CONSEQUENCES OF A DYNAMICAL VOLUME ELEMENT

[COSMOLOGY, THE Λ-problem, DARK MATTER, FERMION FAMILY PROBLEM]

E. GUENDELMAN ISRAEL

IN MEMORY OF MY MOTHER
REGINA ISRAEL
BORN IN THESSALONIKI (1917)
(SALONICA)

LIVED UP TO AGE 12 IN
MEG. ALEXANDROU Str.
(WHERE I AM NOW STAYING)

NORMALLY, WHEN CONSIDERING ACTION OF COORDINATE INVARIANT THEORIES WE TAKE (E.G. + A. KAGANOVICH)

$$S_1 = \int \underbrace{\sqrt{-g}\, d^4x}_{\text{INVARIANT VOLUME ELEMENT}} \mathcal{L}_1 \,, \quad g = \det(g_{\mu\nu})$$

BUT CAN TAKE (INSTEAD OR IN ADDITION)

$$S_2 = \int \underbrace{\Phi\, d^4x}_{\text{ALTERNATIVE INVARIANT VOLUME ELEM.}} \mathcal{L}_2$$

WHERE Φ IS BUILT FROM DEGREES OF FREEDOM INDEPENDENT OF $g_{\mu\nu}$, FOR EXAMPLE

$$\Phi = 4!\,\varepsilon^{\mu\nu\alpha\beta}\, \partial_\mu \varphi_1\, \partial_\nu \varphi_2\, \partial_\alpha \varphi_3\, \partial_\beta \varphi_4$$

φ_a $(a = 1, 2, 3, 4)$ 4-SCALARS.

$$\Phi = \varepsilon^{\mu\nu\alpha\beta}\, \varepsilon_{abcd}\, \partial_\mu \varphi_a\, \partial_\nu \varphi_b\, \partial_\alpha \varphi_c\, \partial_\beta \varphi_d$$

TAKE $\mathcal{L}_1$, $\mathcal{L}_2$ independent of φ_a

If $\mathcal{L}_1$, $\mathcal{L}_2$ independent of φ_a

THERE IS INFINITE DIMENSIONAL SYMMETRY: $\varphi_a \to \varphi_a + f_a(\mathcal{L}_2)$

NOTICE $\Phi = \partial_\mu \Omega^\mu$ IS A TOTAL DERIVATIVE

$$\Phi = \partial_\mu \left(\varepsilon^{\mu\nu\alpha\beta} \varepsilon_{abcd} \varphi_a \partial_\nu \varphi_b \partial_\alpha \varphi_c \partial_\beta \varphi_d \right)$$

So IF $\mathcal{L}_2 \to \mathcal{L}_2 + \text{const.}$

$$S \to S + \int \partial_\mu \Omega^\mu d^4x$$

does not change eqs. of motion.

IF STRUCTURE OF THEORY IS PRESERVED, I.E.

$$S = \int \sqrt{-g}\, \mathcal{L}_1 \, d^4x + \int \Phi \, \mathcal{L}_2 \, d^4x$$

$\mathcal{L}_1$, $\mathcal{L}_2$ φ_a independent. S gives ground state with zero cosmological constant without fine tunning. Also see-saw effect for very (above G. S.) small c.c. is possible

ONCE AGAIN, BUT NOW GIVE EXPLICIT EXAMPLE

$$S = \int \mathcal{L}_1 \Phi \, d^4x + \int \mathcal{L}_2 \sqrt{-g} \, d^4x$$

AND NOW CONSIDER A SCALAR ϕ

ϕ - SCALAR FIELD (NO FERMIONS YET)

$$\Phi = \varepsilon^{\mu\nu\alpha\beta} F_{\mu\nu\alpha\beta} \quad \text{or}$$

$$= \varepsilon^{\mu\nu\alpha\beta} \varepsilon_{abcd} \partial_\mu \varphi_a \partial_\nu \varphi_b \cdots \partial_\beta \varphi_d$$

AND THE CHOICES FOR $\mathcal{L}_1, \mathcal{L}_2$:

$$\mathcal{L}_2 = U(\phi)$$

$$\mathcal{L}_1 = -\frac{1}{\kappa} R(\Gamma, g) + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi)$$

$$R = g^{\mu\nu} R_{\mu\nu}(\Gamma), \quad R_{\mu\nu}(\Gamma) = R^\lambda_{\mu\nu\lambda}$$

$$R^\lambda_{\mu\nu\sigma} = \Gamma^\lambda_{\mu\nu,\sigma} - \Gamma^\lambda_{\mu\sigma,\nu} + \Gamma^\lambda_{\alpha\sigma} \Gamma^\alpha_{\mu\nu} - \Gamma^\lambda_{\alpha\nu} \Gamma^\alpha_{\mu\sigma}$$

IN THE ACTION PRINCIPLE

$g_{\mu\nu}$, $\Gamma^\lambda_{\mu\nu}$, φ_a and ϕ are INDEPENDENT.

