Fractal, multifractal, and generalized scaling in the turbulent solar wind

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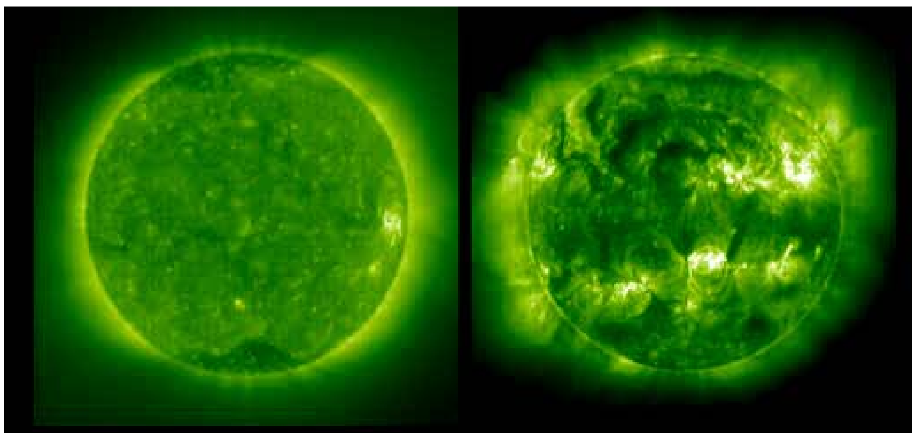
- The quest for *universal* features of turbulence in solar wind
- Multifractal turbulence in the solar wind- the inertial range
- What happens on small scales- dissipation/dispersion range
  on large scales- outer scale

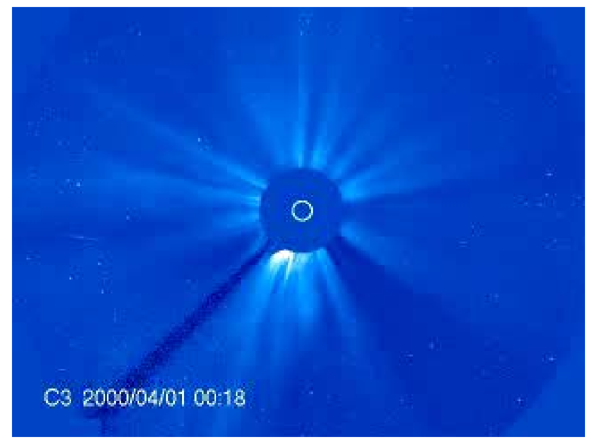
Data thanks to CLUSTER, WIND, ACE, ULYSSES teams

Overview: the solar wind as a turbulence laboratory

SOHO-EIT image of the corona at solar minimum and solar maximum



SOHO- LASCO image of the outer corona near solar maximum



I: coronal signature has scaling properties
II: solar wind has intermittent (multifractal) inertial range of turbulence
III: in-situ observations span inertial range,
dissipation/dispersion range and lower k

Solar wind at 1AU power spectra- suggests inertial range of (anisotropic MHD) turbulence. Multifractal scaling in velocity and magnetic field components..

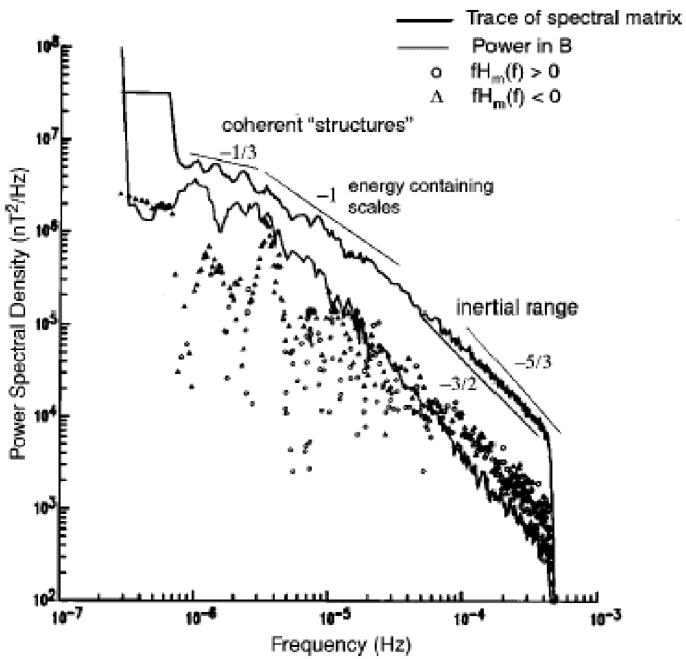


FIG. 1. A power spectrum of the solar wind magnetic field from a time series spanning more than a year. The upper curve is the trace of the power spectral matrix of the three components of B, the lower solid curve is the power in |B|, and the circles and triangles are the positive and negative values of the [reduced (Ref. 91)] magnetic helicity (Refs. 39 and 92) respectively.

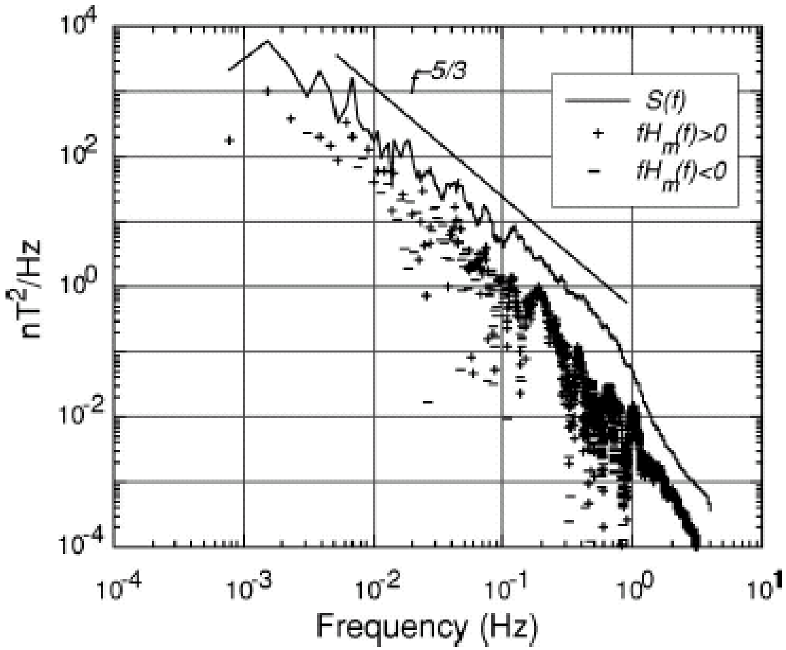


FIG. 2. A power spectrum of Mariner 10 data from 0.5 AU showing the dissipation range of magnetic fluctuations. Also shown are positive and negative values of $fH_m(f)$.

Goldstein and Roberts, POP 1999, See also Tu and Marsch, SSR, 1995

Turbulence and scaling

structures on many length/timescales.

single spacecraft- time interval τ a proxy for space

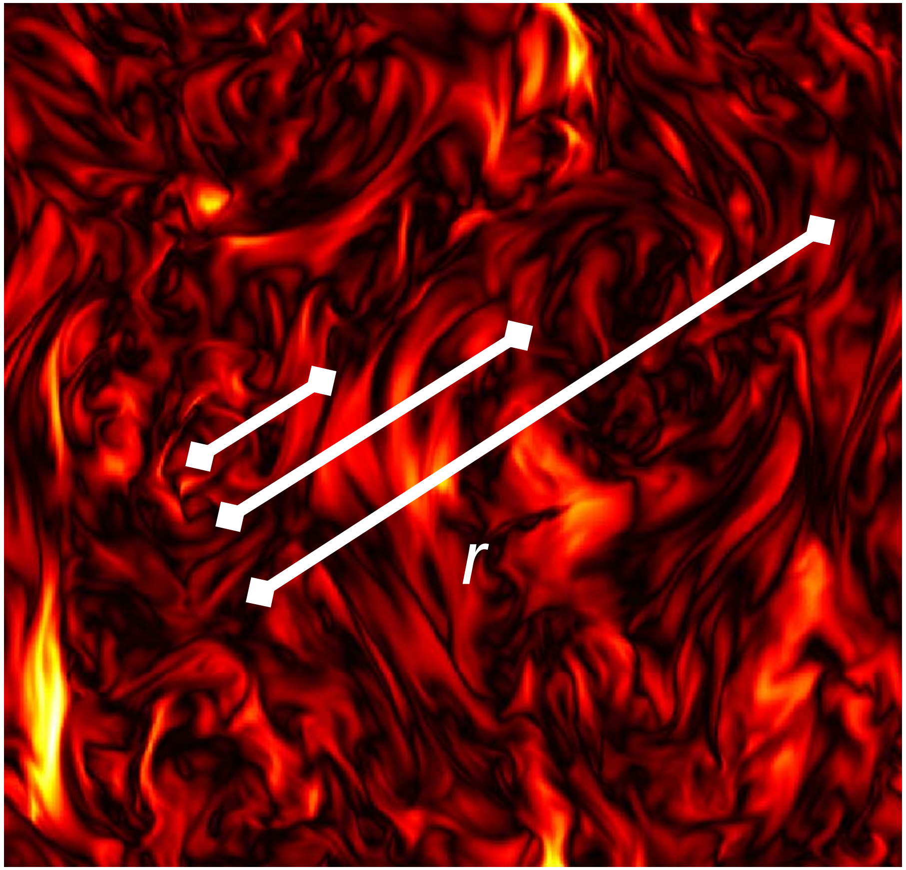
Reproducible, predictable in a *statistical* sense.

to focus on any particular scale r take a difference:

$y(l,r) = x(l+r) - x(l)$

look at the statistics of $y(l,r)$

power spectra- compare power in Fourier modes on different scales r



DNS of 2D compressible MHD turbulence Merrifield, SCC et al, POP 2006,2007

Beyond power spectra- quantifying scaling/turbulence from observations

single spacecraft- time interval τ a proxy for space

look at (time-space) differences:

$y(t,\tau) = x(t+\tau) - x(t)$

for all available t_k of the timeseries $x(t_k)$

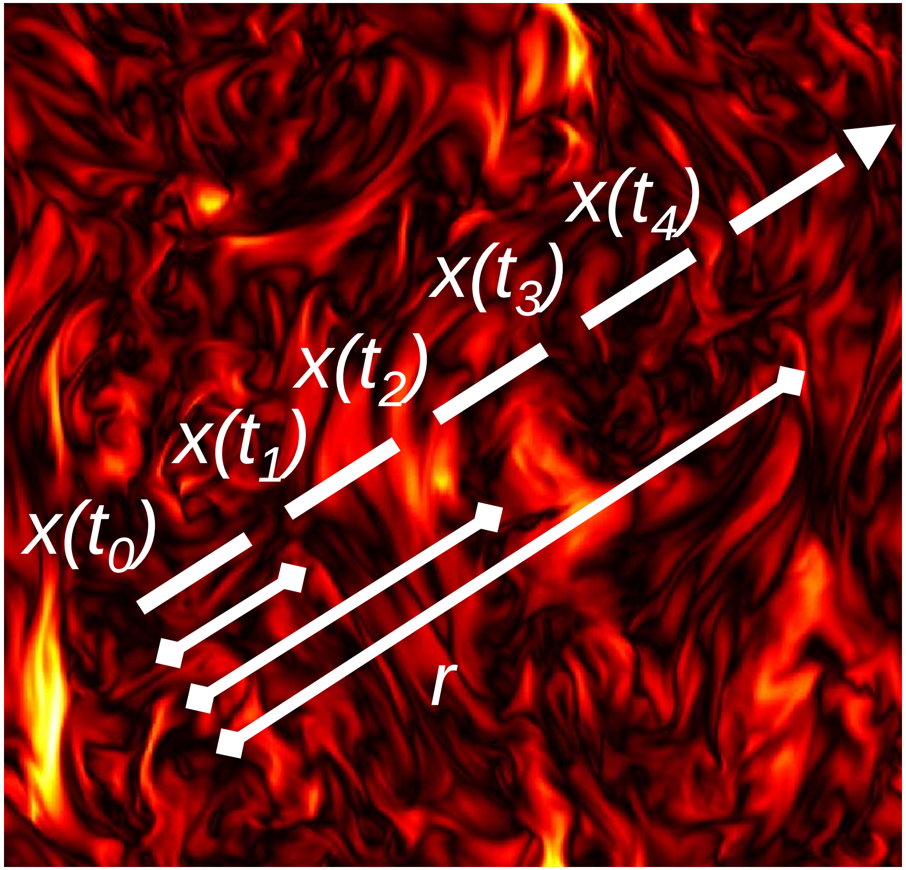
test for statistical scaling i.e

structure functions $S_p(\tau) = \langle |y(t,\tau)|^p \rangle \propto \tau^{\zeta(p)}$

we want to measure the $\zeta(p)$

fractal (self- affine) $\zeta(p) \sim \alpha p$

multifractal $\zeta(p) \sim \alpha p - \beta p^2 + \ldots$



NB structure functions are just one example of a 'scaling measure' cf wavelets, Fourier, SVD/PCA…

All decompose the timeseries as a sum of functions on different scales.

The inertial range- anisotropy and phenomenology from scaling

Multifractal inertial range turbulence- examples

$S_p = <| x(t+\tau) - x(t) |^p > \sim \tau^{\xi(p)}$, plot $\log(S_p)$ vz. $\log(\tau)$ to obtain $\xi(p)$

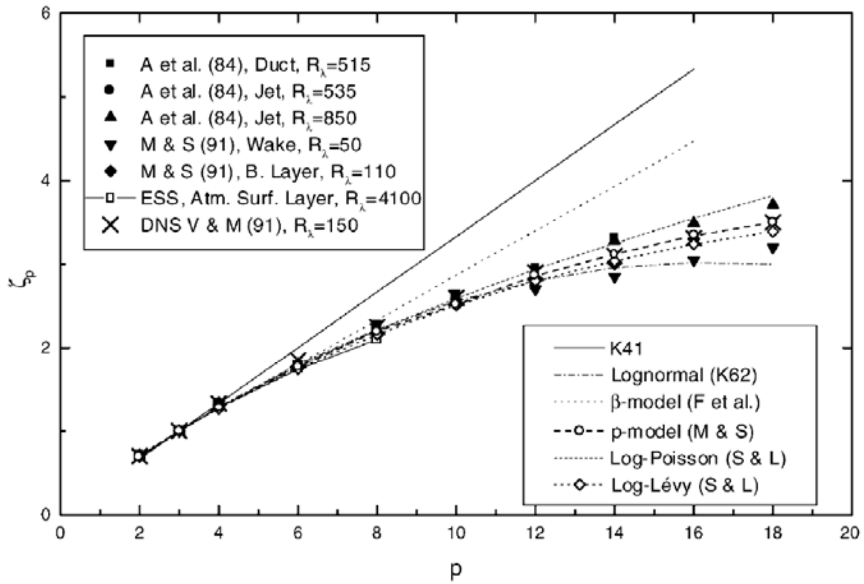
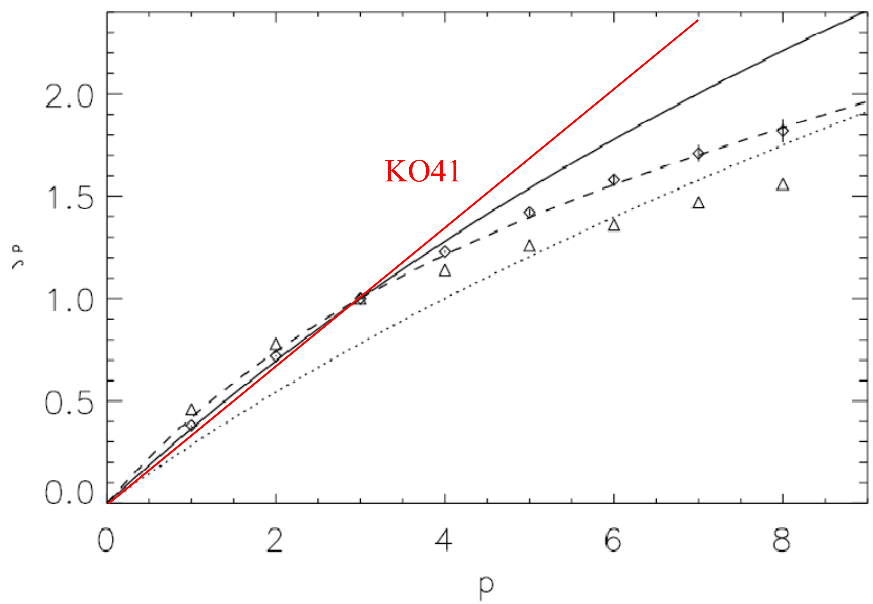


Fig. 11. Power-law exponents ζ_p of the structure functions as a function of the order p, together with the values predicted by K41 and the various intermittency models of Table 1.



IG. 4. Scaling exponents ζ_p^+ for 3D MHD turbulence (diaionds) and relative exponents ζ_p^+/ζ_3^+ for 2D MHD turbulence riangles). The continuous curve is the She-Leveque model ζ_p^{SL}, the dashed curve the modified model ζ_p^{MHD} (7), and the dotted line the IK model ζ_p^{IK}.

