Magneto-Jeans Instabilities in anisotropic homogeneous universe

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A brief outline of the talk:

- We present the general relativistic version of the magnetohydrodynamic(MHD) equations, which we apply to the investigation of perturbation effects and hence to the study of the linearizied stability criteria.

- Using the perturbed MHD equations, we explore the way in which a homogeneous magnetic field influence the Jean's instabilities and examine if pang cake configurations are possible in a homogeneous and anisotropic Universe with homegeneous magnetic field along the z-axis

- Conclusions

- References

1. Introduction:

It is known that the formation of large-scale structures (e.g. galaxies, superclouds in galaxies e.t.c) in cosmological and astrophysical scales is close related to the concept of instabilities. The first serious theory of galaxy formation was proposed by Sir James Jeans early in the twentieth century(Jeans 1902,1928). Jeans supp7osed the universe to be filled with non-relativistic fluid, with mass density ρ,pressure p,velocity $\vec{\upsilon}$ and a gravitational field g, governed by the equation of continuity, the Euler equation and the gravitational field equations(Weinberg 1971). Unfortunately, Jeans's theory is not applicable to the formation of galaxies in an expanding universe, because Jeans assumed a static medium. Nevertheless, the conclusion of Jean's criterion of gravitational instability is that density fluctuations with wave length λ greater than the critical value λ_J, will grow so that the system becomes unstable. This means that an isothermal gaseous sphere with the length scale greater than λ_J is gravitational unstable and is going to contract constantly.

The first satisfactory theory of the instabilities of an expanding universe was given by Lifshitz (Lifshitz 1946). Lifshitz showed that the disturbances at wave number below the Jeans wave number,k_J, grow, not exponentially, by like a power of $R(t)$ where $R(t)$ is the scale factor of the expanding universe. In the scenarios of cosmic fluctuations when gravitation and cosmic expansion are essentially irrelevant, the work by Weinberg(Weinberg 1971) is of particular interest since he had considered the role of dissipation in the survival of protogalaxies, using Eckart's (Eckart 1940) formalism which is different from Landau's and Lifshitz(1959) one. He had found that the protogalactic fluctuations behaving as ordinary sound waves, during the period where $M << M_J$, damp in the acoustic phase due to the viscosity of the considered imperfect fluid.

Gravitational instabilities in the presence of a magnetic field in an expanding universe has been discussed by Hacyan (Hacyan S (1983)). He found closed form solutions for the evolution of linear density and magnetic field fluctuations in some cosmological models, in the Newtonian limit and verified that the uniform magnetic field slows down the growth rate of the unstable perturbations. Fennelly (Fennelly (1980)) has shown that several MHD processes, like pinch effects, hose instabilities, sausage and kink instabilities may contribute to the growth of galaxies eventhought they differ as different harmonics in the eigenfunction solutions of the wave equation for plasma instabilities but their separable time development will all be similar. He found (Fennelly 1980 and references therein) that those instabilities can drive both velocity and density perturbations, but they also require energy to support themselves. In the Newtonian and certain relativistic cases, he found that they extract more energy than they contribute and he suggested a method how can someone to be fed more energy into density and velocity modes than that they extract.

Recently, Crazyna S. et.al.(astro-ph/0402492) discussed the amplification of the cosmological magnetic field associated with forming gravitational structure. They have computed self similar solutions of the MHD equations both in linear and non-linear regime.

In the paper we are intended to discuss the problem in an homogeneous anisotropic cosmological model with homogeneous magnetic field along the z-axis and the considered fluctuation to propagate across the x-axis. In a recent work Papadopoulos,Vlahos and Esposito(Papadopoulos,Vlahos and Esposito (2002);Paper I) have considered the fluctuations propagating in the z- axis and Jean's criterion did not give any new result.

1. Basic equations:

The exact equations governing finite-amplitude wave propagation in hydromagnetic media in the frame of general relativity have been discussed in (Papadopoulos D Esposito F P 1982, Papadopoulos D. et al.2001, Papadopoulos D Vlahos L and Esposito F P 2001). For completeness we recast the relevant equations. We start with the Einstein field equations

$$R_{ab} - \frac{1}{2} g_{ab} R = -\kappa T_{ab}, \tag{1}$$

with

$$T^{ab}_{;b} = 0 \tag{2}$$

For simplicity we obtain $c = 1$ and $\kappa = \frac{8\pi G}{c^4} = 1$. Taking the covariant divergence of the Bianchi identities we obtain

$$(R^{ab} - \frac{1}{2} R g^{ab})_{;ab} = 0 \tag{3}$$

where $R_{ab} = R^{c}_{abc}$ is the Ricci tensor and R_{abcd} is the curvature tensor