GLOBAL SCALE INVARIANCE

$$g_{\mu\nu} \to e^{\theta} g_{\mu\nu} \quad (\theta = \text{const.})$$

AND IF $\boxed{V(\phi) = f_1 e^{\alpha\phi}, \quad U(\phi) = f_2 e^{2\alpha\phi}}$

THEN, IF $\varphi_a \to \lambda_a \varphi_a$ (no sum on a)

WHICH MEANS

$$\Phi = \varepsilon^{\mu\nu\alpha\beta} \varepsilon_{abcd} \partial_\mu \varphi_a \partial_\nu \varphi_b \partial_\alpha \varphi_c \partial_\beta \varphi_d \to \left(\prod_a \lambda_a\right) \Phi$$

SUCH THAT $\left(\prod_a \lambda_a\right) \equiv \lambda = e^{\theta}$

THEN THERE IS SCALE INVARIANCE if

$$\phi \to \phi - \frac{\theta}{\alpha}$$

ϕ - needed to achieve scale invariance call it the "dilaton" (E. G. MOD. PHY. LETT. A14 (1999) 1043)

EQUATIONS OF MOTION

VARIATION WITH RESPECT TO $\varphi_a \Rightarrow$

$$A^\mu{}_a \partial_\mu \mathcal{L}_1 = 0, \quad A^\mu{}_a = \varepsilon^{\mu\nu\alpha\beta} \varepsilon_{abcd} \partial_\nu \varphi_b \partial_\alpha \varphi_c \partial_\beta \varphi_d$$

can check that $\det(A^\mu{}_a) = \frac{4^{-4}}{4!} \Phi^3 \neq 0$

if $\Phi \neq 0$, THEN $A^\mu{}_a \partial_\mu \mathcal{L}_1 = 0 \Rightarrow \partial_\mu \mathcal{L}_1 = 0$

$$\Rightarrow \mathcal{L}_1 = -\frac{1}{\kappa} R(\Gamma, g) + \frac{1}{2} \partial_\mu \phi \partial_\nu \phi g^{\mu\nu} - V = M \; *$$

M = constant. VARIATION W/R TO $g_{\mu\nu} \Rightarrow$

$$\Phi \left(-\frac{1}{\kappa} R_{\mu\nu}(\Gamma) + \frac{1}{2} \phi_{,\mu} \phi_{,\nu}\right) - \frac{1}{2} \sqrt{-g}\, U(\phi) g_{\mu\nu} = 0 \quad **$$

SOLVING FOR $g^{\mu\nu} R_{\mu\nu}(\Gamma) \equiv R$ FROM $**$

AND INSERTING IN $*$ WE GET

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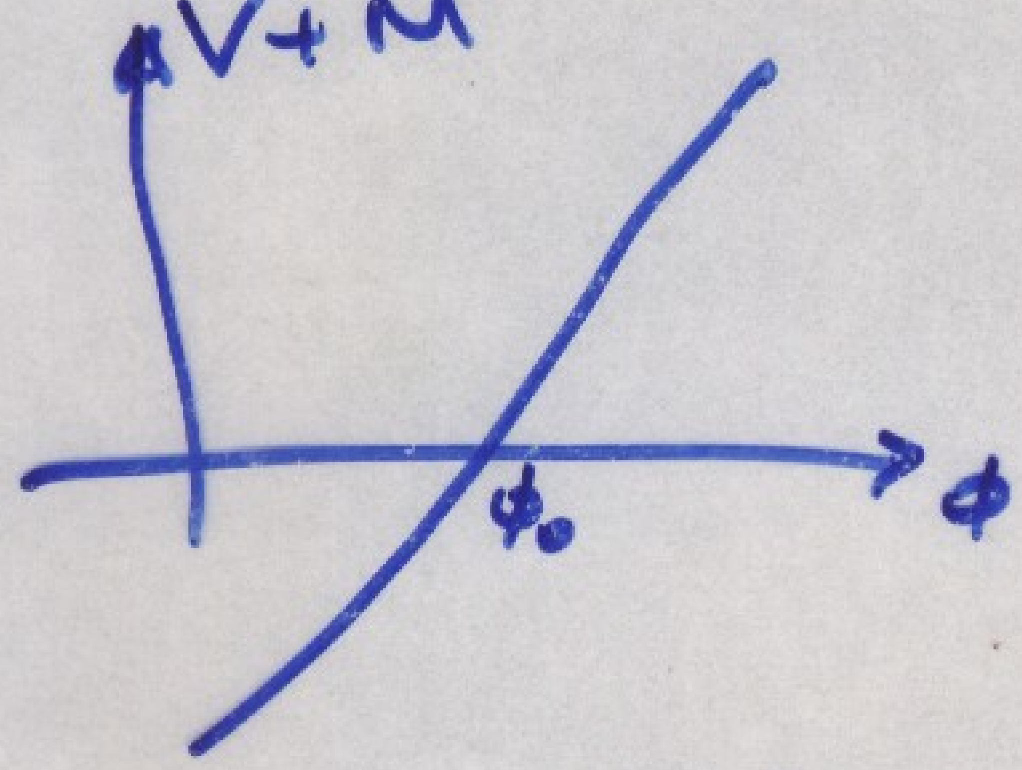
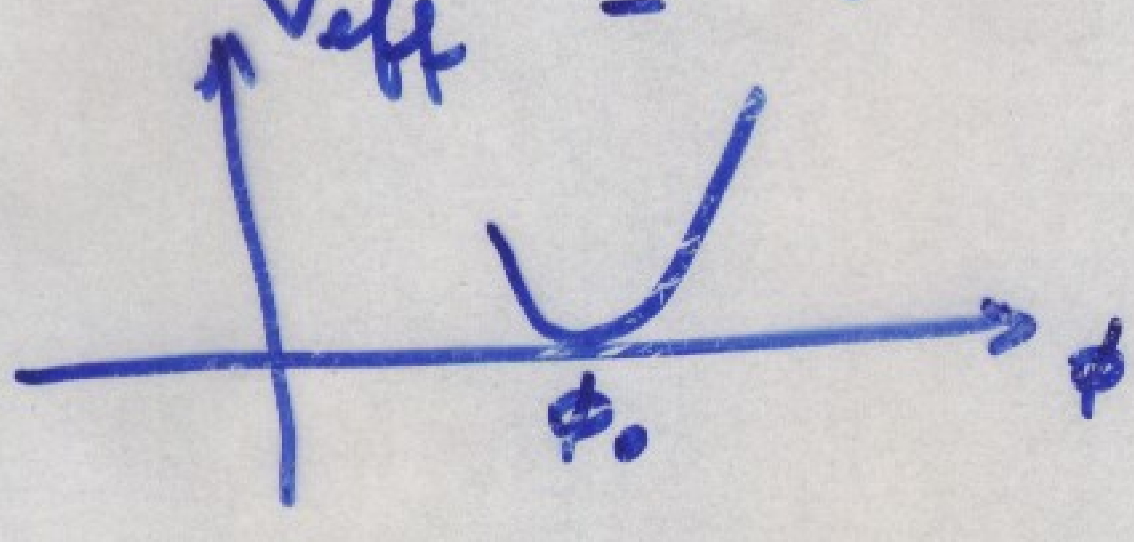
If $V + M = 0$ for $\phi = \phi_0$

