Statistical/Evolutionary Models of Power-laws in Plasmas

Modern Challenges in Nonlinear Plasma Physics

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GREECE

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Heavy Tails

- *Statistics of Energization Processes*
- *Anomalous Transport/Diffusion*
- *Entropy of Super-diffusive Processes*

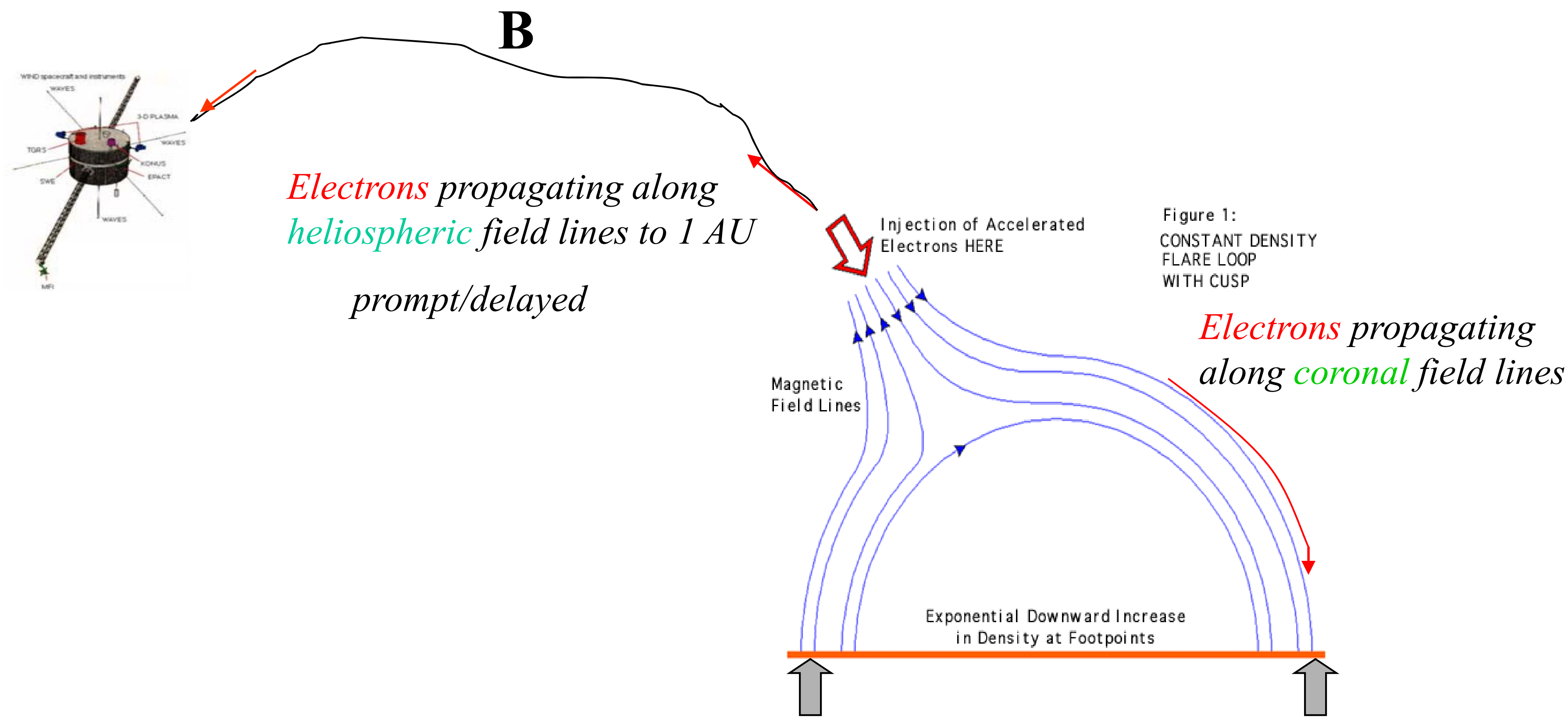
BASICS: Ergodic, weakly interacting system

converges into Boltzmann-Gibbs statistics/function

THEREFORE: particles that interact stochastically with electromagnetic fields and perform Brownian motion in phase space, characterized by short-range deviation and short term microscopic memory will approach asymptotically a Gaussian.

Why is there an ubiquity of (broken) power laws?

Example - Solar/Heliospheric Electrons



Heliospheric Signatures

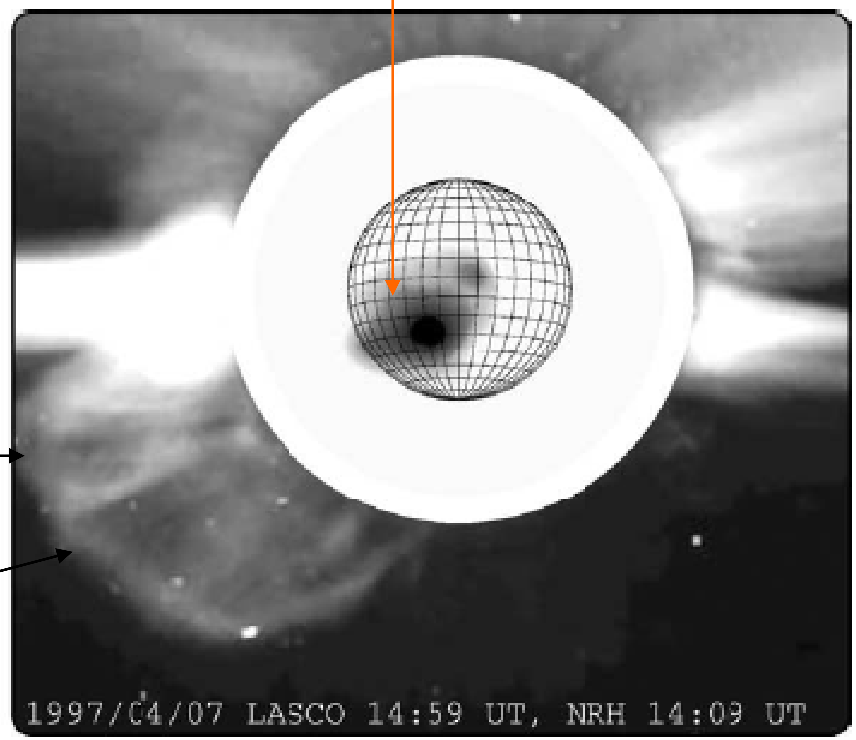
Relation between coronal and IP perturbations

Acceleration Sites?

164 MHz NRH images

Injection of electrons into heliosphere

LASCO/SOHO



CME

Type II signature

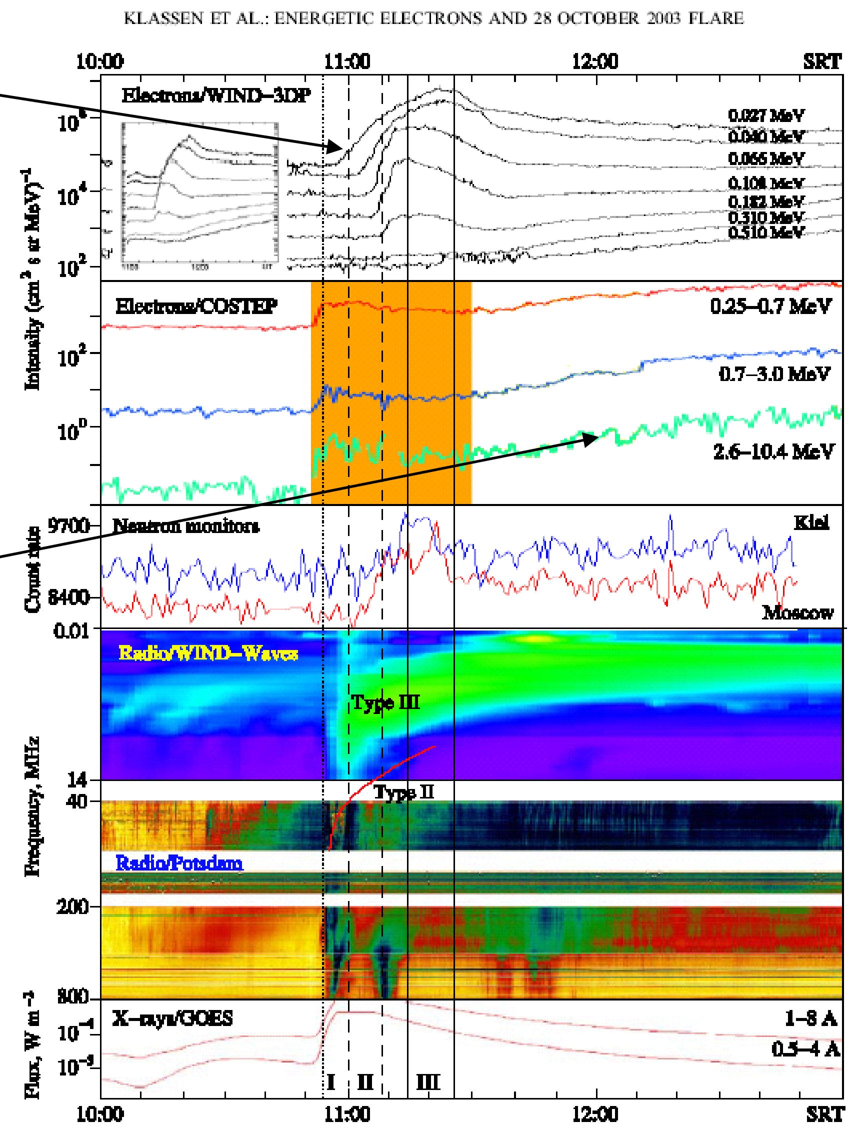
Maia, 2001

Energization - occurs behind the CME?

