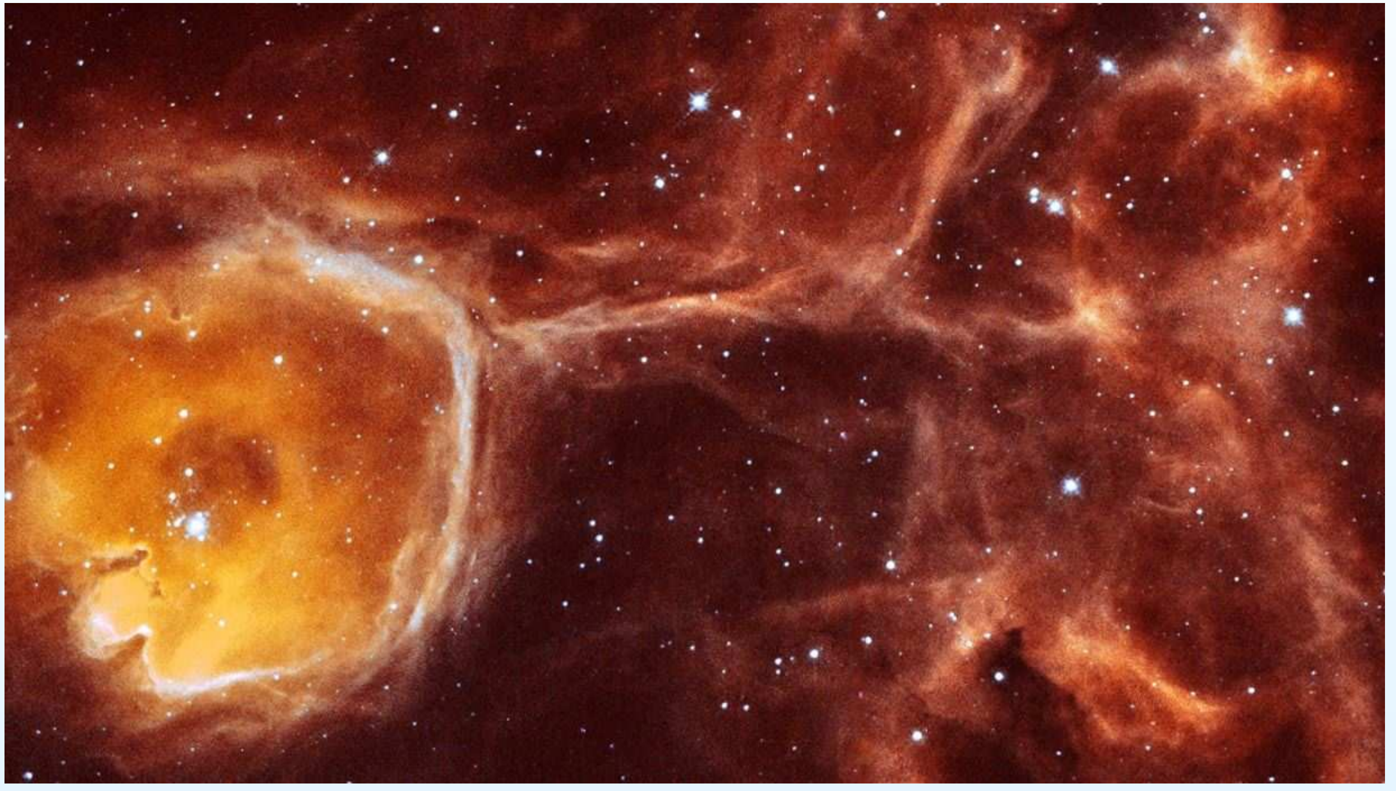
Covariant Approaches to Relativistic Plasmas in Astrophysics and Cosmology

MATTIAS MARKLUND



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- Instabilities, Landau damping, soliton formation, wave steepening ...

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- Cosmology, Friedmann–Lemaître–Robertson–Walker model, standard of cosmology (homogeneous, isotropic).

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- Einstein's field equations $R_{ab} - \frac{1}{2}Rg_{ab} = T_{ab}$ couples gravity (LHS) to matter (RHS). R_{ab} Ricci curvature, R Ricci scalar, g_{ab} metric, T_{ab} energy-momentum tensor.