And If $U(\phi_0) > 0$



$$V_{eff} = (V+M)^2 / 4U(\phi)$$



Zero of V_{eff} and $V'_{eff} = 0$ at the same time without fine tuning!

THAT WE CAN SOLVE FOR $\chi = \frac{\Phi}{\sqrt{-g}}$

$$\chi = \frac{2U(\phi)}{M + V(\phi)}$$ (M ≠ 0 breaks s.i. as can be seen here)

SOLUTION FOR $\Gamma^{\lambda}_{\mu\nu} = \left\{ {\lambda \atop \mu\nu} \right\} \Big|_{\bar{g}_{\mu\nu} = \chi g_{\mu\nu}}$

A SIMPLE WAY TO SEE THIS:

$$\frac{\delta}{\delta \Gamma^{\lambda}_{\mu\nu}} S = -\frac{1}{\kappa} \frac{\delta}{\delta \Gamma^{\lambda}_{\mu\nu}} \int \Phi g^{\mu\nu} R_{\mu\nu}(\Gamma)\, d^4x$$

$$\Phi g^{\mu\nu} = \sqrt{-\bar{g}}\, \bar{g}^{\mu\nu} \quad \left(\bar{g}_{\mu\nu} = \chi g_{\mu\nu} = \frac{\Phi}{\sqrt{-g}} g_{\mu\nu} \right)$$

so $$\frac{\delta}{\delta \Gamma^{\lambda}_{\mu\nu}} \int \sqrt{-\bar{g}}\, \bar{g}^{\mu\nu} R_{\mu\nu}(\Gamma)\, d^4x = 0$$

gives for $\Gamma^{\lambda}_{\mu\nu}$ the same <u>solution</u> <u>familiar</u> to General Relativity $\Rightarrow$ $\boxed{\Gamma^{\lambda}_{\mu\nu} = \left\{ {\lambda \atop \mu\nu} \right\}_{\bar{g}_{\mu\nu}}}$

In terms of $\bar{g}_{\mu\nu} = \chi g_{\mu\nu}$

EQS. HAVE THE EINSTEIN FORM

$$R_{\mu\nu}(\bar{g}_{\alpha\beta}) - \frac{1}{2} \bar{g}_{\mu\nu} R(\bar{g}_{\alpha\beta}) = \frac{\kappa}{2} T^{eff}_{\mu\nu}(\phi)$$

($R_{\mu\nu}(\bar{g}_{\alpha\beta})$ = usual Ricci tensor in terms of $\bar{g}_{\alpha\beta}$)

$$T^{eff}_{\mu\nu} = \phi_{,\mu} \phi_{,\nu} - \frac{1}{2} \bar{g}_{\mu\nu} \phi_{,\alpha} \phi_{,\beta} \bar{g}^{\alpha\beta} + \bar{g}_{\mu\nu} V_{eff}(\phi)$$

