Large-scale peculiar motions and cosmic acceleration

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ABSTRACT

Recent surveys seem to support bulk peculiar velocities well in excess of those anticipated by the standard cosmological model. In view of these results, we consider here some of the theoretical implications of large-scale drift motions. We find that observers with small, but finite, peculiar velocities have generally different expansion rates from the smooth Hubble flow. In particular, it is possible for observers with larger than the average volume expansion at their location, to experience apparently accelerated expansion when the Universe is actually decelerating. Analogous results have been reported in studies of inhomogeneous (non-linear) cosmologies and within the context of the Lemaitre–Tolman–Bondi models. Here, they are obtained within the linear regime of a perturbed, dust-dominated Friedmann–Robertson–Walker cosmology.

Key words: cosmology: theory – large-scale structure of Universe.

1 INTRODUCTION

In idealized Friedmann–Robertson–Walker (FRW) cosmologies, comoving observers simply follow the universal expansion. In more realistic models, the smooth Hubble flow is distorted and matter acquires ‘peculiar’ velocities. The dipolar anisotropy of the cosmic microwave background (CMB) has been traditionally interpreted as the result of our peculiar motion relative to the cosmic rest-frame: the frame that redshifts with the expansion and in which the dipole vanishes. Our Local Group of galaxies drifts with respect to the CMB frame at roughly 600 km s$^{-1}$ (Padmanabhan 1993; Strauss & Willick 1995; Dodelson 2003). Analogous velocities, but for bulk motions on much larger scales, were also recently reported in the surveys of Watkins, Feldman & Hudson (2009); Feldman, Watkins & Hudson (2009) and those of Kashlinsky et al. (2008, 2009, 2010). The latter group, in particular, finds coherent peculiar flows as strong as 1000 km s$^{-1}$ out to scales of 450 and 800 Mpc. Both surveys appear to be in disagreement with the current concordance ΛCDM scenario (e.g. see Perivolaropoulos 2008).

This report considers the theoretical implications of such drift velocities for the kinematics of the associated observers by focusing on the scalars that describe their average volume expansion. The key question is whether observers drifting relative to the CMB and those following the Hubble expansion (in a dust-dominated FRW universe) can ‘measure’ different deceleration parameters. Whether, in particular, it is theoretically possible for a peculiarly moving observer to ‘experience’ accelerated expansion while the universe is decelerating. We show that, even when the peculiar velocities are relatively small, the answer to this question is positive and explain why. Not surprisingly, we also find that the effects of the peculiar motions are local. Nevertheless, the affected scales can be large enough to give the impression that the universe had recently moved into an accelerating phase. Another way of interpreting our results is that accelerated expansion for an observer moving relative to the CMB does not necessarily imply the same for the universe itself.

2 DRIFT MOTIONS IN PERTURBED FRW UNIVERSES

The microwave background introduces a preferred cosmological frame, relative to which large-scale peculiar velocities can be defined and measured. If $u_a$ is the reference 4-velocity of the CMB, typical observers in the universe have (see Fig. 1)

\[ \tilde{u}_a = u_a + v_a, \]

where $v_a$ (with $u_a v^a = 0$ and $v^2 = v_a v_a \ll 1$) is their drift velocity (King & Ellis 1973). The CMB also defines the coordinate system where the universe is a dust-dominated FRW model. The ‘tilded’ frame, on the other hand, corresponds to a typical observer in a galaxy like the Milky Way.

The average kinematics of the tilded observers are determined by the volume scalar ($\Theta = \nabla^a \tilde{u}_a$) of their motion (Ellis & van Elst 1998; Tsagas, Challinor & Maartens 2008).

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1 The $\tilde{u}_a$-field is also time-like, since $\tilde{u}_a \tilde{u}^a = -1$ irrespective of the magnitude of the peculiar velocity. Each frame defines its own time direction and 3-space (parallel and orthogonal to the corresponding 4-velocity, respectively). The tensors $\tilde{h}_{ab} = g_{ab} + u_a u_b$ and $\tilde{h}_{ab} = g_{ab} + \tilde{u}_a \tilde{u}_b$, with $g_{ab}$ representing the spacetime metric, project orthogonal to $u_a$ and $\tilde{u}_a$, respectively. These tensors also define the orthogonally projected covariant derivative operators by means of $D_a = \tilde{h}^{bb} \nabla_b$ and $D_a = \tilde{h}^{bb} \nabla_b$ (V_a is the standard covariant derivative) (Ellis & van Elst 1998; Tsagas, Challinor & Maartens 2008).
1998; Tsagas et al. 2008). Positive values for Θ imply that the mean separation between these observers increases and therefore indicate expansion. Similarly, Θ (with Θ = \nabla\nu u > 0) monitors the expansion of the universe. To first order in \nu, the two scalars are related by (Maartens 1998)