Lab Fluid experiments,
Anselmet et al, PSS, 2001

2 and 3D MHD simulations
Muller & Biskamp PRL 2000

How large can we take p? *See eg Dudok De Wit, PRE, 2004*

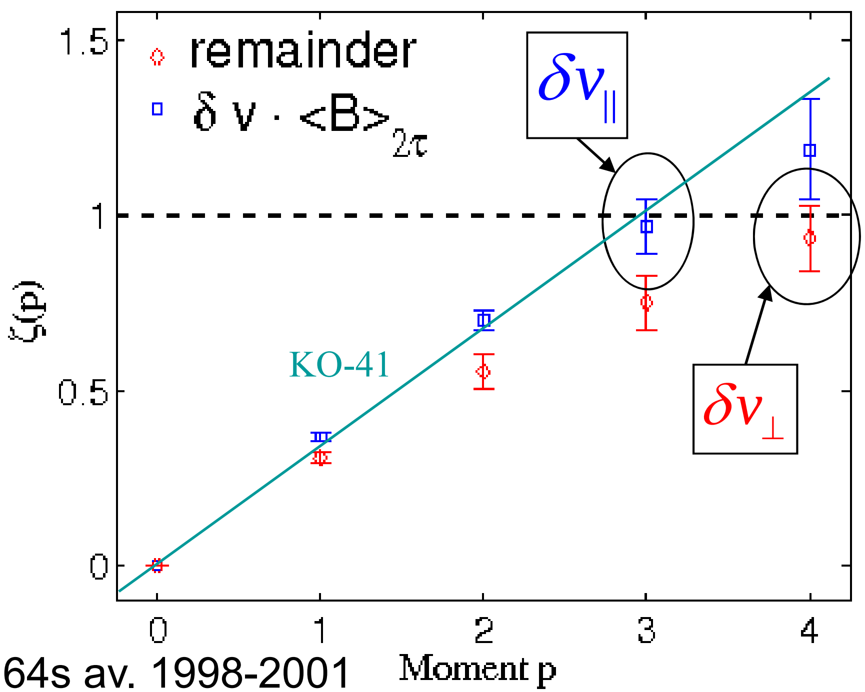
Solar wind anisotropy: Velocity fluctuations parallel and perpendicular to the *local* B field direction

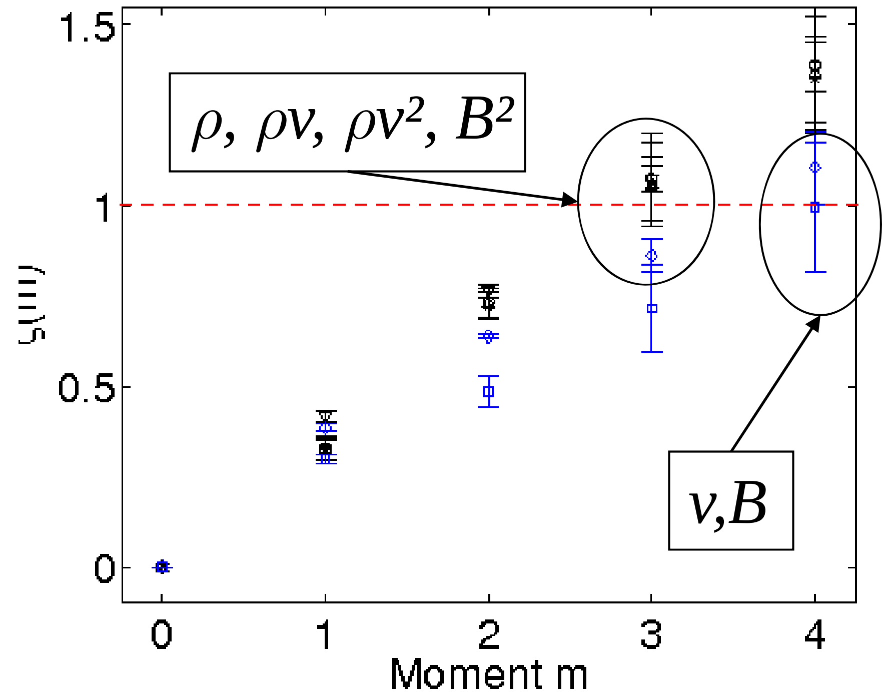
Exponents $\zeta(p)$ for $<\left|\delta v_{\parallel,\perp}\right|^{p}>\sim \tau^{\zeta(p)}$ for

$\delta v_{\parallel} = \delta \mathbf{v}.\hat{\mathbf{b}}$ and its remainder $\delta v_{\perp} = \sqrt{\delta \mathbf{v}.\delta \mathbf{v} - \left(\delta \mathbf{v}.\hat{\mathbf{b}}\right)^{2}}$

$\zeta(3+\alpha)=1$ determines phenomenology

$\overline{\mathbf{B}} = \mathbf{B}(t) + \ldots + \mathbf{B}(t+\tau'),\ \hat{\mathbf{b}} = \frac{\overline{\mathbf{B}}}{\left|\overline{\mathbf{B}}\right|}$, here $\tau' = 2\tau$ and $\delta \mathbf{v} = \mathbf{v}(t+\tau) - \mathbf{v}(t)$



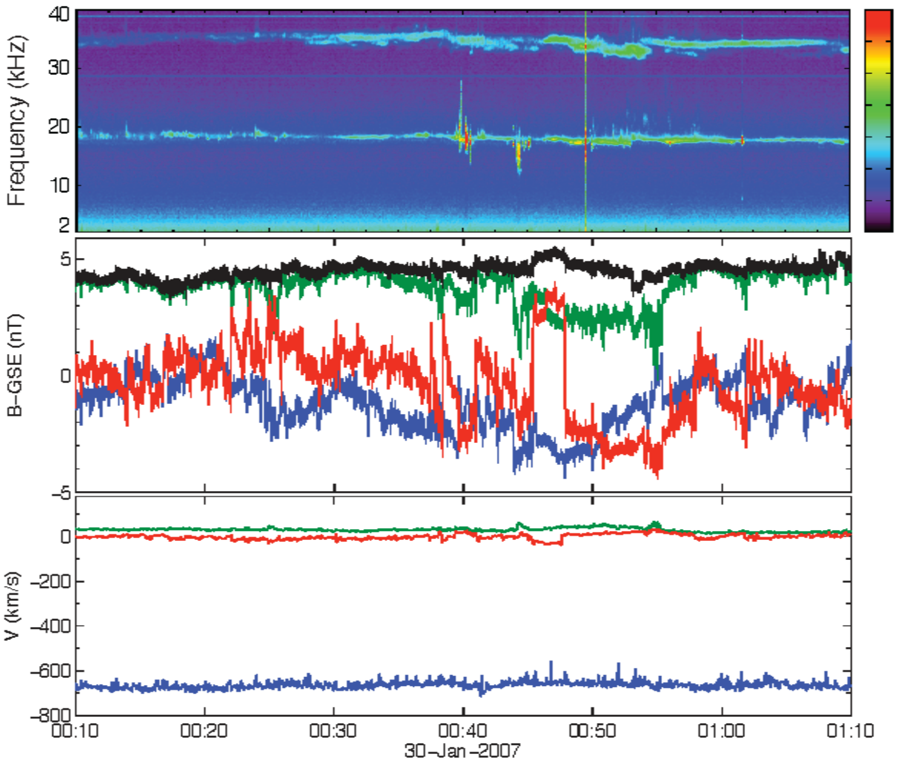


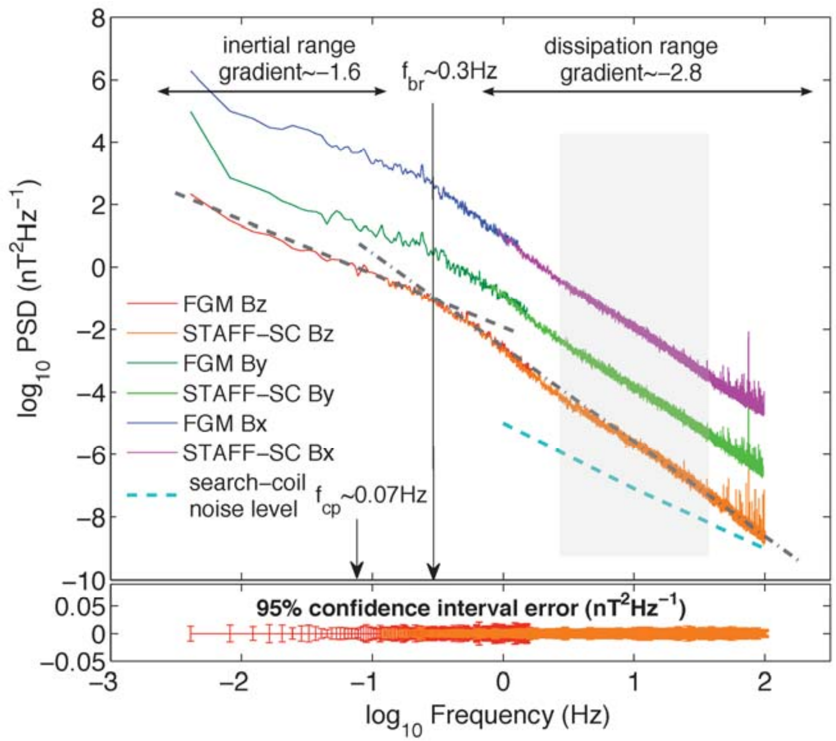
ACE 64s av. 1998-2001

SCC et al GRL 2007, see also Hnat, SCC et al PRL 2005, SCC et al NPG 2008

The ‘dissipation’ range- what happens on kinetic scales

A nice quiet fast interval of solar wind- CLUSTER high cadence B field spanning IR and dissipation range



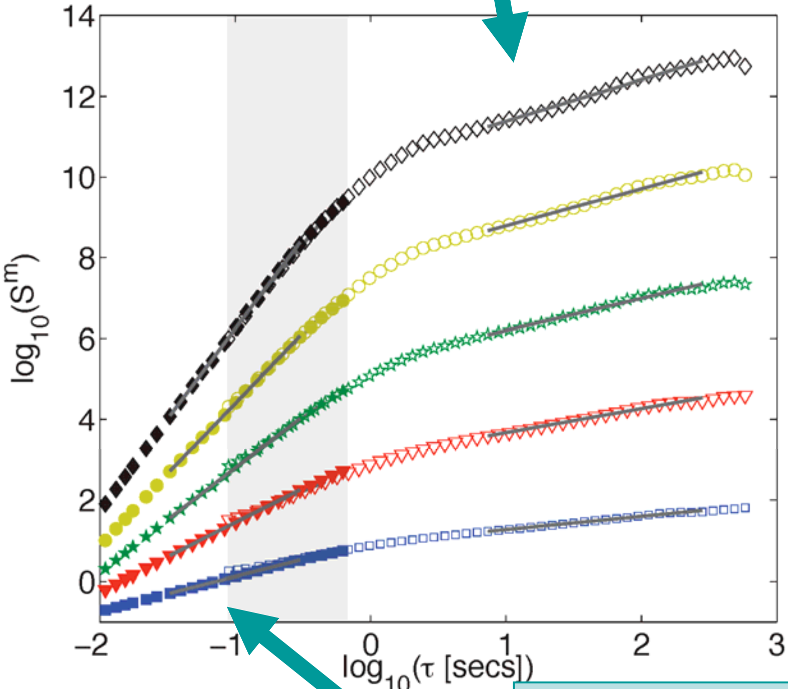


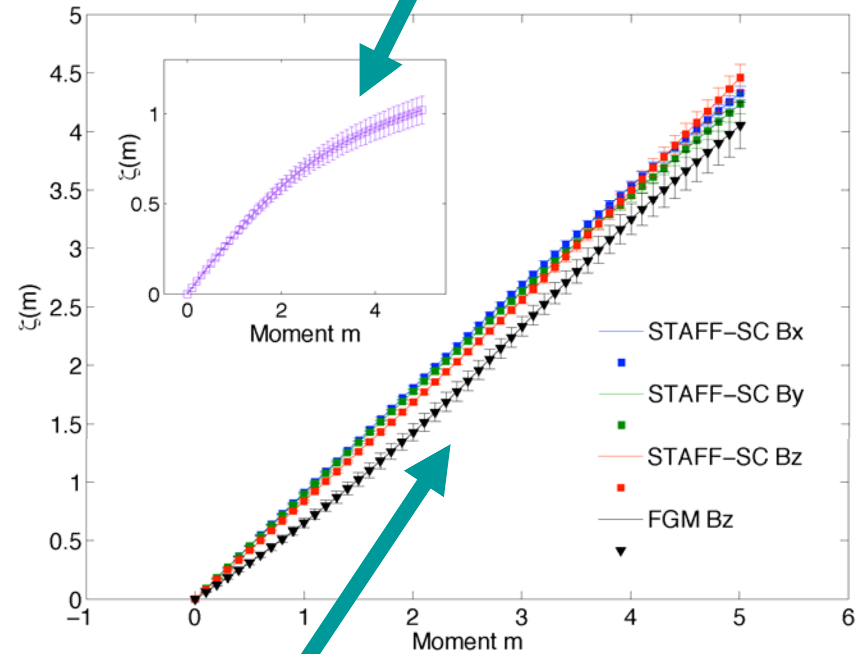
CLUSTER STAFF and FGM shown overlaid.

Kiyani, SCC et al PRL submitted 2009

$$S_p = <| x(t+\tau) - x(t) |^p > \sim \tau^{\xi(p)}, \text{ plot } \log(S_p) \text{ vz. } \log(\tau) \text{ to obtain } \xi(p)$$

Inertial range- multifractal





Dissipation range- fractal

CLUSTER STAFF and FGM shown overlaid.

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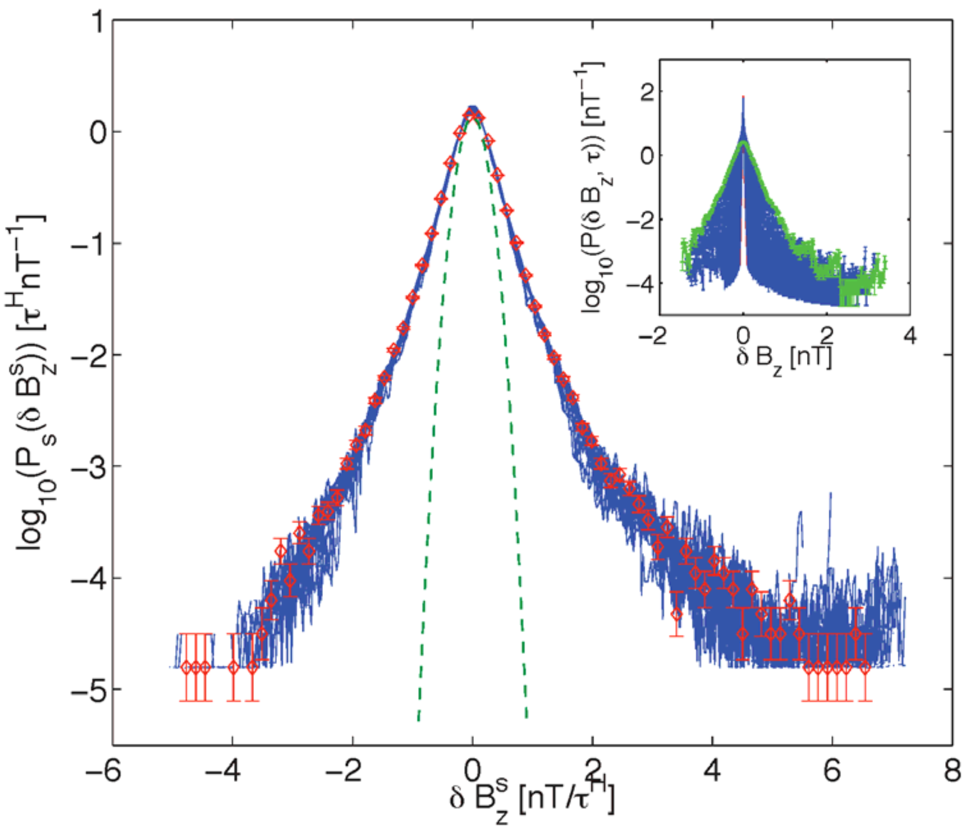


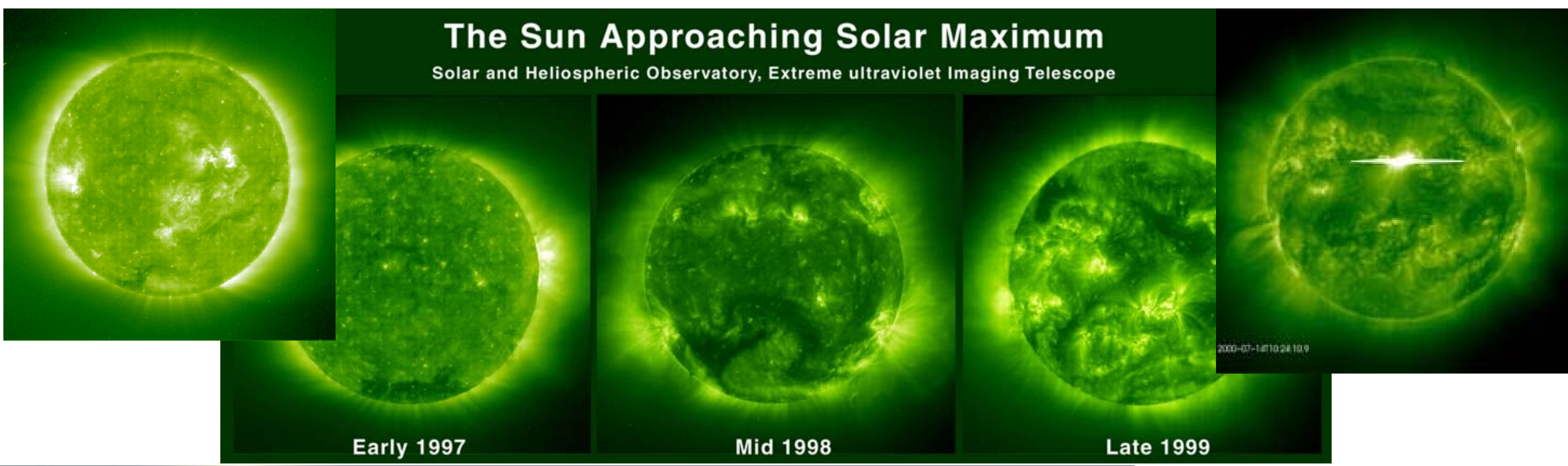
Non- Gaussian PDF in dissipation range,
single exponent scaling collapse

The ‘outer scale’- the end of the inertial range of turbulence at larger scales

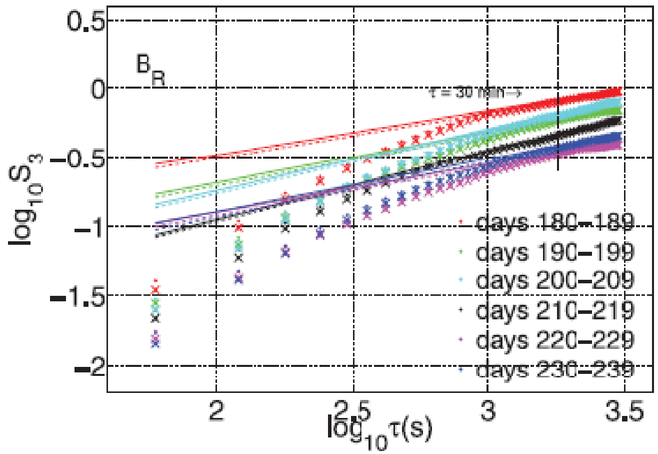
ULYSSES- north and south polar passes at solar minimum

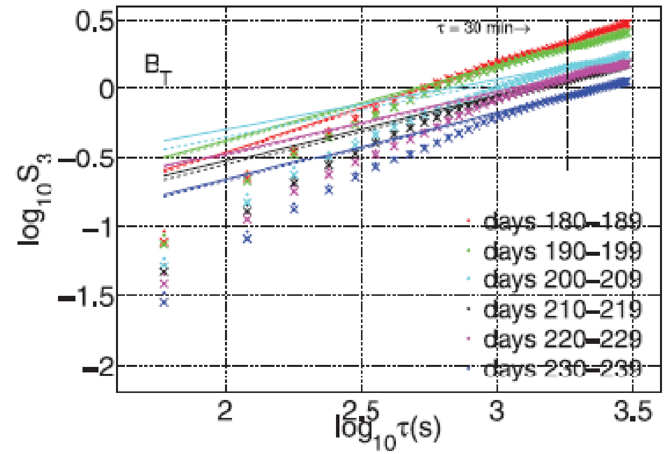
ULYSSES 60s averages B field components
~8.5x10^4 points, selected as a quiet interval
-Multifractal

-See also *Nicol, SCC et al, ApJ (2008), SCC et al ApJL (2009)*
Solar cycle dependence in correlation
Wicks, SCC et al, ApJ (2009)



Evolving turbulence- Quiet, fast polar solar wind: 1995 North polar pass, solar min, ULYSSES





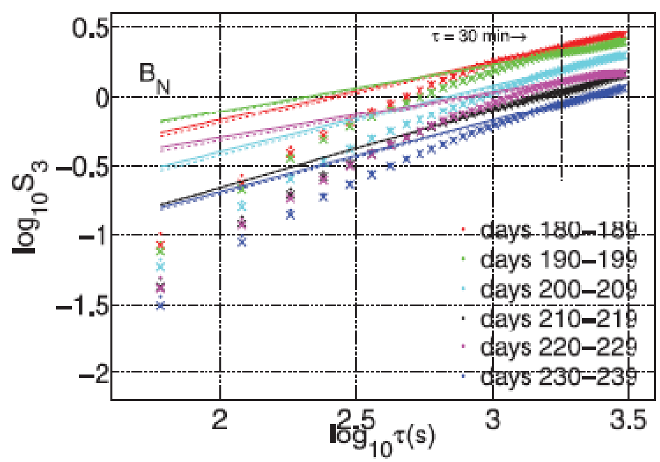
IR turbulence- expect

$S_3 \sim \tau^{\zeta(3)}$

i.e. straight line on log-log plot

not quite seen here!