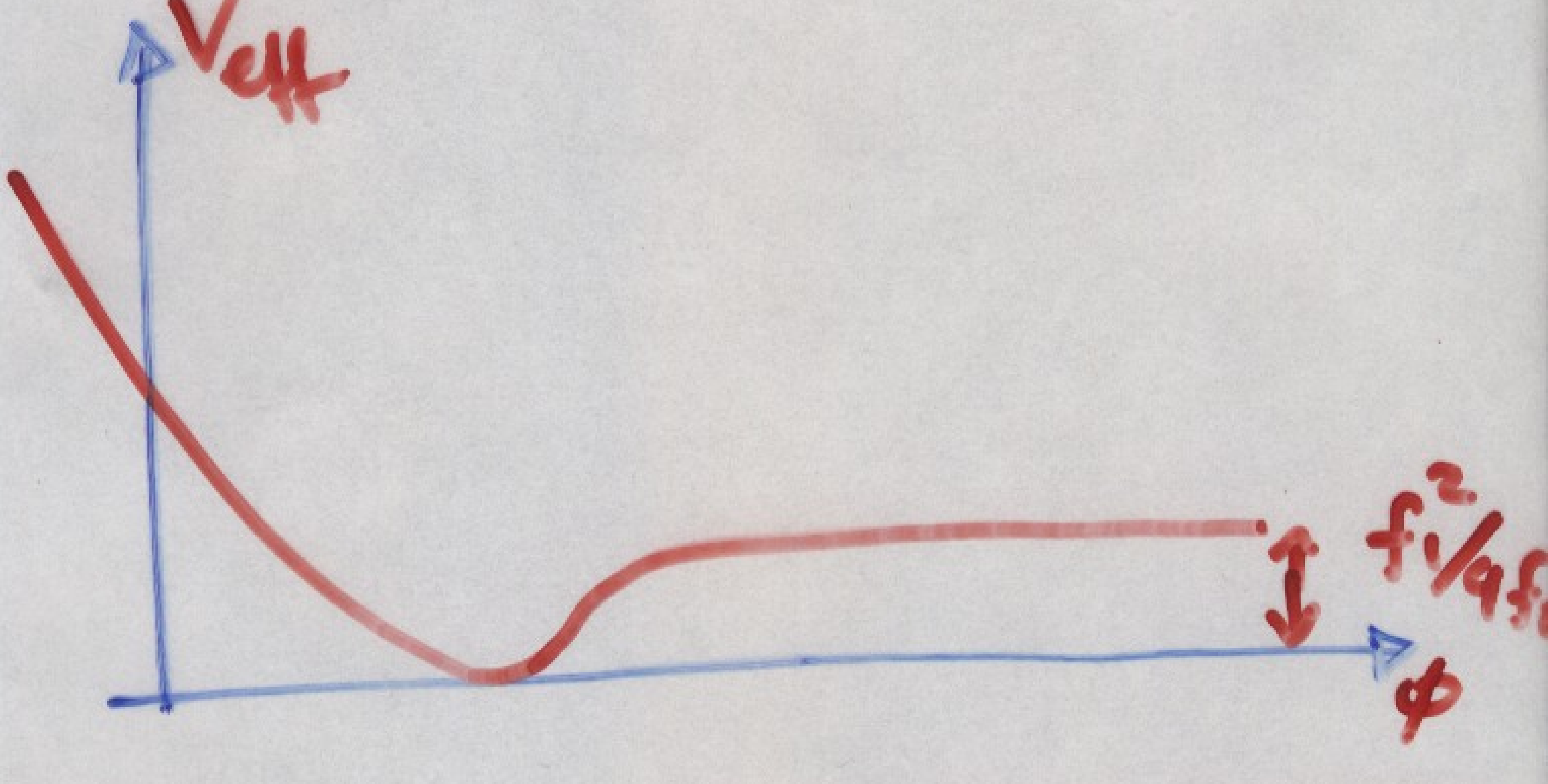
$$V_{eff}(\phi) = \frac{1}{4U(\phi)} (V + M)^2$$

FOR THE SCALE INVARIANT CASE (7

$$V(\phi) = f_1 e^{\alpha\phi}, \quad U(\phi) = f_2 e^{2\alpha\phi}$$

$$V_{eff}(\phi) = \frac{(V+M)^2}{4U} = \frac{1}{4f_2}\left(f_1 + M e^{-\alpha\phi}\right)^2$$

If $\frac{f_1}{M} < 0$, $V_{eff} = 0$ at $\phi_m = -\frac{1}{\alpha}\ln\left|\frac{f_1}{M}\right|$

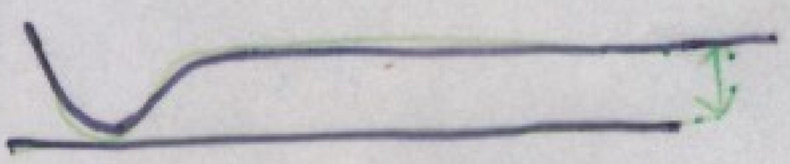


CAN MAKE VACUUM ENERGY
AS $\phi \to \infty$ VERY SMALL
BY TAKING $f_1/f_2 << 1$

SEE SAW EFFECT FOR
VACUUM ENERGY.

SOMETHING ON SEE SAW COSMOLOGY:

CONSIDER THE PREDICTED VACUUM ENERGY DENSITY OF FLAT REGION:



$\frac{f_1^2}{4 f_2}$ CAN IT BE USED FOR PRESENT UNIVERSE?

