

Contents

1	Describing fluids and fluid flows	1
1.1	Introduction	1
1.2	Wood, water, air	2
1.3	Forces: motion and equilibrium	3
1.3.1	MKS-system of physical dimensions	3
1.3.2	Newton's equations of motion	3
1.3.3	Viscosity	5
1.4	Describing a fluid	6
1.4.1	Material properties	6
1.4.2	Fluid variables	6
1.5	Describing fluid flow	8
1.5.1	Static/dynamic and steady/unsteady	8
1.5.2	Analysing fluid motions	9
1.5.3	Vorticity	11
1.5.4	Circulation	12
1.5.5	Fluid element	13
1.5.6	Streamlines, particle paths and streaklines	13
1.6	Modelling fluids	17
1.6.1	Eulerian description	17
1.6.2	Lagrangian description	17
1.6.3	Boundary conditions and initial conditions	18
1.6.4	Computational fluid dynamics	19
2	Mathematical techniques	21
2.1	Overview	21
2.2	Vector calculus in Cartesian coordinates	21
2.3	Vector calculus in orthogonal curvilinear coordinates	22

CONTENTS

2.3.1	Cylindrical polar coordinates	23
2.3.2	Spherical polar coordinates	25
2.4	Vector analytical identities	26
2.5	Integral theorems	27
2.5.1	Gauß' integral theorem	27
2.5.2	Stokes' integral theorem	27
2.6	Cartesian index notation	28
2.7	Matrices and linear transformations	29
2.8	Dirac's delta function	32
2.9	Orthonormal function expansions	32
2.9.1	Fourier series expansions	33
2.9.2	Fourier integral representations	34
3	Introduction to fluid flows	37
3.1	Basic concepts	37
3.1.1	Convective derivative	37
3.1.2	Balance equations	40
3.1.3	Conservation of mass	41
3.1.4	Incompressibility	42
3.1.5	Pressure force	43
3.1.6	Hydrostatics and Archimedes' principle	44
3.2	Ideal fluids	46
3.2.1	Euler's equations of motion	46
3.2.2	Bernoulli's streamline theorem	47
3.2.3	Bernoulli's streamline theorem for irrotational flow	48
3.2.4	Example: fluid streams	49
3.2.5	Vorticity equation	50
3.2.6	Boundary conditions for ideal fluids	52
3.3	Viscous fluids	52
3.3.1	Boundary conditions for viscous fluids	53
3.3.2	Boundary layers	53
3.3.3	Navier–Stokes equations of motion	54
3.3.4	Reynolds number	54
3.3.5	Example: steady viscous flow between fixed parallel plates	55

4	Analysis and classification of fluid motion	57
4.1	Analysis of fluid motion	57
4.1.1	Vorticity	58
4.1.2	Rate of strain tensor	59
4.1.3	Summary	61
4.2	Classification of fluid flows	62
4.2.1	Irrotational flows	62
4.2.2	Incompressible flows	63
4.2.3	Axisymmetric incompressible flows	64
5	Irrotational flows of incompressible fluids	67
5.1	Irrotational and incompressible flows	67
5.1.1	Laplace's equation	67
5.1.2	Existence and uniqueness theorems	68
5.2	Sources, sinks and dipoles	68
5.2.1	Sources and sinks	68
5.2.2	Line source	69
5.2.3	Solid harmonics	69
5.2.4	Dipoles	70
5.3	Examples	72
5.3.1	Sphere moving through fluid at constant velocity . . .	72
5.3.2	Source in a uniform stream	74
5.3.3	Method of images	76
5.4	Solutions of Laplace's equation	77
5.4.1	Spherical harmonics	77
5.4.2	Axisymmetric case: Legendre's equation	79
5.4.3	Cylindrical harmonics	81
6	Fluid equations of motion	85
6.1	Forces in fluids	85
6.1.1	Stress vector	86
6.1.2	Stress tensor	86
6.2	Fluid equations of motion	89
6.2.1	Cauchy's equations of motion	89
6.2.2	Stress tensor for Newtonian viscous fluids	91
6.3	Navier–Stokes equations	93

CONTENTS

6.3.1	Navier–Stokes equations for compressible viscous fluids	93
6.3.2	Navier–Stokes equations for incompressible viscous fluids	93
6.4	Kelvin’s circulation theorem	95
6.5	Helmholtz’s vortex theorems	97
6.6	Bernoulli’s streamline theorem (again!)	99
6.7	Hydrostatics	100
6.8	Aerofoils and lift	100
6.8.1	Forces on objects in inviscid, incompressible and irrotational flows	101
7	Incompressible viscous flows	105
7.1	One-dimensional flow	105
7.1.1	Steady flow between fixed parallel plates	106
7.1.2	Steady flow through circular pipe	107
7.1.3	Steady flow under gravity down an inclined plane	108
7.1.4	Flow due to an impulsively moved plane boundary	110
7.2	Flow with circular streamlines	112
7.2.1	Viscous decay of a line vortex	114
8	Waves in fluids	117
8.1	Describing waves	117
8.1.1	Terminology	117
8.1.2	Group speed and dispersion relation	119
8.2	Surface gravity waves on deep water	120
8.2.1	Surface conditions and linearisation	121
8.2.2	Dispersion relation	123
8.2.3	Particle paths in water waves on deep water	124
8.3	Dispersion and group speed	124
8.4	Capillary waves	125
8.5	Sound waves	128
8.5.1	Euler’s equations in symmetric hyperbolic form	129
8.5.2	Small-amplitude sound waves: linearisation	131