

Contents

1	Derivation of the Navier-Stokes equations	7
1.1	Notation	7
1.2	Kinematics	8
1.3	Reynolds transport theorem	14
1.4	Momentum equation	15
1.5	Energy equation	19
1.6	Navier-Stokes equations	20
1.7	Incompressible Navier-Stokes equations	22
1.8	Role of the pressure in incompressible flow	24
2	Flow physics	29
2.1	Exact solutions	29
2.2	Vorticity and streamfunction	32
2.3	Potential flow	40
2.4	Boundary layers	48
2.5	Turbulent flow	54
3	Finite volume methods for incompressible flow	59
3.1	Finite Volume method on arbitrary grids	59
3.2	Finite-volume discretizations of 2D NS	62
3.3	Summary of the equations	65
3.4	Time dependent flows	66
3.5	General iteration methods for steady flows	69
4	Finite element methods for incompressible flow	71
4.1	FEM for an advection–diffusion problem	71
4.1.1	Finite element approximation	72
4.1.2	The algebraic problem. Assembly.	73
4.1.3	An example	74
4.1.4	Matrix properties and solvability	76
4.1.5	Stability and accuracy	76
4.1.6	Alternative Elements, 3D	80
4.2	FEM for Navier–Stokes	82
4.2.1	A variational form of the Navier–Stokes equations	83
4.2.2	Finite-element approximations	85
4.2.3	The algebraic problem in 2D	85
4.2.4	Stability	86
4.2.5	The LBB condition	87
4.2.6	Mass conservation	88
4.2.7	Choice of finite elements. Accuracy	88
A	Background material	95
A.1	Iterative solutions to linear systems	95
A.2	Cartesian tensor notation	98
A.2.1	Orthogonal transformation	98
A.2.2	Cartesian Tensors	99
A.2.3	Permutation tensor	100
A.2.4	Inner products, crossproducts and determinants	100
A.2.5	Second rank tensors	101

A.2.6	Tensor fields	102
A.2.7	Gauss & Stokes integral theorems	103
A.2.8	Archimedes principle	104
A.3	Curvilinear coordinates	106
B	Recitations 5C1214	109
B.1	Tensors and invariants	109
B.2	Euler and Lagrange coordinates	113
B.3	Reynolds transport theorem and stress tensor	117
B.4	Rankine vortex and dimensionless form	120
B.5	Exact solutions to Navier-Stokes	124
B.6	More exact solutions to Navier-Stokes	129
B.7	Axisymmetric flow and irrotational vortices	132
B.8	Vorticity equation, Bernoulli equation and streamfunction	136
B.9	Flow around a submarine and other potential flow problems	140
B.10	More potential flow	146
B.11	Boundary layers	148
B.12	More boundary layers	152
B.13	Introduction to turbulence	155
B.14	Old Exams	157
C	Study questions 5C1212	173