$f_1 \sim$ coupling of ϕ to measure Φ

$f_2 \sim$ coupling of ϕ to $\sqrt{-g}$

Physics can be different. Take as natural scales (i) the Electroweak Scale M_{EW} and associate this to f_1

$\Rightarrow \quad f_1 \sim (M_{EW})^4$

(ii) The Planck scale and associate this to $f_2 \Rightarrow f_2 \sim (M_{Pl})^4$

So $\Rightarrow$ Vacuum energy density

$$= \frac{f_1^2}{4 f_2} \sim \frac{(M_{EW})^8}{M_{Pl}^4} = \left(\frac{M_{EW}}{M_{Pl}}\right)^8 M_{Pl}^4 = 10^{-120} M_{Pl}^4$$ ✓

If vacuum energy $\sim M_{EW}^8 / M_{Pl}^4$ interesting COSMIC COINCIDENCES ARE explained. Discussion by N. Arkani-Hamed, L. Hall, C. Kolda and H. Murayama.

(e.g. existence of 3-coincidence point. $\rho_\Lambda \sim \rho_m \sim \rho_R$) MANY OTHER WORKS WHERE VACUUM ENERGY $\sim M_{EW}^8 / M_{Pl}^4$ HAVE APPEARED.

CORRESPONDENCE WITH

Induced Gravity Model of Zee.

$$S = \int \sqrt{-g} \left(-\frac{1}{2} \epsilon \varphi^2 R + \frac{1}{2} \partial_\mu \varphi \partial_\nu \varphi g^{\mu\nu} - \frac{\lambda}{8} (\varphi^2 - \eta^2)^2 \right) d^4x$$

where

$R = R(g)$ = usual Riemannian scalar curvature defined in terms of $g_{\mu\nu}$. If $\eta = 0$ model is invariant under global scale invariance

$$g_{\mu\nu} \to e^{\theta} g_{\mu\nu}$$

$$\varphi \to e^{-\theta/2} \varphi$$

However we look at $\eta \neq 0$.

Define $\bar{g}_{\mu\nu} = k^2 \epsilon \varphi^2 g_{\mu\nu}$ ($2k^2 = \kappa$)
and the scalar field

$\phi = \frac{1}{k}\sqrt{6 + \frac{1}{\epsilon}}\, \ln \varphi$. Then

Zee's model is equivalent to GR coupled minimally to the scalar field ϕ which has a potential

$$V_{eff} = \frac{\lambda}{8k^4\epsilon^2}\left(1 - \eta^2 e^{-2\sqrt{\frac{\epsilon}{1+6\epsilon}}\, k\phi}\right)^2$$

which is to be compared with the form obtained here

$$V_{eff} = \frac{1}{4f_2}\left(f_1 + M e^{-\alpha\phi}\right)^2$$

$\Rightarrow \quad \alpha = 2\sqrt{\frac{\epsilon}{1+6\epsilon}}\, k$ (correspondence to Zee's model)

$M \propto -\eta^2$, etc.

PROBLEM: SUCH POTENTIAL CONTAINS ONLY <u>ONE</u> FLAT REGION; EITHER GOOD FOR INFLATION OR SLOWLY ACCELERATED PHASE, BUT NOT FOR BOTH. SOMETHING IS MISSING TO DESCRIBE BOTH PERIODS OF THE EVOLUTION OF THE UNIVERSE

A SIMPLE ADDITION OF A SCALE INVARIANT $\epsilon \int \sqrt{-g}\, R^2 d^4x$ TERM WILL DO IT.

(E. I. GUENDELMAN & O. KATZ, CLASS. & QUANT. GRAV. 20, 1715 (2003).)

NOTICE THAT $S_{R^2} = \epsilon \int \sqrt{-g} \left(R_{\mu\nu}(\Gamma) g^{\mu\nu}\right)^2 d^4x$

IS INVARIANT UNDER $g_{\mu\nu} \to e^{\theta} g_{\mu\nu}$.

IN FIRST ORDER FORMALISM (WHERE $g_{\mu\nu}$, $\Gamma^{\lambda}_{\mu\nu}$ are independent) even for $\theta = \theta(x^{\nu})$

<u>**NOTE:**</u> R^2 theory in 1^{st} order formalism not equivalent to R^2 theory in 2^{nd} order formalism

VARIATION OF S w/r to $g_{\mu\nu}$ gives

$$R_{\mu\nu}(\Gamma)\left(-\frac{\Phi}{\kappa} + 2\epsilon R\sqrt{-g}\right) + \Phi \frac{1}{2}\phi_{,\mu}\phi_{,\nu}$$

$$-\frac{1}{2}\left(\epsilon R^2 + U(\phi)\right)\sqrt{-g}\, g_{\mu\nu} = 0$$

UPON CONTRACTION ϵ-TERMS (WITH $g^{\mu\nu}$) GO AWAY.