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- Tensor field (Landau & Lifshitz gravity pseudo energy-momentum tensor).

General relativistic plasmas

- Gravity modifies plasma dynamics through geometry: Vlasov equation

$$p^a \frac{\partial f}{\partial x^a} - \Gamma^a{}_{bc} p^b p^c \frac{\partial f}{\partial p^a} = 0,$$

or energy-momentum conservation equations (follows from twice contracted Bianchi identities)

$$\nabla_b T^{ab} = 0,$$

and Maxwell's equations

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- Rotation coefficients capture influence of gravity.

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- Nonlinear effects due to EM fields in Einstein's equations may give many interesting effects, and should not *a priori* be discarded.

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  - Allows for covariant gauge-invariant perturbation theory (see Christos Tsagas’ lecture).

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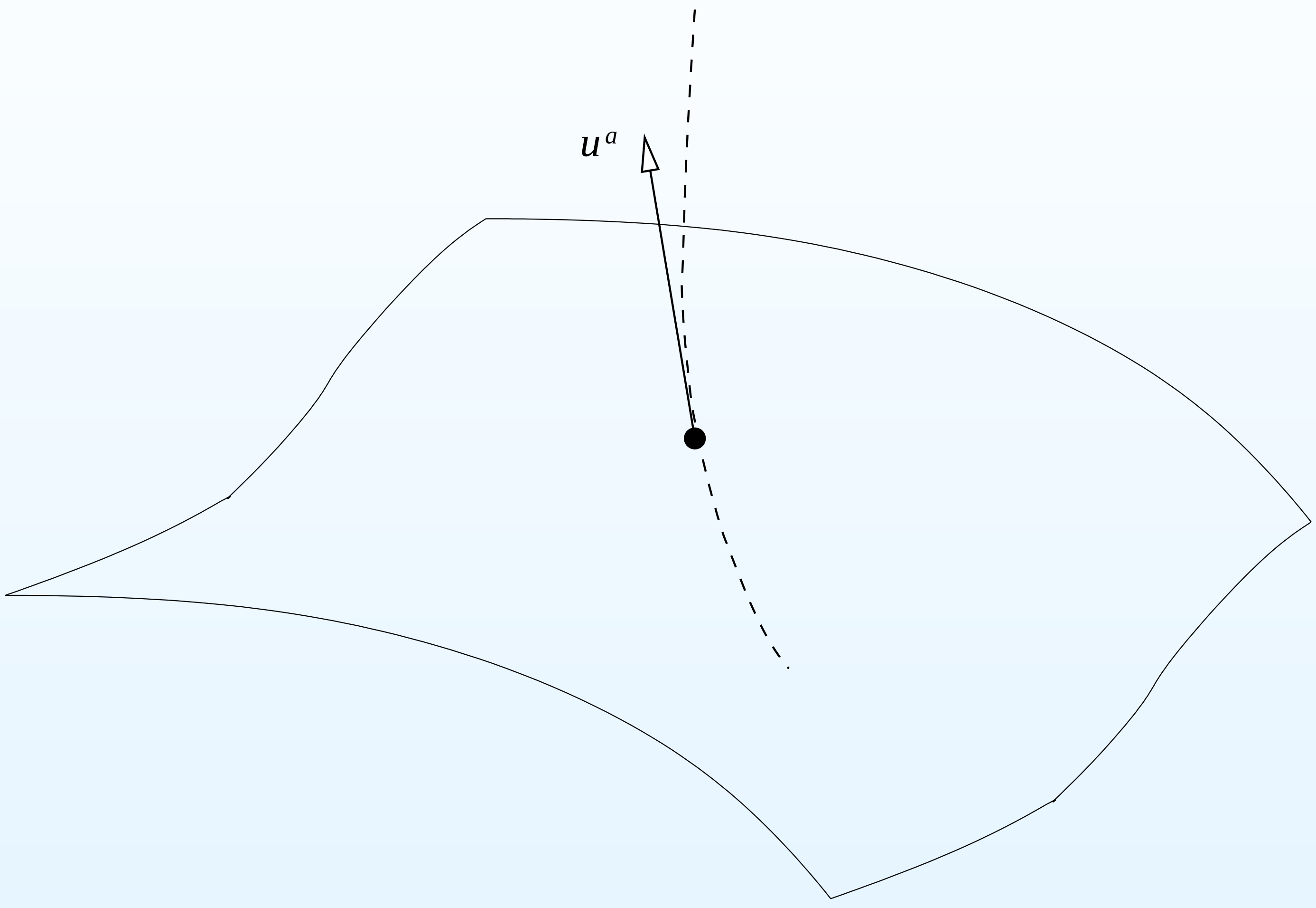

