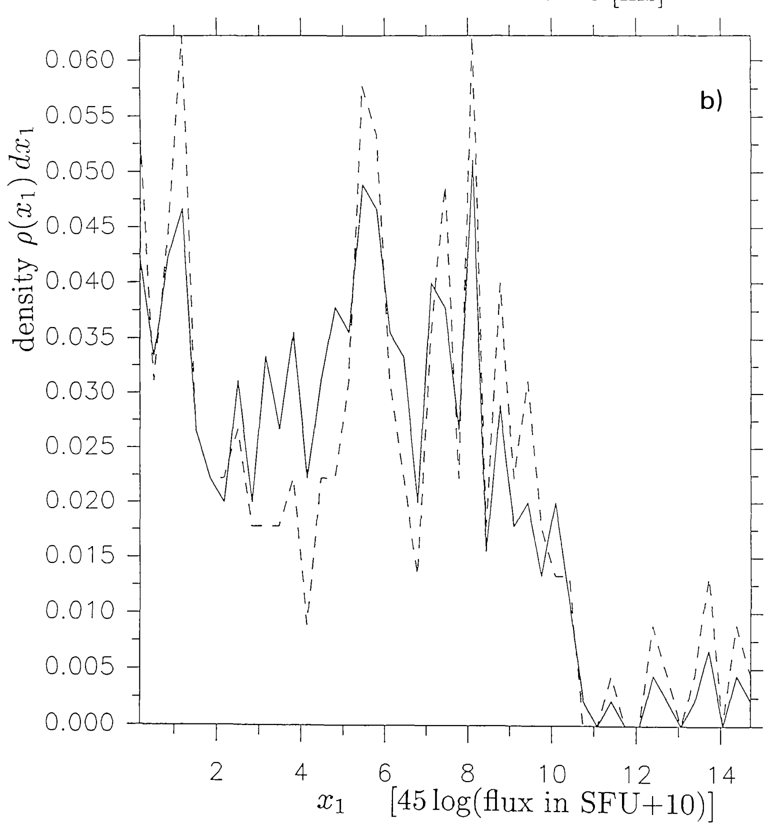


Fig. 1. (a) The time profile of the section 10:06:35.5–10:06:36.4 out of the ms-spike event of 17 July, 1982 (see Table I). Times are in milliseconds after 10:06:35.5 UT. (b) The invariant densities, $\rho(x_1)\,\mathrm{d}x_1$, are based on the entire times series (solid line) and on the first half of the same time series, respectively (broken line). We used 45 bins on the x_1-axis. (For a better visualization we plot the midpoints of the intervals against the density instead of the histograms.)