IN ADDITION COUPLING TO Φ UNCHANGED $\Rightarrow$ $L_1 = M$ SAME

$\Rightarrow$ ONCE AGAIN $\chi = \frac{\Phi}{\sqrt{-g}} = \frac{2U(\phi)}{V+M}$

EINSTEIN FRAME:

$$\Gamma^{\lambda}_{\mu\nu} = \left\{ {\lambda \atop \mu\nu} \right\} \Big|_{\bar{g}_{\alpha\beta} = (\chi - 2\kappa\epsilon R) g_{\alpha\beta}}$$

$$\bar{R}_{\mu\nu} - \frac{1}{2}\bar{g}_{\mu\nu}\bar{R} = \frac{\kappa}{2} T^{eff}_{\mu\nu}$$

$$T^{eff}_{\mu\nu} = \frac{\chi}{\chi - 2\epsilon\kappa R}\left(\phi_{,\mu}\phi_{,\nu} - \frac{1}{2}\bar{g}_{\mu\nu}\phi_{,\alpha}\phi_{,\beta}\bar{g}^{\alpha\beta}\right)$$
$$+ g_{\mu\nu} V_{eff}$$

$$V_{eff} = \frac{\epsilon R^2 + U}{(\chi - 2\epsilon\kappa R)^2}$$

(NOTE V_{eff} contains $\partial_\mu \phi$ ...)

HERE WE CAN SOLVE R:

FROM $R_{\mu\nu} \ldots$ eq. UPON CONTRACTION

$$-\frac{1}{\kappa} R(\Gamma, g) + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V = M$$

or $$-\frac{1}{\kappa} R + (\chi - 2\epsilon R \kappa) \bar{g}^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V = M$$

or $$R = \frac{-\kappa(V+M) + \frac{\kappa}{2} \bar{g}^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \chi}{1 + \kappa^2 \epsilon \bar{g}^{\mu\nu} \partial_\mu \phi \partial_\nu \phi}$$

which should be inserted back into V_{eff}

V_{eff} COMPLICATED FUNCTION OF ϕ, $\partial_\mu \phi \partial_\nu \phi \bar{g}^{\mu\nu}$. FOR SLOW-ROLLING, TAKE $\partial_\mu \phi \simeq 0$.

$V_\phi = f_1 e^{\alpha \phi}$, $U = f_2 e^{2\alpha\phi}$

$$V_{eff} = \frac{(f_1 e^{\alpha\phi} + M)^2}{4(\epsilon \kappa^2 (f_1 e^{\alpha\phi} + M)^2 + f_2 e^{2\alpha\phi})}$$

$$e^{\alpha\phi} \to \infty, \quad V_{eff} \to \frac{f_1^2}{4(\epsilon \kappa^2 f_1^2 + f_2)}$$

CORRECTION $\epsilon \kappa^2 f_1^2$ SMALL IF

$f_1 \sim M_{EW}^4$, $f_2 \sim M_{PL}^4$, $\kappa^2 = \frac{1}{M_{PL}^4}$

$$\epsilon \kappa^2 f_1^2 \sim \epsilon \frac{1}{M_{PL}^4} M_{EW}^4 M_{EW}^4 \ll f_2 \sim M_{PL}^4$$

$\alpha\phi \to \infty$ LIMIT UNCHANGED!

NOW IN

$$V_{eff} = \frac{(f_1 e^{\alpha\phi} + M)^2}{4(\epsilon\kappa^2(f_1 e^{\alpha\phi} + M)^2 + f_2 e^{2\alpha\phi})}$$

TAKE LIMIT $e^{\alpha\phi} \to 0$ $(\alpha\phi \to -\infty)$

IF $M \neq 0$ (S.S.B OF S.I.)

$$V_{eff} \to \frac{\cancel{M^2}}{4\epsilon\kappa^2 \cancel{M^2}} = \frac{1}{4\epsilon\kappa^2} \sim \frac{M_{Pl}^4}{4\epsilon}$$