$1/f$ is actually $1/f^{\gamma}$



Nicol, SCC et al ApJ 2008, SCC et al, ApJL 2009

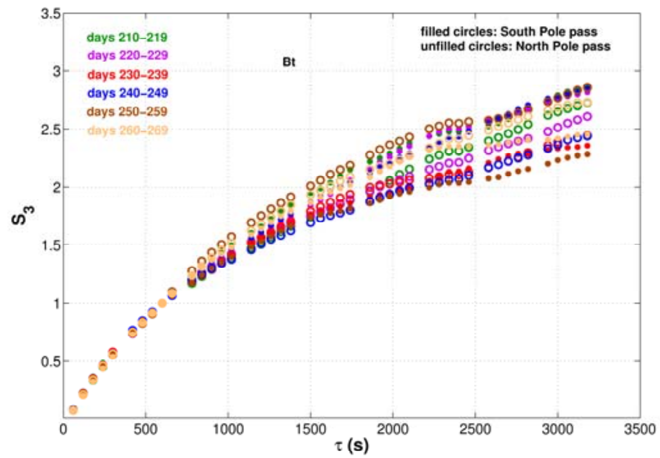
Generalized similarity (scaling)- turbulence at the outer scale- universal behaviour?

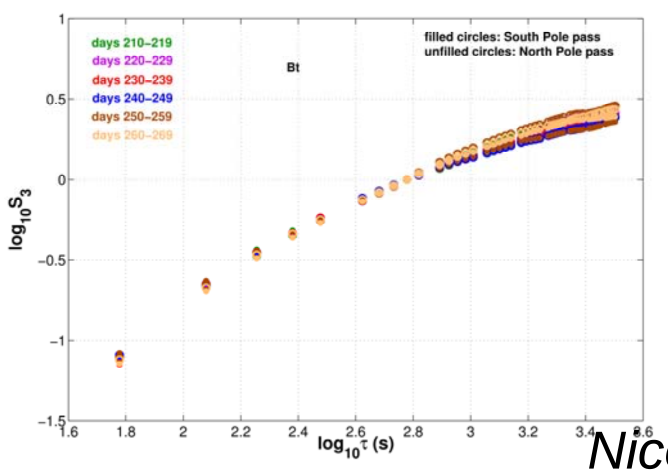
South pass 1994, North pass 1995, solar min

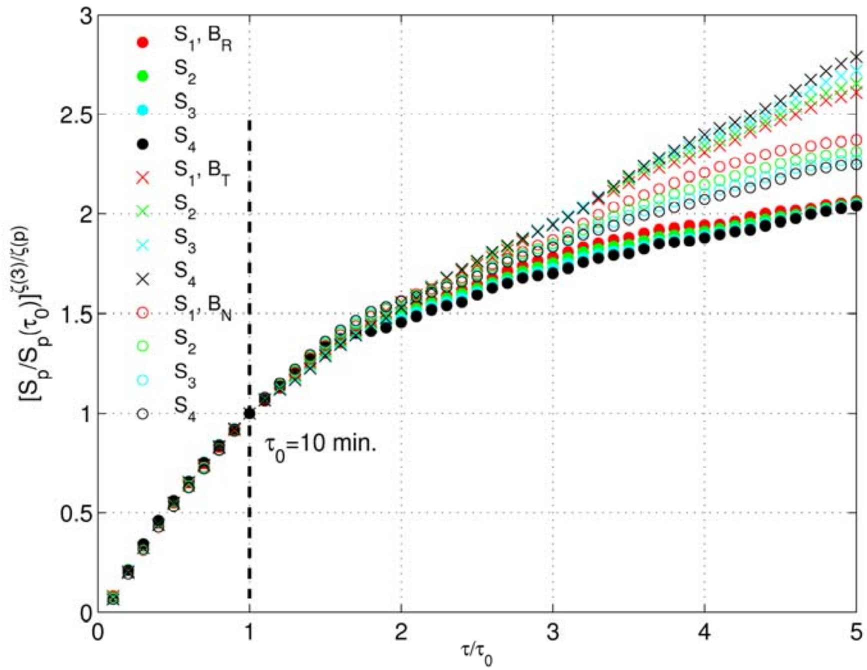
$$S_p \sim g(\tau)^{\xi(p)}$$

invert to obtain $g(\tau)$

same $g(\tau)$ seen





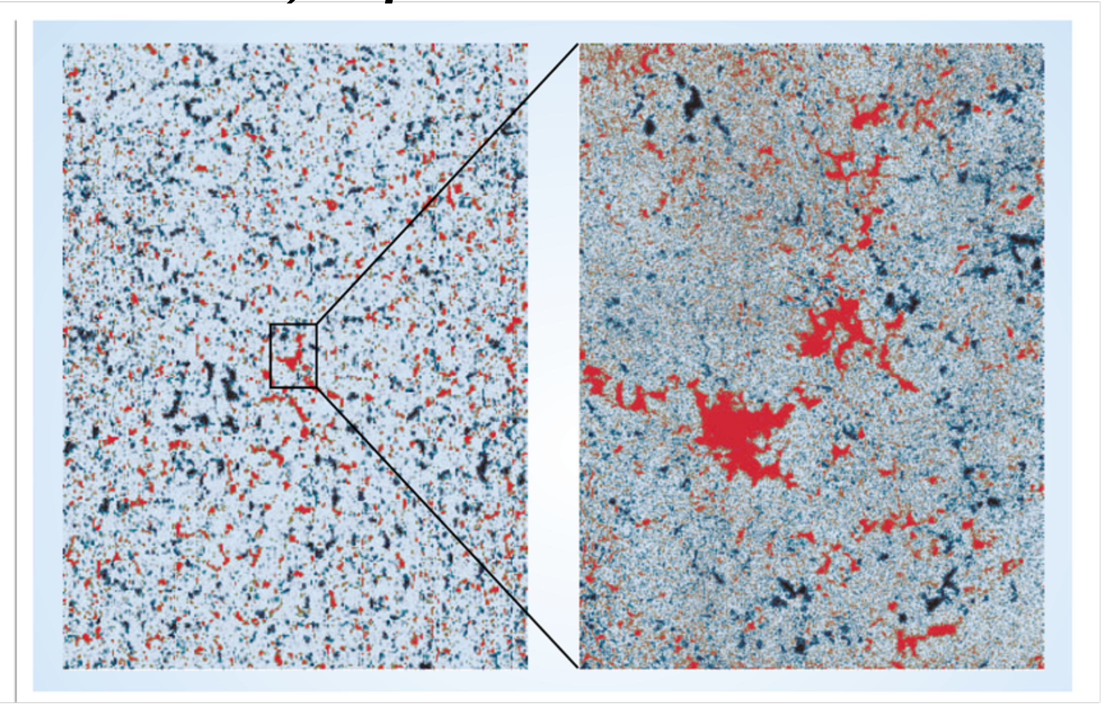


Nicol, SCC et al ApJ 2008, SCC et al, ApJL 2009

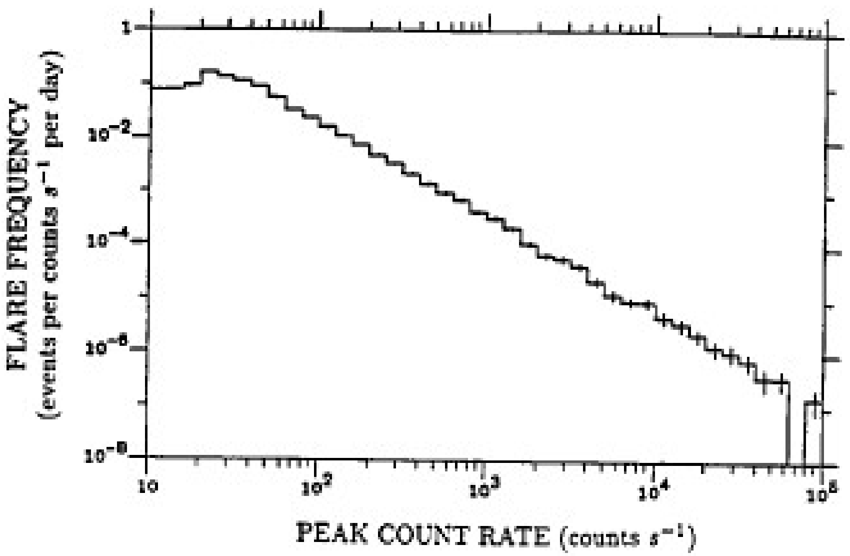
Scaling from the corona?

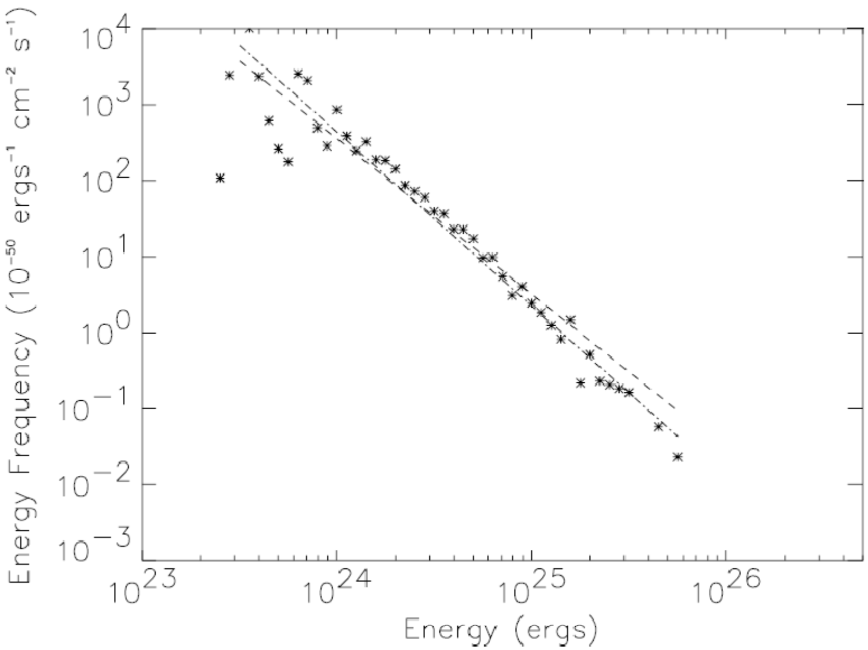
Scaling from the sun: Fractal patches of magnetic polarity on the quiet sun

Patches of opposing polarity –
Zeeman effect photosphere, quiet sun,
(Stenflo, Nature 2004, See eg Janssen et al A&A 2003, Bueno et al Nature 2004+..) - ***spatial***



Scaling from the sun: power law flare statistics



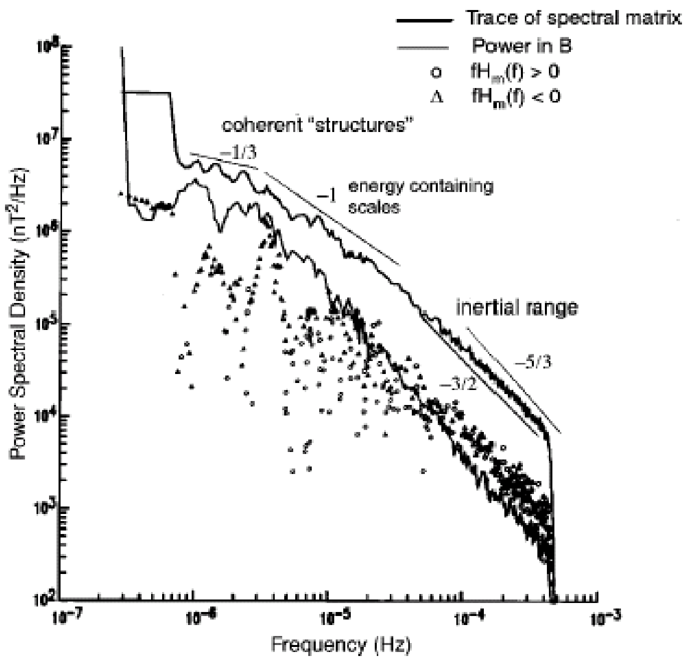


Peak flare count rate *Lu&Hamilton ApJ 1991*

TRACE nanoflare events *Parnell&Judd ApJ 2000*

-temporal

Solar wind at 1AU power spectra- B magnitude shows a single power law range, inertial range of (anisotropic MHD) turbulence seen in components



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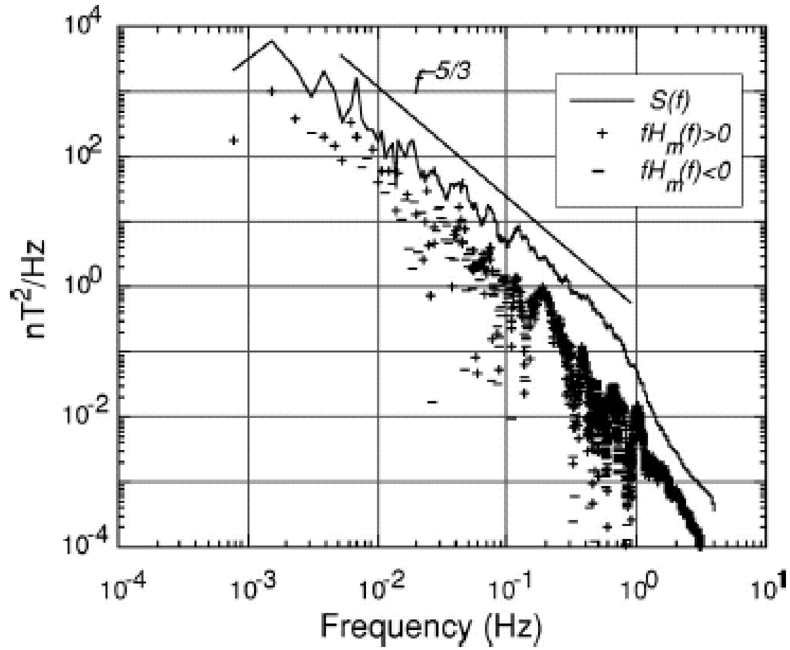

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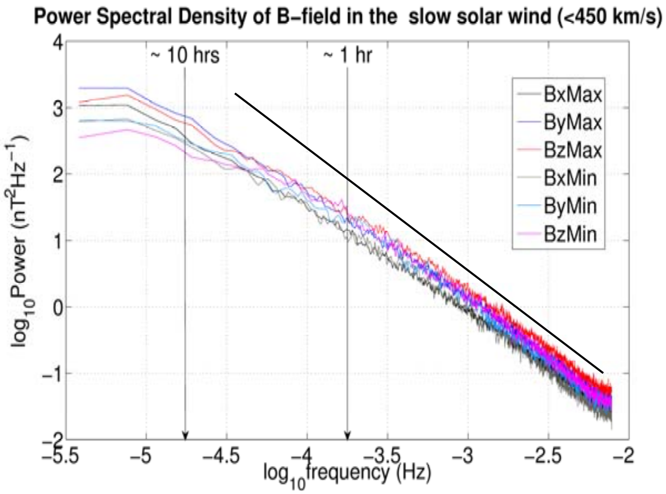
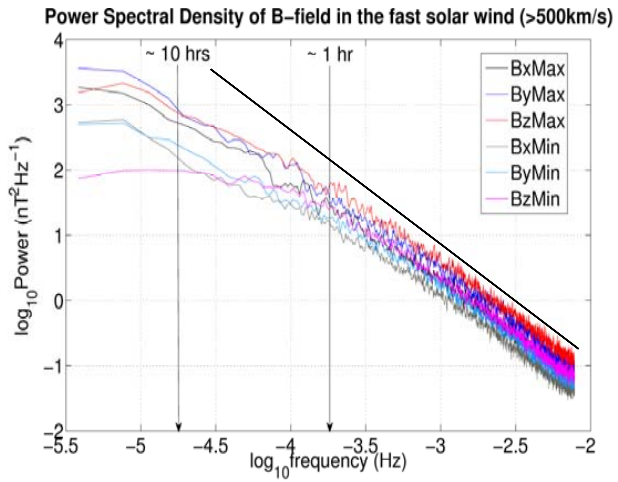
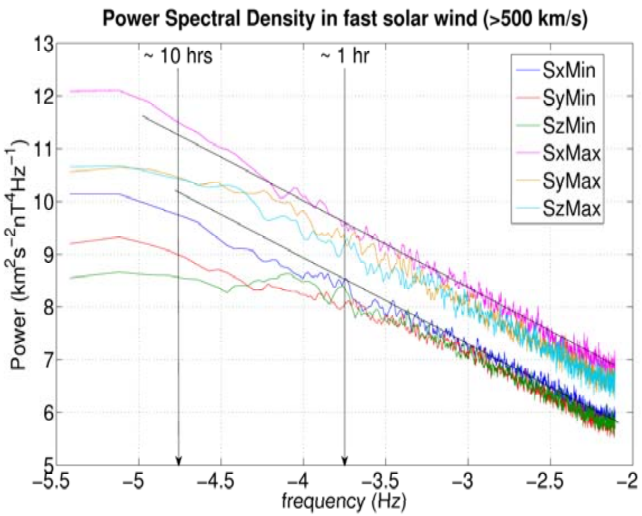