For a unit time-like vector we choose $u_a u^a = -1$ and our hydromagnetic system will be specified by the following choice for the energy-momentum tensor

$$T^{ab} = (\epsilon + \frac{H^2}{2}) u^a u^b + (p + \frac{H^2}{2}) h^{ab} - H^a H^b \quad (4)$$

with

$$h^{ab} = g^{ab} + u^a u^b, \ \ \epsilon = \rho + \rho \Pi \quad (5)$$

where u^a is the fluid velocity, ρ the mass density, $\rho\Pi$ the internal energy density, p the pressure of the fluid and H^a is the prevailing magnetic field as measured by an observer co-moving with u^a. Furthermore we introduce the expansion $\theta = u^a_{;a}$, the shear $\sigma_{ab} = h^c_a h^d_b u_{(c;d)} - \frac{1}{3}\theta h_{ab}$ and the twist $\omega_{ab} = h^c_a h^d_b u_{[c;d]}$, where the round bracket denotes symmetrization while the square bracket antisymmetrization.

From (1) and (4) we find

$$R^{ab} = -\{\frac{1}{2}(\epsilon + 3p + H^2)u^a u^b$$
$$+ \ \frac{1}{2}(\epsilon - p + H^2)h^{ab} - H^a H^b\} \qquad (6)$$

Substituting (6) into (3) we have

$$\ddot{x} + \Box(p + \frac{H^2}{2}) + 2\dot{x}\theta + x_{;a}\dot{u}^a$$
$$+ \ x(\dot{\theta} + \theta^2 + \dot{u}^a_{;a}) - (H^a H^b)_{;ab} = 0 \qquad (7)$$

where $\Box$ is the covariant d'Alembertian operator, $\dot{u}^a = u^a_{;c}u^c$ and $x = \epsilon + p + H^2$ (Notice,that (7) can be obtained directly from (2) and (4)).

Raychaudhuri's equation in the form

$$\begin{aligned} \dot{u}^a_{;a} &= \dot{\theta} + \frac{\theta^2}{3} + 2(\sigma^2 \\ &- \omega^2) + \frac{1}{2}(\epsilon + 3p + H^2) \end{aligned} \tag{8}$$

where $2\sigma^2 = \sigma^{ab}\sigma_{ab}$, $2\omega^2 = \omega^{ab}\omega_{ab}$ and dot means covariant derivative along u^a allows us to write the (7) as

$$\begin{aligned} & \ddot{x} + \Box(p + \frac{H^2}{2}) + 2\dot{x}\theta + 2x\dot{\theta} + x_{;a}\dot{u}^a \\ +& 2x(\frac{2\theta^2}{3} + \sigma^2 - \omega^2) + \frac{1}{2}x(\epsilon + 3p + H^2) \\ -& (H^a H^b)_{;ab} = 0 \end{aligned} \tag{9}$$

In order to make further progress with (9) we must use the equations of motion for the fluid and Maxwell's equations. These equations are

$$
\begin{aligned}
& T^{ab}_{;b} = \dot{x}u^a + x\dot{u}^a + x\theta u^a \\
+ \quad & (p + \frac{H^2}{2})_{;b} g^{ab} - (H^a H^b)_{;b} = 0 \qquad (10)
\end{aligned}
$$

1.a. Time component:

$$
\begin{aligned}
& T^{ab}_{;b} u_a = 0 = -\dot{x} - x\theta + \dot{p}^* \\
- \quad & u_a (H^a_{;b} H^b + H^a H^b_{;b}) \\
= \quad & \dot{\epsilon} + (\epsilon + p)\theta \qquad (11)
\end{aligned}
$$

where $p^* = p + \frac{H^2}{2}$. From the last equation we easily have

$$
\ddot{\epsilon} + (\dot{\epsilon} + \dot{p}) + (\epsilon + p)\dot{\theta} = 0 \qquad (12)
$$

From (9) and (12) and recalling that $x = \epsilon + p + H^2$, we obtain

$$
\begin{aligned}
&(\epsilon - \frac{H^2}{2})_{;ab} u^a u^b = h^{ab}(p + \frac{H^2}{2})_{;ab} \\
+ \;& 2(\dot{H^2\theta}) - (H^a H^b)_{;ab} + 2x(\frac{2\theta^2}{3} + \sigma^2 - \omega^2 \\
- \;& \dot{u}^a \dot{u}_a) + \frac{x}{2}(\epsilon + 3p + H^2) \\
+ \;& 2\dot{u}_a (H^a H^b)_{;b} + (H^2)_{;a} \dot{u}^a \qquad (13)
\end{aligned}
$$

1.b.The space-component:

$$h^{\gamma}_{\alpha} T^{\alpha\beta}_{;\beta} = 0 \qquad (14)$$

or

$$x \dot{u}^{\gamma} = h^{\gamma}_{\alpha} [H^{\alpha} H^{\beta} - g^{\gamma\beta} p^*]_{;\beta} \qquad (15)$$