Impulsive

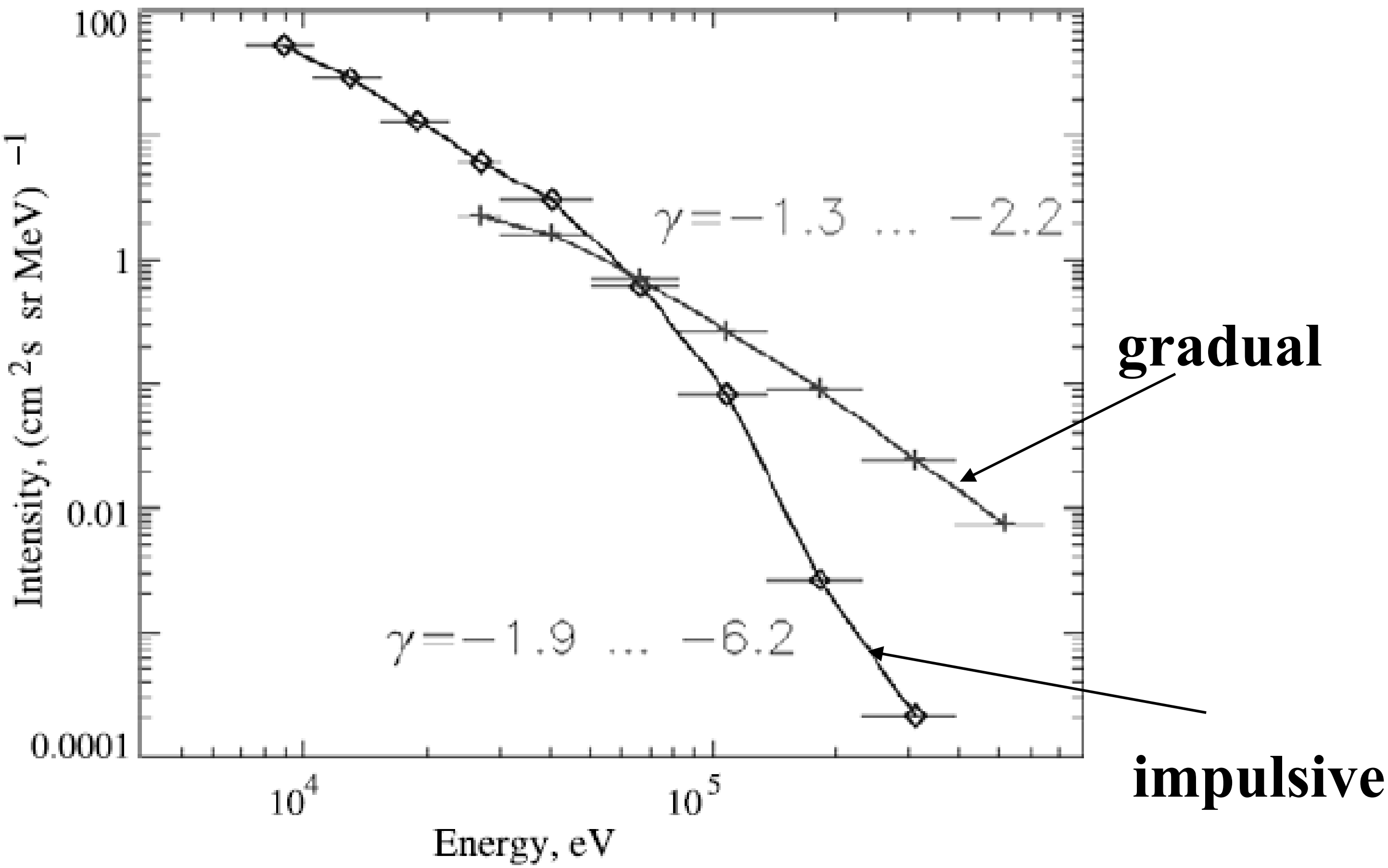
Gradual

Distinct delay between Type III and energetic electron injections



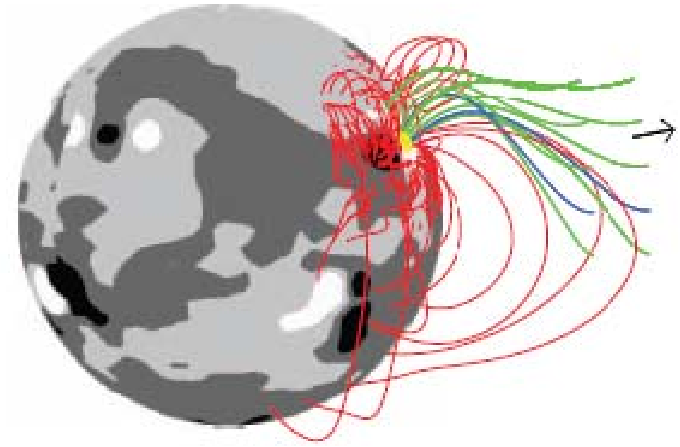
Post-flare, post-CME: Gradual electrons

Electron Spectra

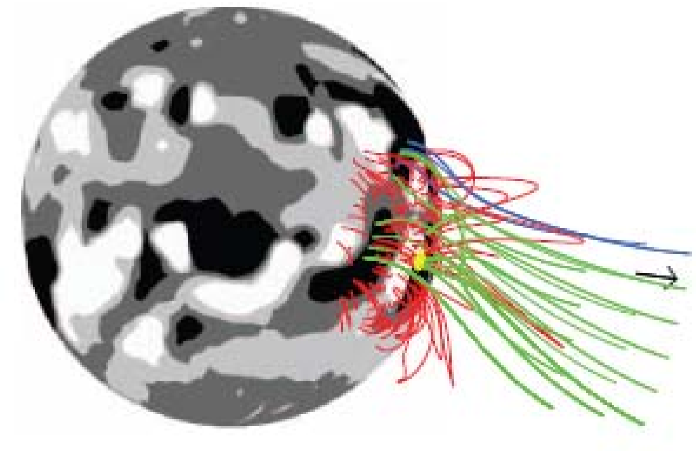


Wang, 2006

Post CME reconstruction



(a) 1997 NOVEMBER 24



(b) 2000 MARCH 7

Yellow – flare site

Black arrow – CME direction

Red – closed (stretched) magnetic field lines

Blue - open, in ecliptic plane

green – open, non-ecliptic

Yellow – flare site *Black arrow – CME direction*

Red – closed (stretched) magnetic field lines

Blue - open, in ecliptic plane *green – open, non-ecliptic*

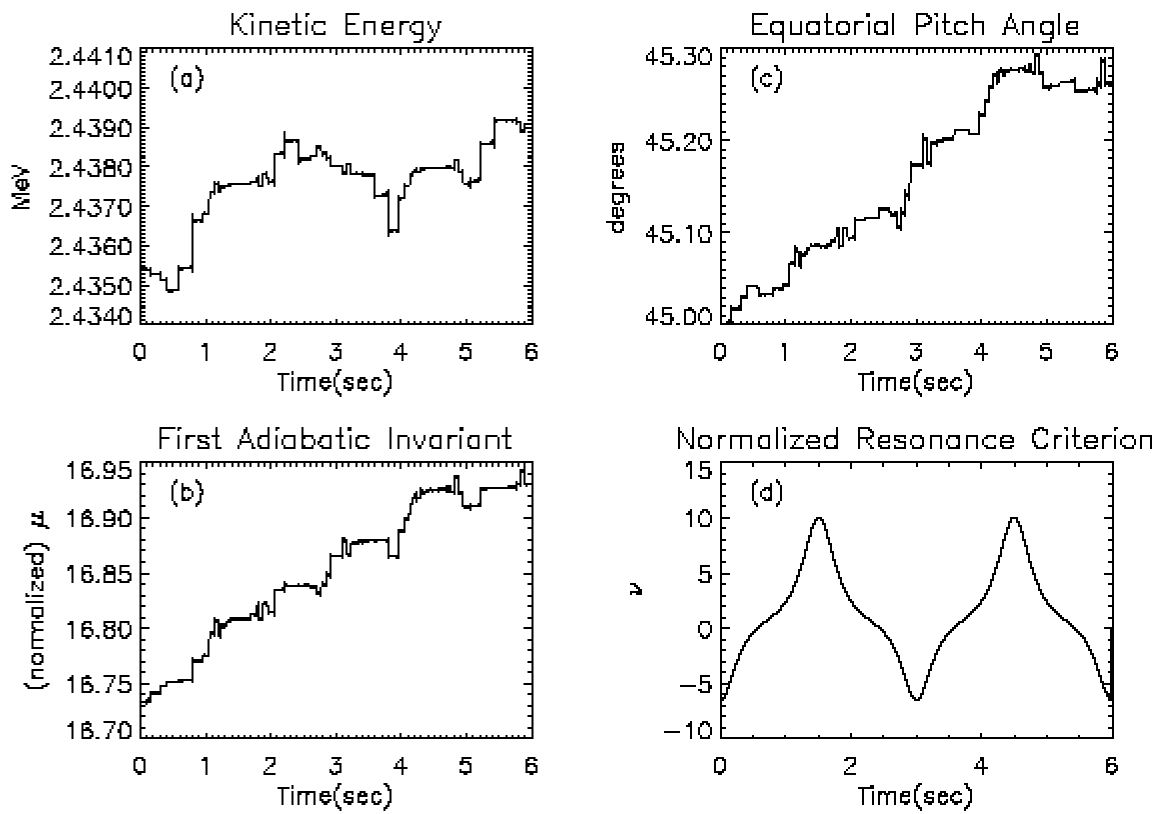
Foot-points Radio emissions

Whistlers Interact on closed field lines

CME propagates along the arrow

(f) 2003 MARCH 17

Characteristics of electron trajectory in the presence of (oblique) whistler waves



Statistics of Energization Processes

- *Anomalous Transport/Diffusion*

- *Entropy of Super-diffusive Processes*

Statistical Analysis – Particle Distribution

Energization is determined by an increment of an independent, identically distributed (iid) random variable (in the language of statistical mathematics), with an assigned probability distribution.

Central Limit Theorem (CLT):

Sum of random variables with a finite variance will converge (weakly) to a normal or Gaussian distribution.

No Heavy Tail.

Stable Distribution: addition of a large number of random variables preserves the shape of the distribution (infinitely divisible).

Normal Distribution is an example of a Stable Distribution ("attractor distribution").