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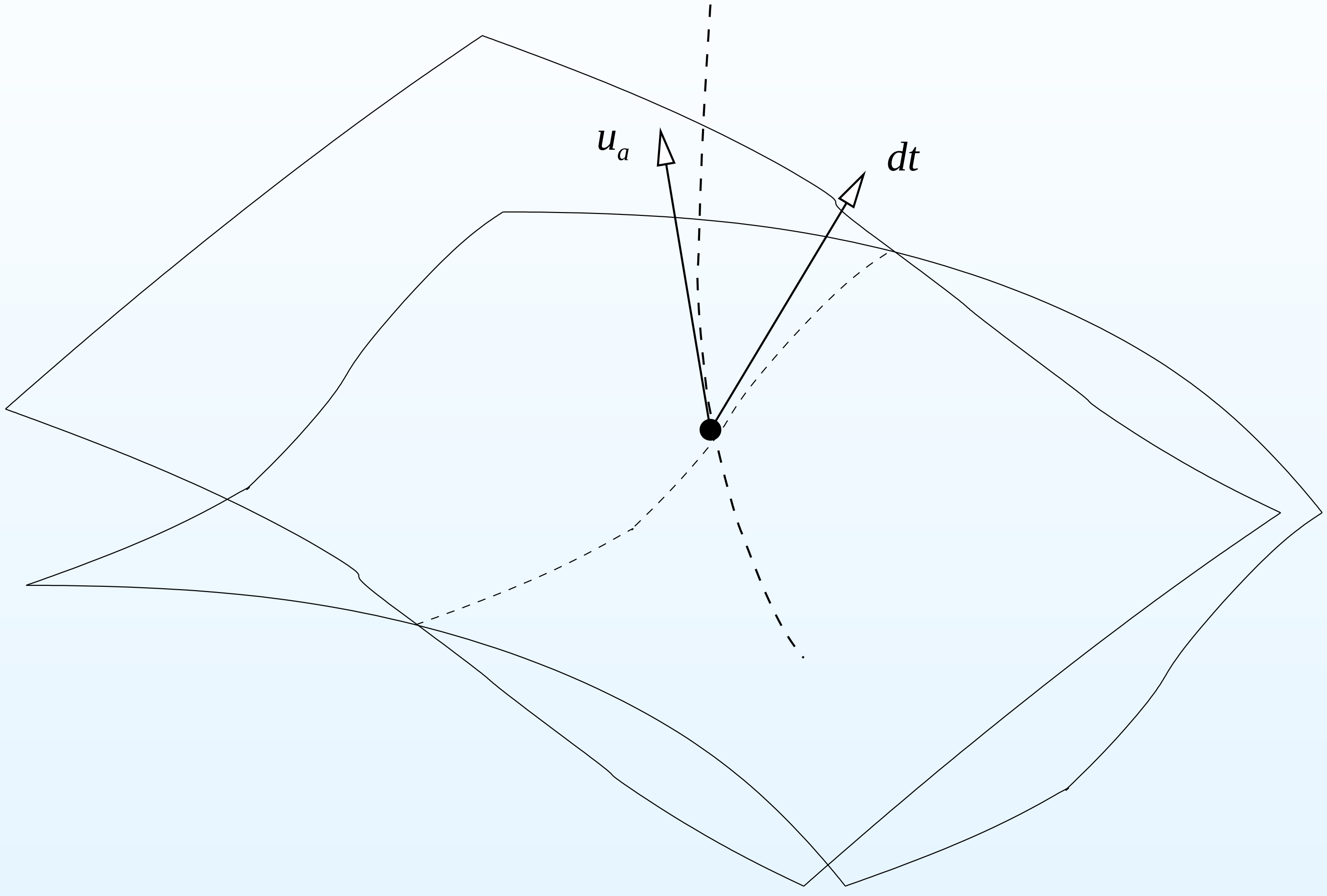

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- That's it!