From (13) and (15) we have:

$$
\begin{aligned}
& (\epsilon - \frac{H^2}{2})_{;ab} u^a u^b = h^{ab}(p + \frac{H^2}{2})_{;ab} + 2(\dot{H^2 \theta}) \\
- & \quad (H^a H^b)_{;ab} + 2x(\frac{2\theta^2}{3} + \sigma^2 - \omega^2 \\
- & \quad \dot{u}^a \dot{u}_a) + \frac{x}{2}(\rho + 3p + H^2) + 2\dot{u}_a (H^a H^b)_{;b} \\
+ & \quad (H^2)_{;a} \dot{u}^a \qquad (16)
\end{aligned}
$$

1.c. The Maxwell equation:

$$
\begin{aligned}
& (u^\alpha H^\beta - u^\beta H^\alpha)_{;\alpha} = 0 \Rightarrow \\
& \theta H^\beta + \dot{H}^\beta - u^\beta_{;\alpha} H^\alpha - u^\beta H^\alpha_{;\alpha} = 0 \qquad (17)
\end{aligned}
$$

Because of the (8) and $h_\alpha^\beta H^\alpha = H^\beta$ the equation (17) becomes:

$$\dot{H}^a = (\sigma_b^a + \omega_b^a - \frac{2}{3}\delta_b^a\theta)H^b$$
$$+ \quad \frac{1}{\epsilon + p} p_{;b} H^b u^a \tag{18}$$

The last equation (18) may be written in the following form

$$\frac{\mu \dot{H}^2}{8\pi} = \frac{\mu}{4\pi}\sigma_{ij}H^i H^j - \frac{4\theta}{3}(\frac{\mu H^2}{8\pi}) \tag{19}$$

where μ is the permeability.

Summary: The general relativistic version of the magnetohydrodynamic(MHD) equations, which will be applied to the investigation of perturbation effects and hence to the study of the linearizied stability criteria, are:

$$
\begin{aligned}
& (\epsilon - \frac{H^2}{2})_{;ab} u^a u^b = h^{ab}(p + \frac{H^2}{2})_{;ab} + 2(\dot{H^2\theta}) \\
- & (H^a H^b)_{;ab} + 2x(\frac{2\theta^2}{3} + \sigma^2 - \omega^2 \\
- & \dot{u}^a \dot{u}_a) + \frac{x}{2}(\rho + 3p + H^2) + 2\dot{u}_a (H^a H^b)_{;b} \\
+ & (H^2)_{;a} \dot{u}^a \qquad (20)
\end{aligned}
$$

$$
\dot{x} u^a + x \dot{u}^a + x\theta u^a + (p + \frac{H^2}{2})_{;b} g^{ab} - (H^a H^b)_{;b} = 0 \qquad (21)
$$

$$
\dot{H}^\mu = (\sigma^\mu_\nu + \omega^\mu_\nu - \frac{2}{3}\delta^\mu_\nu \theta) H^\nu + \frac{1}{\epsilon + p} p_{;\nu} H^\nu u^\mu \qquad (22)
$$

and

$$
\dot{u}^a_{;a} = \dot{\theta} + \frac{\theta^2}{3} + 2(\sigma^2 - \omega^2) + \frac{1}{2}(\epsilon + 3p + H^2) \qquad (23)
$$

where μ is the permeability, u^a is the fluid velocity, ρ is the mass density, $\epsilon = \rho + \rho\Pi$, $\rho\Pi$ is the internal energy, $\dot{u}^a = u^a_{;c}u^c$, $\theta = u^a_{;a}$ is the expansion velocity, $x = \epsilon + p + H^2$, H the magnetic field, $h^{ab} = g^{ab} + u^a u^b$ and $G = c = 1$.

We perturb (20),(21), (22) and (23) using the condition $\delta g_{ab} = 0$ and keeping only first order terms in pressure, density, velocity, and magnetic field.

We linearize the above equations and search for the amplification or damping of small amplitude hydromagnetic waves in the early universe described by the anisotropic cosmological model due to Thorne (Thorne (1967); Jacobs (1968)),

The metric:

$$ds^2 = -dt^2 + A^2(dx^2 + dy^2) + W^2 dz^2 \quad (24)$$

Thorne's model has the following characteristics: (i) The model contains perfect fluid obeying the equation of state $p = \gamma\rho$ where $\frac{1}{3} < \gamma \leq 1$. (ii) The fluid comoves with the coordinate system and therefore $u^\mu = (-1, 0, 0, 0)$ and (iii) as seen in the rest frame of the fluid there is a magnetic field of strength H pointing in the z-direction but no electric field. The functions A and W which enter into line element are: $A = A(t) = t^{1/2}$ and $W = W(t) = t^l$, while $\rho = \frac{3-\gamma}{16\pi t^2(1+\gamma)^2}$ and $H = \frac{(1-\gamma)^{1/2}(3\gamma-1)^{1/2}}{2t(1+\gamma)}$ and $l = (1-\gamma)/(1+\gamma)$. Notice, that for $\gamma = 1/3$ or $\gamma = 1$ the strength of the magnetic field vanish.