\begin{equation}
\tilde{\Theta} = \Theta + \vartheta,
\end{equation}

where \vartheta = \tilde{D}\nu. This scalar measures the average separation between neighbouring peculiar-flow lines. Expression (2) implies that, in regions where \vartheta is positive, the peculiarly moving observers expand faster than the universe (i.e. \tilde{\Theta} > \Theta). For our purposes it is crucial that the drift motion ‘adds’ to the background expansion and the reasons should become clear as we proceed. We will therefore always consider sections where \vartheta is positive.

In multi-systems, each group of observers has its own time direction. So, in our case, time can be measured relative to the CMB frame and along that of the tilded observers. The rate of the expansion along a given time direction is determined by the associated Raychaudhuri equation (Ellis & van Elst 1998; Tsagas et al. 2008). When the universe is a dust-dominated FRW model and the drift velocities are small, the Raychaudhuri equations in the CMB and the tilded frames are

\begin{equation}
\Theta' = -\frac{1}{2} \Theta^2 - \frac{1}{2} \rho \quad (3a)
\end{equation}

and

\begin{equation}
\tilde{\Theta}' = -\frac{1}{2} \tilde{\Theta}^2 - \frac{1}{2} \tilde{\rho} + \tilde{D}\nu \tilde{A}_u, \quad (3b)
\end{equation}

respectively. Here, primes indicate time differentiation along \nu and overdots are time derivatives with respect to the \tilde{u}_u-field. In other words, \Theta' = \nabla\nu \Theta and \tilde{\Theta} = \tilde{\nabla}\nu \tilde{\Theta}, with the 4-velocity vectors related through equation (1). Also, \rho and \tilde{\rho} are the matter densities in the CMB and the tilded frames, respectively (with \tilde{\rho} = \rho to linear order in \nu – see Maartens 1998). Finally, \tilde{A}_u is the 4-acceleration in the tilded frame. This vector vanishes in the CMB frame by definition (i.e. \tilde{A}_u = 0) but is non-zero in every other relatively moving coordinate system. In particular, to linear order in \nu, we find that \tilde{A}_u = \nu_u + (\Theta/3)\nu_u (Maartens 1998). The 4-acceleration term in equation (3b) is central to our analysis. Its presence means that expressions (3a) and (3b) are different, even when matter is in the form of pressureless dust and the peculiar velocities are small. In other words, observers drifting relative to the CMB have expansion rates different than that of the actual universe simply because of their relative motion. This represents a significant theoretical deviation from the conventional single-fluid studies (e.g. see Hirata & Seljak 2005).

**3 THE DECELERATION PARAMETER OF THE DRIFTING OBSERVER**

Expressed in terms of their volume scalars, the deceleration parameters associated with the \nu and \tilde{u}_u frames are respectively given by

\begin{equation}
q = -\left(1 + \frac{3\Theta'}{\Theta^2}\right) \quad (4a)
\end{equation}

and

\begin{equation}
\tilde{q} = -\left(1 + \frac{3\tilde{\Theta}}{\tilde{\Theta}^2}\right). \quad (4b)
\end{equation}

Our main question is whether \tilde{q} can take negative values while q is still positive. If so, the tilded observers will experience accelerated expansion in a decelerating universe. To investigate this possibility, we first use definitions (4) to recast expressions (3) into

\begin{equation}
(1 + q)\Theta^2 = \Theta^2 + \frac{3}{2} \rho \quad (5a)
\end{equation}

and

\begin{equation}
(1 + \tilde{q})\tilde{\Theta}^2 = \tilde{\Theta}^2 + \frac{3}{2} \tilde{\rho} - 3\tilde{D}\nu \tilde{A}_u, \quad (5b)
\end{equation}