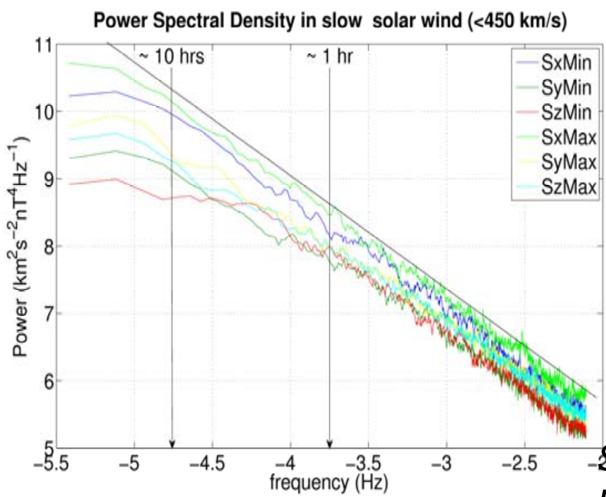


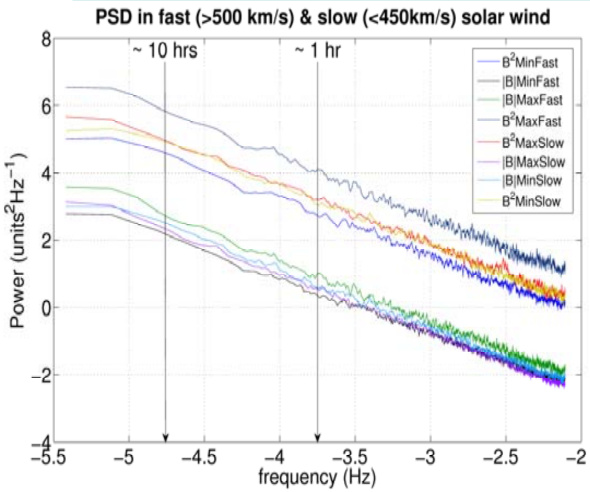
Components show 2 regions inertial range and '1/f'

x- component of Poynting flux B magnitude one single region

Shown: *log-log* plots of PSD of 3 day intervals averaged over 1 year
ACE solar max (2000); solar min (2007)
Plotted: **|B|**, B^2 and normalized **S**=-[**B**(**v.B**)-**v**B^2]
Fast $v>500kms^{-1}$ and slow $v<450kms^{-1}$



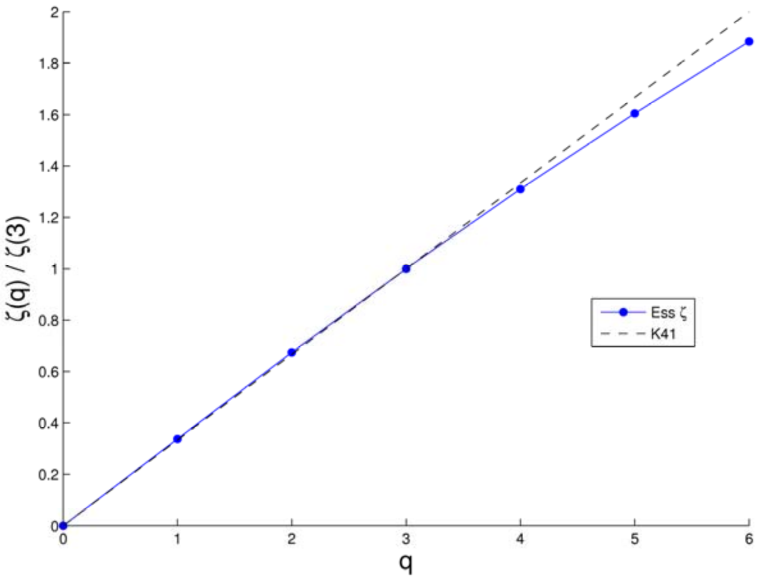


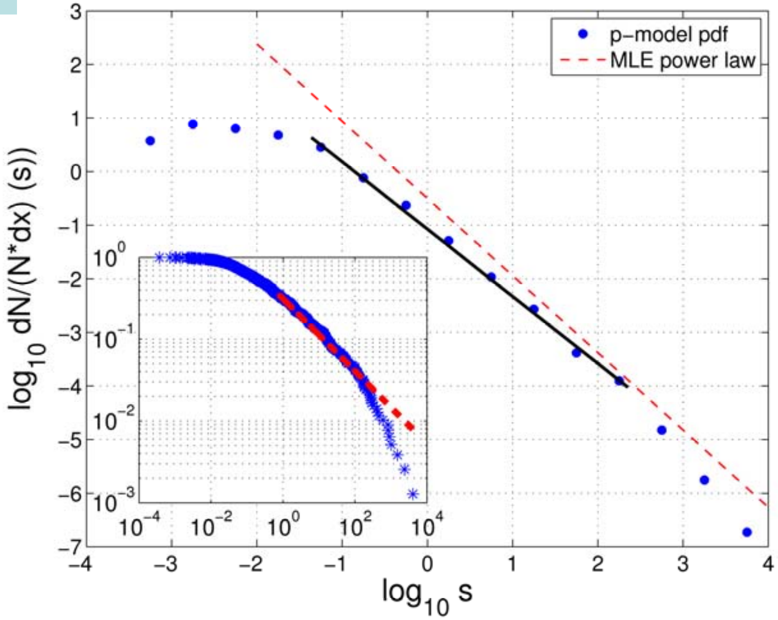


Signature of coronal fields within IR- Kiyani, SCC et al PRL, 2007

p-model for intermittent turbulence- shows finite range power law avalanches

p-model timeseries shows multifractal behaviour in structure functions as expected





Thresholding the timeseries to form an avalanche distribution- finite range power law
Watkins, SCC et al, PRL, 2009, SCC et al, POP 2009

Summary

- The quest for *universal* features of turbulence in solar wind
- The corona contributes some scaling- dominates x component of Poynting flux and B magnitude
- inertial range: Components- Multifractal anisotropic MHD turbulence
- dissipation/dispersion range- **fractal (monoscaling)** distinct from fluid/MHD phenomenology
- outer scale- a universal function for the largest scales in finite range turbulence?

Turbulence, fractals and multifractals (intermittency)

velocity difference across an eddy $d_r v = v(l+r) - v(l)$

eddy time $T(r)$ and energy transfer rate $\varepsilon_{\mathrm{r}} \propto \dfrac{d_r v^2}{T}$

now $T \propto \dfrac{\mathrm{r}}{d_r v}$ so that $\varepsilon_{\mathrm{r}} \propto \dfrac{d_r v^3}{r}$ or $\langle d_r v^3 \rangle \propto \langle \varepsilon_{\mathrm{r}} \rangle r$ and $\langle d_r\, v^p \rangle \propto r^{p/3} \langle \varepsilon_r^{p/3} \rangle$

If the flow is non- intermittent $\langle {\varepsilon_r}^p \rangle = \bar{\varepsilon}^p, r$ independent

$\Rightarrow \langle d_r\, v^p \rangle \propto r^{p/3} \bar{\varepsilon}^{p/3} \sim r^{\zeta(p)}$ -$\zeta(p) = \alpha p$ linear with p – *selfsimilar(fractal)*

intermittency correction- $\langle {\varepsilon_r}^p \rangle$ is r dependent- $\zeta(p)$ quadratic in p – *multifractal*

$\langle \varepsilon_r \rangle = \bar{\varepsilon}$ independent of r (steady state) so $\zeta(3) = 1$

cannot distinguish fractal from multifractal from power spectrum (only fixes $\zeta(2)$)

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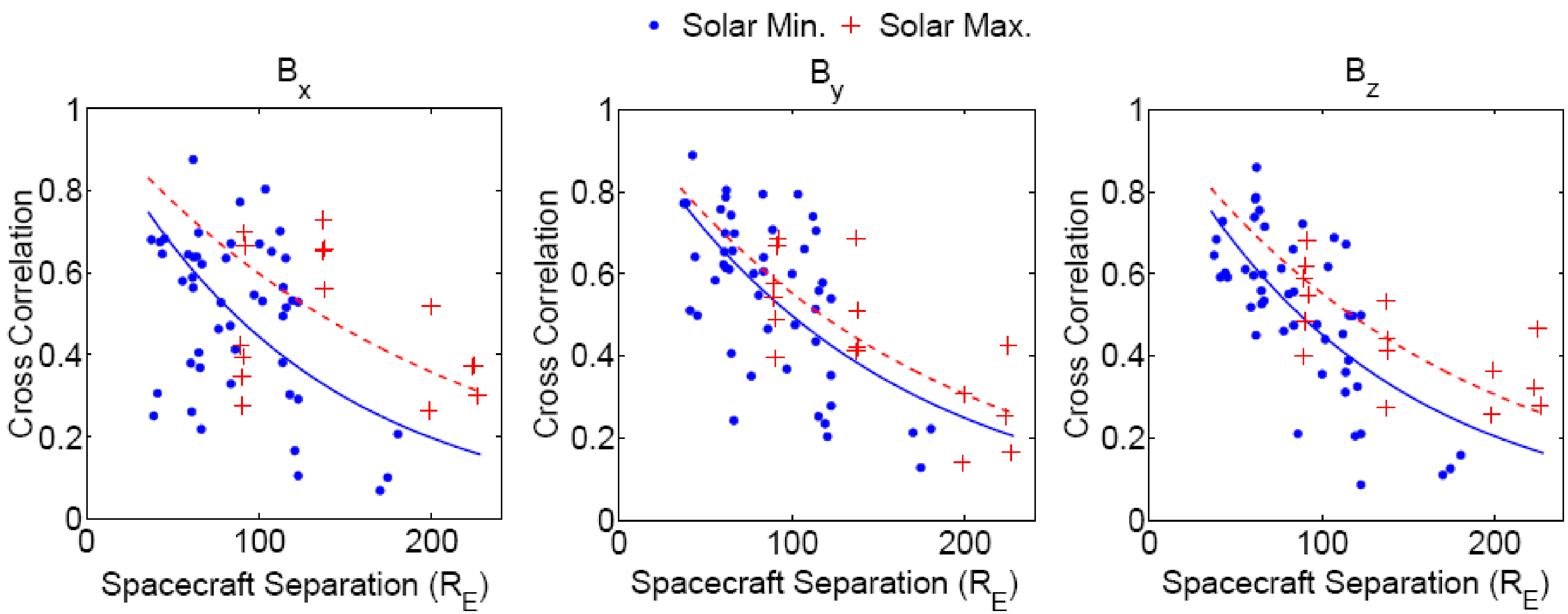
MHD: now T is due to (say) Alfvenic collisions $T \sim \dfrac{r}{d_r v}\left(\dfrac{v_0}{d_r v}\right)^{\alpha}$ giving $\varepsilon_r \sim \dfrac{d_r v^{3+\alpha}}{r}$

MHD: same with $p/3 \rightarrow p/(3+\alpha)$ and $\zeta(\alpha+3) = 1$

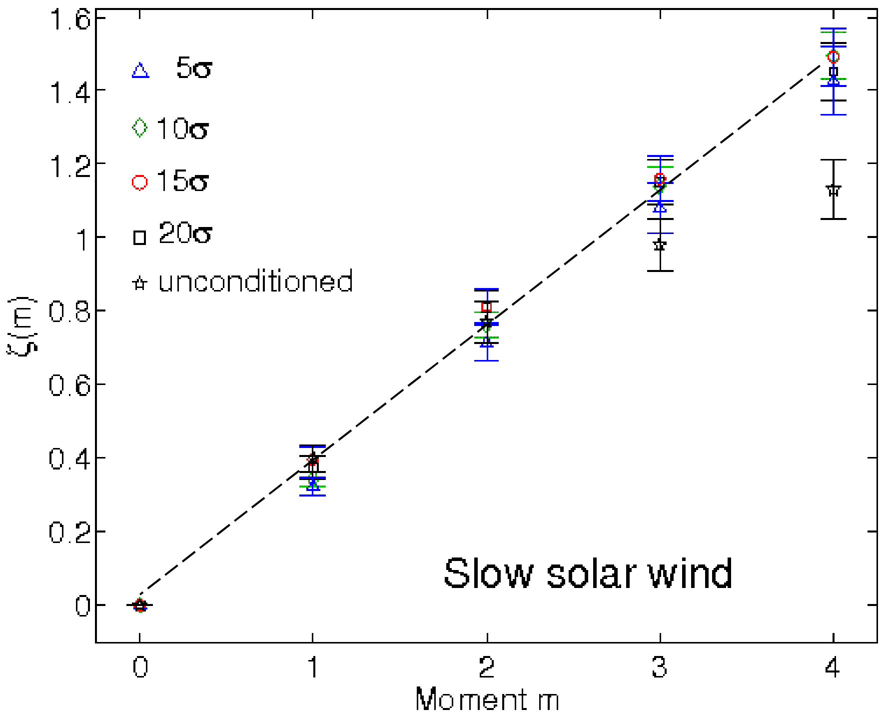
Much less (if any) solar cycle variation in components.



Summary- what we have learned..

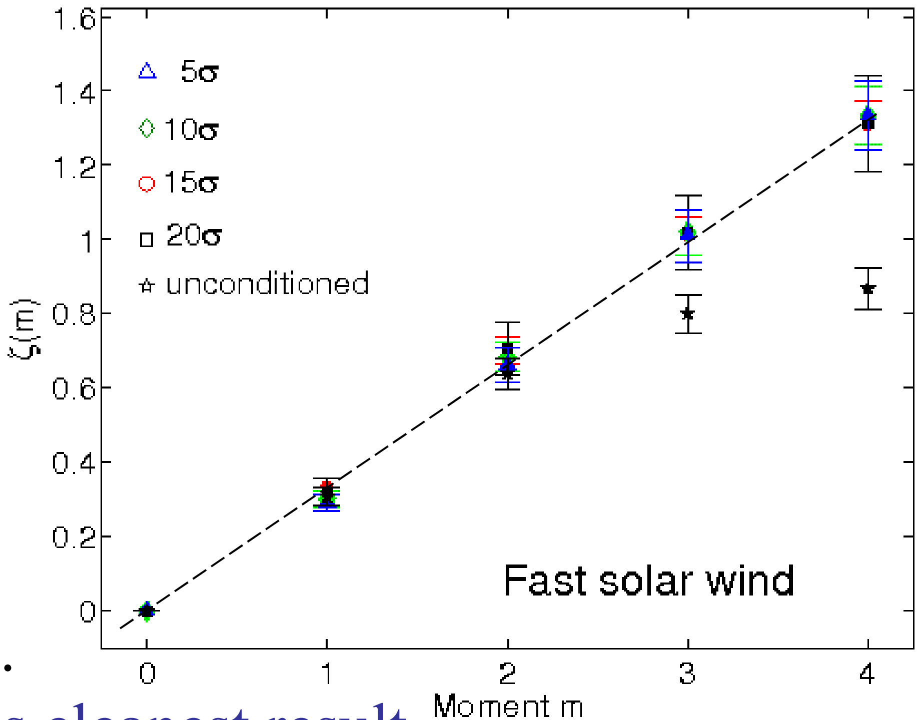
- ‘Turbulence’ is a sub- class of ‘scaling’- and we observe scaling
- 2 types of ‘scaling and bursty’ in time (i) fractal and heavy tailed (non- Gaussian) (ii) multifractal (and both are sometimes called intermittent)
- Method to distinguish these proposed
- The ‘straightforward’ solar wind- scaling coronal signature within the inertial range of turbulence
- Coronal signatures in magnetic energy density, Poynting flux, density?
- Physical insights flow from universality (asking: are the observed exponents the same as…?) to determine the physics- so the *error bars* are important!
- Finite size data sets, time stationarity!
- SDE models as a bridge between scaling (turbulence) and critical phenomena, as a method for quantifying ‘anomalous transport’
- Evolving and boundary layer turbulence- universal(?) functions we can measure

Scaling- ρ in slow and fast solar wind



compare ρ for raw and conditioned GSF

ρ quickly converges to $\zeta(m) \sim m/3$, for small m

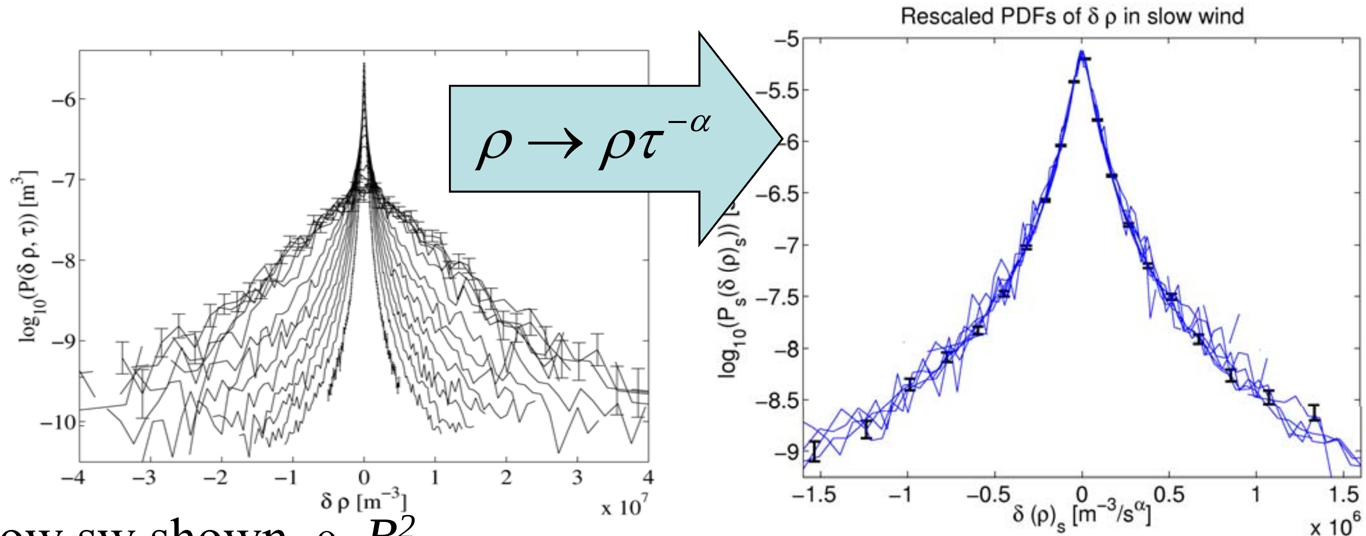


ACE 98-01 (4years)

Slow: 1x10^6, fast: 6x10^5 samples.