Normal Diffusion:

Brownian (random walk) motion – CLT applies, if

a) distribution with finite moments (variance)

b) no long range interaction

c) short term memory - Markovian process

Anomalous Diffusion:

CLT does not hold in its regular form

Result: **Sub-diffusion or extended tails**

Normal Distribution

$$X \sim N(\mu, \sigma^2); P(X \le x) = \frac{1}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{x} \exp[-(x-\mu)^2 / 2\sigma^2] dx$$

$$X_1, X_2 - independent, \ \ (not) identically - distributed\ r.v,$$

$$aX_1 \sim N[a\mu, (a\sigma)^2], \ \ bX_2 \sim N[b\mu, (b\sigma)^2]$$

$$\exists c, d: \ \ aX_1 + bX_2 = \ \ cX + d \ \ = Y \quad (in \ \ distribution)$$

$$c^2 = a^2 + b^2, \ \ d = (a+b-c)\mu; \quad Y \sim N[c\mu + d, (c\sigma)^2]$$

Convolution preserves the (normal) probability

This (stable) Distribution is not unique

Generalization: the sum of a large number of time-homogeneous, independent, identically distributed random variables with an infinite variance will tend to a symmetric (skew) stable Levy distribution.

$$P\ (X_1 < Z_N < X_2) \mapsto \int_{X_1}^{X_2} L_{\alpha,\beta}(x)dx$$

(Gedanenko-Kolgomorov-Levy)

Lévy (α stable) Probability Distribution

The stable distribution functions are completely defined by their Characteristic Function (Fourier Transform of a single event probability)

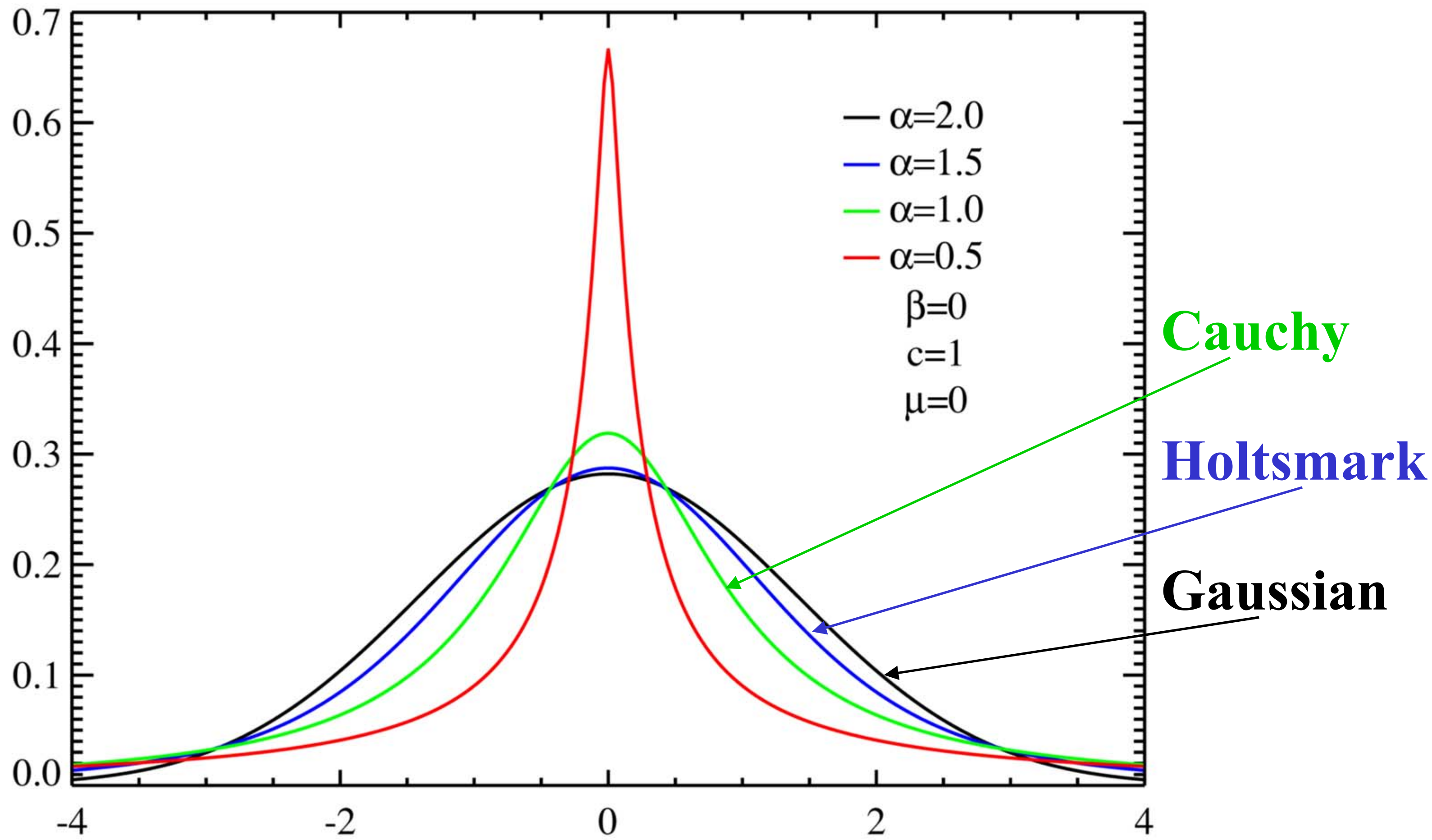
$$\hat{l}_\alpha(\lambda) = \exp[-c|\lambda|^\alpha (1 - i\beta \tan(\pi\alpha/2)\,\mathrm{sgn}(\lambda))]$$

$0 \le \alpha \le 2$ – Lévy exponent, c – scale, β - skewness

(Gaussian : α = 2; Lorentzian /Cauchy: α = 1)

α=2 – "phase transition"

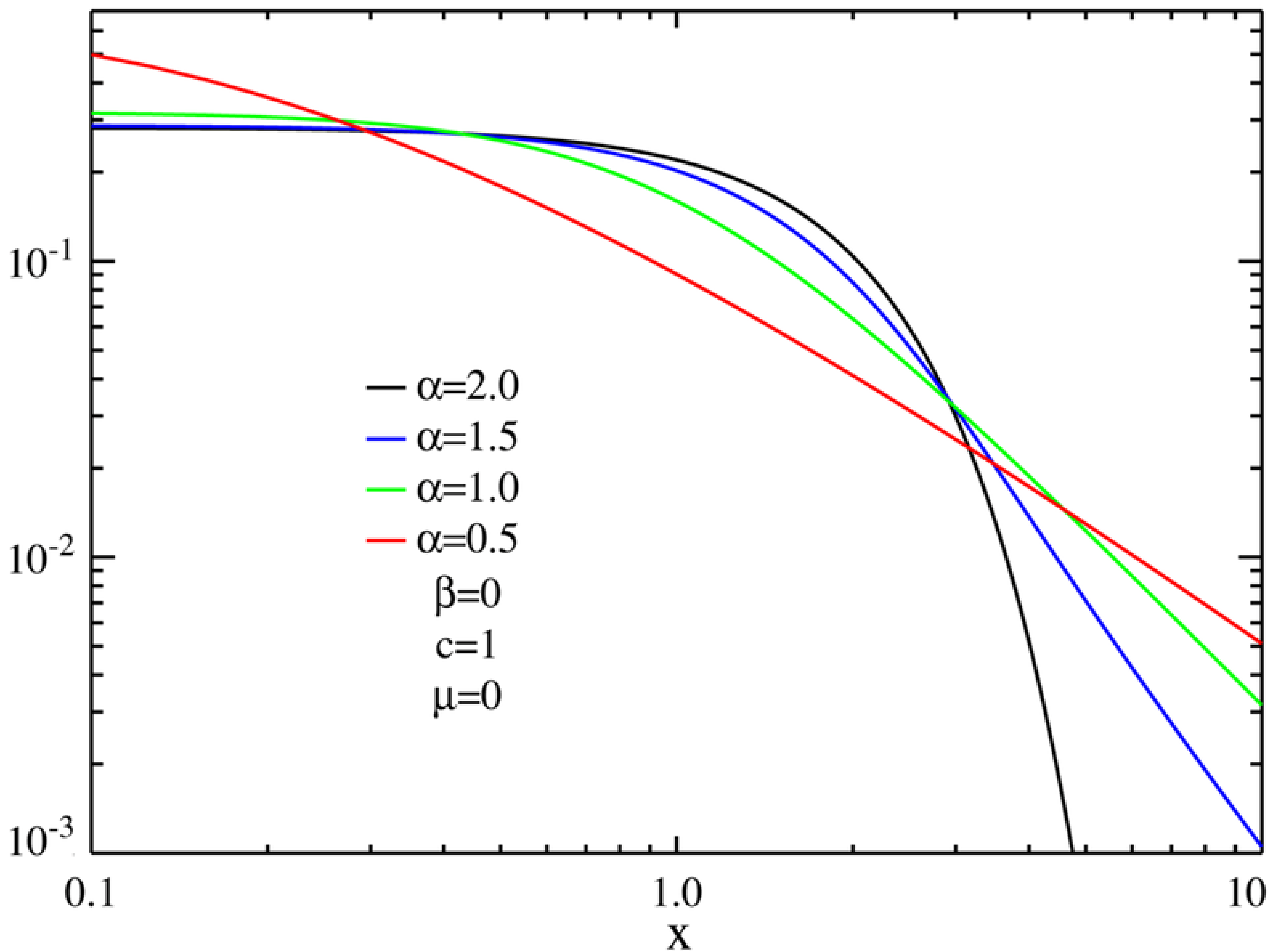
Normal law attracts all distributions with α > 2