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- Multi-fluids (Ellis, Dunsby, Marklund, Betschart, Servin, Tsagas): Observers with 4-velocity $u^a \to h^{ab} \equiv g^{ab} + u^a u^b$ metric on local rest space of u^a.

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- Relative to u^a:

$$T^{ab}_{(i)} = \hat{\mu}_{(i)} u^a u^b + \hat{p}_{(i)} h^{ab} + 2u^{(a}\hat{q}^{b)}_{(i)} + \hat{\pi}^{ab}_{(i)};$$

where

$$\hat{\mu}_{(i)} \equiv \gamma^2_{(i)}(\mu_{(i)} + p_{(i)}) - p_{(i)} \ ,$$

$$\hat{p}_{(i)} \equiv p_{(i)} + \tfrac{1}{3}\gamma^2_{(i)}(\mu_{(i)} + p_{(i)})v^2_{(i)} \ ,$$

$$\hat{q}^a_{(i)} \equiv \gamma^2_{(i)}(\mu_{(i)} + p_{(i)})v^a_{(i)} \ ,$$

$$\hat{\pi}^{ab}_{(i)} \equiv \gamma^2_{(i)}(\mu_{(i)} + p_{(i)})(v^a_{(i)}v^b_{(i)} - \tfrac{1}{3}v^2_{(i)}h^{ab}) \ .$$

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- Fluid equations Let $\dot{f} \equiv u^a \nabla_a f$ and D_a 3-D covariant derivative $\rightarrow$ energy conservation

$$\dot{\mu}_{(i)} = -\left(\mu_{(i)} + p_{(i)}\right)\left(\Theta + \mathrm{D}_a v^a_{(i)}\right)$$
$$-\gamma^{-1}_{(i)}\left(\mu_{(i)} + p_{(i)}\right)\left(\dot{\gamma}_{(i)} + \gamma_{(i)}\dot{u}_a v^a_{(i)} + v^a_{(i)}\mathrm{D}_a\gamma_{(i)}\right)$$
$$-v^a_{(i)}\mathrm{D}_a\mu_{(i)},$$

momentum conservation

$$\left(\mu_{(i)} + p_{(i)}\right)\left(\dot{u}^a + \dot{v}^{\langle a\rangle}_{(i)}\right) = -\gamma^{-2}_{(i)}\mathrm{D}^a p_{(i)} - \tfrac{1}{3}\Theta\left(\mu_{(i)} + p_{(i)}\right)v^a_{(i)} - \dot{p}_{(i)}v^a_{(i)}$$
$$-\left(\mu_{(i)} + p_{(i)}\right)\left(v^b_{(i)}\mathrm{D}_b v^a_{(i)} + \sigma^a{}_b v^b_{(i)} + \epsilon^{abc}\omega_b v_{(i)c}\right)$$
$$+\gamma^{-1}_{(i)}\left(\mu_{(i)} + p_{(i)}\right)\left(v^a_{(i)}\dot{\gamma}_{(i)} + v^a_{(i)}v^b_{(i)}\mathrm{D}_b\gamma_{(i)}\right)$$
$$-v^a_{(i)}v^b_{(i)}\mathrm{D}_b p_{(i)} + \gamma^{-1}_{(i)}\rho_{c(i)}(E^a + \epsilon^{abc}v_{(i)b}B_c),$$

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- Maxwell's equations (see Marklund et al. for notational details)

$$\dot{E}^{\langle a\rangle} = -\tfrac{2}{3}\Theta E^a + \sigma^a{}_b E^b + \epsilon^{abc}\omega_b E_c + \epsilon^{abc}\dot{u}_b B_c + \mathrm{curl} B^a - \sum_s j^{\langle a\rangle}_{(s)}\,,$$

$$\dot{B}^{\langle a\rangle} = -\tfrac{2}{3}\Theta B^a + \sigma^a{}_b B^b + \epsilon^{abc}\omega_b B_c - \epsilon^{abc}\dot{u}_b E_c - \mathrm{curl} E^a\,,$$

$$\mathrm{D}_a E^a = \sum_s \rho_{(s)} + 2\omega_a B^a\,,$$

$$\mathrm{D}_a B^a = -2\omega_a E^a\,,$$

where $\mathrm{curl} B^a \equiv \epsilon^{abc}\mathrm{D}_b B_c$.