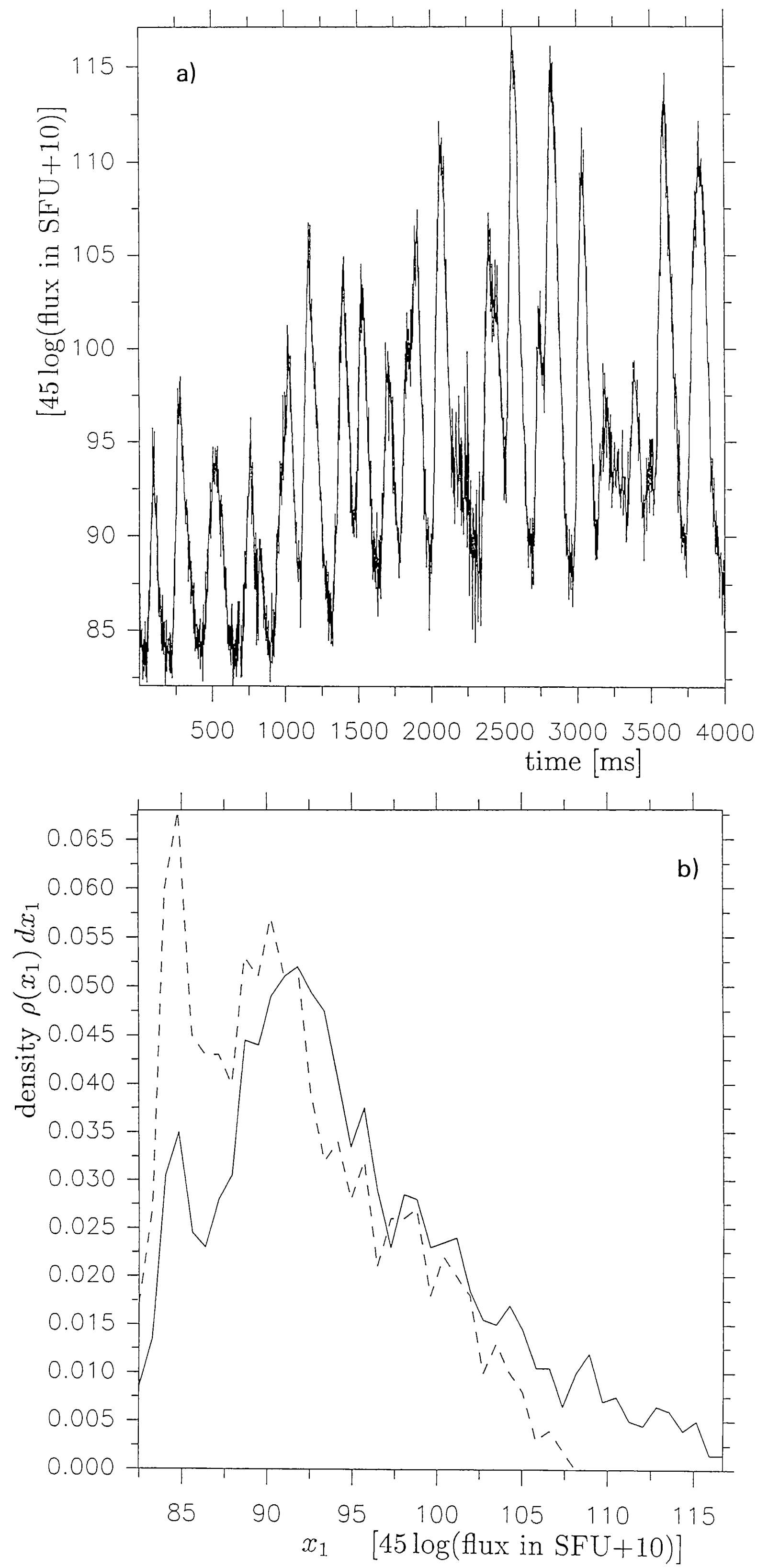


Fig. 2. (a) The time profile of the section 13:38:46.0–13:38:50.0 out of the ms-spike event of 4 June, 1982 (see Table I). Times are in milliseconds after 13:38:46.0 UT. (b) The invariant densities, $\rho(x_1)\,dx_1$, are based on the entire times series (solid line) and on the first half of the same time series, respectively (broken line). We used 45 bins on the x_1-axis. (For a better visualization we plot the midpoints of the intervals against the density instead of the histograms.)

TABLE I

The times, durations, frequencies, and time resolutions of the three investigated ms-spike events. In the third box, the times of the parts are listed for which a correlation dimension was found. '∞' for the correlation dimension $D^{(2)}$ means that the correlation integral did not converge, the system is stochastic or high-dimensional deterministic chaotic ($D^{(2)} \gtrsim 8$). The last column gives the significance levels of the χ^2-test for stationarity, based on the invariant measures (see text). All times are in universal time (UT).

Data			Freq.	time res.	Section		Corr. dim.	Stationarity significance
Date	Start	Points	(MHz)	τ (ms)	from	to	$D^{(2)}$	
4 June, 1982	13:38:41	16'500	363	2	13:39:02.0	13:39:12.0	5.7 ± 0.4	95%
					rest		∞	–
17 July, 1982	10:06:26	10'000	770	2	all		∞	–
16 Dec., 1982	10:04:22	12'000	1010	10	all		∞	–
16 Dec., 1982	10:04:22	12'000	870	10	all		∞	–
16 Dec., 1982	10:04:22	12'000	730	10	all		∞	–

ms-spikes see Güdel and Benz (1990). Temporal high resolution is a necessary condition for a dimensional analysis to be applicable, for the measurements have to contain information about the structures of least interest. The data satisfy this condition, having about 10 to 40 points per single spike. Furthermore, the signal-to-noise ratio is about 20 : 1. These two properties make the data especially appropriate for the search of dimensions.

This search has to be done in a systematic way: not just the entire time series must be analyzed, but smaller sections of it. This has to be continued until either a stationary part with a dimension is identified, or a limit of about 50 spikes is reached – smaller divisions are not possible for technical reasons, as stated above. The reason is that the system need not be on an attractor during the entire time of measurement, if at all. For the system may suffer a disturbance on its attractor, leave the current attractor, go to a new one or remain in a transient phase. The shape of the attractor can change, too, as it depends on the very same exterior parameters as do the underlying equations of a dissipative system.

We did not find any dimensions for the whole time series, just for parts. They are listed in Table I. In the event of 4 June, 1982, one part of 10 s duration was found with a correlation dimension between 5 and 6. Figure 3 shows the event, the part with dimension marked by a horizontal bar. The event of 17 July, 1982 showed no part with correlation dimension, it is represented in Figure 4. It is interrupted by two gaps of 4 and 2 s where there is no spike emission, only noise. In the 16 December, 1982 event there are no parts with dimension in all three frequency channels which lie far apart in the spectrum. This may partly be due to the poorer time resolution of 10 ms for this recording. The number of points per spike is between 5 and 10, which is the lower limit for detecting any structures by a dimensional analysis.