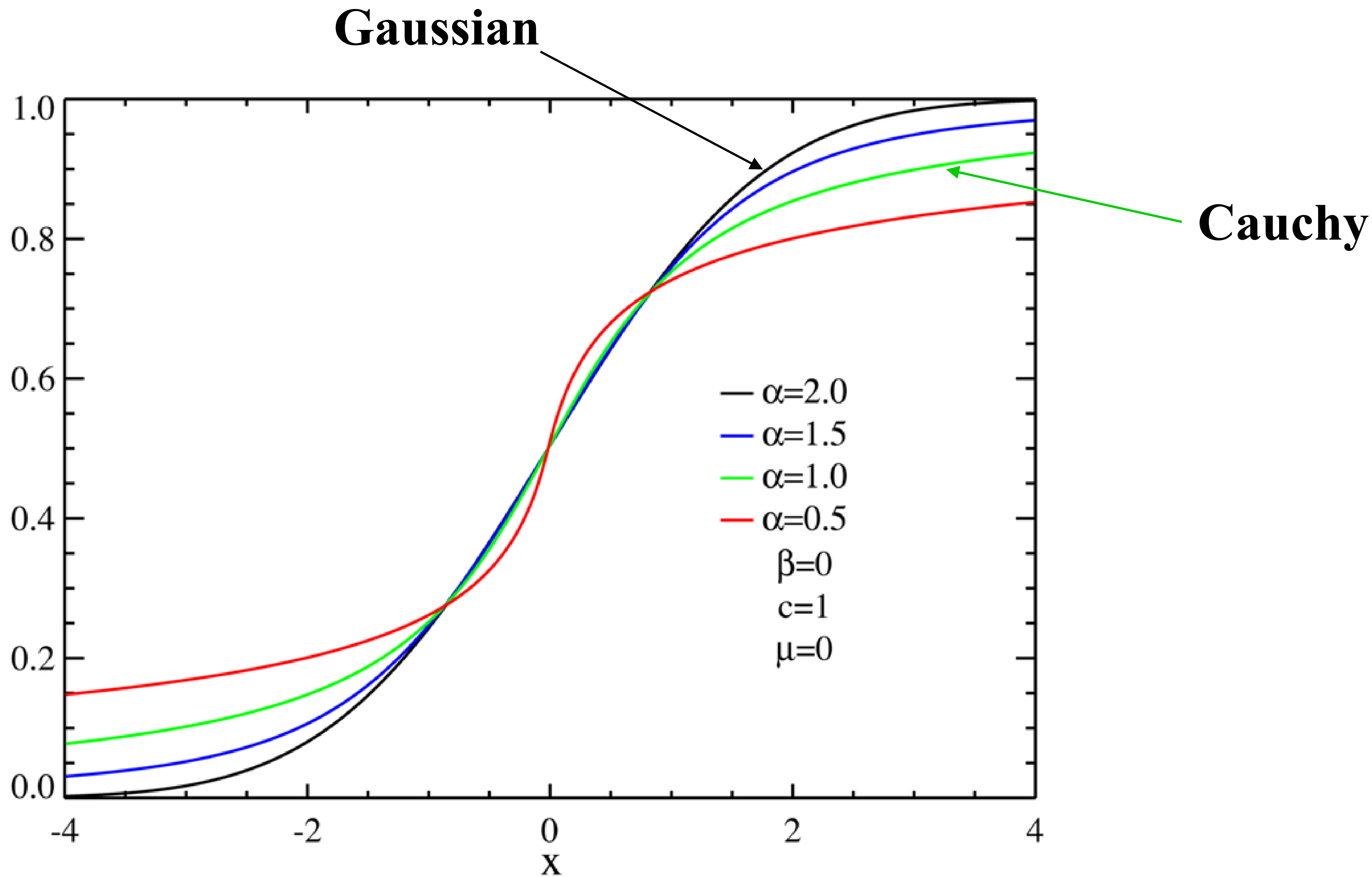
Probability density function of stable Levy distributions

α-exponent; μ-shift; c=σ-width/scale; β-skewness

As α decreases, longer tail and narrower core→δ function.



10-1
10-2
10-3
0.1
1.0
10
x
α=2.0
α=1.5
α=1.0
α=0.5
β=0
c=1
μ=0



Cumulative Distribution Function

Procedure:

Characteristic function – Fourier of one event probability

Multiplication in Fourier space (instead of convolution)

Inverse Fourier integration

a. Pdf with Finite moments

[Gram (1883) - Charlier (1905) - Edgeworth (1905)]

$$P(\xi)\sim\exp(-\xi^{2})/(2\pi)^{1/2}[1+\lambda_3 H_3(\xi)/(3!\,n^{1/2})+\lambda_4 H_4(\xi)/(4!\,n)+\ldots]$$

core **Corrections to Gaussian**

$\xi=(r/\sigma n^{1/2})$, λ_i cumulants normalized to the second moment σ
H_n Hermite polynomials.

The Gaussian core of the distribution extends to very high values

b. Leptokurtotic probability function (diverging high (>2) moments)

$$One \quad event \quad probability : p(r) = \frac{2/\pi^{1/2}}{(1+r^4)}$$

$$Characteristic\ \ Function : \hat{p}(\eta) = \exp\left(\frac{-|\eta|}{\sqrt{2}}\right)\left[\cos\frac{\eta}{\sqrt{2}} + \sin\frac{|\eta|}{\sqrt{2}}\right]$$

$$\hat{p}(\eta) = \exp[\log(\hat{p}(\eta))]$$

$$P_n(r) = \int e^{ir\eta}\ e^{n\log[\hat{p}(\eta)]}\frac{d\eta}{2\pi}$$

Cumulant expansion - log p(η) around η =0

$$\log \hat{p}(\eta) = -\eta^2/2 + |\eta|^3/3\sqrt{2} + 11\eta^4/48 + \ldots.$$

$$\Phi_n(r) = \int e^{ir\eta}\; e^{-\eta^2/2}\;\; [1 + \frac{a|\eta|^3}{\sqrt{n}} + \ldots]\frac{d\eta}{2\pi} \sim$$

$$\sim \Phi_G(r) + \frac{1}{2^{1/2} 6\pi\sqrt{n})}\frac{\partial^3}{\partial r^3}\{\exp(-r^2/2)\int \exp[-(\eta - ir)^2 d\eta\}$$

$$\boxed{\Phi_n(r) \sim \Phi_G(r) + \frac{(-1)}{2^{1/2} 6\pi\sqrt{n}} D'''(\frac{r}{2^{1/2}}) + O[\frac{D^{(4)}}{n}] + \ldots}$$

$$\Phi_G(r) - Gaussian; \qquad D - Dawson\;\; Integral : D(r) = e^{-r^2}\int_0^r e^{y^2} dy$$

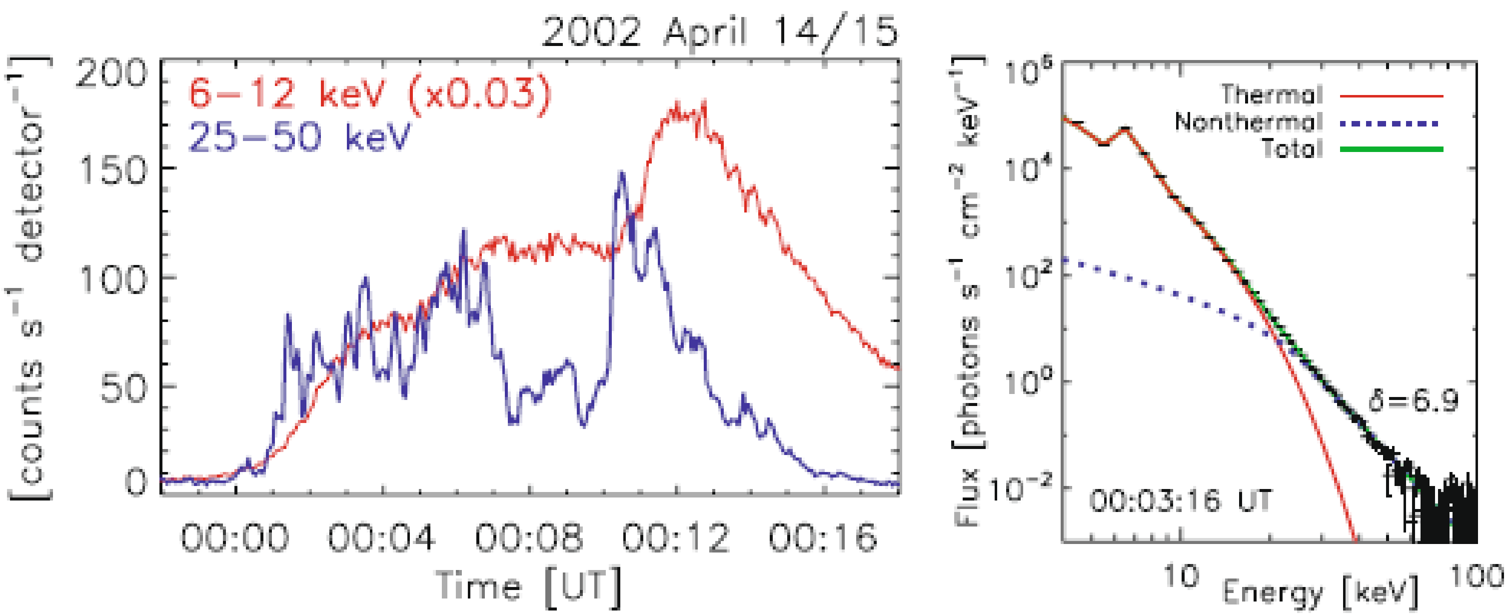
$$D(x) \sim \frac{2}{2x}[1 + \frac{1}{2x^2} + \frac{3}{4x^4} + \sum_{m=3}^{\infty} \frac{(2m-1)!!}{2^m x^{2m}}]$$

$$D'''(x) \sim -1/3x^4 \qquad as \quad x -> \infty$$

$$\Phi_n(\xi) \sim \Phi_G(\xi) + \frac{1}{2^{1/2} 6\pi\sqrt{n})}[\xi^{-4}/12 + \xi^{-6}/288n]$$

The edge of the central core ~ (log n)$^{1/2}$

Due to limited interaction time, the observed distribution converges extremely slowly to a Gaussian and exhibits heavy tails.