For the metric (24) and a comoving observer with a 4-velocity $u^\mu = [1, 0, 0, 0]$ we have:

$$\dot{u}^\mu = 0, \quad , \omega_{\mu\nu} = 0, \quad \omega^2 = 0 \tag{25}$$

$$\theta = \frac{2}{(1+\gamma)t} \tag{26}$$

The non-zero components of the $\sigma_{\mu\nu}$ are:

$$\sigma_{11} = \sigma_{22} = \frac{3\gamma - 1}{6(1+\gamma)}, \ \sigma_{33} = -\frac{(3\gamma - 1)t^m}{3(1+\gamma)t} \tag{27}$$

with $m = -2\frac{\gamma-1}{\gamma+1}$ and

$$\sigma^2 = \frac{(3\gamma - 1)^2}{6(1+\gamma)^2 t^2} \tag{28}$$

2. The Perpendicular case:

We assume that all the perturbed quantities propagate in the x-axis e.g. they have the form $(\delta\rho, \delta H^{\mu}, \delta u^{\mu}) \sim exp^{i(nt-kx)}$,and the linearized perturbed equations, in the metric (24), now read:

$$\delta u^0 = \delta u^2 = \delta u^3 = 0 \quad (29)$$

$$\delta u^1 [2x\Gamma^1_{01} - (H^3)^2 \Gamma^0_{33} + p^* + inx] = ik\delta p + ikH_3 \delta H^3 (3$$

$$\delta H^1 = \delta H^2 = 0 \quad (31)$$

$$\delta H^3_{,0} = \theta \delta H^3 + H^3 \delta\theta \quad (32)$$

where $p^* = p + \frac{H^2}{2}$,

The perturbed Raychaudhuri's equation gives:

$$\delta\theta = \frac{1}{\epsilon + p}[\delta\epsilon_{,0} - \theta(\delta\epsilon + \delta p)] \tag{33}$$

$$\delta\theta_{,0} = \frac{8}{3}\theta\delta\theta + \frac{1}{3}(\delta\epsilon + \delta p + 2H_3\delta H^3) \tag{34}$$

and

For the density fluctuations we have:

$$
\begin{aligned}
& \delta\epsilon_{,00} - \delta p_{;ab}h^{ab} - 4(\epsilon + p)\theta\delta\theta \\
- & (\frac{4}{3}\theta^2 + 2\sigma^2)(\delta\epsilon + \delta p) \\
- & (\frac{\epsilon + 3p}{2})(\delta\epsilon + \delta p) - \frac{1}{2}(\epsilon + p)(\delta\epsilon + 3\delta p) \\
= & H_3(\delta H^3_{;ab}h^{ab}) + (H_3\delta H^3)_{,00} + 4[\theta(H_3\delta H^3)]_{,0} - \\
- & 2\delta H^3[H^3_{,0}\Gamma^0_{33} + H^3(\Gamma^0_{33})_{,0} \\
+ & H^3\Gamma^b_{b0}\Gamma^0_{33} \\
- & H^3\Gamma^0_{33}\Gamma^3_{03}] - H^3\delta H^3_{,0}\Gamma^0_{33} \\
+ & 4H^2\theta\delta\theta + 4(H_3\delta H^3)(\frac{2}{3}\theta^2 + \sigma^2) \\
+ & H^2(\delta\epsilon + 2\delta p) + 2(\epsilon + 2p + H^2)(H_3\delta H^3)
\end{aligned}
$$

Because of the propagation of the perturbation in the x-axis,the equation (35), with the aid of (29),(30),(31),(32),(33) and (34) gives:

$$\begin{aligned} -n^2 & +k^2c_s^2 + J_1 - 4in\theta \\ = & \quad [H_3\frac{\delta H^3}{\delta\epsilon}][-n^2 - k^2 + R_1 - in\frac{\theta}{2}(3\gamma - 7)] \\ + & \quad \frac{u_A^2}{(1+\gamma)}[R_2 - in\frac{\theta}{3}(6\gamma - 28)] \end{aligned} \quad (36)$$

where $u_A^2 = \frac{H^2}{\epsilon}$,

$$J_1 = (1+\gamma)(\frac{8}{3}\theta^2 - 2\sigma^2) - (1+3c_s^2)(1+c_s^2)\epsilon \quad (37)$$

$$\begin{aligned} R_1 = & \frac{\theta^2}{2}(6\gamma^2 - 15\gamma + 13) + 4\sigma^2 + 4\theta_{,0} \\ + & \quad 2(1+2\gamma)\epsilon + \frac{10}{3}H^2 \end{aligned} \quad (38)$$

$$R_2 = \frac{1}{3}[(7\gamma+3)(\gamma+1)\epsilon + (6\gamma - 28)(\gamma+1)\theta^2] \quad (39)$$

From (32) and (33) we find:

$$H_3\frac{\delta H^3}{\delta\epsilon} = \frac{u_A^2}{(1+\gamma)}\frac{[\theta(\gamma+1)-in][\theta+in]}{\theta^2+n^2} \quad (40)$$

