Investigating the dynamics of the magnetosphere using various complexity measures

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Motivation

- ***Dynamical complexity detection*** for output time series of complex systems:

  one of the foremost problems in physics, etc.

- In space plasma physics: ***accurate detection of the dissimilarity between normal and abnormal states (e.g. pre-storm activity and magnetic storms)*** can vastly improve space weather diagnosis and, consequently, the mitigation of space weather hazards.

Outline

- From pre-storm activity to magnetic storms: a transition described in terms of fractal dynamics

- Dynamical complexity in D_{st} time series using non-extensive Tsallis entropy

Power-laws

- If a time series is a temporal fractal then a power law of the form:

$$S(f) \sim f^{-\beta}$$

is obeyed,

$S(f)$ - power spectral density

f - frequency

β - spectral scaling exponent, a measure of the strength of time correlations

r - linear correlation coefficient: represents the fit of the time series to a power-law

Wavelets

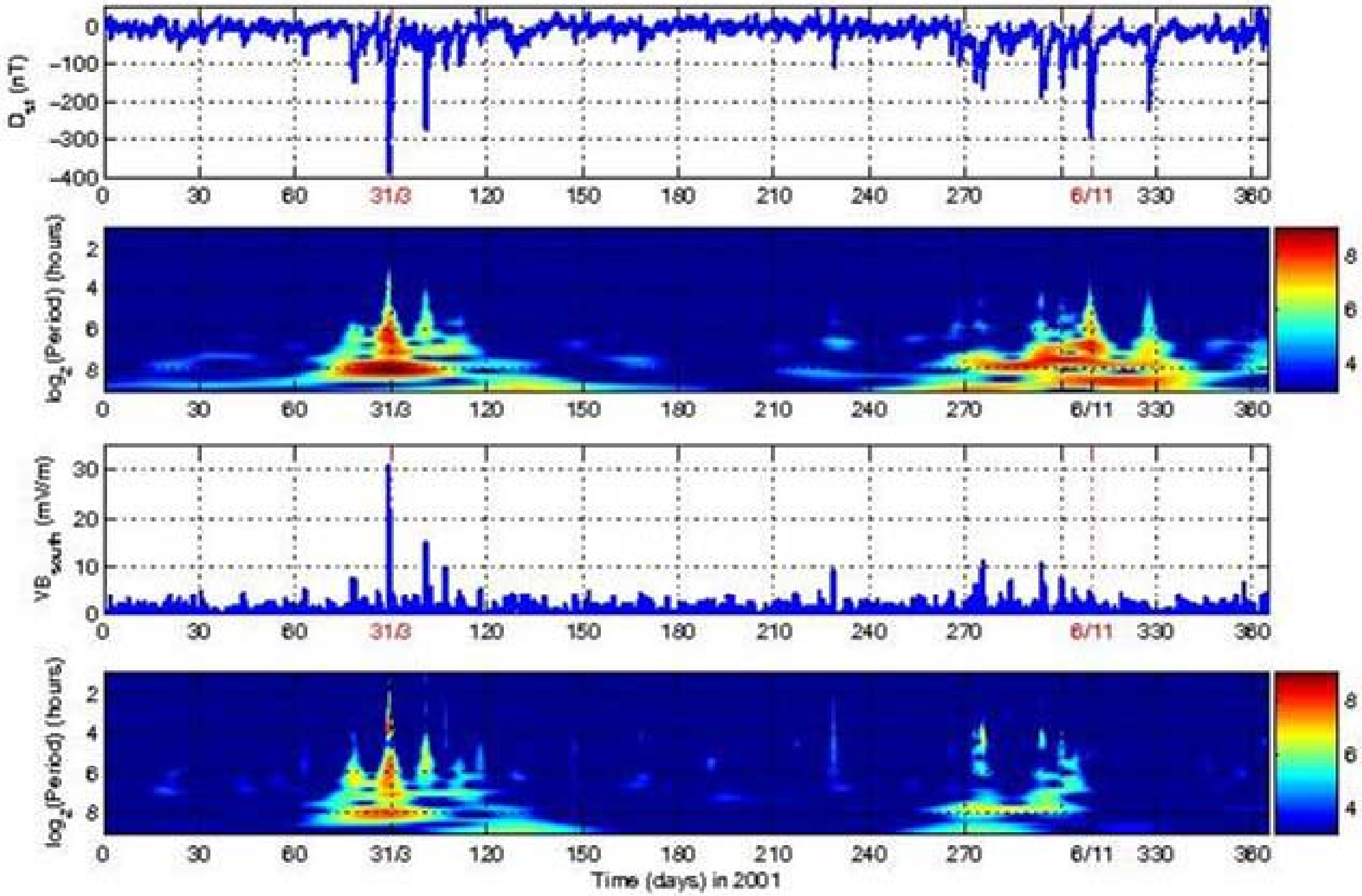
- The wavelet transform is useful for the *Dst* time series because *Dst* is non-stationary and has a time-varying frequency content.
- The wavelet analysis technique has been applied to the *Dst* time variations in order to derive the coefficients of its power spectrum.
- The continuous wavelet transform has been used, with the Morlet wavelet as basis function.