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$$\mathrm{D}_a E^a = \sum_s \rho_{(s)} + 2\omega_a B^a \,,$$

$$\mathrm{D}_a B^a = -2\omega_a E^a \,,$$

where $\mathrm{curl} B^a \equiv \epsilon^{abc}\mathrm{D}_b B_c$.

- Gravitational equations given in Ellis & van Elst!

Example

Magnetic field generation in cosmology (see also Christos Tsagas' talk).

- We may define gauge-invariant fluid- and electromagnetic perturbations using the above formalism.

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  - Wave equations for the electric and magnetic fields.

Example

- The magnetic field as a function of the redshift:

$$|B| \approx \mathscr{K}_i h \left(\frac{1+z}{1+z_i} \right)^{1/4} (1+z)^{3/2} \times 10^{-24}\,\mathrm{G}.$$

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- Dynamo mechanism requires $|B| \sim 10^{-30}\,\mathrm{G}$. Thus we are well within reach of this!

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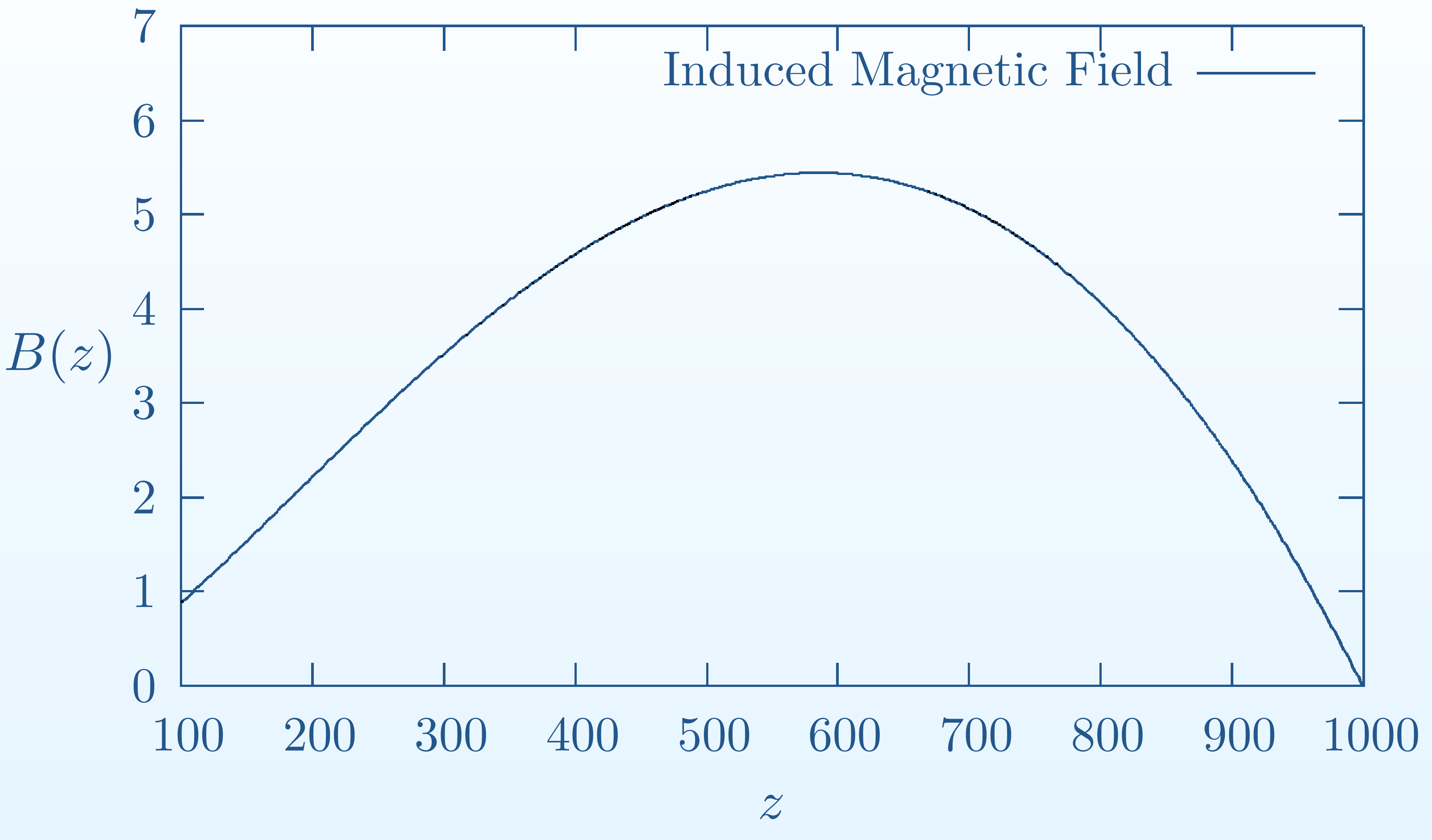