Combining the (40) and (36), we find the following dispersion relation:

$$\begin{aligned} & -n^4 + n^2(k^2c_s^2 + J_1 - \theta^2) + \theta^2(k^2c_s^2 + J_1) \\ - & \ 4in\theta(n^2+\theta^2) = \frac{u_A^2}{(1+\gamma)}\{-n^4 \\ + & \ n^2[-k^2 + \frac{1}{2}\theta^2(3\gamma^2 - 9\gamma - 2) + R_1 + R_2] \\ - & \ k^2\theta^2(1+\gamma) + \theta^2[R_1(1+\gamma) + R_2] \\ + & \ in\theta[(-n^2 - k^2 + R_1)\gamma - \frac{1}{6}\theta^2(9\gamma^2 - 77) \\ - & \ \frac{1}{6}n^2(27\gamma - 77)]\} \end{aligned} \quad (41)$$

The real part of (41) gives:

$$
\begin{aligned}
& -n^4[1-\frac{u_A^2}{(1+\gamma)}]+n^2\{k^2[c_s^2+\frac{u_A^2}{(1+\gamma)}] \\
+ & J_1-\theta^2-\frac{u_A^2}{(1+\gamma)}[\frac{\theta^2}{2}(3\gamma^2-9\gamma-2) \\
+ & R_1+R_2]\}+\theta^2(k^2c_s^2+J_1)-\frac{u_A^2\theta^2}{(1+\gamma)} \\
\times & [-k^2(1+\gamma)+R_1(1+\gamma)+R_2]=0 \quad (42)
\end{aligned}
$$

From the (42) we obtain the solutions:

$$
\begin{aligned}
& n_{1,2}^2=\frac{1}{1-\frac{u_A^2}{(1+\gamma)}}\{k^2[c_s^2+\frac{u_A^2}{(1+\gamma)}] \\
+ & J_1-\theta^2-\frac{u_A^2}{(1+\gamma)}[\frac{\theta^2}{2}(3\gamma^2-9\gamma-2) \\
+ & R_1+R_2]\}\pm\sqrt{\Delta} \qquad (43)
\end{aligned}
$$

$$
\begin{aligned}
\Delta = & \{k^2[c_s^2 + \frac{u_A^2}{(1+\gamma)}] + J_1 - \theta^2 \\
- & \frac{u_A^2}{(1+\gamma)}[\frac{\theta^2}{2}(3\gamma^2 - 9\gamma - 2) \\
+ & R_1 + R_2]\}^2 + 4[1 - \frac{u_A^2}{(1+\gamma)}]\{\theta^2(k^2c_s^2 + J_1) \\
- & \frac{u_A^2\theta^2}{(1+\gamma)}[-k^2(1+\gamma) + R_1(1+\gamma) + R_2]\}
\end{aligned} \quad (44)
$$

From the (43) we take the $k_\perp^2$ of the following form:

$$
\begin{aligned}
& k_\perp^2[c_s^2 + u_A^2] = (1+3c_s^2)(1+c_s^2)\epsilon \\
- & (1+\gamma)(\frac{8}{3}\theta^2 - 2\sigma^2) + \frac{u_A^2}{(1+\gamma)}[R_1(1+\gamma) \\
+ & R_2]
\end{aligned} \quad (45)
$$

It is easy to verify that if $\gamma = \frac{1}{3}$, $H^2 = 0, \sigma = 0$ and (45) reduce to a dispersion relation corresponding to the FRW space time with a scale factor $S(t) = t^{(1/2)}$.

3. The parallel case:

In this case, we assume that the perturbations propagate along the z-axis e.g. $(\delta\epsilon, \delta u^{\mu}, \delta H^{\mu}) \sim$ exp $i(nt - kz)$. The obtained perturbed MHD equations are very complicate. However, they simplify considerably at certain limits(e.g at very large(small) values of t). Thus at very large values of t, we end up with a dispersion relation of the following form:

$$-n^2 \quad [1 - \frac{u_A^2}{(1+\gamma)}] + k^2[c_s^2 + \frac{u_A^2}{(1+\gamma)}]$$
$$= \quad -J_1 + \frac{u_A^2}{(1+\gamma)}[R_1 + R_2 + \frac{1}{2}\gamma(3\gamma - 7)\theta^2], \quad a$$

and

$$k_{\|}^2 \quad [c_s^2 + \frac{u_A^2}{(1+\gamma)}] = -J_1 + \frac{u_A^2}{(1+\gamma)}[R_1 + R_2$$
$$+ \quad \frac{1}{2}\gamma(3\gamma - 7)\theta^2], \quad as \ \ t \to \infty \qquad (47)$$

For small values of t, real part of the dispersion relation gives:

$$-n^2[1 - \frac{u_A^2}{(1+\gamma)}] + k^2[c_s^2 + u_A^2]$$
$$= u_A^2[R_1 + \frac{R_2}{(1+\gamma)}] \qquad (48)$$

and

$$k_{\|}^2[c_s^2 + u_A^2] = -J_1 + \frac{u_A^2}{(1+\gamma)}[R_1(1+\gamma)$$
$$+ R_2], \quad as \;\; t \to 0 \qquad (49)$$

The ratio of the (45),(47) reveals that $k_\perp^2 < k_\|^2$, which means that in this case, pang cakes configurations are possible.