respectively. These already show that \tilde{q} and \tilde{q} are generally different, but it helps to relate the two deceleration parameters directly. Recall that \tilde{\rho} = \rho and \tilde{A}_u = \nu_u + (\Theta/3)\nu_u to linear order in \nu. Then, employing definition \tilde{\vartheta} = \tilde{D}\nu, relation (2) and keeping up to \nu\_order terms, expressions (5a) and (5b) combine to

\begin{equation}
(1 + \tilde{q})\tilde{\Theta}^2 = (1 + q)\Theta^2 + \Theta \vartheta - 3\tilde{D}\nu \nu_u, \quad (6)
\end{equation}

where \Theta, \vartheta > 0 always. We may also involve the volume scalar of the peculiar motion further by using the (linear in \nu) relation

\[\frac{\rho}{a^3} = \frac{\rho}{\Theta^3} = \frac{\tilde{\rho}}{\tilde{\Theta}^3},\]

\[\frac{\tilde{D}\nu}{a^3} = \frac{\tilde{D}\nu}{\Theta^3} = \frac{\tilde{D}\nu}{\tilde{\Theta}^3},\]

\[\frac{\partial}{a^3} = \frac{\partial}{\Theta^3} = \frac{\partial}{\tilde{\Theta}^3},\]

\[\frac{\partial^2}{a^3} = \frac{\partial^2}{\Theta^3} = \frac{\partial^2}{\tilde{\Theta}^3},\]

\[\frac{\partial^3}{a^3} = \frac{\partial^3}{\Theta^3} = \frac{\partial^3}{\tilde{\Theta}^3},\]

where \partial = \nabla\nu and \partial = \tilde{\nabla}\nu, respectively.

2 A more familiar form for equation (2) is the Newtonian expression \tilde{u}_u = H\nu_u + \nu_u, where \tilde{u}_u and \nu_u are respectively the physical and the peculiar velocities of an observer with physical coordinates \nu_u (H = \Theta/3 is the Hubble parameter). The (physical) divergence of the above leads to equation (2), with \Theta and \vartheta corresponding to \nabla\nu \tilde{u}_u and \tilde{\nabla}\nu \nu_u, respectively.

3 We assume non-relativistic peculiar velocities and therefore drop terms of order \nu^2 and higher from (3b) and the rest of our equations. Also, throughout this paper we use geometrized units with \epsilon = 1 = 8\pi G.
\[ \dot{\vartheta} = \dot{D} v_0 - \Theta \dot{\vartheta}/3 \] (Ellis & Tsagas 2002). Then, equation (6) leads to

\[ 1 + \ddot{q} = (1 + q) \left( 1 + \frac{\vartheta}{\Theta} \right)^{-2} - \frac{3 \ddot{\vartheta}}{\Theta^2} \left( 1 + \frac{\vartheta}{\Theta} \right)^{-2} . \tag{7} \]

given that \( \hat{\vartheta} = \Theta + \vartheta \). The above relates the deceleration parameter in the tilded frame to that of the actual universe and it is our main result. It should now be clear that \( q \) and \( \hat{q} \) are generally different. Moreover, as long as the right-hand side of (7) remains below unity, positive values for \( q \) do not a priori guarantee the same for \( \hat{q} \). In other words, it is theoretically possible for the tilded observer to experience accelerated expansion in a decelerating universe.\(^4\)

Putting it differently, one could say that measuring negative deceleration parameter in a frame drifting relative to the CMB (such as that of our Local Group, for example) does not necessarily imply an accelerating universe.

At this point it is worth noting that, according to (4b), condition \(-1 < \ddot{q} < 0\) is equivalent to \(-\Theta^2/3 < \hat{\Theta} < 0\). This means that both \( \hat{q} \) and \( \hat{\Theta} \) can be simultaneously negative. Analogous relations also hold between \( q, \Theta^2 \) and \( \vartheta \). Given that, one should distinguish between accelerated expansion with simply \(-1 < \ddot{q} < 0\) and that with \( \hat{\Theta} > 0 \). We may therefore view \(-1 < \ddot{q} < 0\) and \( \hat{\Theta} > 0 \) (equivalently \( \hat{q} < -1 \)) as the conditions for ‘weakly’ and ‘strongly’ accelerated expansion, respectively. Then, it is important to recognize that, as long as we only require \( \ddot{q} \) to lie in the \((-1,0)\) range, the 4-acceleration term in equations (3b) and (5b) does not need to dominate the right-hand side of these expressions. This implies that peculiar motions can lead to weakly accelerated expansion within the limits of the linear (the almost FRW) approximation. Given that, we will focus on the \(-1 < \ddot{q} < 0\) case for the rest of this paper. Note that the supernovae results put the deceleration parameter close to \(-0.5\) (Turner & Riess 2002; Riess et al. 2004).