Induced Magnetic Field
B(z)
z
0
1
2
3
4
5
6
7
100
200
300
400
500
600
700
800
900
1000

$1 + 1 + 2$ split

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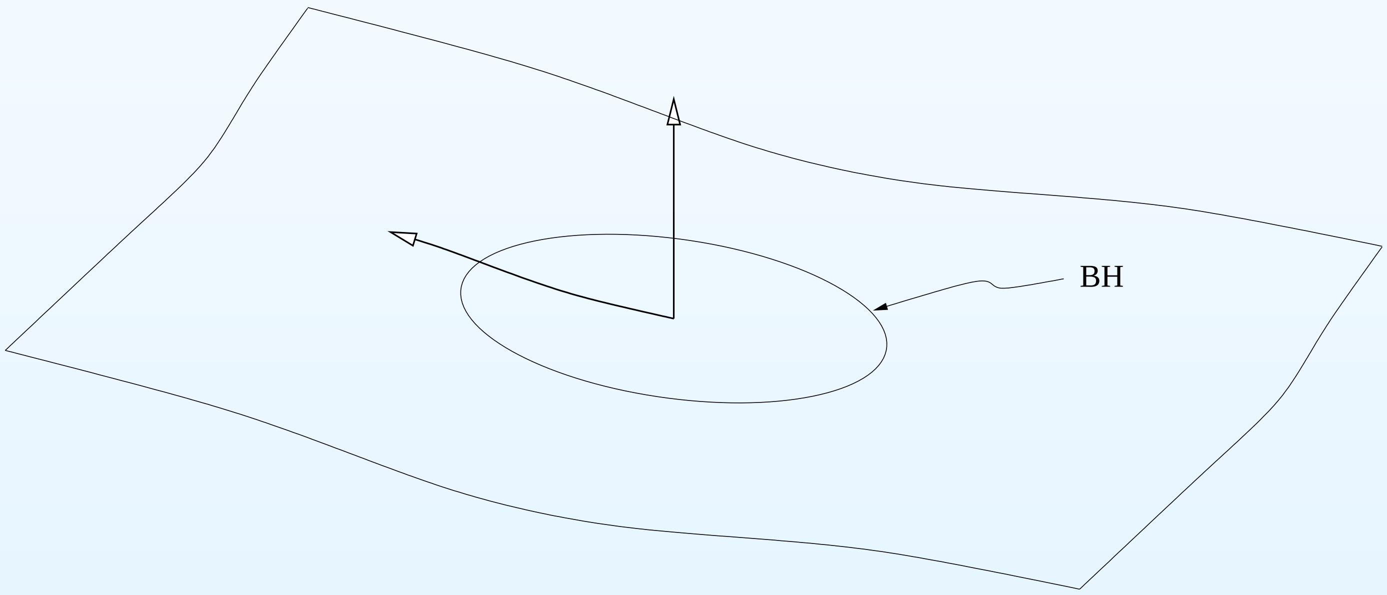

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- Perturbations of Locally Rotationally Symmetric Spacetimes (anisotropic cosmologies), black holes, and neutron stars (see also talk by Kostas Kokkotas).

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- Problem: Gravitational wave from oscillating BHs surrounded by strong magnetic fields (see Clarkson et al.):

  EM field around a BH $=$ induced 'currents' of the form $($*GW* $\times$ *strong static magnetic field*$)$.