This section with a finite dimension was tested for stationarity. We usually subtracted a minimum envelope from the time series with a time constant much greater than the

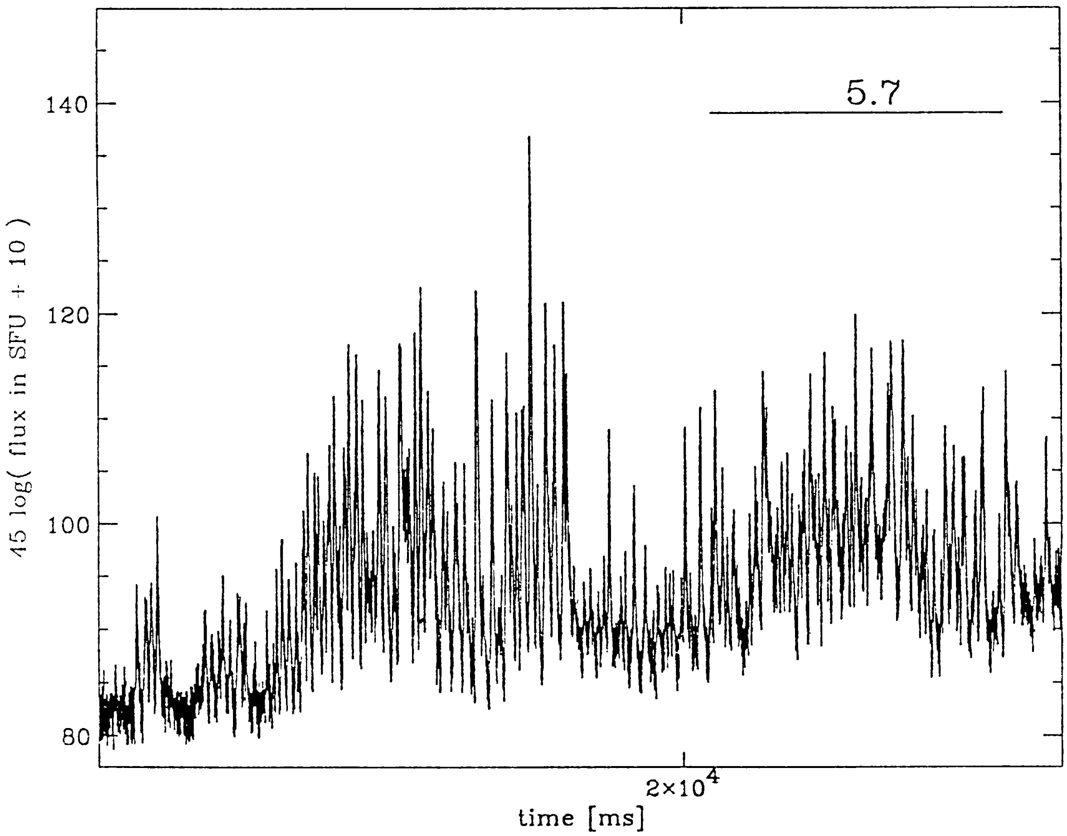


Fig. 3. The time profile of the entire ms-spike event on 4 June, 1982. Precise time, frequency, and time resolution are given in Table I. Times are in milliseconds after 13:38:41.0 UT. The horizontal bar marks the section in which a correlation dimension was found – see text for its discussion. The number above the bar is the derived correlation dimension $D^{(2)}$.

characteristic time of the process (the autocorrelation time). In other words, we allow for the possibility that the ms-spike events are superimposed upon a varying background, not deterministically connected with the ms-spike phenomenon. This does not affect the evaluation of the correlation dimensions.

Table I lists the significance level of the interesting part: in the 4 June, 1982 event, there is no doubt about the section, its confidence level is rather high (95%). By the way, in all three events, there are many stationary sections without a finite correlation dimension.

4. Conclusion

One of the three investigated ms-spike events shows a correlation dimension in a stationary state. It must be considered as deterministic chaotic. Its dimension is rather high, between 5 and 6. At least six nonlinear equations are necessary to describe the process – many usually-studied model equations for chaos are below that, e.g., the Lorenz or the Rössler equations.