### 4 APPARENT ACCELERATION IN PERTURBED FRW UNIVERSES

Let us now consider an extended spatial region \( A \) – see Fig. 2, which largely complies with the FRW symmetries and expands with the Hubble flow, but is still endowed with a bulk peculiar velocity field that ‘adds’ to the background expansion (i.e. with \( \vartheta > 0 \)). Typical observers inside \( A \) have peculiar velocities close to the mean bulk flow of the patch. To linear order in \( V_u \), the deceleration parameter for those observers obeys equation (7). The simplest case corresponds to \( 3 \dot{\vartheta}/\Theta^2 \sim 0 \), which occurs when \( \vartheta \) varies very slowly with time (for example). Then, when the Hubble expansion dominates the kinematics, \( \vartheta/\Theta \ll 1 \) and a straightforward Taylor expansion reduces equation (7) to

\[ 1 + \ddot{q} = \left( 1 + \frac{\vartheta}{\Theta} \right) \left[ 1 - 2 \left( \frac{\vartheta}{\Theta} \right) \right] . \tag{8} \]

Recall that \( q = \vartheta/\Theta/2 \) in dust-dominated FRW models, with \( \vartheta \equiv 3 \rho/\Theta^2 \) and \( \rho \) representing the density of the matter in the \( u_\alpha \)-frame. Noting that \( \vartheta \) may also be seen as the effective density parameter of patch \( A \), the tilded observers will experience accelerated expansion if

\[ \left( 1 + \frac{\vartheta}{\Theta} \right) \left[ 1 - 2 \left( \frac{\vartheta}{\Theta} \right) \right] < 1 . \tag{9} \]

Whether this condition is satisfied or not and the affected scale (i.e. the size of patch \( B \) in Fig. 2), depends on the value of \( \vartheta \) and on the ratio \( \vartheta/\Theta \). To estimate the latter we need to know the bulk velocities of drift motions on scales far beyond that of our Local Group.

Peculiar velocities are difficult to measure, since direct measurements only provide their radial component. One also needs to subtract the Hubble expansion, which requires independent knowledge of the galaxy’s distance. As a result, bulk peculiar velocities are estimated by means of statistical methods (Strauss & Willick 1995). Recent independent reports have claimed large-scale coherent drift velocities significantly higher than those anticipated by the concordance \( \Lambda \)CDM model. These surveys extend to lengths of 100 h\(^{-1}\) Mpc (Feldman et al. 2009; Watkins et al. 2009), 300 h\(^{-1}\) Mpc and 500 h\(^{-1}\) Mpc (Kashlinsky et al. 2008, 2009, 2010), with \( h \) being the Hubble parameter in units of 100 km s\(^{-1}\) Mpc\(^{-1}\). The results show bulk velocities as large as 500 km s\(^{-1}\) (Feldman et al. 2009; Watkins et al. 2009) and up to 1000 km s\(^{-1}\) (Kashlinsky et al. 2008, 2009, 2010) on the corresponding scales. On smaller lengths (between 30 and 60 Mpc) the work of Li & Schwarz (2008) suggests a (positive) variance in the local Hubble rate up to 10 per cent. With the possible exception of the last survey, there is currently no way of knowing whether the reported bulk flows are of the desired type (i.e. with \( \vartheta > 0 \)). Nevertheless, in the absence of better data, we will use the magnitudes of the aforementioned peculiar velocities to infer reasonable (order-of-magnitude) estimates for \( \vartheta \). In addition, mainly for algebraic simplicity and illustration purposes, we will also consider the intermediate value of 700 km s\(^{-1}\) as a yardstick.