2002 April 14/15
6–12 keV (x0.03)
25–50 keV
[counts s^{-1} detector^{-1}]
200
150
100
50
0
00:00
00:04
00:08
00:12
00:16
Time [UT]
Thermal
Nonthermal
Total
Flux [photons s^{-1} cm^{-2} keV^{-1}]
10^{6}
10^{4}
10^{2}
10^{0}
10^{-2}
δ=6.9
00:03:16 UT
10
Energy [keV]
100

c. Diverging second pdf moment

Characteristic function with α<2

$$\hat{l}_\alpha(\lambda) = \exp(-c|\lambda|^\alpha)$$

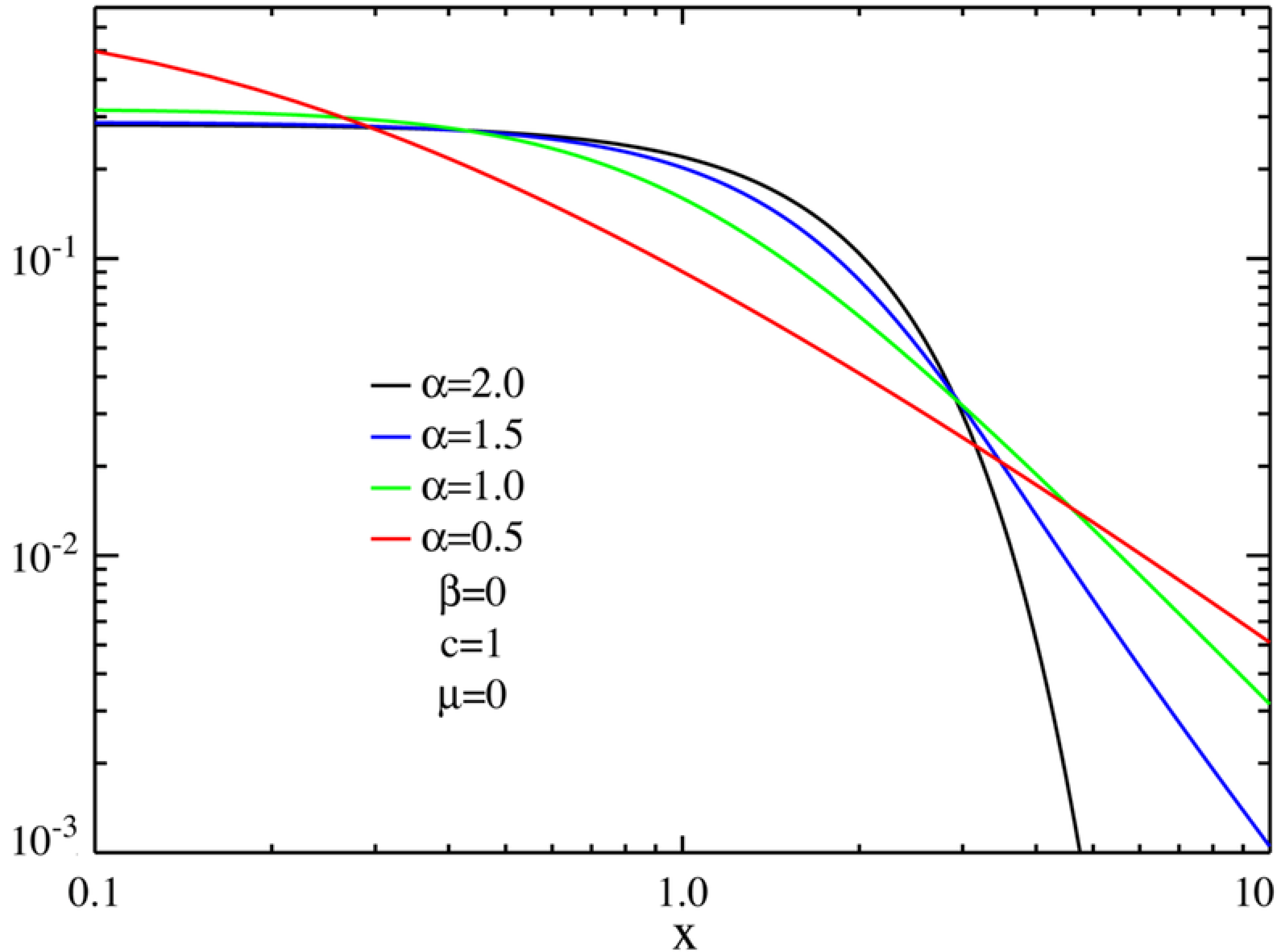
$$L_{\alpha,N}(r) = \int_{\infty}^{\infty} \exp(i\lambda r - cn|\lambda|^\alpha)d\lambda = n^{-1/\alpha} l_\alpha(r/n^{1/\alpha})$$

asymptotically $$\boxed{\Phi_\alpha(\zeta) = \frac{c\sin(\alpha\pi/2)\Gamma(1/\alpha)\alpha}{\pi|\zeta|^{1+\alpha}}} \quad for \quad \zeta \to \infty$$

And a Gaussian which narrows as α -->0 at the central core region

$$\Phi_\alpha(\zeta) = \frac{1}{\pi\alpha}\sum_m \frac{(-\zeta^2)^m}{(2m)!}\frac{\Gamma(2m+1/\alpha)}{c^{(2m+1/\alpha)}}$$

$$\Phi_\alpha(\zeta) \sim \frac{\Gamma(1/\alpha)}{\pi\alpha c^{1/\alpha}}\exp(-\zeta^2/\sigma^2); \quad \sigma \sim \sqrt{\pi/2}\alpha^{3/2} \qquad for \quad \zeta \to 0$$



α=2.0
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c=1
μ=0
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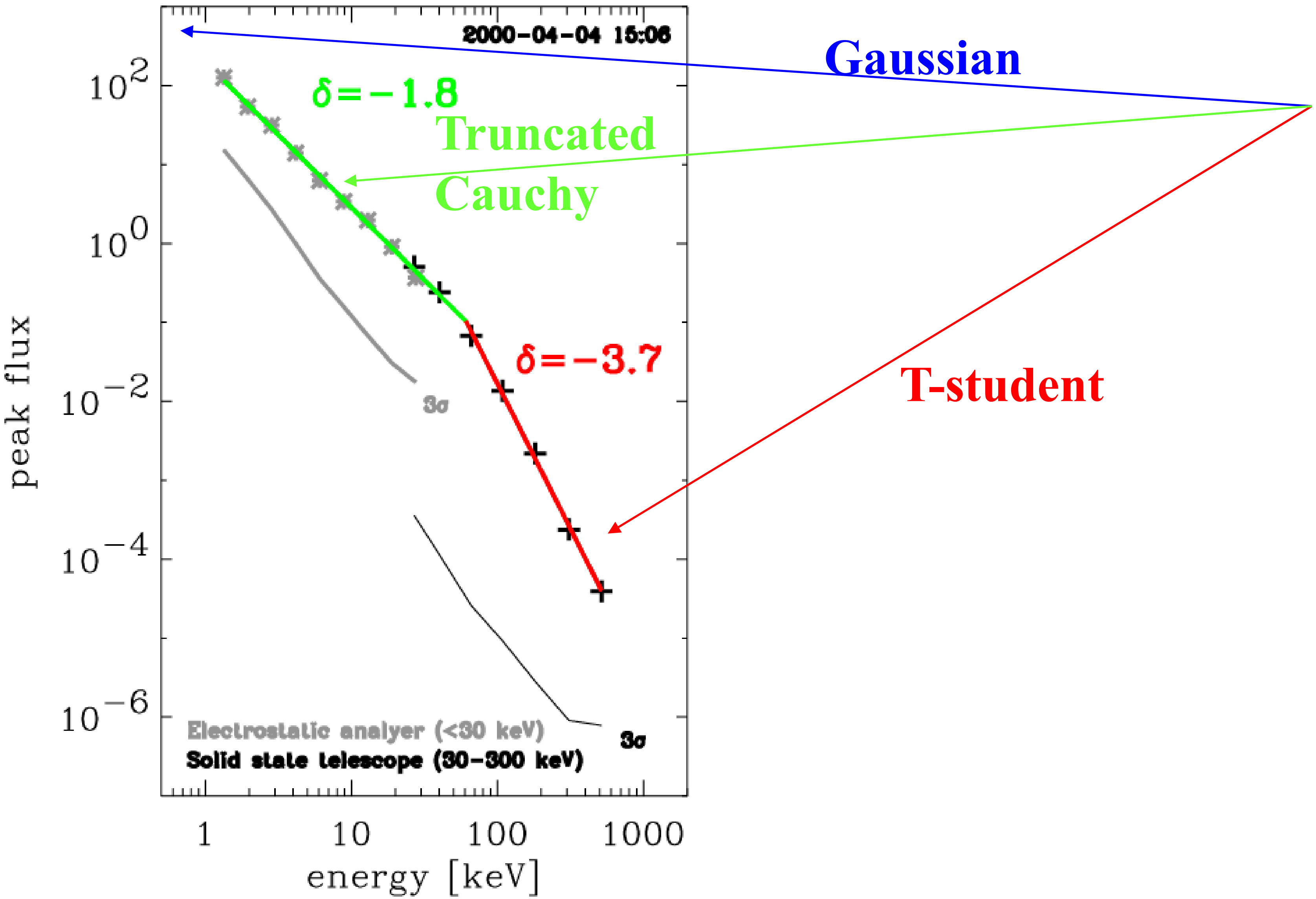


2000-04-04 15:06
Gaussian
δ=−1.8
Truncated
Cauchy
δ=−3.7
T-student
3σ
3σ
Electrostatic analyer (<30 keV)
Solid state telescope (30-300 keV)
peak flux
energy [keV]