Threshold is 450 km/sec. fast wind is cleanest result

Rescaled PDF- ρ,B^2 in the solar wind



slow sw shown, ρ, B^2

selfsimilar or

weakly multifractal scaling up to τ~few hrs

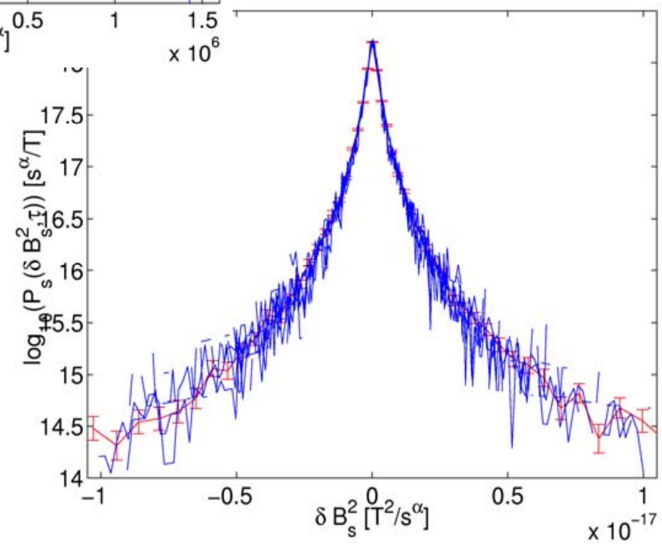
WIND 46/98s

Key Parameters ’95-’98

Approx 10^6 samples

Verified with ACE

Hnat, SCC et al GRL,2002, POP 2004



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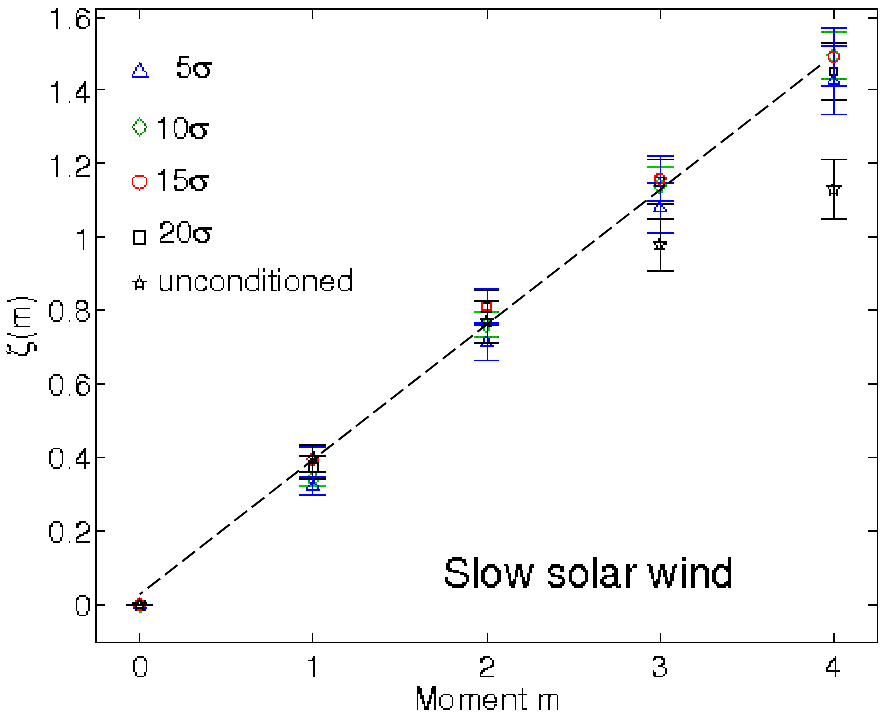

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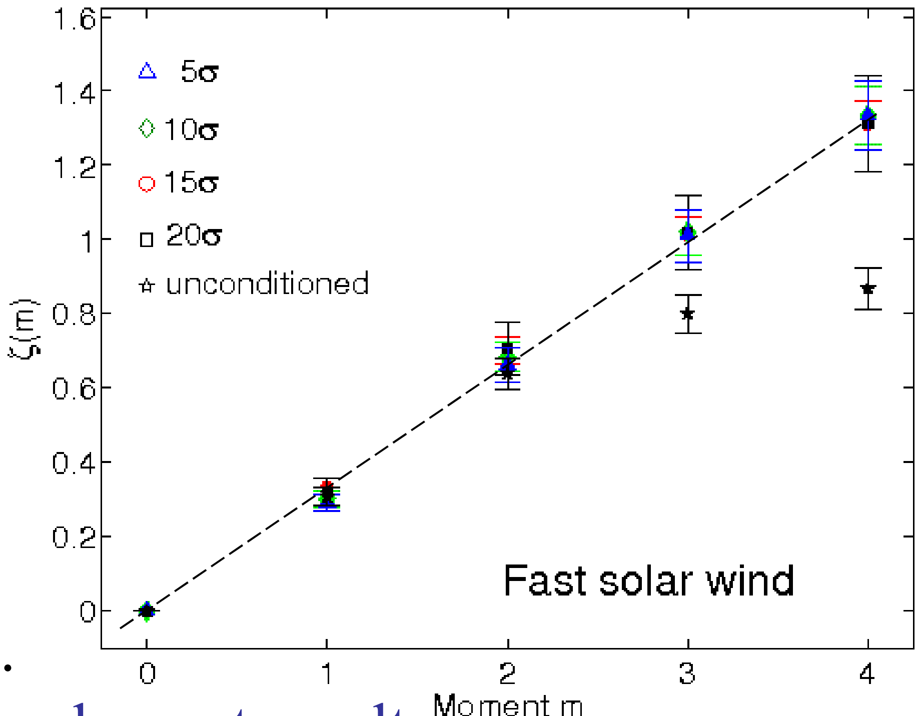

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Passive scalars and incompressibility

Bershadskii and Sreenivasan PRL '04 argued that |B| is passive scalar..
Appeal to ***universality*** in scaling exponents (same physics, same scaling)

$$\frac{DQ}{Dt} = \frac{\partial Q}{\partial t} + v \bullet \nabla Q = 0$$

e.g.

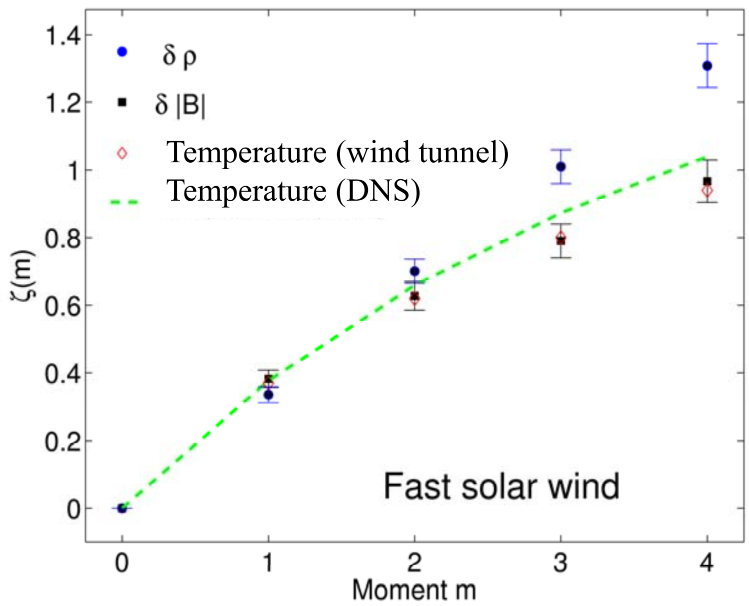
$$\frac{\partial \rho}{\partial t} + \nabla \bullet (\rho v) = 0 = \frac{\partial \rho}{\partial t} + v \bullet \nabla \rho$$

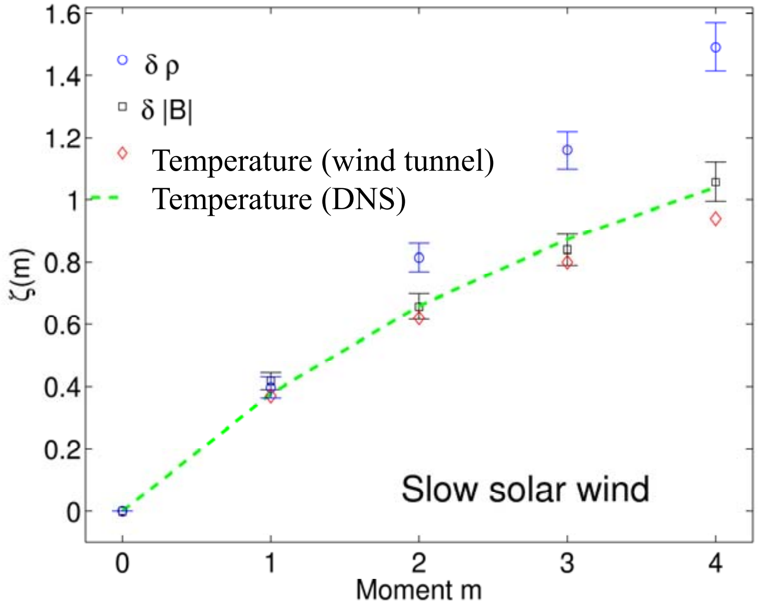
with $\nabla \bullet v = 0$ incompressible flow

if the flow is incompressible- ρ must be a passive scalar-

Passive scalars comparison

does not need to be so precise..





ρ is not passively advected with the flow?

Hnat, SCC et al PRL '05

1 year ACE data (1998)

Compare ρ with passive scalars:

Conditioned |B| (same dataset), + others

Argued that |B| is passive scalar..

Bershadskii and Sreenivasan PRL '04

Fokker- Planck models
(see also fractional kinetics and Lévy flights)

Langevin equation

$$\frac{dx}{dt} = \beta(x) + \gamma(x)\eta$$

η stochastic iid

Fokker- Planck equation

$$\frac{\partial P(y,t)}{\partial t} = \nabla(A(y)P(y,t) + B(y)\nabla P(y,t))$$

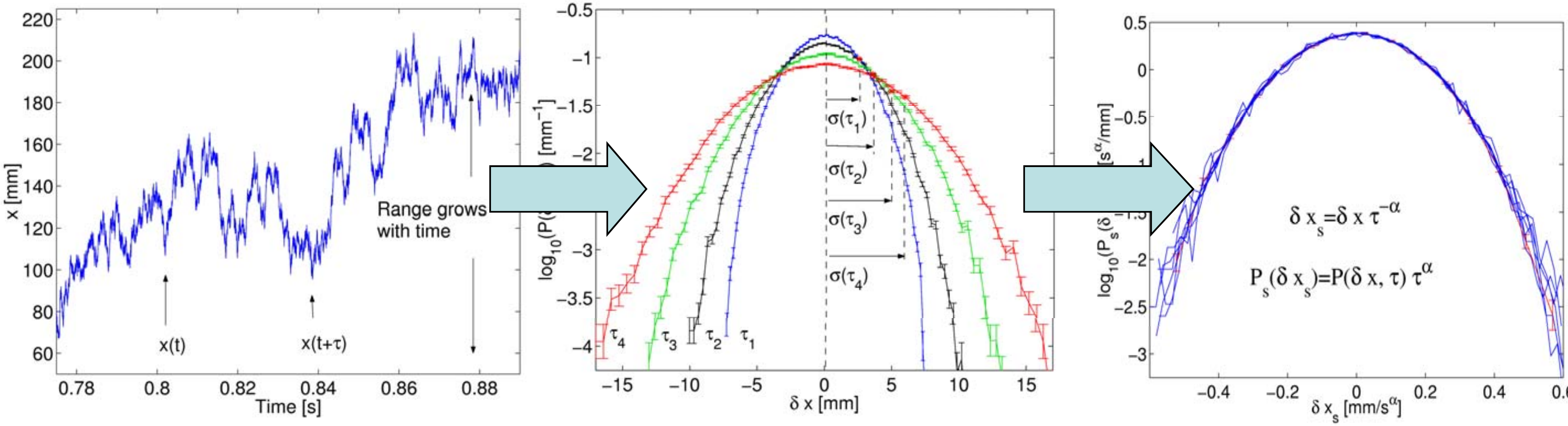
can solve for $P(y,t)$

Note: $y(t)$ is distance travelled in interval $t = \tau$ –a differenced variable

Renormalization-scaling system looks the same under

$$t' = \frac{t}{\tau},\ y' = \frac{y}{\tau^{\alpha}} \text{ and } \alpha \neq \frac{1}{2} \text{..........which implies } P(y',t') = \tau^{\alpha} P(y,t)$$

How 'differences' tell us about scaling - Brownian walk ('fractal')



1) difference the timeseries $x(t)$ on timescale τ to obtain $y(t,\tau) = x(t+\tau) - x(t)$

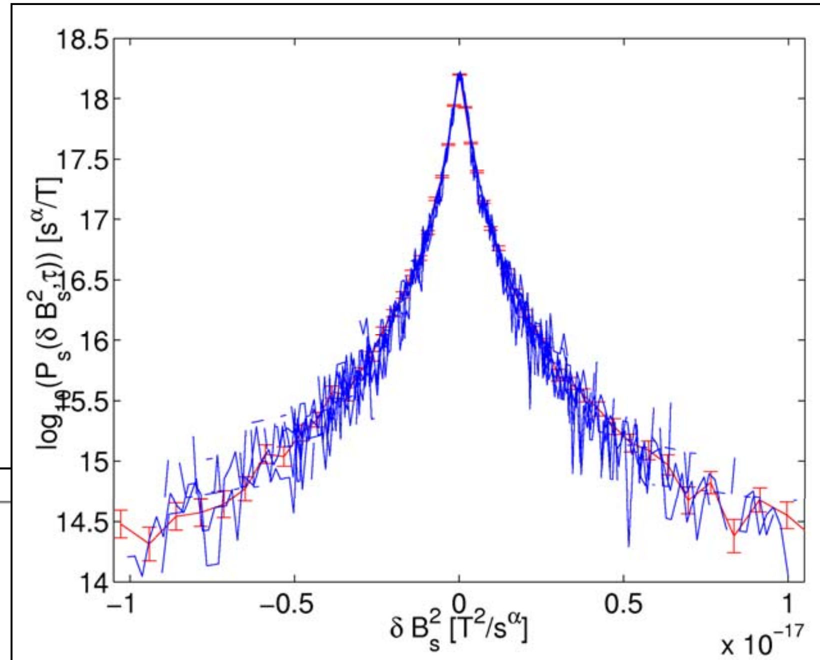
2) $P(y,\tau)$ are self- similar (fractal) - *if* same function under single parameter rescaling

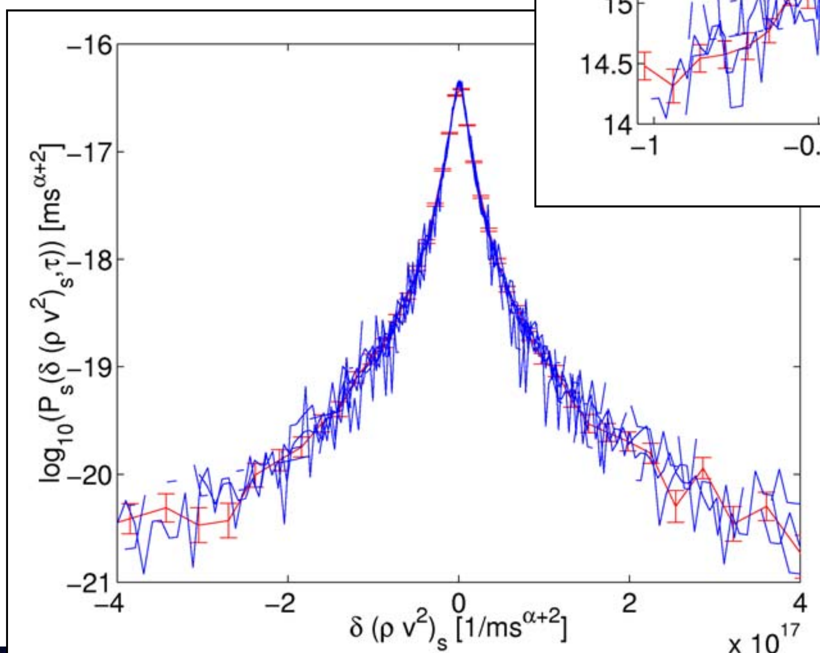
3) rescaling parameter comes from the data eg $\sigma(\tau) \sim \tau^{\alpha}, \alpha = \frac{1}{2}$ here

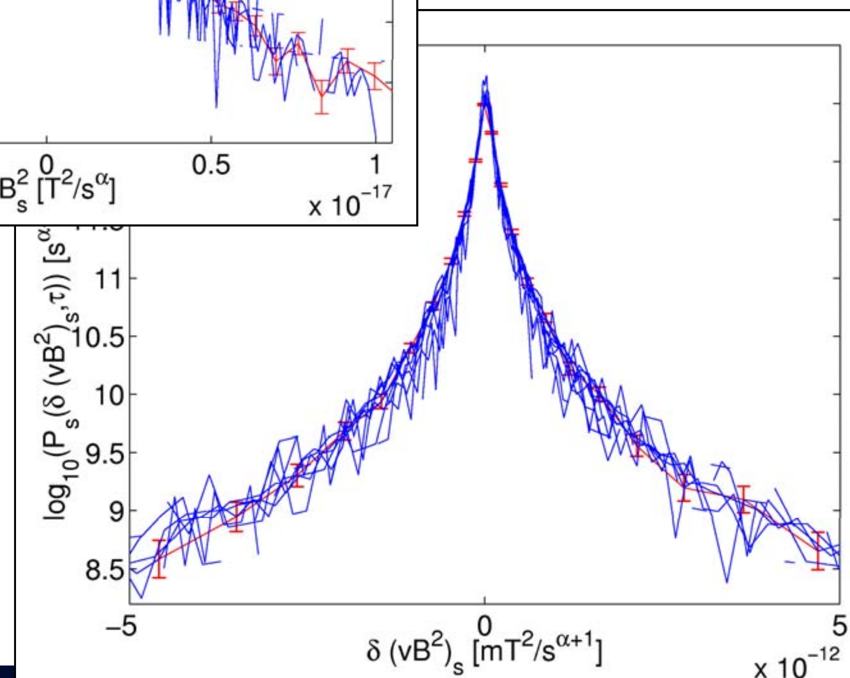
4) so moments of the PDF: $\left\langle y(t,\tau)^{p} \right\rangle_{t} \sim \tau^{\alpha p}$

Rescaled PDF of S=vB^2,ρv^2,B^2 ..