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\(^4\) Expression (7) also implies that two decelerating expansions can combine to give an accelerating one. Another way of showing this is by writing equation (2) as \( \ddot{a}/a = (\dot{a}/a) + (a/a) \), where \( \dot{a}, a \) and \( a \) are the three scale factors (with \( \dot{a}, a > 0 \)). Then, \( \ddot{a}/a = (\dot{a}/a) + (a/a) + 2(\dot{a}/a)(\dot{a}/a) \), meaning that negative values for \( \dot{a} \) and \( a \) do not guarantee the same for \( \ddot{a} \). Note that for simplicity we have used overdots for both time derivatives.
peculiar velocity. Note that this value is very close to the drift velocity of our Local Group.

Setting the Hubble parameter at 70 km s$^{-1}$ Mpc and extrapolating to 50, 100 and 1000 Mpc, we find that $q = \dot{\Omega}$ is close to 0.5, 1, 10 and 1/100, respectively. Then, following condition (9), the tilted observer will ‘measure’ negative deceleration parameter within a region of up to 50 Mpc (in an otherwise decelerating universe) if $0 \leq \Omega < 1.3$. This condition strengthens to $0 \leq \Omega < 0.5$ at 100 Mpc, while further out, near the 1000 Mpc mark for instance, $\dot{q}$ will remain positive unless $0 < \Omega < 0.04$. Inserting these numbers into equation (8), we obtain a range of values for the deceleration parameter of the tilted observer on the corresponding scales. Thus, provided (9) is satisfied, $\dot{q}$ varies within $(-0.4, 0)$ on scales of 50 Mpc, between $(-0.2, 0)$ when we move to 100 Mpc and within $(-0.02, 0)$ near the 1000 Mpc threshold. So, in this example the size of the accelerated region (i.e. that of patch B in Fig. 2) ranges between 50 and 1000 Mpc. Within these scales $\dot{q}$ lies in the $(-0.4, 0)$ range, taking its minimum value in small-scale regions of low density and approaching zero as we move on to larger lengths. These estimates are not far from those inferred by the supernovae data, which value the deceleration parameter close to $-0.5$ and put the transition to deceleration near $z = 0.5$ (Turner & Riess 2002; Riess et al. 2004). The picture does not change much when we adopt the results of Li & Schwarz (2008), the surveys of Watkins et al. (2009) and Feldman et al. (2009), or those of Kashlinsky et al. (2008, 2009, 2010). Substituted into expressions (8) and (9), the former give $-0.2 < \dot{q} < 0$ in regions of 50 Mpc when $\Omega < 0.5$ there. Similarly, close to 150 Mpc, the measurements of Watkins et al. (2009) and Feldman et al. (2009) put $\dot{q}$ in the range $(-0.1, 0)$, provided $\Omega < 0.2$ there. Finally, on lengths of 450 and 800 Mpc, the results of Kashlinsky et al. (2008, 2009, 2010) suggest that $\dot{q}$ varies the range $(-0.07, 0)$ and $(-0.04, 0)$, respectively, when $\Omega < 0.15$ and $\Omega < 0.07$ on the corresponding scales. Note that the same survey indicates bulk flows of 1500 km s$^{-1}$ on scales close to 150 Mpc. Inserted into equations (8), (9), these values lead to $-0.3 < \dot{q} < 0$ when $\Omega < 0.8$. One should keep in mind, however, that on relatively small scales the peculiar-velocity error bars are large (see Kashlinsky et al. 2008).

Let us now turn to the last term of equation (7). Qualitatively speaking, a positive $\dot{q}$ will assist the acceleration, relax the above-given conditions and lead to lower values of $\dot{q}$. So, here, we will assume that $\dot{q}$ is negative. We will also demand that $\dot{q}/\dot{\Omega} \approx \dot{\theta}/\theta < 1$, to ensure that both $\dot{q}$ and $\dot{q}$ are small perturbations relative to their background associates. The next step is to recast Raychaudhuri’s formula (see equation 3a) in the form

$$\Theta' = -\frac{1}{3} \Theta^2 \left( 1 + \frac{1}{2} \Omega \right),$$

with $\Theta > 0$. Solving the above for $\Theta'$, substituting into equation (7) and employing some straightforward algebra, we arrive at

$$1 + \dot{q} = \left( 1 + \frac{1}{2} \Omega \right) \left( 1 - \frac{q}{\theta^2} \right).$$