However, as $t \to 0$,the (45) is the same as (49), indicating that, in a universe described by (24), at early times,pang-cake configurations are not possible.

Results:

- The right hand sides of the (45),(47) and (49) are functions of t and γ. In those equations, the coefficients of t^{-2} are functions of γ, which take values in the interval $(1/3, 1]$

- It is easy to verify that if $\gamma = 1/3$, then $H^2 = 0, u_A^2 = 0, \sigma^2 = 0$ and the equations (45),(47) and (49),take the known simple form:

$$k_\perp^2 c_s^2 = k_\parallel^2 c_s^2 = \epsilon(1+c_s^2)(1+3c_s^2) - \frac{32}{9}\theta^2 \quad (50)$$

- If $\gamma = 1$ then $H^2 = u_A^2 = 0$,but $\sigma^2 \neq 0$ and the equations (45),(47) are equal e.g:

$$k_\perp^2 c_s^2 = k_\parallel^2 c_s^2 = \epsilon(1 + c_s^2)(1 + 3c_s^2) - \frac{4}{3}(\frac{8}{3}\theta^2 - 2\sigma^2) \quad (51)$$

- In the case where $k_{<}k_{||}$, the $\lambda_{||} < \lambda_{\perp}$,indicating the existence of a possible pang cage configuration.

- Nevertheless, if the universe is static, $\theta = 0 \rightarrow k_{\perp} = k_{||}$ and we end up with one equation for the wave number known from the past(Jacson (1972))

- For any value of γ, from the above mentioned interval,at very late times of the universe,the right-hand sides of the (45),(47) are different, indicating the existence of a possible pang cage configuration. The results, forces us to think that only at late times, e.g. as t approaches to very large values and for any value of γ belonging in the interval (1/3,1), pang cakes configurations are possible.

4. The MHD Equations with conductivity:

The equations governing the hydrodynamics of a conductive magnetofluid with energy density ϵ, pressure p, 4-velocity u^μ, resistivity $\eta = \frac{1}{4\pi\sigma}$(Here σ is the conductivity) and magnetic field $H^\mu = (0, 0, 0, H^3)$ are as follows :

$$\dot{x}u^a + x\dot{u}^a + x\theta u^a + (p + \frac{H^2}{2})_{;b}g^{ab} - (H^a H^b)_{;b} = 0 \quad (52)$$

$$\begin{aligned}
(\epsilon - \frac{H^2}{2})_{;ab}u^a u^b &= h^{ab}(p + \frac{H^2}{2})_{;ab} \\
&+ 2(\dot{H^2\theta}) - (H^a H^b)_{;ab} + 2x(\frac{2\theta^2}{3} + \sigma^2 \\
&- \omega^2 - \dot{u}^a \dot{u}_a) + \frac{x}{2}(\rho + 3p + H^2) \\
&+ 2\dot{u}_a (H^a H^b)_{;b} + (H^2)_{;a}\dot{u}^a \qquad (53)
\end{aligned}$$

$$\dot{H}^\alpha = (\sigma^\alpha_\beta + \omega^\alpha_\beta)H^\beta - \frac{2}{3}\theta H^\alpha + u^\alpha H^\gamma_{;\gamma} + \frac{1}{4\pi\sigma}h^{\gamma\beta}H^\alpha_{;\gamma\beta} \quad (54)$$

The equations (52),(53) and (54) are perturbed. The obtained equations are complicate again. We examine them at certain limits. Thus, in the case that the perturbations propagate in the x-direction, e.g. $(\delta\epsilon, \delta u^{\mu}, \delta H^{\mu}) \sim e^{i(\omega t - kx)}$, we have the dispersion relation:

$$a_4\omega^4 - a_2\omega^2 - a_0 = 0 \tag{55}$$

where

$$a_4 = [1 - \frac{u_A^2}{(1+\gamma)} - 4\eta\theta - \frac{\eta\theta u_A^2}{2(1+\gamma)}] \tag{56}$$

$$
\begin{aligned}
a_2 = \{k^2 c_s^2 + J_1 - \theta^2 + \frac{u_A^2}{(1+\gamma)}[k^2 \\
- \quad & (R_1 + R_2) - \frac{\theta^2}{2}(3\gamma^2 - 9\gamma - 2)] \\
- \quad & 2\eta\theta[k^2 + 2(k^2 c_s^2 + J_1) \\
- \quad & \frac{\theta^2}{4}(\gamma^2 - 1)] + \eta\theta\frac{u_A^2}{(1+\gamma)}[-k^2(3\gamma - 9) \\
- \quad & \frac{1}{4}\theta^2(\gamma^2 - 1)(\gamma + 1) + R_1(2+\gamma) + 4R_2 \\
- \quad & \frac{\theta^2}{8}(3\gamma - 7)(\gamma^2 - 4\gamma - 5)]\} \qquad (57)
\end{aligned}
$$