PDF shown to 10σ
(raw data)
Hnat et al PRE 2003



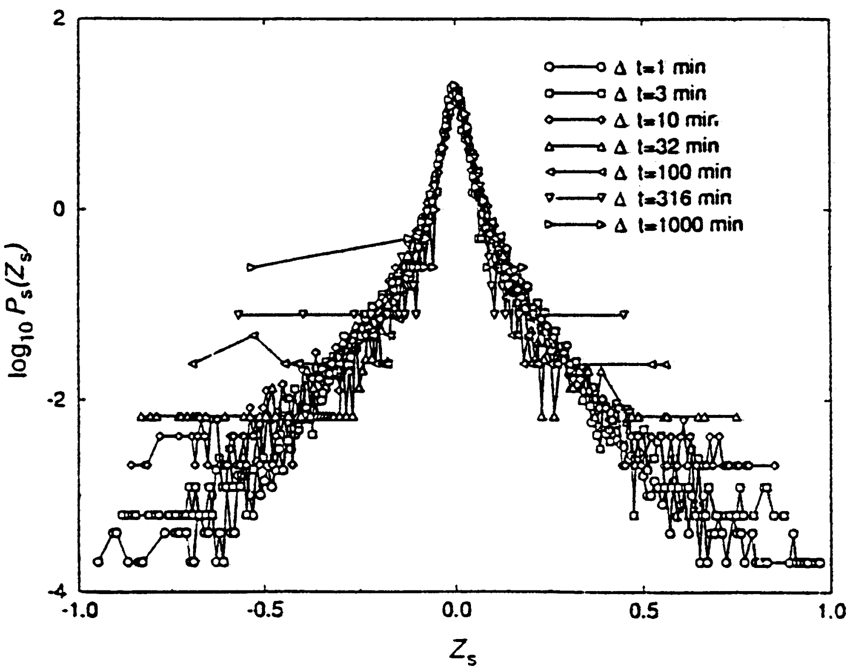




A not so simple fractal timeseries- financial markets

- *Mantegna and Stanley- Nature,1995*
- S+P500 index
- 'heavy tailed' distributions
- Brownian walk in log(price) is the basis of Black Scholes (FP model for price dynamics)
- Non- Gaussian PDF, fractal scaling- Fractional Kinetics or non- linear FP

in solar wind: Hnat, SCC et al PRE 2003, SCC et al NPG 2005



Structure functions-estimating the $\zeta(p)$ from data

Define structure function (generalized variogram) S_p for differenced timeseries:

$$y(t,\tau) = x(t+\tau) - x(t)$$

$S_p(\tau) = <|y(t,\tau)|^p> \propto \tau^{\zeta(p)}$ if scaling

We would like to calculate $S_p(\tau) = <|y(t,\tau)|^p> = \int_{-\infty}^{\infty} |y|^p \, P(y,\tau) dy$

then $S_p(\tau) = \tau^{\zeta(p)} \int_{-\infty}^{\infty} y_s^p P_s dy_s$

Conditioning- an estimate is:

$<|y|^p> = \int_{-A}^{A} |y|^p \, P(y,\tau) dy$ where $A = [10-20]\sigma(\tau)$

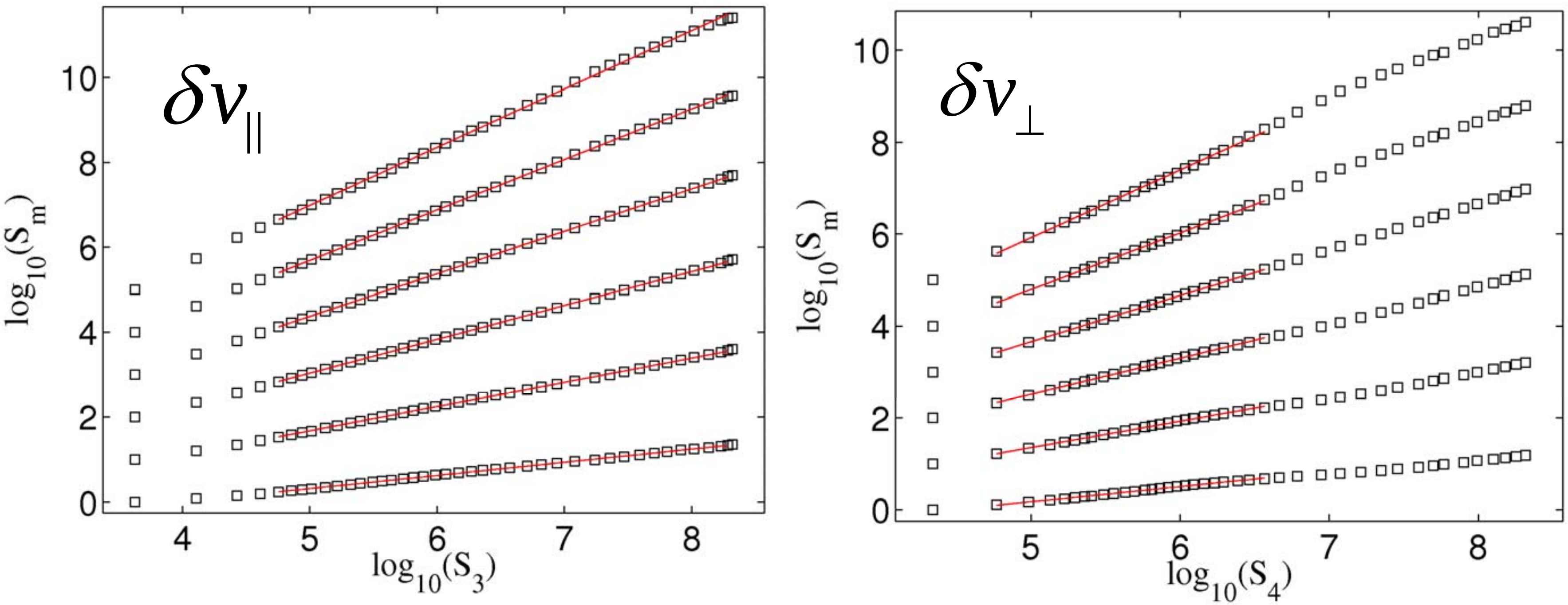
strictly ok if selfsimilar: $y \to y_s \tau^{\alpha}, P \to P_s \tau^{-\alpha}$, $\zeta(p) = p\alpha$

if ξ(p) is quadratic in p (multifractal)- weaker estimate

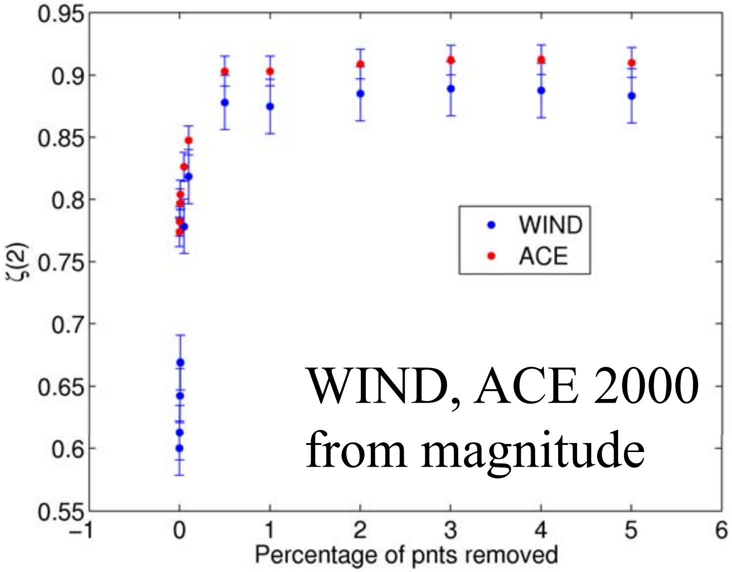
Confirmation of a scaling range- ESS plots:

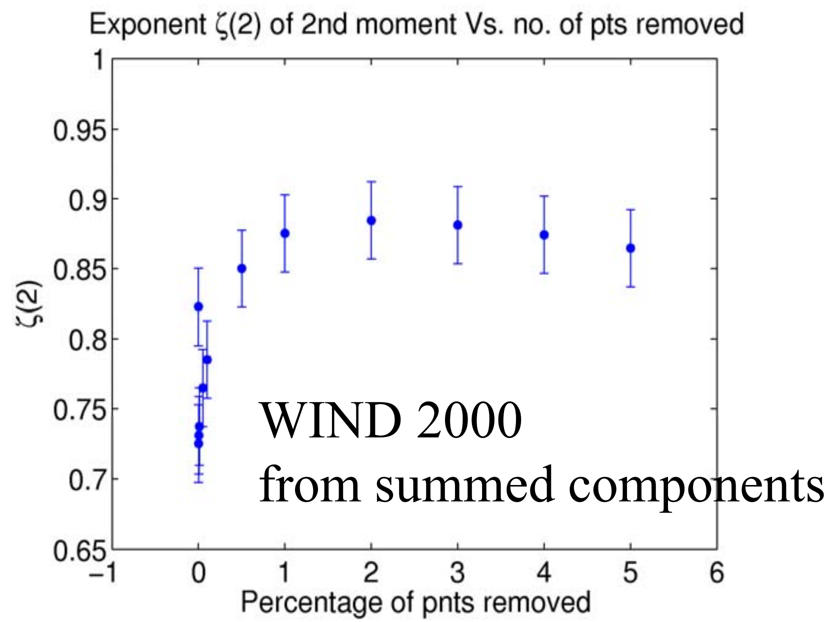
$S_p = <\left|\delta \mathbf{v}\bullet\hat{\mathbf{b}}\right|^p> \sim \tau^{\zeta(p)}$ and its remainder versus S_3, S_4 where $\zeta(3), \zeta(4) \approx 1$ respectively

ESS tests $S_p = G(\tau) S_q^{\zeta(p)/\zeta(q)}$



Seen both in WIND and ACE





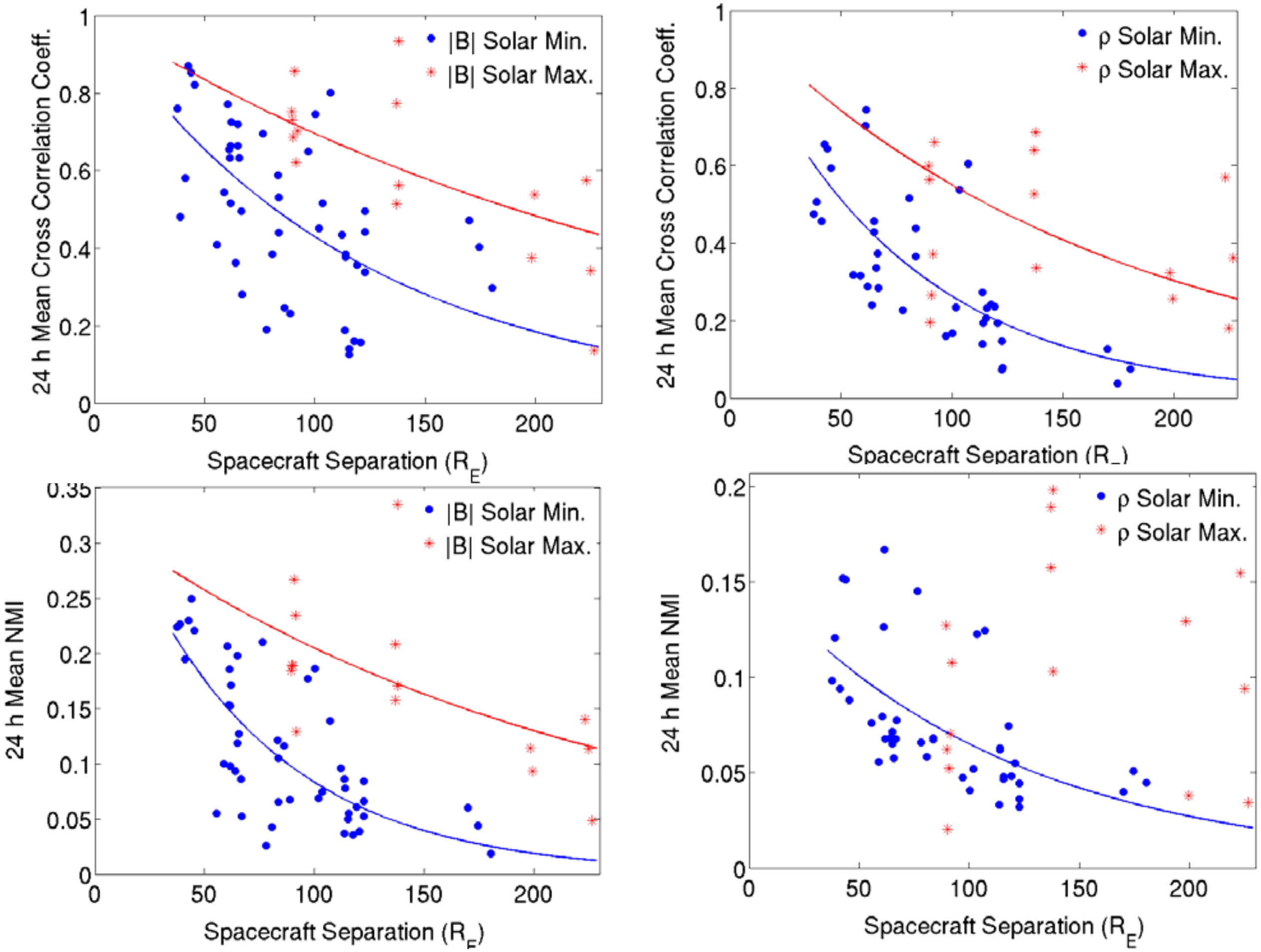
Scaling is sensitive to calibration?

Shown B^2

WIND: from summed components,
 and from magnitude

ACE: from magnitude

Solar Cycle Dependence of Solar Wind Correlation Length- between WIND-ACE



- Fits are of the form :

  $y = A \exp(-x/\lambda)$
- A = 1 for cross correlation fitting.
- Each point represents 24 hours of data
- Running mean subtracted.

Wicks, SCC et al, ApJ(2009)

Diffusion- random walk

Brownian random walk

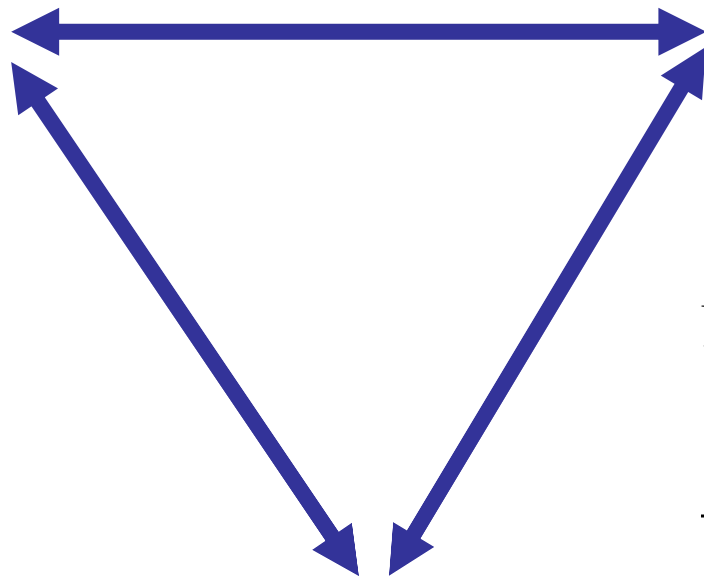
$$\frac{dx}{dt} = \eta$$

η is stochastic iid

diffusion equation

$$\frac{\partial P(y,t)}{\partial t} = D\nabla^2 P(y,t)$$

$\Rightarrow P(y,t)$ is Gaussian



Note: $y(t)$ is distance

travelled in interval $t = \tau$

–a differenced variable

Renormalization-scaling system looks the same under

$t' = \dfrac{t}{\tau}$, $y' = \dfrac{y}{\tau^{\alpha}}$ and $\alpha = \dfrac{1}{2}$..........which implies $P(y',t') = \tau^{\alpha} P(y,t)$

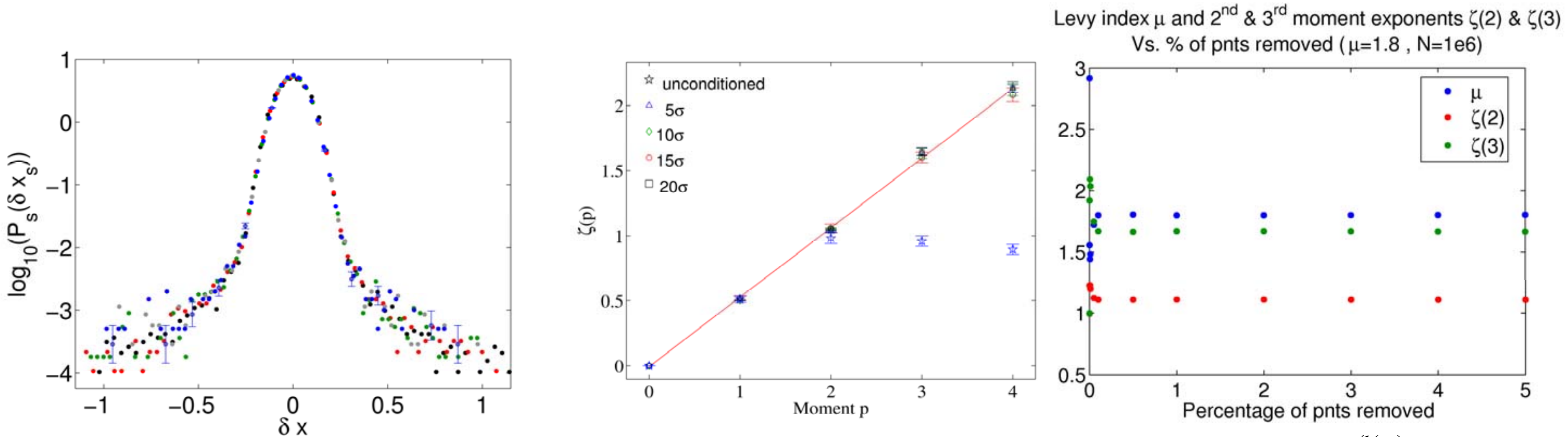
$\Rightarrow P(y,t)$ is Gaussian, the fixed point under RG

A more precise test for fractality-
outliers and convergence: example-Lèvy flight ('fractal')

$P(x) \sim \frac{C}{x^{1+\mu}}, x \to \pm\infty, 1 < \mu < 2$ power law tails, self similar

for a finite length flight $(x - \langle x \rangle)^2 \sim t^{2/\mu}$

so $\mu = 2$ is Gaussian distributed, Brownian walk



PDF rescaling $x \to x_s \tau^{\alpha}, P \to P_s \tau^{-\alpha}$

Structure functions: $S_p(\tau) = \langle |x(t,\tau)|^p \rangle \propto \tau^{\zeta(p)}$

expect $\zeta(p) \sim \alpha p, \alpha = 1/\mu$

SCC et al, NPG, 2005,Kiyani, SCC et al PRE (2006)

Velocity fluctuations parallel and perpendicular to the *local* B field direction

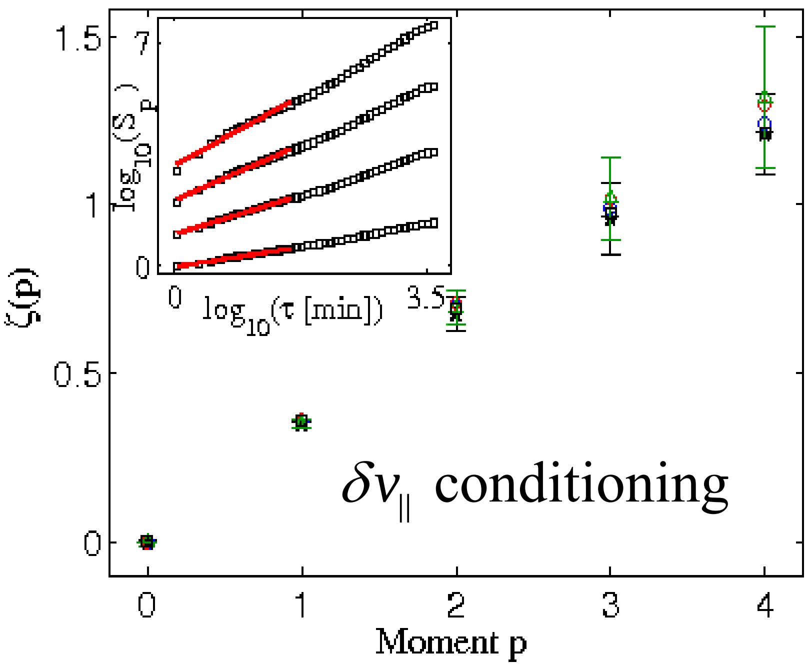
Exponents $\zeta(p)$ for $<\left|\delta v_{\parallel,\perp}\right|^{p}>\sim \tau^{\zeta(p)}$ for