Using the previous values of $\theta/\dot{\theta}$, we find that negative $\dot{q}$s on $\sim 50$ Mpc scales need $\Omega < 0.5$. Similarly, expression (11) translates into $\Omega < 0.2$ close to 100 Mpc and into $\Omega < 0.02$ near the 1000-Mpc mark, if $\dot{q}$ is to become negative there. Under these conditions,

5 Recall that $\Theta = 3H$ and that $\dot{q} = \dot{\Theta}/\Theta \approx \partial \Theta / \Theta \sim 3\Theta / r$, where $r$ is the magnitude of the bulk velocity in a given region and $r$ the size of that region. Then, $\dot{q}/\dot{\theta} \sim v/Hr$.

the accelerated patch extends from 50 to 1000 Mpc, with $\dot{q}$ varying within $(-0.2, 0)$. So, even with the last term of (7) accounted for (and in an unfavourable way), negative values for $\dot{q}$ are still possible. Conventional almost FRW kinematics can accommodate accelerated expansion.

5 SUMMARY AND DISCUSSION

To summarize, suppose that in a dust-dominated FRW universe a sufficiently large region $A$ is endowed with a weak bulk peculiar velocity of positive divergence (i.e. $\dot{q} > 0$). When the right-hand side of (7) drops below unity, around every point in $A$ there is an essentially spherically symmetric patch $B$, where the expansion is ‘weakly’ accelerated (i.e. $-1 < \dot{q} < 0$ there). As a result, nearly every observer in $A$ will experience accelerated expansion, although region $A$ and the universe itself may be actually decelerating. The accelerating effect, in a given region, depends on the magnitude of the peculiar velocity and the density of the region in question. Overall, the larger the drift velocity and the lower the density, the faster the acceleration.

Little more than a decade ago, observations of high-redshift supernovae indicated that our universe was expanding at an accelerating pace (Riess et al. 1998; Perlmutter et al. 1999). This conclusion was reached after applying the observed luminosity distances of the supernovae to the distance–redshift relation,

$$D_L = (1 + z)H_0^{-1} \int_0^z e^{-\left[\frac{1}{2} \int_0^y [1 + \dot{q}(1+y)] dy \right]} dx,$$

(12)
of an FRW model. Note that in the above, $q$ is the deceleration parameter of the universe and not that of an observer moving relative to the CMB. The results have repeatedly given negative values to $q$, indicating an accelerated expansion for our universe. In particular, the deceleration parameter was estimated to be close to $-0.5$. The same measurements also suggested that the accelerated phase was a relatively recent event, putting the transition from deceleration to acceleration around $z = 0.5$ (i.e. between 2000 and 3000 Mpc – Turner & Riess 2002; Riess et al. 2004). The supernovae results were so unexpected that they have since dominated almost every aspect of contemporary cosmology. The main problem is that negative values for the deceleration parameter appear theoretically impossible in FRW (as well as in perturbed, almost FRW) cosmologies, unless new physics or drastic changes to the matter content of the universe were introduced. Dark energy, an unknown and elusive form of matter with negative gravitational mass, has so far been the most popular answer.

6 This conclusion has been based on the average peculiar kinematics without incorporating anisotropies. For instance, the symmetry of region $A$ and the observer’s position in it can induce anisotropy in the spatial distribution of $\dot{q}$. Generally, the higher the spherical symmetry of $A$ and the closer the observer at the centre the better. These matters are less of an issue, however, when $A$ is considerably larger than $B$, namely as long as patch $B$ lies well within region $A$. The direction of the peculiar motion can also introduce an anisotropy in the $\dot{q}$-distribution. This effect is maximized when the peculiar velocity maintains the same magnitude and direction throughout the integration period (i.e. from $z \simeq 0.5$ to the present – see expression 12). In the opposite case, when the $v_p$-field has been sufficiently randomized, the anisotropy will be negligible. Estimating effects like these is currently impossible, however, as it requires detailed data on the distribution of peculiar velocities within regions of several hundred megaparsecs.