$$
\begin{aligned}
a_0 &= \theta^2(k^2c_s^2 + J_1) + \frac{\theta^2 u_A^2}{(1+\gamma)} + 2\eta\theta(k^2c_s^2 \\
+ &\quad J_1)[-k^2 + \frac{\theta^2}{4}(\gamma^2 - 1)] - \eta\theta\frac{u_A^2}{} \\
\times &\quad (1+\gamma)[k^4(1+\gamma) - \frac{k^2}{4}\theta^2(\gamma^2-1)(\gamma+1) \\
- &\quad k^2R_1(\gamma+1) - 2k^2R_2 + \frac{1}{4}R_1\theta^2(\gamma^2-1) \\
\times &\quad (\gamma+1) + \frac{1}{2}R_2\theta^2(\gamma^2-1)\} \qquad (58)
\end{aligned}
$$

Thus,neglecting terms of the order η^2,in (55) we obtain a fourth order algebraic equation in terms of k:

$$
\begin{aligned}
& 2\eta\theta k_\perp^4(c_s^2 + \frac{u_A^2}{2}) + k_\perp^2[\theta^2(c_s^2 + u_A^2) \\
+ & \eta\theta B] + \{\theta^2 J_1 - \theta^2 \frac{u_A^2}{(1+\gamma)} \\
\times & [R_1(\gamma+1) + R_2] + \eta\theta\Gamma\} = 0
\end{aligned}
\tag{59}
$$

where $B = 2J_1 - \theta^2\gamma(\gamma^2 - 1) - \frac{u_A^2}{(1+\gamma)}[\frac{\theta^2}{4}(\gamma^2 - 1)(\gamma+1) + R_1(\gamma+1) + 2R_2]$ and $\Gamma = -\frac{1}{2}\theta^2(\gamma^2 - 1)J_1 + u_A^2[\frac{\theta^2}{4}(\gamma^2 - 1)R_1 + \frac{1}{2}\theta^2 R_2]$

From (59), we obtain two exact solutions:

$$k^2_{\perp(1)} = \frac{1}{(c_s^2 + u_A^2)}\{-J_1 + \frac{u_A^2}{(1+\gamma)}[R_1(\gamma+1) + R_2]\} \quad (60)$$

and

$$k^2_{\perp(2)} = -\frac{\theta^2(c_s^2 + u_A^2)}{2\eta\theta(c_s^2 + \frac{u_A^2}{2})} - \frac{B}{2(c_s^2 + \frac{u_A^2}{2})} + \frac{M}{c_s^2 + u_A^2} \quad (61)$$

where $M = J_1 - \frac{u_A^2}{(1+\gamma)}[R_1(\gamma+1) + R_2]$. The equation (60) is exactly the same as equation (45). The Eq.(61) exhibits an $\frac{1}{\eta}$, dependence which means that at the late stages of the discussed model of the universe, the value of $k_\perp^2$ is very small, since, by that time, η is very small.

Now, assuming that the perturbations are propagating in the z-axis,for large values of t, we obtain the dispersion relation :

$$\begin{aligned}
&- \omega^2\{[1-\frac{u_A^2}{(1+\gamma)}]-\eta\theta\frac{u_A^2}{(1+\gamma)} \\
&\times \frac{(3\gamma-7)}{2}\}+k^2\{[c_s^2+\frac{u_A^2}{(1+\gamma)}] \\
&- \eta\theta\frac{u_A^2}{(1+\gamma)}\frac{(3\gamma-7)}{2}\} \\
&= -J_1+\frac{u_A^2}{(1+\gamma)}[R_1+R_2], \quad t\rightarrow\infty \qquad (62)
\end{aligned}$$

and

$$
\begin{aligned}
& k_{\|}^{2}\{[c_s^2 + \frac{u_A^2}{(1+\gamma)}] - \eta\theta\frac{u_A^2}{(1+\gamma)} \\
\times & \quad \frac{(3\gamma - 7)}{2}\} = -J_1 \\
+ & \quad \frac{u_A^2}{(1+\gamma)}[R_1 + R_2], \ t \to \infty
\end{aligned}
\tag{63}
$$