- *Statistics of Energization Processes*

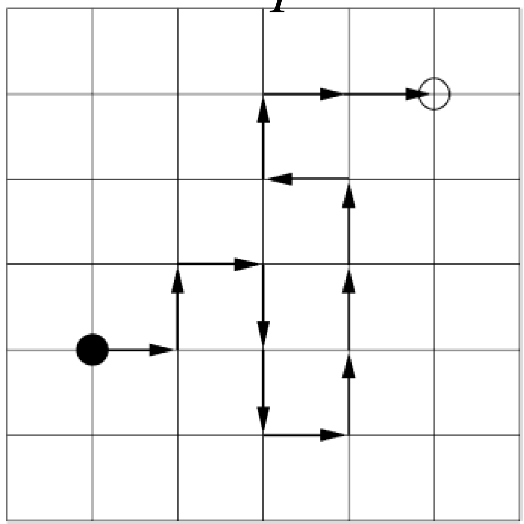
Anomalous Transport/Diffusion

- *Entropy of Super-diffusive Processes*

Random walk: continuum description → FP

$$P_i(t+\Delta t) = \frac{1}{2} P_{i+1}(t) + \frac{1}{2} P_{i-1}(t);$$

Phase space



$$r_{i\pm1} = r_i \pm \Delta r; \quad \textit{isotropic} \quad \textit{jumps}$$

Local in space and time

$$\frac{\partial P(r,t)}{\partial t} = \lim_{dr\to 0, dt\to o} \frac{(dr)^2}{dt} \frac{\partial^2}{\partial r^2} P(r,t) \equiv K \frac{\partial^2}{\partial r^2} P(r,t)$$

Continuum limit

$$P(r,t) = (4\pi K t)^{-1/2} \exp(-r^2 / 4Kt)$$

Green propagator

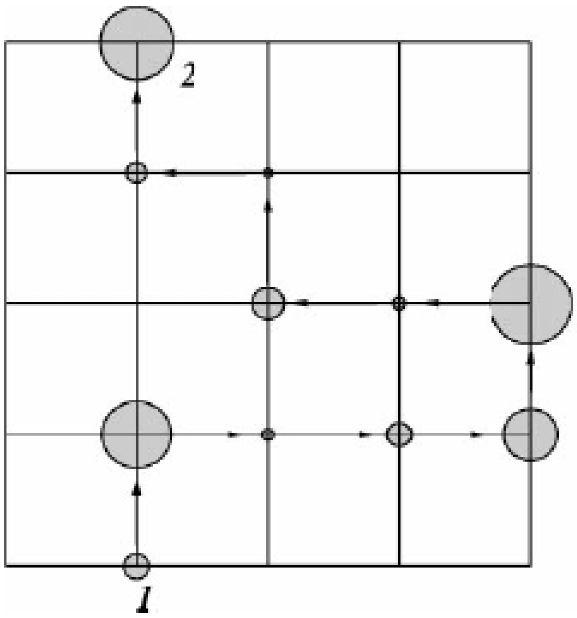
$<r^2> \sim t^\alpha ;\ \alpha=1$

Central limit theorem

Continuous time Random walk

Non-locality in phase space

$P_j(t+dt)=\sum_{n=1} [A_{j,n}P_{j-n}(t)+ B_{j,n}P_{j+n}(t)]$



Generalization

Length of jump, waiting time

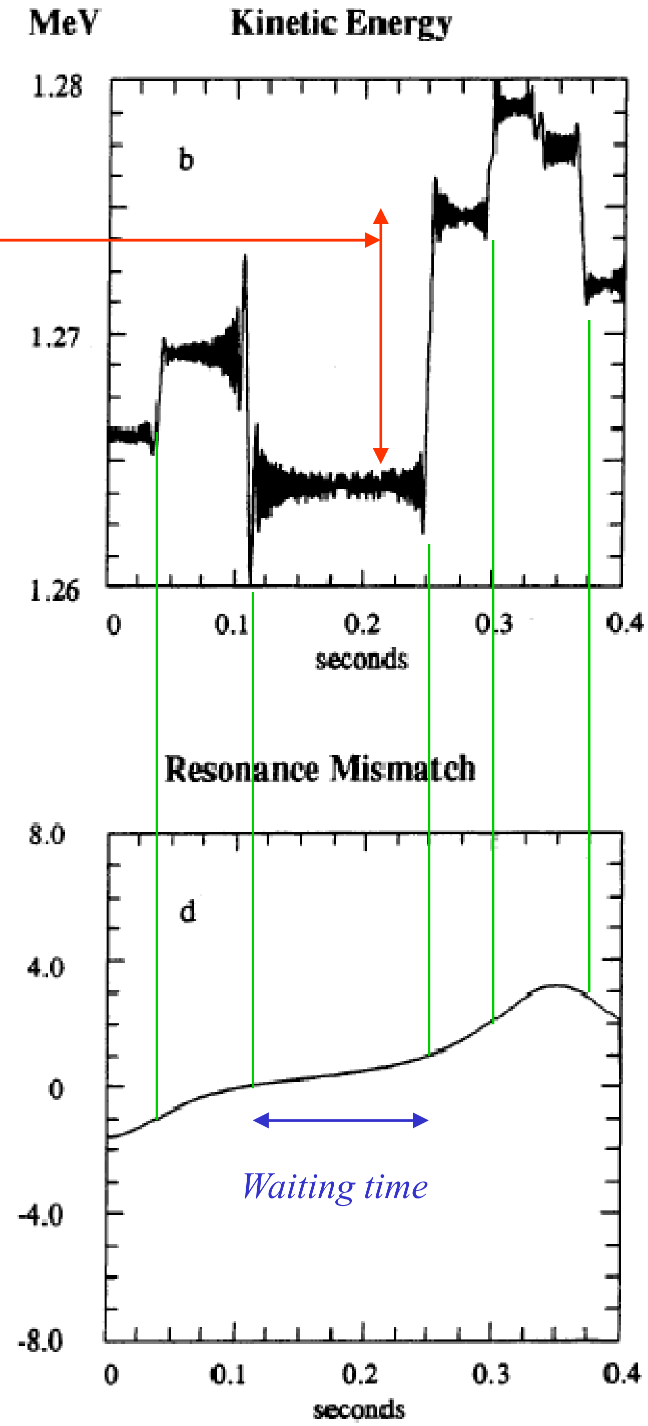
$$\chi(r, t) = \lambda(r)\tau(t)$$

Decoupled temporal/spatial memory

Probabilities

$\lambda(r)dr$ *phase space jump*

$\tau(t)dt$ *waiting time between jumps*



$$\nu = \frac{\omega - k_{\parallel}\,(x,t)\;v_{\parallel}\,(x,t)}{\Omega(x,t)}$$

ν=n

Probability P(r,t) - phase space position r at time t probability $\zeta(r,t')$ of arrival at r at an earlier time t' and staying there without a jump:
"survival" probability $\varphi(t)=1-\int dt\ \tau(t)$;

$$P(r,t) = \int dt'\ \zeta(r,t')\ \varphi(t-t')$$

Master equation

$$\zeta(r,t) = \int dr' \int dt'\ \zeta(r',t')\ \chi(r-r',t-t') + P_o(r)\ \delta(t)$$

Fourier-Laplace
(*Montroll-Weiss)*

$$\boxed{P(k,u) = \frac{1-\tau(u)}{u} \frac{P_0}{1-\chi(k,u)}}$$

A. NORMAL Brownian motion - short memory, short range

$\tau(t) = [\exp(-t/t_0)/ t_0]$; Poissonian (finite) characteristic time

$\lambda(r)=(4\pi\sigma\ )^{-1}\exp[-(r/2\sigma)^2]$ finite jump length variance

F-L: $\tau(u)\sim 1-ut_0$ and $\lambda(k)\sim 1-\sigma^2 k^2$ as $(k,u)\to(0,0)$

$$\boxed{P(k,u)=\frac{1}{u+Kk^2}}$$

Inverse Fourier-Laplace →diffusion limit; FP

$$\frac{\partial P}{\partial t}=K\frac{\partial^2}{\partial x^2}P$$

B. SUBDIFFUSIVE PROCESS

Long-tailed waiting time pdf with finite jump variance

$\tau(t) = A\,(t/t_0)^{-1-\alpha}$, $\alpha<1$; $\lambda(r)=(4\pi\sigma\ )^{-1}\exp[-(r/2\sigma)^2]$

$\tau(u)\sim 1-(ut_0)^{\alpha}$; $\lambda(k)\sim 1-\sigma^2k^2$ as $(k,u)\text{->}(0,0)$

$$\boxed{P(k,u) = \frac{P_0(k)/u}{1+K_\alpha u^{-\alpha}k^2}}$$

$$K_\alpha = \sigma^2 / t^\alpha$$

Riemann-Liouville $${}_0D_t^{-\alpha}G(x,t) = \frac{1}{\Gamma(\alpha)}\int_0^t dt' \frac{G(t')}{(t-t')^{1-\alpha}}$$

$$\pounds\{D^{-\alpha}P(r,t)\} = u^{-\alpha}P(r,u)$$

Inverse F-L -→ Fractional FP

$$\frac{\partial P(r,t)}{\partial t} = D_t^{\boxed{1-\alpha}} K_\alpha \frac{\partial^2}{\partial r^2} P(r,t)$$

Temporal Integration emphasizes the non-Markovian, long term memory

Solution converges to Brownian as α-->1

Normal diffusive vs anomalously sub-diffusive

$$P(k,u) = \frac{P_0(k)}{u + Kk^2} \qquad P(k,u) = \frac{P_0(k)/u}{1 + K_\alpha u^{-\alpha} k^2}$$

Single mode decay

$$P(k,t) = \exp(-Kk^2 t) \qquad P(k,t) = E_\alpha(-K_\alpha k^2 t^\alpha)$$

Exponential relaxation

Stretched exponential

Mittag-Leffler

$$E_\alpha(-\, t^\alpha) = \sum_{n=0}^{\infty} \frac{(-t^\alpha)^n}{\Gamma(1+\alpha n)} \sim [t^\alpha \Gamma(1-\alpha)]^{-1}$$

$\langle r^2 \rangle = [K/\Gamma(\alpha+1)]\, t^\alpha,\ \alpha<1$

C. SUPERDIFFUSIVE PROCESS

Short memory waiting time - long jumps variance

Markovian time pdf

Levy diverging jumps: $\mu<2$

$$\tau(\mathrm{u}) \sim 1 - \mathrm{ut}_0; \qquad \lambda(\mathrm{k}) = \exp(-\sigma^{\mu}|\mathrm{k}|^{\mu}) \sim 1 - \sigma^{\mu}|\mathrm{k}|^{\mu}$$

$$\boxed{P(k,u) = \frac{P_0(k)}{u + K^{\mu}|k|^{\mu}};}$$

$$K^{\mu} = \sigma^{\mu} / t_0$$

Riess/Weyl fractional operator

$$ {}_{-\infty}D_z^{\mu} G(z) = {}_{-\infty}D_z^{n-(n-\mu)} G(z) = \frac{1}{\Gamma(n-\mu)} \frac{d^n}{dz^n} \int_{-\infty}^{z} dz' \frac{G(z')}{(z-z')^{1-(n-\mu)}} $$

$$ Fourier: \mathbf{F}\{{}_{-\infty}D_z^{\mu} G(z)\} = -|k|^{\mu} \mathbf{F}\{G(z)\} \qquad \mu \in (n-1, n) $$