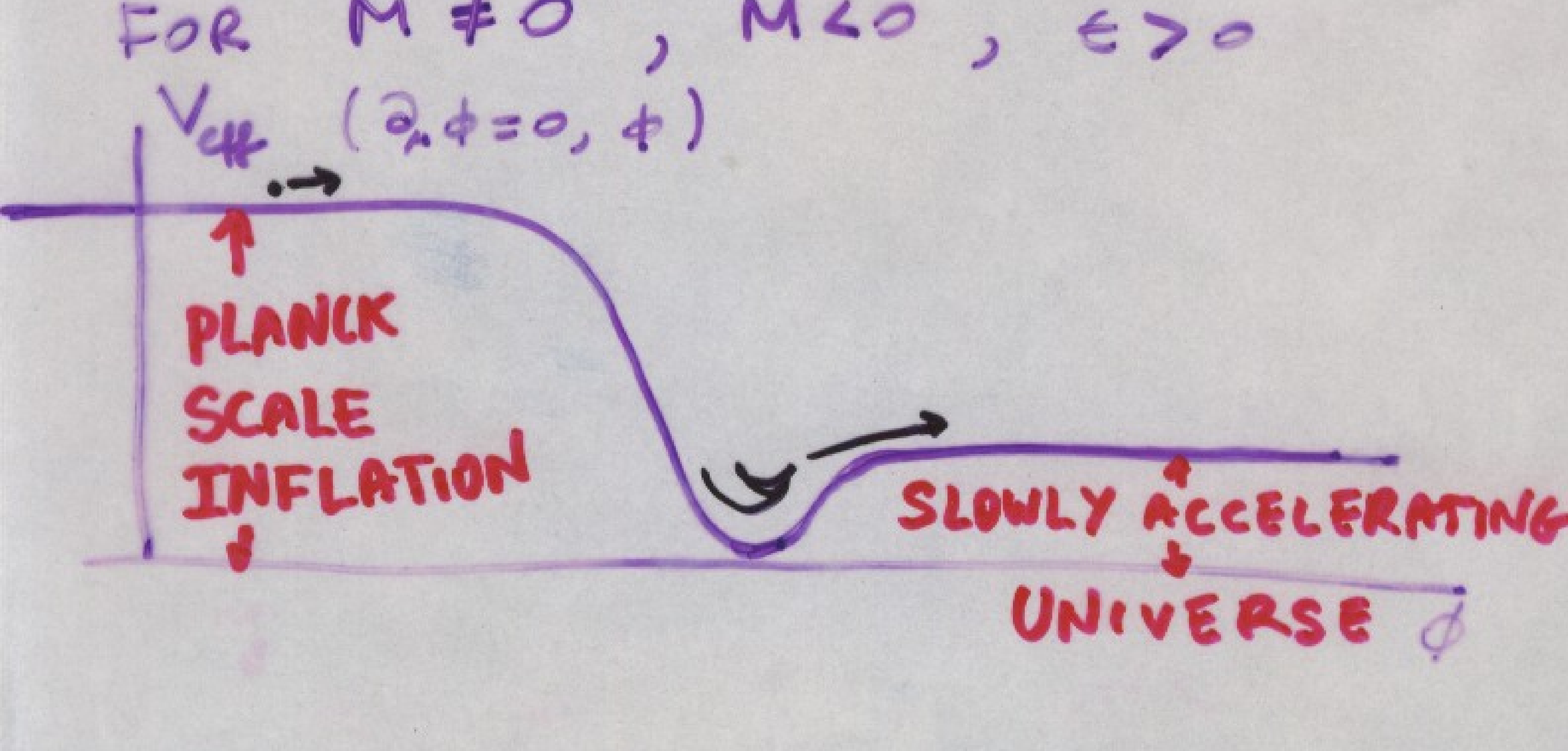
IF $M = 0$, V_{eff} is TRIVIAL. (NO S.S.B. OF S.I.)

$$V_{eff} = \frac{f_1 \cancel{e^{2\alpha\phi}}}{4(\epsilon\kappa^2 f_1^2 + f_2)\cancel{e^{2\alpha\phi}}}.$$