7 To account for the effects of our peculiar motion on the right-hand side of equation (12), one should replace $q$ with $\dot{q}$. To linear order in $v_p$, the latter is given by expression (7), or by its simplified counterpart (8).
The implications of peculiar velocity perturbations on the luminosity distance of distant galaxies, within the context of perturbed FRW models, has been investigated in the past (e.g. Vanderveld et al. 2007), in an attempt to reconcile expression (12) with positive values for the deceleration parameter. That work has investigated the impact of peculiar motions on $D_L$. Here, we have followed a different approach. Turning our attention to the deceleration parameter, the aim was to examine whether peculiar motions can ‘make’ the latter negative. Our results show that this is theoretically possible. Peculiar motions can locally mimic the kinematic effects of dark energy. Observers moving relative to the smooth Hubble flow can have local expansion rates appreciably different than that of the actual universe. This reflects the fact that the Raychaudhuri equations in the two coordinate systems (that of the CMB and that of a drifting observer) are not the same. The difference is due to a 4-acceleration term, which vanishes in the CMB frame but takes non-zero values in any other relatively moving reference system. As a result, accelerated expansion is possible even when the drift velocities are small and matter is simple pressure-free dust, namely within the limits of the linear (almost FRW) approximation.

Extrapolating our drift velocity relative to the CMB frame, we found that peculiarly moving observers can measure negative deceleration parameter on scales between (roughly) 50 and 1000 Mpc, with $q$ varying in the range ($-0.4, 0$). Based on the surveys of Watkins et al. (2009), Feldman et al. (2009) and particularly those of Kashlinsky et al. (2008, 2009, 2010), the deceleration parameter was confined within ($-0.3, 0$). These results are qualitatively in the right direction, though quantitatively stop short of fully reproducing the current supernovae data. In particular, the largest scale considered here is between a half and a third of the typical scale of the accelerated domain (Turner & Riess 2002; Riess et al. 2004). Also, typically, the larger the scale the lower the required (effective) density parameter, putting the latter potentially at odds with the current observational constraints. On these grounds, this work should be seen as a proof of principle, rather than a full fit to the current supernovae data. Nonetheless, it is important to know that (based on estimates inferred from peculiar-flow observations) apparently accelerating expansion is possible in linearly perturbed FRW models with conventional (dust) matter.

Given that peculiar velocities are a by-product of structure formation, their role can be seen as a ‘backreaction’ effect (e.g. see Rasanen 2004, 2006; Barausse et al. 2005; Coley et al. 2005; Kolb et al. 2006; Wiltshire 2009 and also Buchert 2008 for a recent review). These scenarios consider the overall impact of inhomogeneity and anisotropy, go beyond the linear regime and usually employ an averaging scheme (Zalaletdinov 1992; Buchert 2000). Averaging also raises issues related to the ‘fitting problem’ and the choice of a ‘background’ (Ellis 1984; Kolb, Marr & Matarrese 2009), the ‘dressing’ of cosmological parameters (Buchert & Carfora 2002, 2003), the propagation of light (Rasanen 2008, 2009) and the ‘synchronisation of clocks’ (Wiltshire 2007a,b, 2008). Here, we have focused on peculiar motions without introducing any spatial averaging. We have also remained within the linear approximation. Nevertheless, the effects are of the same nature. Also, since we have looked at peculiar velocities that increase the volume expansion at the observer’s location, our model is analogous to a local void. The possibility of apparent acceleration in the void scenario has been studied by many authors both qualitatively and quantitatively (e.g. see Mustapha et al. 1997, Celerier 2000, Tomita 2001a,b, Iguchi et al. 2002, Alnes et al. 2006, 2007, Zibin et al. 2008, Clifton et al. 2008, Bolejko & Anderson 2008, Sussman 2008 for a representative though incomplete list). Whereas the effects of large voids have been generally studied in the context of the Lemaître–Tolman–Bondi (LTB) solution$^8$, our analysis has been performed within a perturbed FRW model. The analogies between the two approaches found here, also seem to support the claim by Enqvist, Mattsson & Rigopoulos (2009): that LTB models fitting the supernovae data (with appropriate initial conditions) are equivalent to perturbed FRW spacetimes along the observer’s past light-cone. Although single void models appear unrealistic, given the complexity of the observed structure, the simple analysis and the results presented here suggest that (when more realistic averages are performed) identifying the deceleration parameter measured in the frame of a drifting observer with that of the universe itself could be misleading.

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$^8$ The reader is directed to Plebanski & Krasinski (2006) for a general discussion of LTB cosmologies and to Zibin (2008) and Clarkson, Clifton & February (2009) for perturbative studies of these models.
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