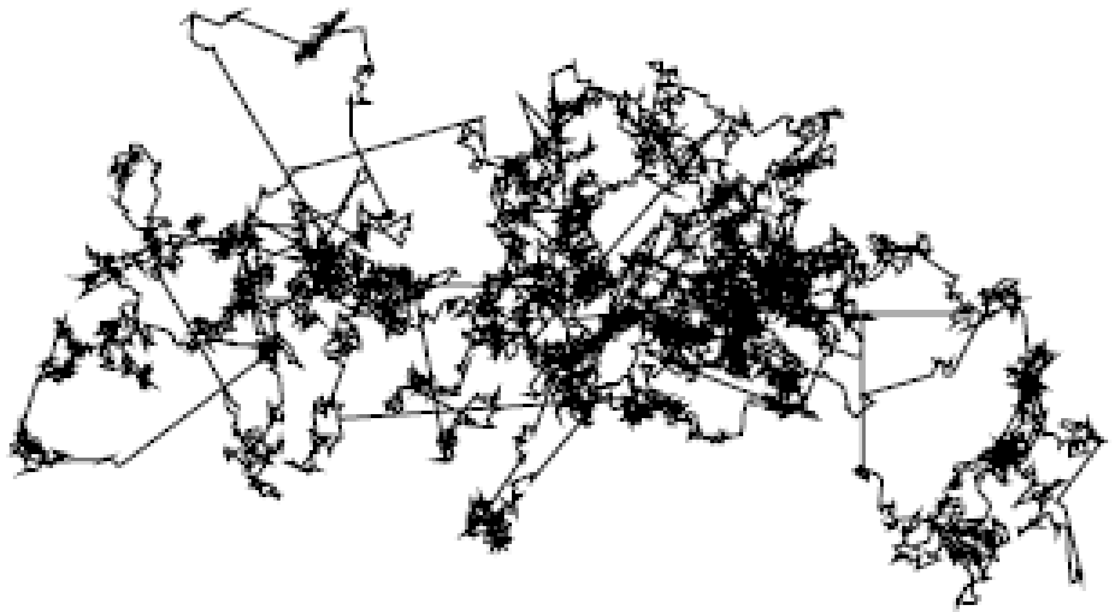
$$ \frac{\partial P(r,t)}{\partial t} = K^{\mu}\, {}_{-\infty}D_r^{\mu} P(r,t) $$

Spatial Integration emphasizes the probability for a long range interaction

Solution converges to Brownian as μ-->2

Gauss walk

Levy flight



Normal diffusive vs anomalously super-diffusive

$$P(k,u) = \frac{P_0(k)}{u + Kk^2}$$

$$P(k,u) = \frac{P_0(k)}{u + K^{\mu}k^{\mu}}$$

Single mode decay

$$P(k,t) = \exp(-Kk^2 t)$$

$$P(k,t) = \exp(-K^{\mu}|k|^{\mu} t)$$

Exponential relaxation

Extended tail

Symmetric Levy

Solution: Mellin Transform

$$\mathrm{M}[F(t)] = F(s) = \int_0^\infty F(t) t^{s-1} dt$$

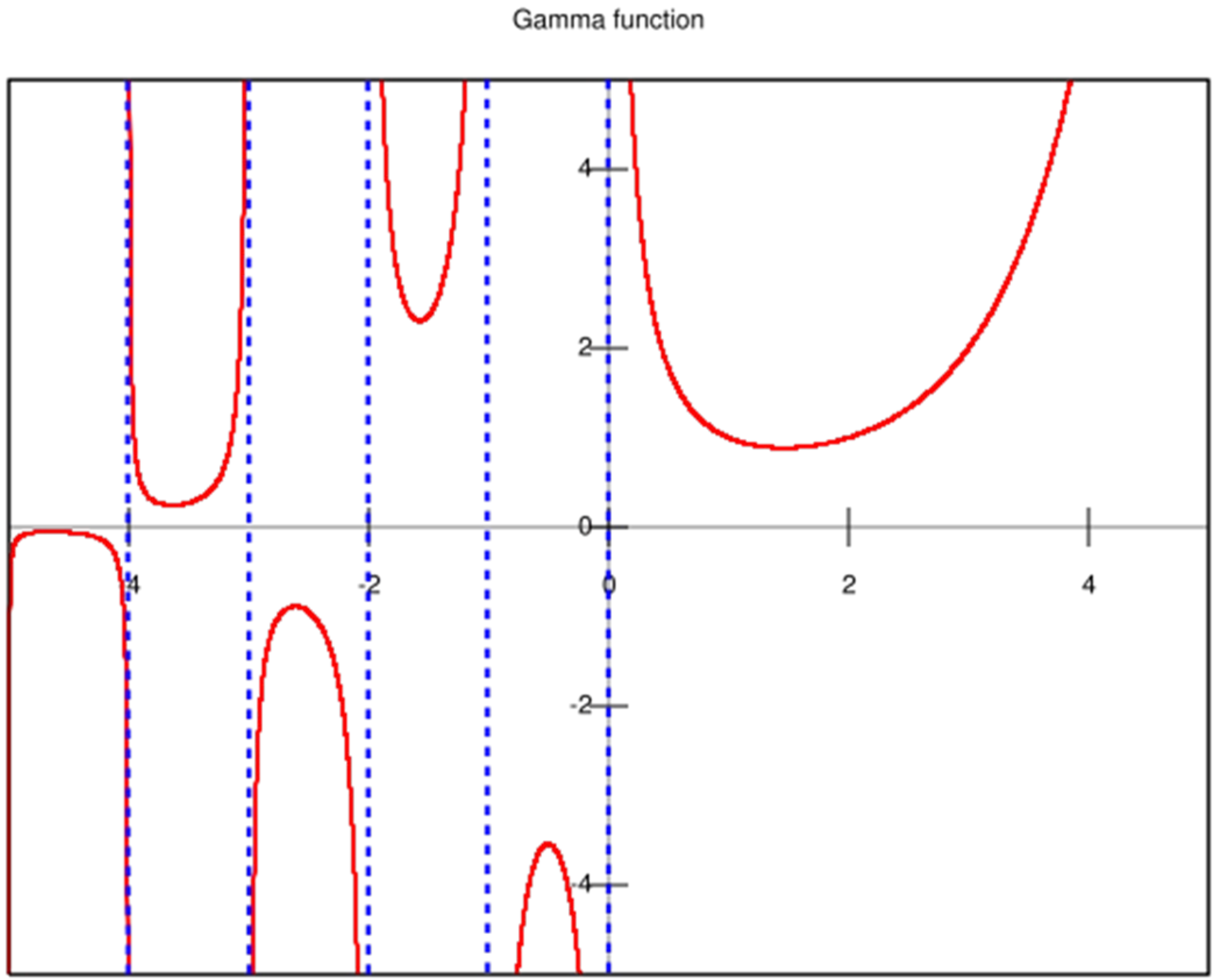
$$\mathbf{M}\{L(F(t), p), s\} = \Gamma(s)\mathbf{M}[F(t), 1-s]$$

Resulting in a set of Γ functions

$$\Gamma(s) = \int_0^\infty e^{-t} t^{s-1} dt$$

$$_0D_r^q t^p = \frac{\Gamma(1+p)}{\Gamma(1+p-q)} t^{p-q} \Rightarrow\Rightarrow {}_0D_r^q 1 = \frac{1}{\Gamma(1-q)} t^{-q}$$

Gamma function – holomorphic with simple poles at all the non-positive integers; the residue at -n is $(-1)^n/n!$



Fox (H) : generalized Mellin-Barnes (Meijer G) Functions

$$H(z) = H_{pq}^{mn}[z;{}^{(a,\alpha)}_{(b,\beta)}] = \frac{1}{2\pi i}\int_C \frac{\prod_{i=1}^{m}\Gamma(b_i-\beta_i s)\prod_{i=1}^{n}\Gamma(1-a_i-\alpha_i s)}{\prod_{i=m+1}^{q}\Gamma(1-b_i+\beta_i s)\prod_{i=n+1}^{p}\Gamma(a_i-\alpha_i s)} z^s ds$$

$\alpha_j,\beta_j > 0$; a_j,b_j –complex; roots of (a, α)/ (b,β) left/right of C

Series expansion with residue (-1)/l! at the poles $b_k-\beta_k s=-l$, $l=0,\ldots$

$$H_{pq}^{mn}[z;{}^{(a,\alpha)}_{(b,\beta)}] = \sum_{k=1}^{m}\sum_{l=0}^{\infty} \frac{\prod_{i=1,\neq k}^{m}\Gamma[b_i-\frac{\beta_i(b_k+l)}{\beta_k}]\prod_{i=1}^{n}\Gamma[1-a_i+\frac{\alpha_i(b_k+l)}{\beta_k}]}{\prod_{i=m+1}^{q}\Gamma[1-b_i+\frac{\beta_i(b_k+l)}{\beta_k}]\prod_{i=n+1}^{p}\Gamma[a_i-\frac{\alpha_i(b_k+l)}{\beta_k}]} \frac{(-1)^l z^{\frac{(b_k+l)}{\beta_k}}}{l!\,b_k}$$