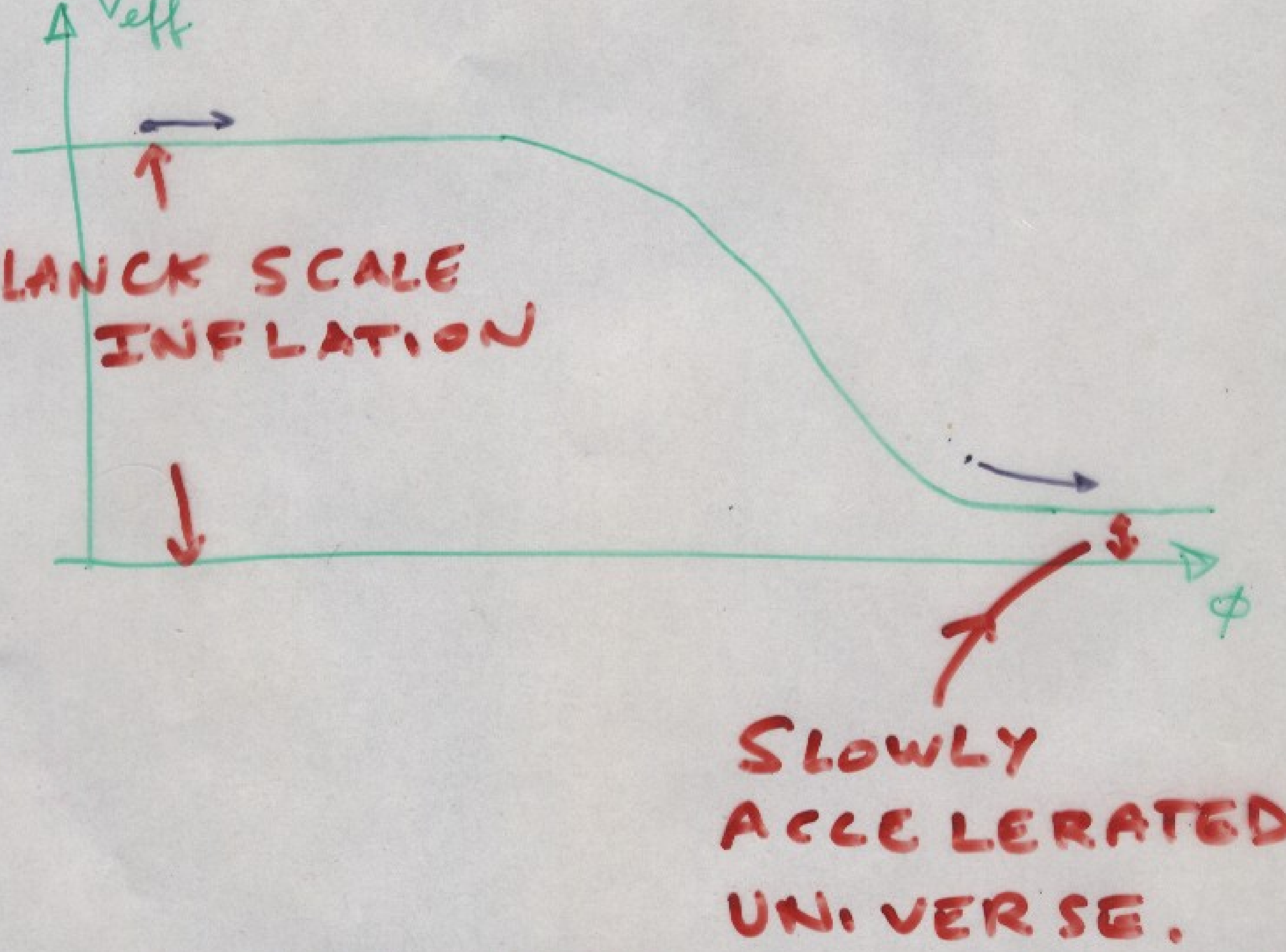
FLAT

NO STRUCTURE.

FOR $M \neq 0$, $M < 0$, $\epsilon > 0$



FOR $M \neq 0$, $M > 0$, $\varepsilon > 0$



INTERPOLATES BETWEEN AN INFLATIONARY PERIOD DRIVEN BY R^2 TERM (STAROBINSKI, 80) AND A SLOWLY ACCELERATED PHASE

FOR THOSE WHO LIKE POT. WITH $\partial_\mu \phi$ DEPEND:

NOTICE THAT IN THIS ANALYSIS NO USE WAS MADE OF RICHER STRUCTURE (CONCERNING $\partial_\mu \phi$ DEPENDENCE) OF $T_{\mu\nu}^{eff}$

CONSEQUENCES OF TMT's

Cosmological Constant Problem

INFLATION

Cosmic Coincidences
Quintessence
+ DARK MATTER

↑ (Cosmology direction)

$$\boxed{\begin{array}{c}\text{TMT}\\ \sqrt{-g}\ \text{and}\ \Phi\end{array}}$$

(particle physics direction)
↓

DARK MATTER
(LOW ENERGY PART. PHYS.)

The origin of three families of fermions

Origin of mixing between families (work in progress).

MIXED RESULTS
(COSMO - PARTICLE RESULTS)

→ i) DARK MATTER AS LOW DENSITY STATE OF NORMAL PART.

CONCERNING COUPLING OF QUINTESSENCE SCALAR TO ORDINARY MATTER : IN THIS CASE

ii) QUINTESSENCE SCALAR ≡ DILATON FIELD

iii) COUPLING OF Q.S. TO FERMIONIC MATTER DISSAPPEARS AS F. M. DENSITY DOMINATES

AVOIDANCE OF 5-th FORCE PROBLEM

FAMILIES BIRTH EFFECTS & DARK MATTER: TWO OPPOSITE LIMITS OF THE SAME THEORY.

FROM THE ACTION ABOVE WE GET CONSTRAINT THAT DETERMINES $\zeta = \Phi / \sqrt{-g}$

(WHEN USING FIRST ORDER FORMULATION, CONNECTIONS INDEPENDENT OF METRIC)

FORM OF CONSTRAINT

$$\left\{ \begin{array}{l} \text{VACUUM ENERGY} \\ \text{LIKE CONTRIBUTION} \end{array} \right\} + \left\{ \begin{array}{l} \text{FERMION} \\ \text{CONTRIBUTION} \end{array} \right\} =$$

LOOKS LIKE "COSMIC COINCIDENCE

FOR HIGH DENSITY 1$^{\text{st}}$ TERM IS NEGLIGIBLE

$\Longrightarrow$ NON LINEAR (CUBIC) EQUATION FOR $\zeta = \Phi / \sqrt{-g}$

$\Longrightarrow$ THREE FERMIONIC FAMILIES AT HIGH DENSITY.

SINCE CUBIC EQ. IMPLIES 3 SOLUTIONS FOR $\frac{\Phi}{\sqrt{-g}}$ inserting back into fermion eq $\Rightarrow$ 3 types of fermions $\Rightarrow$ 3 families. (THIS WE CALL "NORMAL" MATTER)

ALSO EQ.

$$\left\{\begin{matrix}\text{VACUUM TYPE}\\ \text{CONTRIBUTION}\end{matrix}\right\} + \left\{\begin{matrix}\text{FERMIONS}\\ \text{CONTR.}\end{matrix}\right\} = 0$$

IS GUARANTEED TO BE SATISFIED EVEN IF DENSITY OF FERMIONS IS SMALL.

IMPLIES AUTOMATICALLY AN EQUILLBRIUM BETWEEN FERMIONS & VACUUM ENERGY $\Rightarrow$ A COSMIC COINCIDENCE AND AS WE WILL SEE DARK MATTER APPEARS TO BE ORDINARY FERMIONS (NEUTRINOS) AT VERY LOW DENSITIES

<u>WE DISCUSS FIRST HIGH DENSITIES</u>