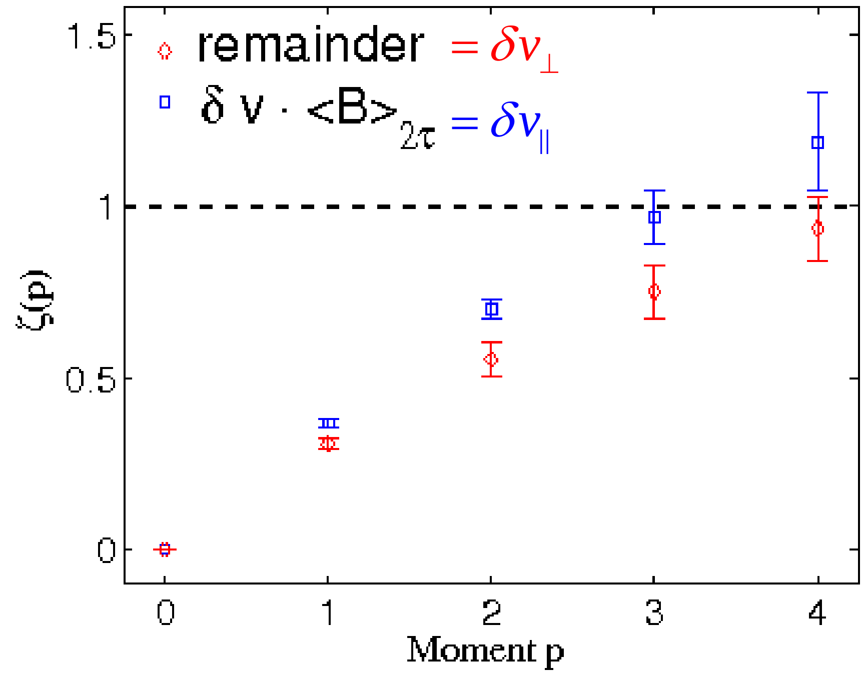
$\delta v_{\parallel} = \delta \mathbf{v}.\hat{\mathbf{b}}$ and its remainder $\delta v_{\perp} = \sqrt{\delta \mathbf{v}\bullet\delta \mathbf{v} - \left(\delta \mathbf{v}.\hat{\mathbf{b}}\right)^{2}}$

$\overline{\mathbf{B}} = \mathbf{B}(t) + \ldots + \mathbf{B}(t+\tau'),\ \hat{\mathbf{b}} = \dfrac{\overline{\mathbf{B}}}{\left|\overline{\mathbf{B}}\right|}$, here $\tau' = 2\tau$ and $\delta \mathbf{v} = \mathbf{v}(t+\tau) - \mathbf{v}(t)$

ACE 64s av. 1998-2001

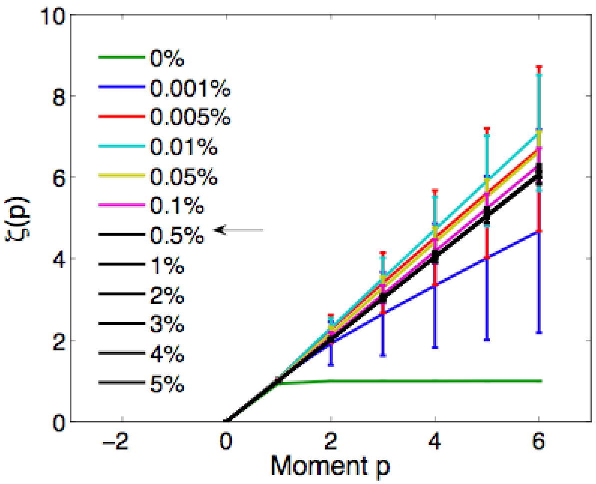
Chapman et al GRL (2007)

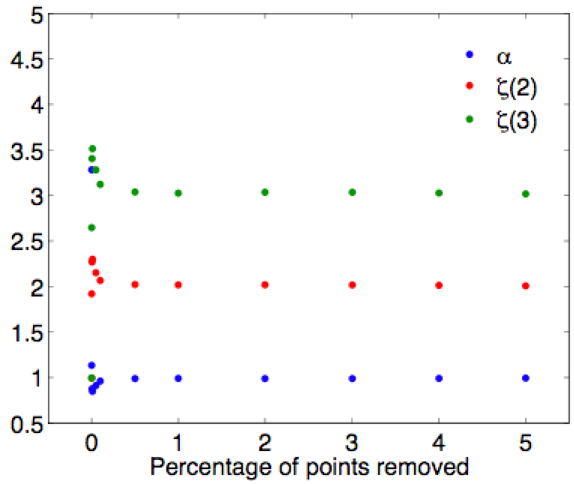




Distinguishing self- affinity (fractality) and multifractality

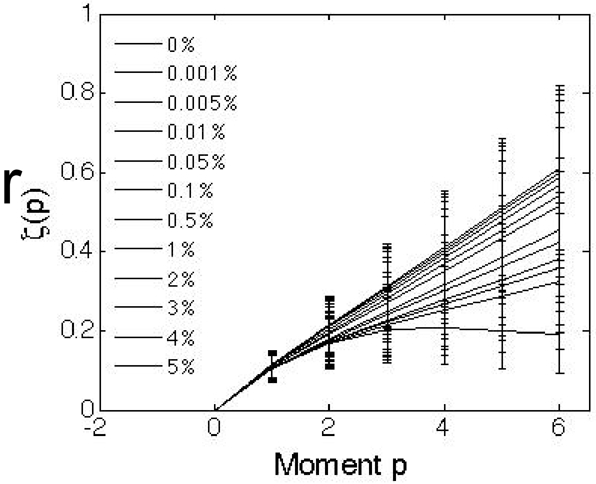
Levy flight -- Fractal

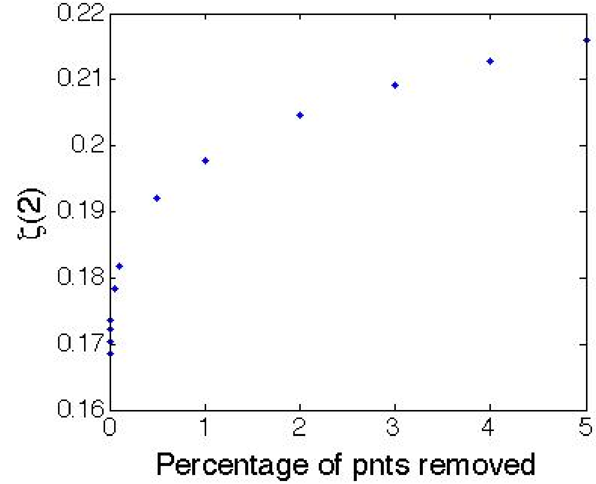




‘rank –order CDF test’

P-model -- Multifractal

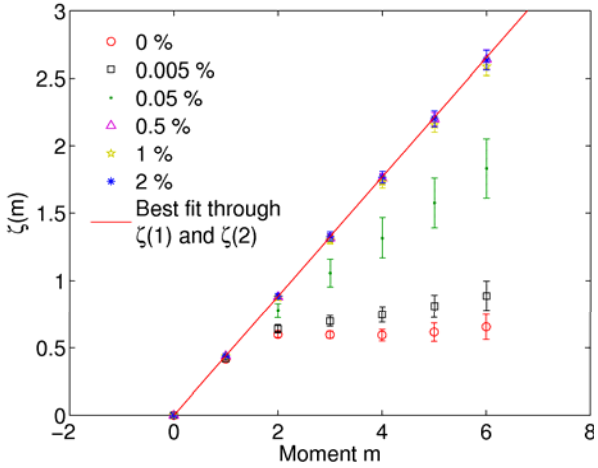




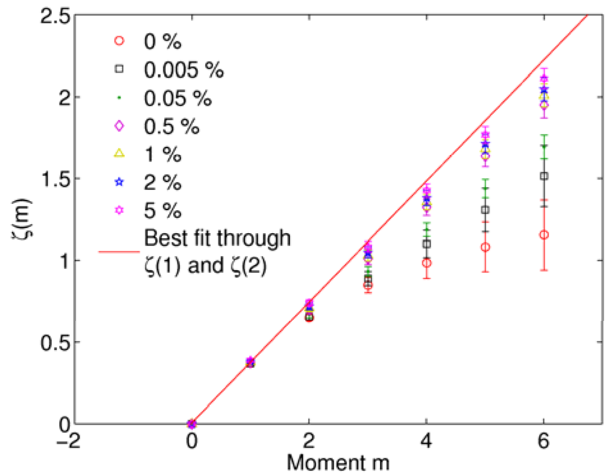
Kiyani, SCC et al, PRL (2007)

Solar cycle variation WIND Inertial Range-- $|B|^2$

2000 - Solar max



1996 - Solar min

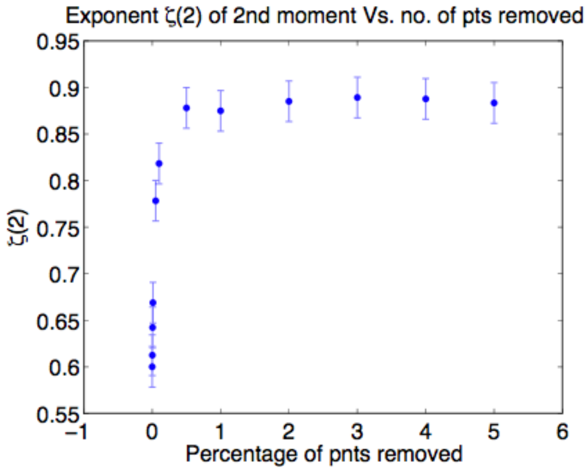


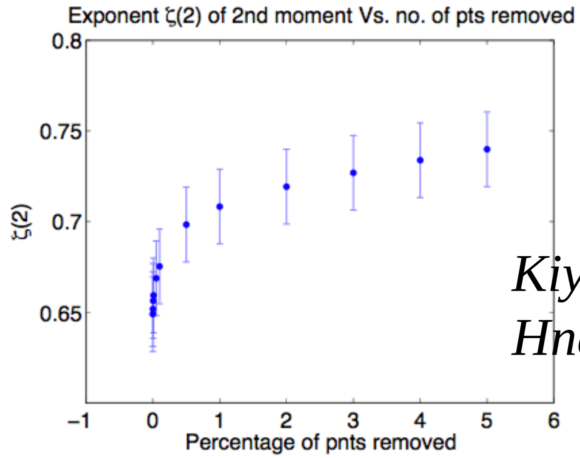
Fractal signature 'embedded' in (multifractal) solar wind inertial range turbulence
-coincident with complex coronal magnetic topology

Max/min:

Distinct topology of coronal fields?

Distinct fast wind?





Kiyani, SCC et al, PRL (2007), Hnat, SCC et al, GRL, (2007)

Anisotropy and intermittency free parameters-Kolmogorov vz MHD scaling

velocity difference $d_r v = v(l+r) - v(l)$, energy transfer rate $\varepsilon_{\mathrm{r}} \sim \dfrac{d_r v^2}{T}$

Kolmogorov: simply have T as the eddy turnover time $T \sim r/d_r v$ so that $\varepsilon_{\mathrm{r}} \sim \dfrac{d_r v^3}{r}$

MHD: now T is due to (say) Alfvenic collisions $T \sim \dfrac{r}{d_r v}\left(\dfrac{v_0}{d_r v}\right)^{\alpha}$ giving $\varepsilon_{\mathrm{r}} \sim \dfrac{d_r v^{3+\alpha}}{r}$

intermittency $\left\langle \varepsilon_r{}^p \right\rangle \sim \bar{\varepsilon}^p \left(r/L\right)^{\tau(p)}$

$\Rightarrow$ Kolmogorov: $\left\langle d_r v^p \right\rangle \sim r^{p/3} \bar{\varepsilon}^{p/3} \left(L/r\right)^{\tau(p/3)} \sim r^{\zeta(p)}$

$\Rightarrow$ MHD: same with $p/3 \to p/(3+\alpha)$ intermittency free $E(k) \sim \left\langle dv^2 \right\rangle / k \sim k^{-(5+\alpha)/(3+\alpha)}$

$\left\langle \varepsilon_r \right\rangle = \bar{\varepsilon}$ independent of r (steady state) so $\tau(1) = 0$ and $\zeta(\alpha+3) = 1$

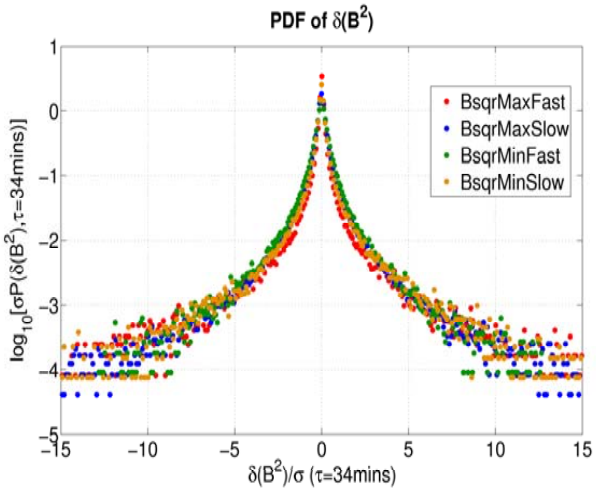
what is α?

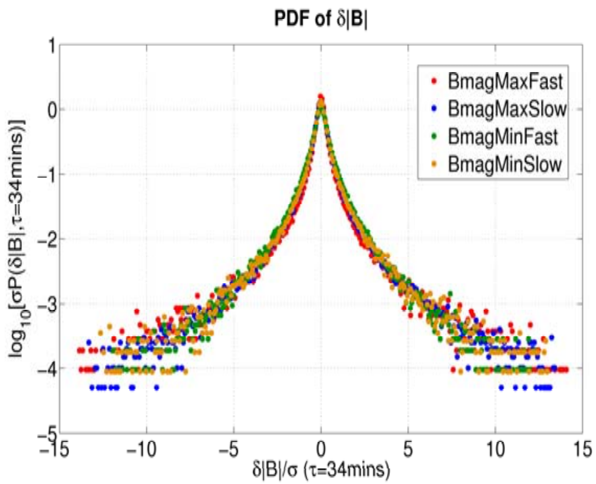
Kolmogorov Obukhov (1941) hydrodynamic: $\alpha=0$

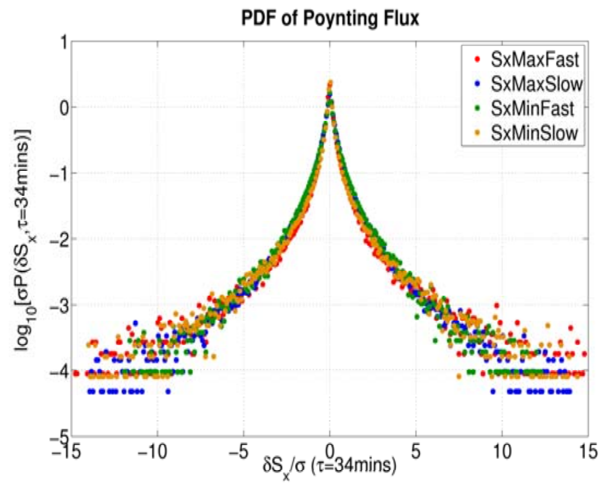
Irosnikov Kraichnan (1964) weak isotropic MHD $\alpha=1$,

Goldreich Sridhar (1994-5) strong MHD $\alpha_\perp = 0$

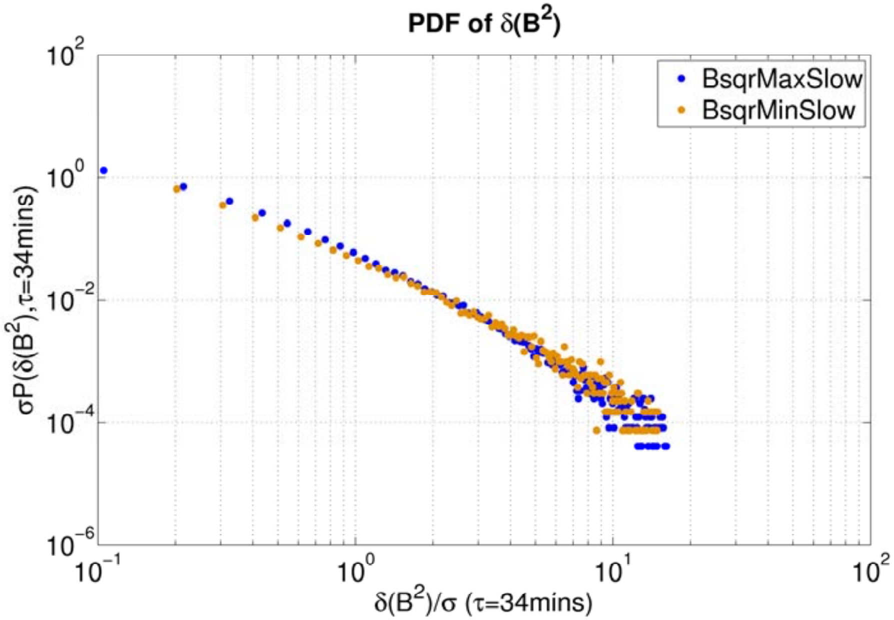
Boldyrev (2005) strong, background field anisotropic MHD $\alpha_\perp = 1$

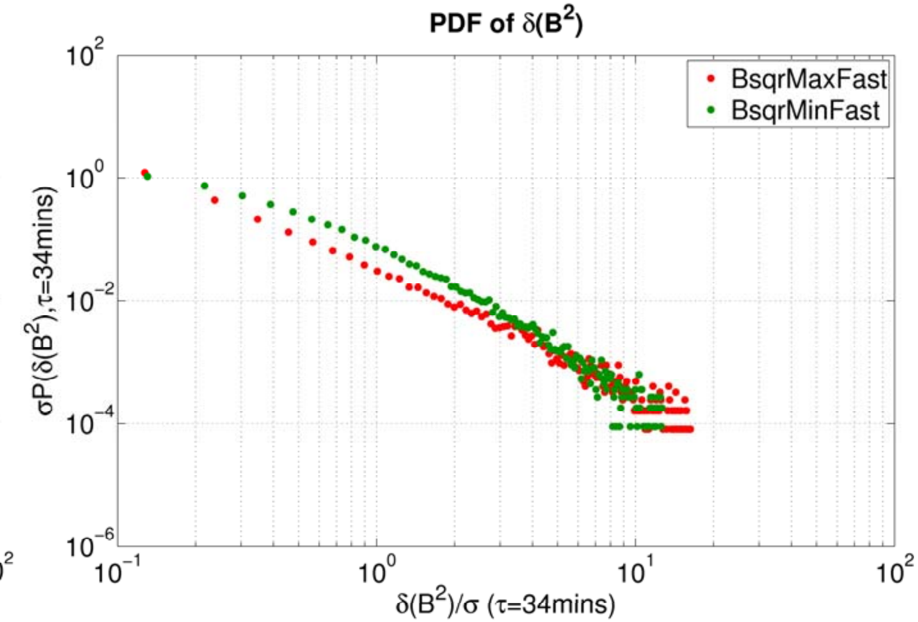






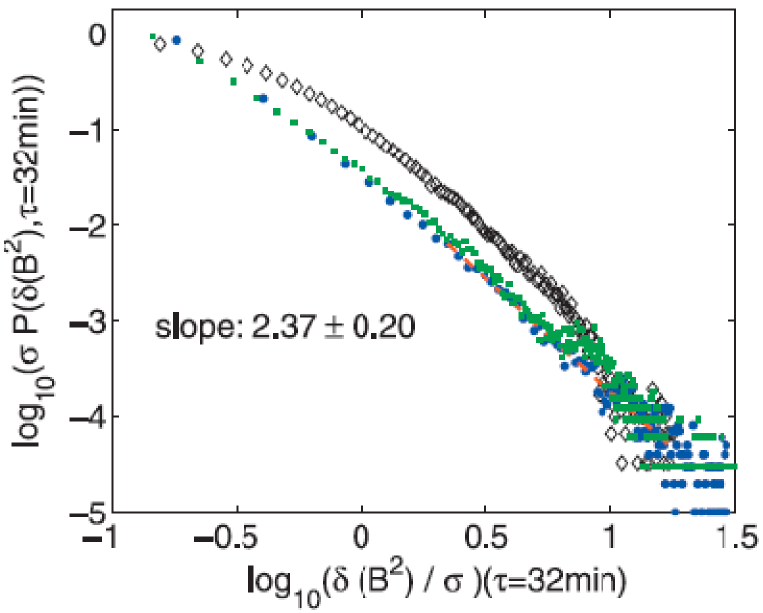
PDF functional form of fluctuations- require a more careful look..