Alternating signs – slow convergence

Explicit solution of the FFPE

poles

$$P(r,t) = \frac{1}{\mu|r|} H_{22}^{11}\left[\xi \left|^{(1,1/\mu)(1,1/2)}_{(1,1)(1,1/2)}\right.\right] = \frac{1}{\mu|r|}\int_C ds \frac{\Gamma(1-s)\Gamma(\frac{s}{\mu})}{\Gamma(\frac{s}{2})\Gamma(1-\frac{s}{\mu})}\xi^s$$

$$\xi = \frac{r}{(K^\mu t)^{1/\mu}}$$

Roots at s/μ=-m; $$P(r,t) = \frac{1}{\mu|r|}\sum_{m=0}^{\infty}\frac{\Gamma(m\mu)}{\Gamma(-m\mu/2)}\frac{(-1)^m}{m!}\xi^{-m\mu} \sim \frac{Kt}{r^{1+\mu}}$$

Slow convergence

$$P(r,t) = (Kt)^{-1/\mu} L_\mu[|r|/(Kt)^{1/\mu}]$$

General: Long waiting times and large jumps

$$\frac{\partial P(r,t)}{\partial t} = D_t^{1-\alpha} K_\alpha^\mu \frac{\partial^\mu}{\partial r^\mu} P(r,t)$$

$$< r^2 >= t^{\alpha/\mu}$$

- *Statistics of Energization Processes*

- *Anomalous Transport/Diffusion*

Entropy of Super-diffusive processes

***Thermodynamic properties - from microscopic states**

***System in contact with a large reservoir**

***Short temporal/spatial interaction – random jumps p(r)**

Boltzmann-Gibbs-Shannon entropy-*extensive* ~system size

$$S_{BG} = S_1 = -\int p(r) \ln p(r) dr$$

Constrains: $\int p(r) dr = 1; \qquad < r^2 > = \int r^2 p(r) dr = \sigma^2 < \infty$

Variational Optimization; β-Lagrange multiplier

$$p_1(r) = \frac{\exp(-\beta r^2)}{Z_1(\beta)} = \frac{\exp(-\beta r^2)}{\int dr e^{-\beta r^2}}; \qquad \beta = T^{-1} = 1/(2\sigma^2)$$

Gaussian – attractor in distribution space

Long range interaction makes the entropy non-extensive

→formation of tails in the distribution.

q-statistics/entropy (Tsallis):

$$S_q = [1 - \int p^q(r)dr]/(q-1) \quad \to S_{BG} = S_1 \; as \; q \to 1$$

Optimization of S_q with q-expectation constraint $<r^2> = \int r^2[p(r)]^q dr$

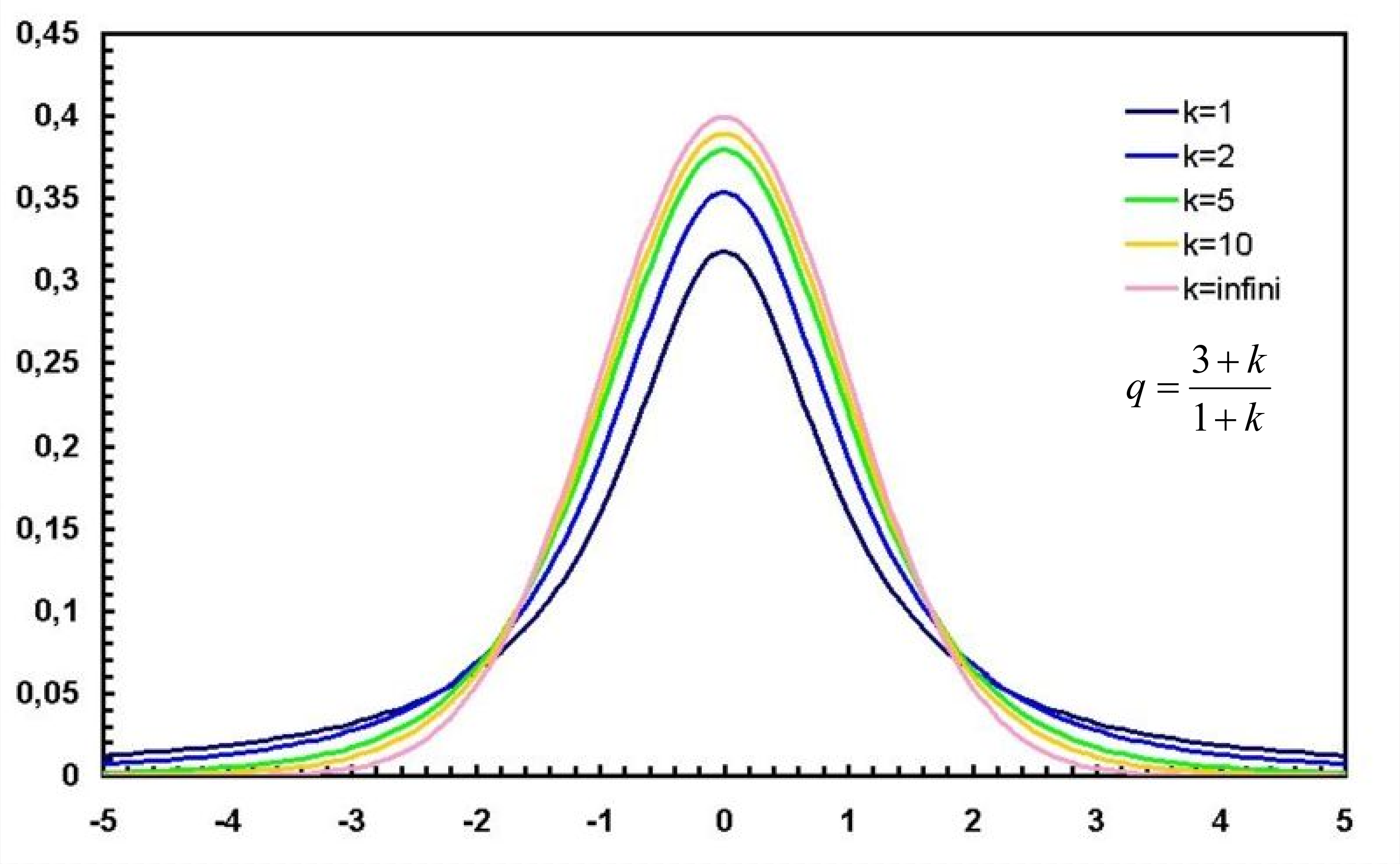
and β as Lagrange multiplier leads to heavy tail distributions

$$p_q(r) = [1-(1-q)\beta r^2]^{1/(1-q)} / Z_q \quad \xrightarrow[q \to 1]{} \frac{\sqrt{\beta}}{\sqrt{\pi}} \exp(-\beta r^2)$$

$$e_q^r = [1+(1-q)r]^{1/(1-q)} \Longrightarrow p_q(r) = \frac{\exp_q(-\beta r^2)}{Z_q(\beta)} = \frac{\exp_q(-\beta r^2)}{\int dr \exp_q(-\beta r^2)}$$

1<q<3 - Student t distribution

$$f(r) = \frac{\Gamma[(k+1)/2]}{\sqrt{\pi k}\,\Gamma[k/2]}(1+\frac{r^2}{k})^{-(k+1)/2}$$



$$p_q(r) = [1-(1-q)\beta r^2]^{\frac{1}{(1-q)}} \Big/ \int [1-(1-q)\beta r^2]^{\frac{1}{(1-q)}} dr$$

$$\infty < q < 3$$

Converges to Gaussian for q=1.

Converges to Cauchy for q=2

Variance – finite for q < 5/3 , diverges for 5/3 <q <3

q-variance: $\int dx x^2 p(x)^q$ **finite for q < 3,**

$$p_q(r) \sim r^{-2/(q-1)} \quad i.e. \quad \alpha = (3-q)/(q-1);\ 0 < \alpha < 2$$

$$p_q(r,N) = [\beta^{1/2}/N^{1/\alpha}] L_\alpha[(\beta^{1/2}/N^{1/\alpha})r]: \quad 5/3 < q < 3$$

“Phase Transition” at q=5/3 (α=2)

q-entropy converges to stable laws (distributions)

Comparison with the data allows to assess

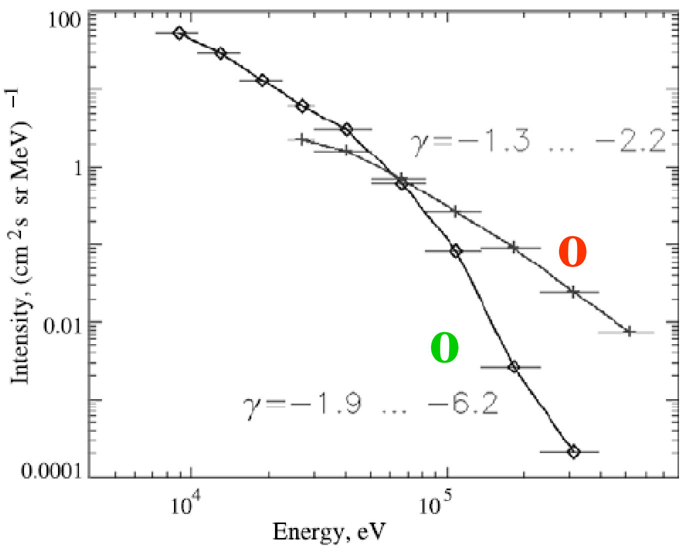
a) **the validity of the single event probability density function**

b) **the relative importance of the different expansion terms – power law transition**

c) **the “duration” of the interaction**

SUMMARY

Electron Spectra



One-event Impulsive probability

O $$p(E)=\frac{A}{(1+E^4)}$$

with an incomplete asymptotic Gaussian evolution

One-event Gradual probability

α=0.5-1.0 (Lévy)

O $$p(E)=\frac{A}{(1+E^\delta)};\delta=1.5-2.0$$

with a truncated Levy index α

at the range of Holtsmark-Cauchy

ΕΠΙΛΟΓΟΣ

Distribution function of electrons and ions in space plasmas due to random, resonant interactions with electromagnetic turbulence displays numerous characteristic forms, combining Gaussians with (broken) power laws. The basic single event interaction probability determines the asymptotic distribution with a phase transition at the index of the characteristic function α=2, while the global interaction time determines the observed non-asymptotic distribution. The distribution function of the injected particles may be construed via general arguments of stochastic processes, and parameterized via non-extensive entropy.