However, as t approaches to small values, the real part of the dispersion relation reads:

$$\begin{aligned}
& -\omega^2[1 - u_A^2 - \eta\frac{\theta}{4}(\gamma^2 - 1)u_A^2] \\
+ & \ k^2[c_s^2 + u_A^2 + \eta\frac{\theta}{4}(\gamma^2 - 1)u_A^2] + J_1 \\
= & \ \frac{u_A^2}{1+\gamma}[R_1(1+\gamma) + R_2 \\
+ & \ \eta R_1(\gamma^2 - 1)(1+\gamma)\frac{\theta}{4}], \ \ as \ \ t \to 0 \qquad (64)
\end{aligned}$$

and

$$\begin{aligned}
& k_{||}^2\{[c_s^2 + u_A^2 + \eta\frac{\theta}{4}(\gamma^2 - 1)u_A^2] \\
= & \ -J_1 + \frac{u_A^2}{(1+\gamma)}[R_1(1+\gamma) \\
+ & \ R_2 + \eta\frac{\theta}{4}(\gamma^2 - 1)(\gamma + 1)R_1], \ t \to 0 \ (65)
\end{aligned}$$

Results:

- In Eqs.(59),(60) and (63), if $\sigma = 0$, we end up with the Eq.(45) and (47),respectively.

- If the resistivity $\eta \neq 0$, but $\gamma = 1/3$, then Eqs.(61) and (63) exhibit an interesting dependence from η, presented below:

$$k_{\perp}^{2} \sim -\frac{\theta}{2\eta} \tag{66}$$

$$k_{\|}^{2} c_{s}^{2} = -J_{1} \tag{67}$$

Summary: We summarize the obtained equations for the k^2 in the cases we do not have conductivity and we do have conductivity:

I. Without conductivity

a.Perpendicular case.

$$\begin{aligned} k_\perp^2[c_s^2 + u_A^2] &= (1+3c_s^2)(1+c_s^2)\epsilon \\ - & (1+\gamma)(\frac{8}{3}\theta^2 - 2\sigma^2) + \frac{u_A^2}{(1+\gamma)}[R_1(1+\gamma) \\ + & R_2] \end{aligned} \quad (68)$$

b. parallel case:

$$\begin{aligned} k_\parallel^2 & [c_s^2 + \frac{u_A^2}{(1+\gamma)}] = -J_1 + \frac{u_A^2}{(1+\gamma)}[R_1 + R_2 \\ + & \frac{1}{2}\gamma(3\gamma - 7)\theta^2], \ \ as \ \ t \to \infty \end{aligned} \quad (69)$$

$$k_{\|}^2[c_s^2 + u_A^2] = -J_1 + \frac{u_A^2}{(1+\gamma)}[R_1(1+\gamma)$$

$$+ \quad R_2], \quad as \quad t \to 0 \tag{70}$$

II.With conductivity:

a. Perpendicular case.

$$k^2_{\perp(1)} = \frac{1}{(c_s^2 + u_A^2)}\{-J_1 + \frac{u_A^2}{(1+\gamma)}[R_1(\gamma+1) + R_2]\} \tag{71}$$

and

$$k^2_{\perp(2)} = -\frac{\theta^2(c_s^2 + u_A^2)}{2\eta\theta(c_s^2 + \frac{u_A^2}{2})} - \frac{B}{2(c_s^2 + \frac{u_A^2}{2})} + \frac{M}{c_s^2 + u_A^2} \tag{72}$$

b.Parallel case.

$$k^2_{\parallel}\{[c_s^2 + \frac{u_A^2}{(1+\gamma)}] - \eta\theta\frac{u_A^2}{(1+\gamma)} \times \frac{(3\gamma - 7)}{2}\} = -J_1$$

$$+ \quad \frac{u_A^2}{(1+\gamma)}[R_1 + R_2],\ t \to \infty \qquad (73)$$

$$\begin{aligned} & k_{\|}^2\{[c_s^2 + u_A^2 + \eta\frac{\theta}{4}(\gamma^2 - 1)u_A^2] \\ = \quad & -J_1 + \frac{u_A^2}{(1+\gamma)}[R_1(1+\gamma) \\ + \quad & R_2 + \eta\frac{\theta}{4}(\gamma^2 - 1)(\gamma + 1)R_1],\ t \to 0 \end{aligned} \qquad (74)$$

5. Conclusions:

In the paper we have used the MHD equations(Papadopoulos and Esposito (1982); Papadopoulos,Vlahos and Esposito (2001)) in an homogeneous and anisotropic expanding universe (Thorne (1967);Jacobs (1968)) to derive a Magneto-Jeans formula. We verified the following:

- In the case where the perturbations propagate perpendicularly to the magnetic field e.g. in the x-axis, the wave number $k_\perp$ is given by the Eq.(45) .

- In the case that the perturbations propagate parallel to the magnetic field which is oriented to the z-axis the wave number $k_{||}$ is given by the Eq.(47) for very large values of t, indicating a possible existence of a pang-cake configuration,since the ratio of $\frac{k_\perp^2}{k_{||}} < 1$. For very small values of t we have Eq.(49) which is identical to Eq.(45).

- Upon the consideration of a conductive magnetofluid the Eqs.(61), (63) and (65) exhibit the influence of the fluid conductivity in the so called Magneto-Jeans wave numbers in the parallel and perpendicular cases.

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