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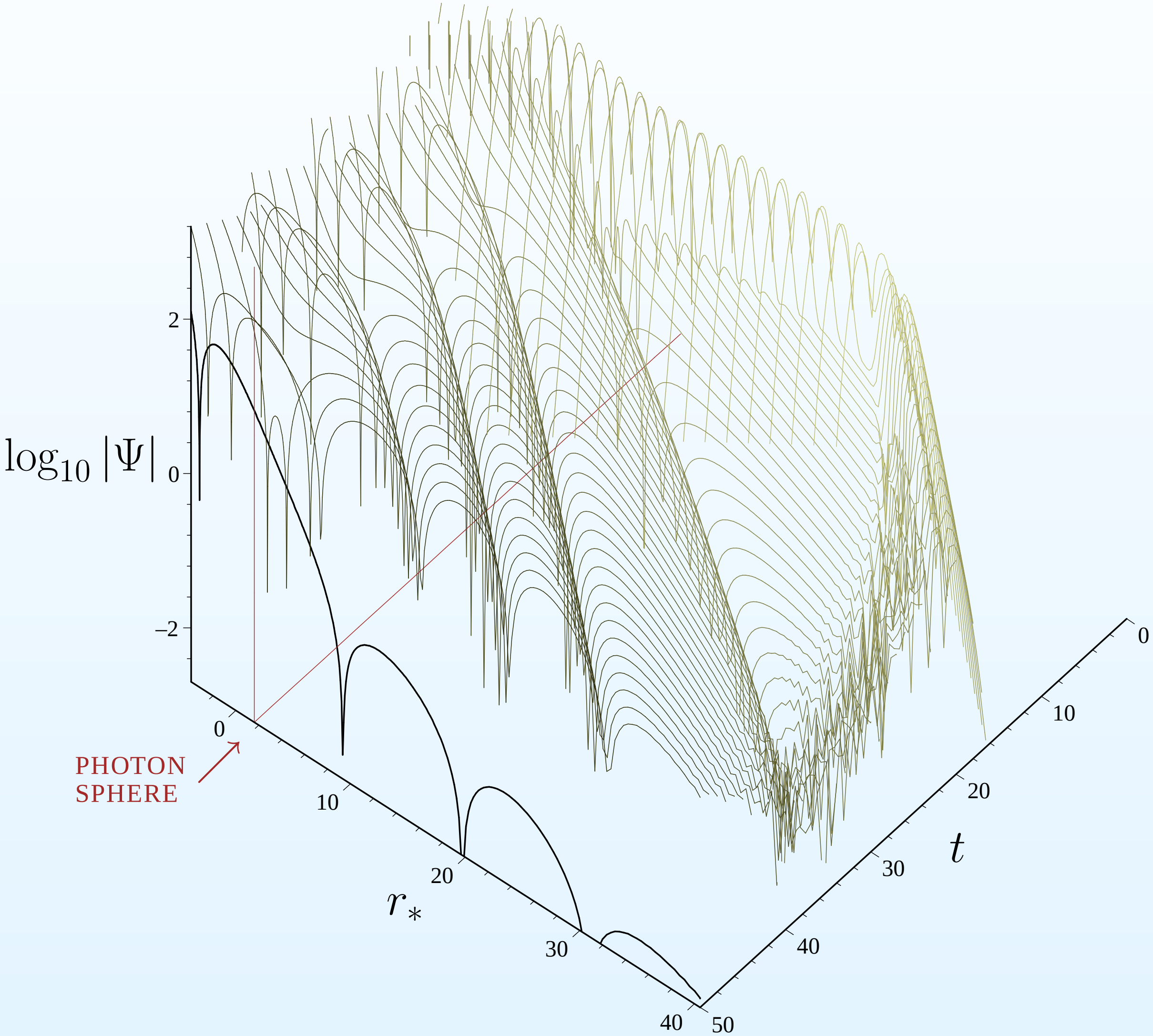
- Pick out the electromagnetic signature from this system.

Example



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- Formulate a MHD theory within the $1 + 1 + 2$ formalism for BHs.
- The covariant formalism is alive and well!