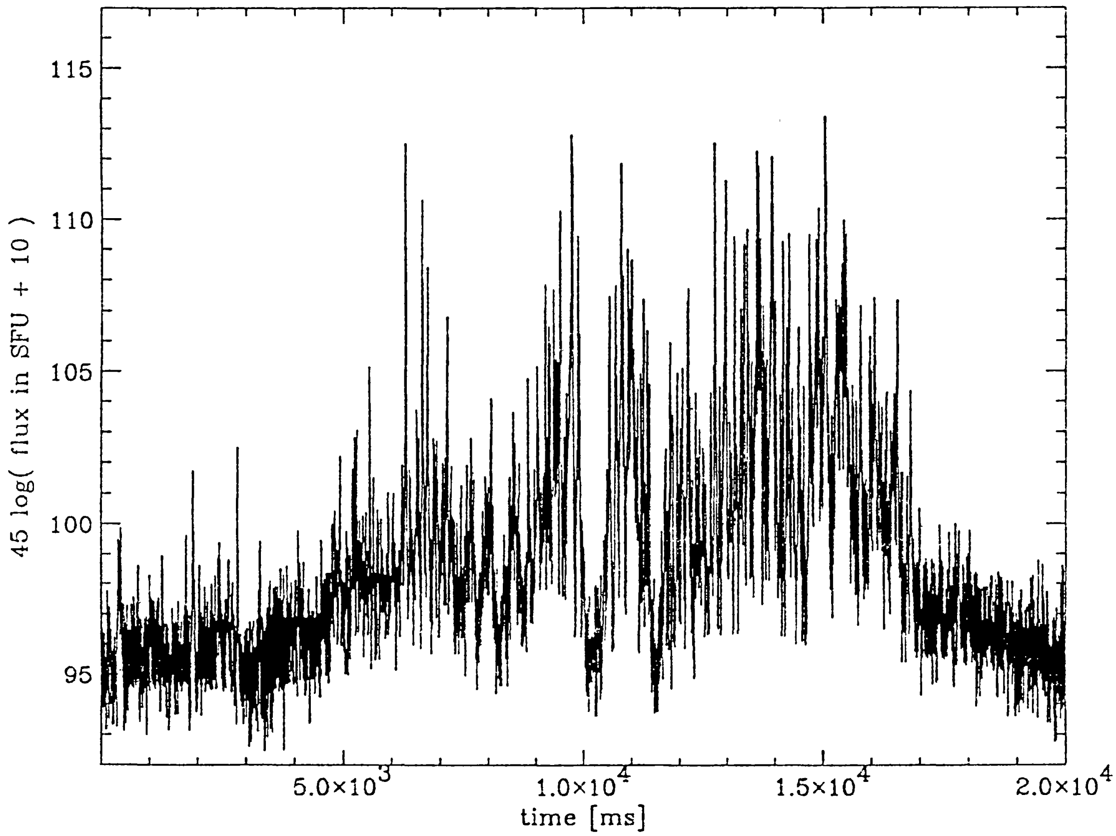


Fig. 4. The time profile of the entire ms-spike event on 17 July, 1982. Precise time, frequency, and time resolution are given in Table I. Times are in milliseconds after 10:06:26.0 UT.

This has an impact on the understanding of the radio source fragmentation at a given frequency: it has deterministic, however chaotic, phases. The single ms-spikes are related to each other by an evolution equation, they do not occur at random for one fixed frequency during such a phase. There must be either one single source, bursting out many times, following a nonlinear model. Or else, there must be several sources emitting with the same frequency in a strongly coherent way: they are connected in a non-random manner, which could be modeled by nonlinear interaction equations. In both cases, the number of independent radio sources is smaller than the number of ms-spikes in an event.

Outside these deterministic chaotic phases the data show two kinds of behaviour: (i) There occur non-stationary sections, probably transient states. (ii) There are many stationary parts.

In case (ii), during a stationary state, the system might be:

(a) in a *stochastic*, stationary phase;

(b) in a *high-dimensional deterministic chaotic* phase (dimension greater than about 8). The complexity (from low- to high-dimensional deterministic chaos) can change for two reasons:

(1) The global or boundary conditions of the source change, correspondingly the

exterior parameters (constants) of the system change, leading to a different shape of the attractor.

(2) There are several independent sources, some of them accidentally emit on the same frequency, each in a deterministic chaotic way (as in the sections with finite correlation dimension). Their superposition yields a new high-dimensional dynamic structure.

The data do not allow for a decision between the two possibilities (a) and (b). We would, however, consider the scenario of high-dimensional deterministic chaos (case b) as the more likely ones. Equations governing a system during a certain time will be present in the system the whole time. It would be difficult to conceive that a system can change from deterministic to stochastic.

Since narrowband ms-spikes are likely signatures of the primary energy release, the deterministic character of the spike sources suggests that the flare fragmentation is not only in space (as indicated by the frequency variation of spikes). There is also a fragmentation in time, meaning that the same source can burst several times. Its behaviour then is controlled by a set of equations and therefore deterministic.

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