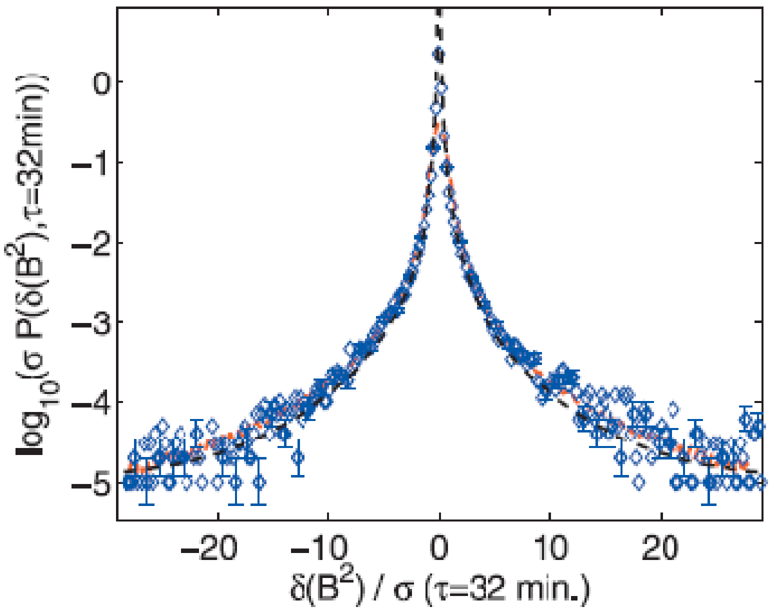




Fast and slow wind have different PDFs of fluctuations. For fast wind, these also vary with solar cycle.

Left: B² fluctuation PDF solar max and solar min
Right: solar max, FP and Lévy fit

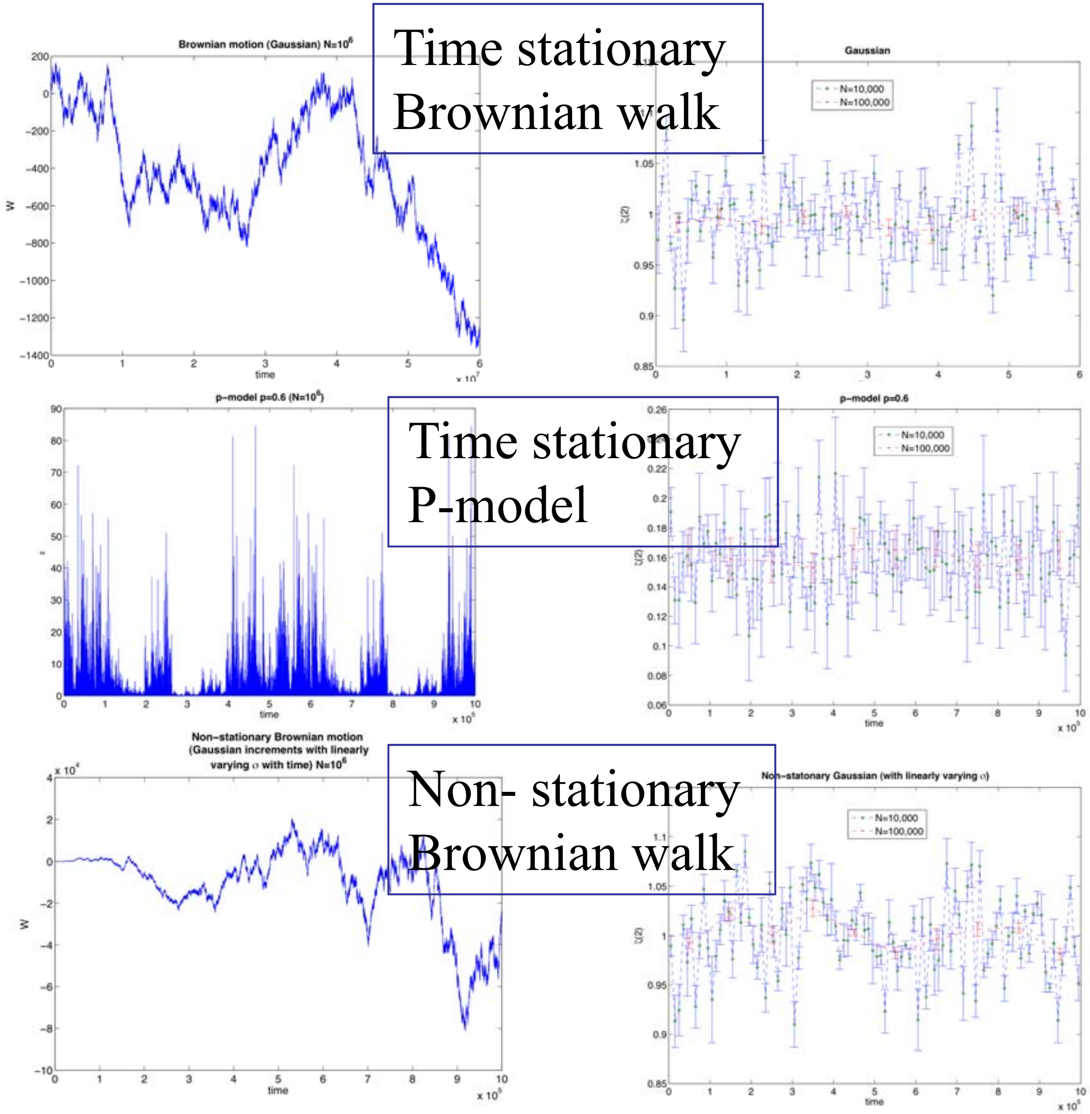




WIND 1996 min (◊), 2000 max (◦), ACE 2000 max (□)

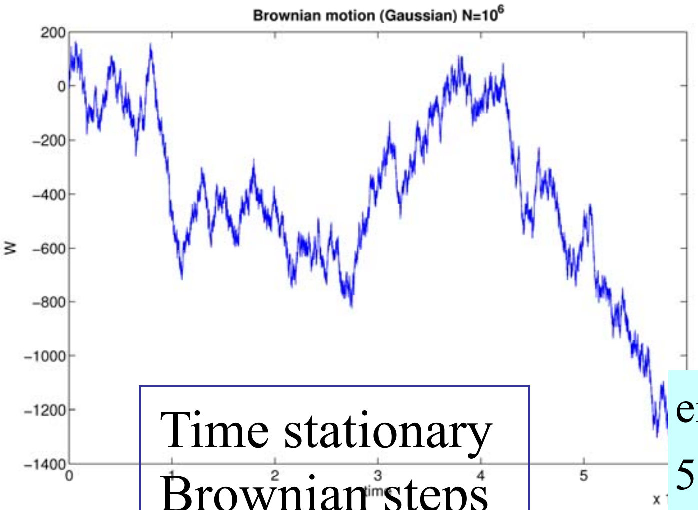
Hnat, SCC et al, GRL, (2007)

Finite sample effect- error on exponent ζ(2) as a function of sample size N

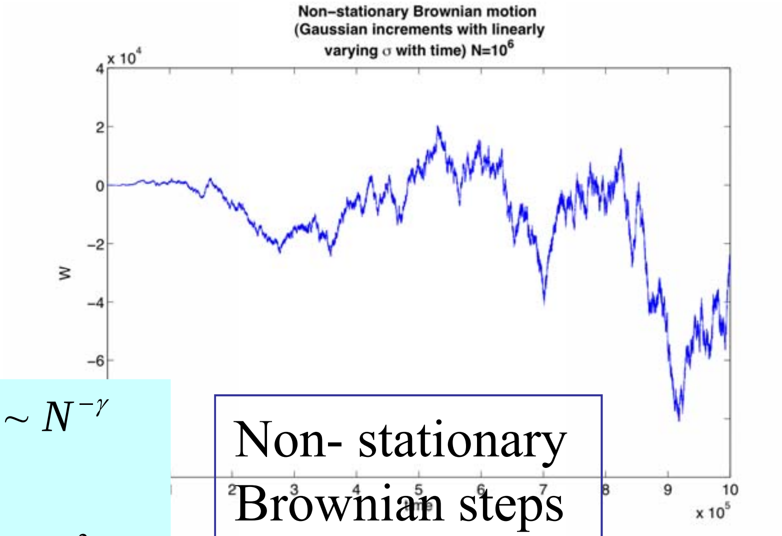


Kiyani, SCC et al, PRE submitted (2008)

Finite sample effect- error on exponent ζ(2) as a function of sample size N



Time stationary Brownian steps



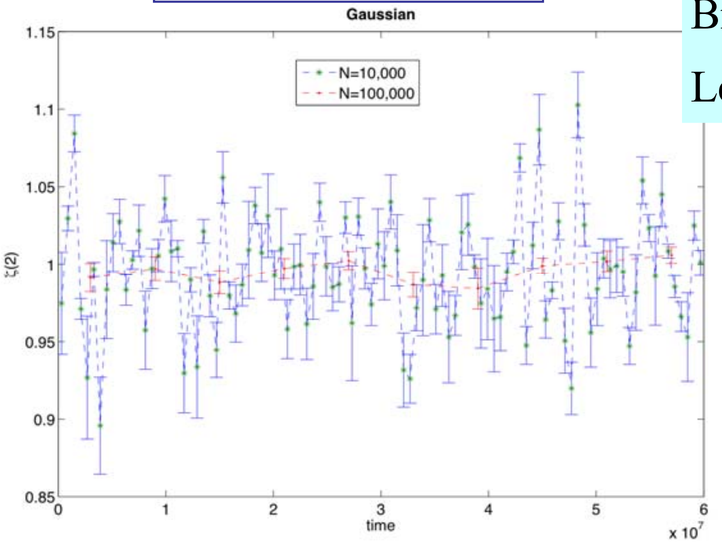
Non- stationary Brownian steps

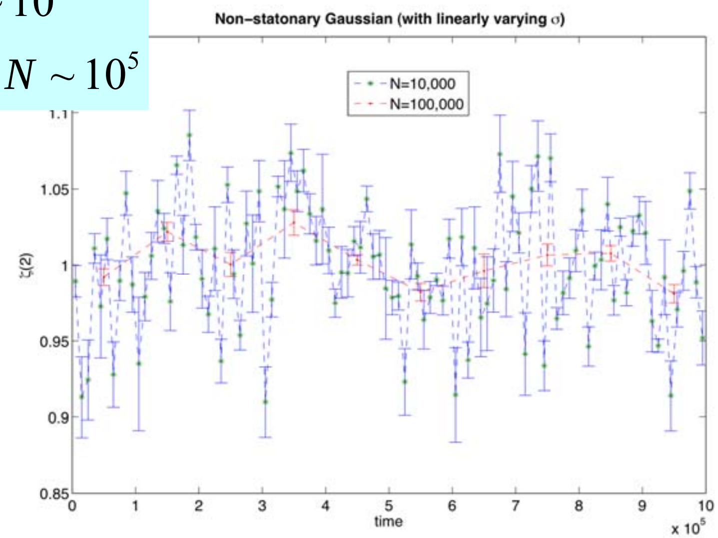
error on $\zeta(2) \sim N^{-\gamma}$

5% error:

Brownian: $N \sim 10^3$

Levy, p-model $N \sim 10^5$

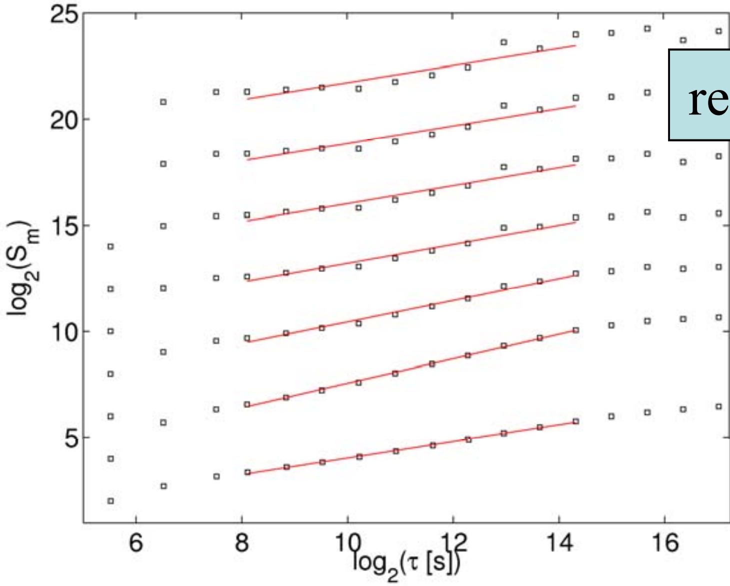




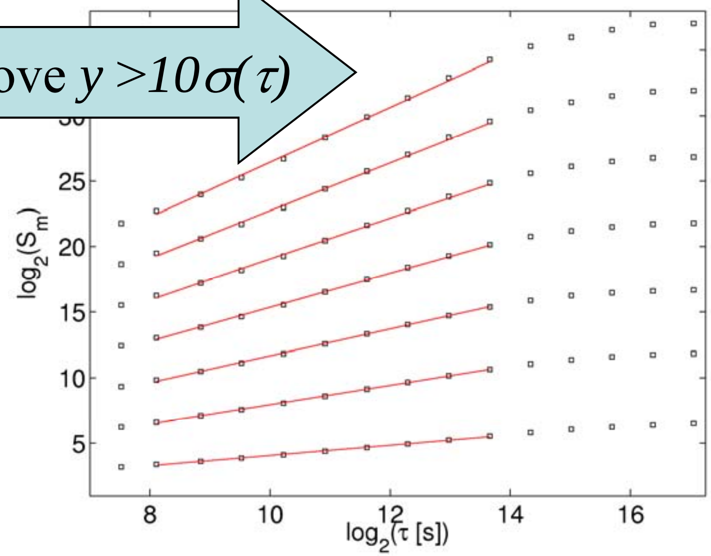
Kiyani, SCC et al, PRE submitted, 2008. See also Dudok De Wit, PRE, 2004

Structure functions- uncertainties (example - ρ in slow solar wind)

$$y(t,\tau) = x(t+\tau) - x(t) \text{ test for scaling - } S_m(\tau) = <|y(t,\tau)|^m> \propto \tau^{\zeta(m)}$$

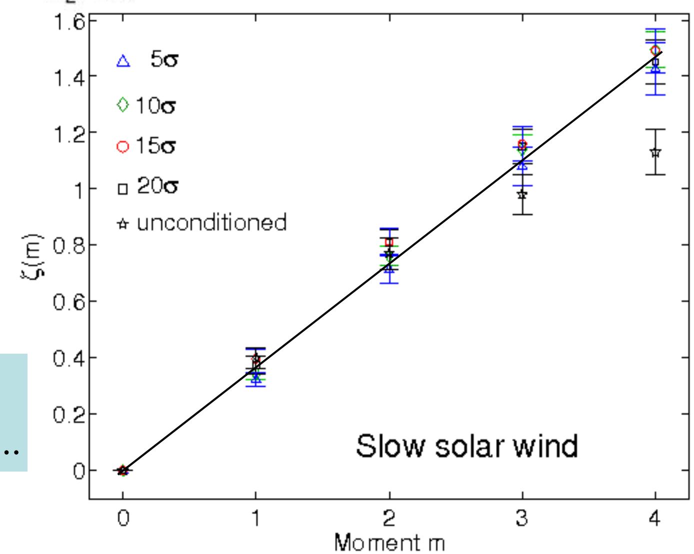


remove $y > 10\sigma(\tau)$



2 sources of uncertainty in exponent

1) Fitting error of lines (error bar estimates)
2) Outliers- Shown: removed < 1% of the data
   ACE 98-01 (4years)-10^6 samples.
   Threshold 450 km/sec.



fractal or multifractal?

fractal (self- affine) $\zeta(p) \sim \alpha p$

multifractal $\zeta(p) \sim \alpha p - \beta p^2 + \ldots$

cf Fogedby et al PRE 'diffusion in a box'

Dynamical model for self similar fluctuations

If the PDF of fluctuations $y = x(t+\tau) - x(t)$ on timescale τ is selfsimilar:

$$P(y,\tau) = \tau^{-\alpha} P_s(y\tau^{-\alpha})$$

P is then a solution of a Fokker- Planck equation:

$$\frac{\partial P}{\partial \tau} = \nabla\left[AP + B\nabla P\right],$$ where transport coefficients $A = A(y), B = B(y)$

with $A \propto y^{1-1/\alpha}, B \propto y^{2-1/\alpha}$ we solve the Fokker- Planck for P_s

This corresponds to a Langevin equation: $\frac{dx}{dt} = \beta(x) + \gamma(x)\xi(t)$

and we can obtain β, γ via the Fokker- Planck coefficients

see Hnat, SCC et al. Phys. Rev. E (2003), Chapman et al, NPG (2005)