Fractal Spectral Analysis

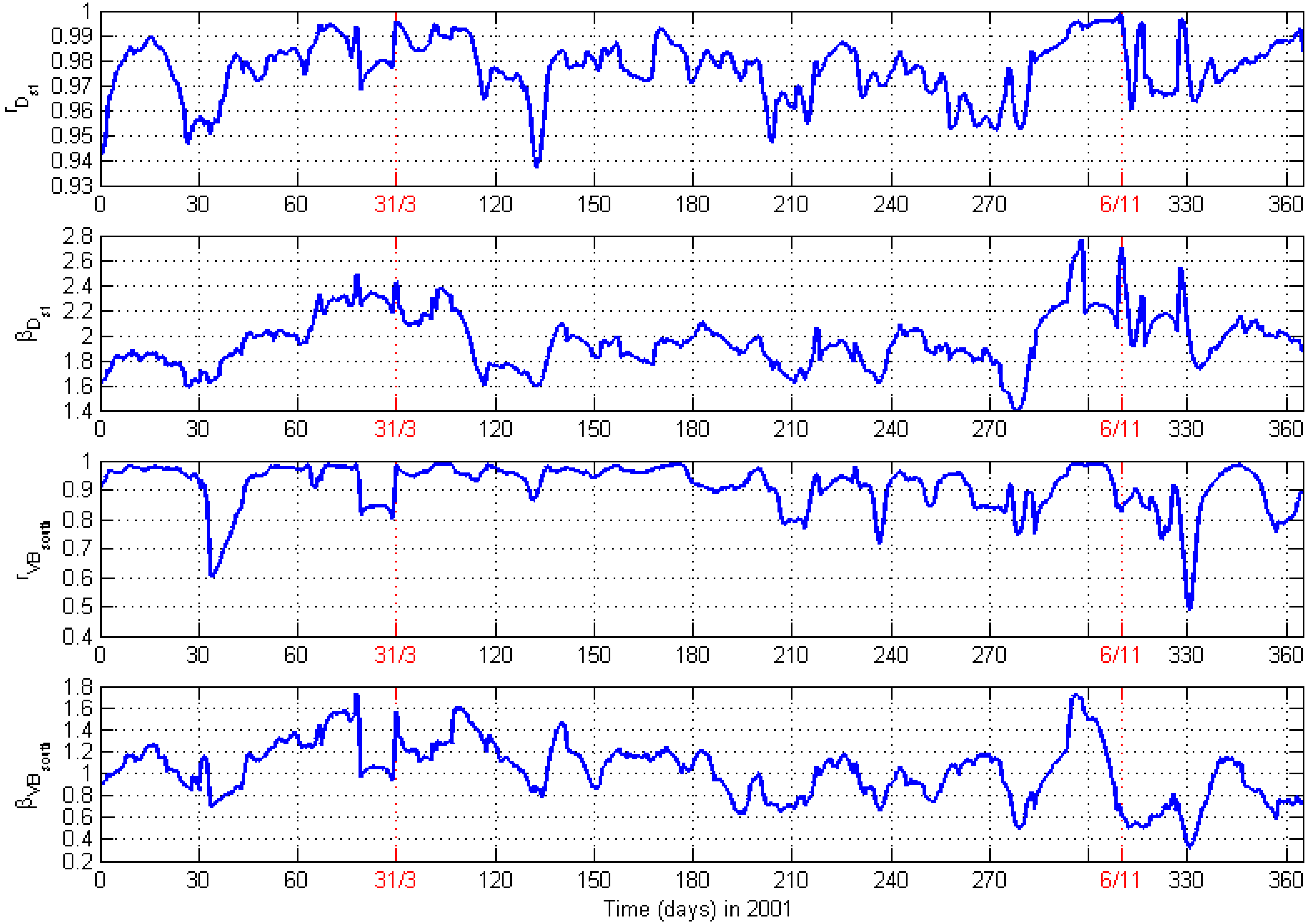
- Hourly Dst values of year 2001
- Wavelet transform to a matrix with 65 x (365 x 24) elements, where 65 is the number of frequencies.
- Power spectral densities are estimated in the frequency range from 2 to 128 hours using a moving window of 256 samples.
- The number of samples by which the moving window sections overlap is 255.
- Spectral parameters *r* and *β* were calculated for each window.

- First figure shows the *Dst* time series and its wavelet power spectrum
- Second figure shows the temporal evolution of its spectral parameters *r* and *β*

D_{st} index time series



Scaling parameters of the D_{st} index



Transition from anti-persistent to persistent behavior

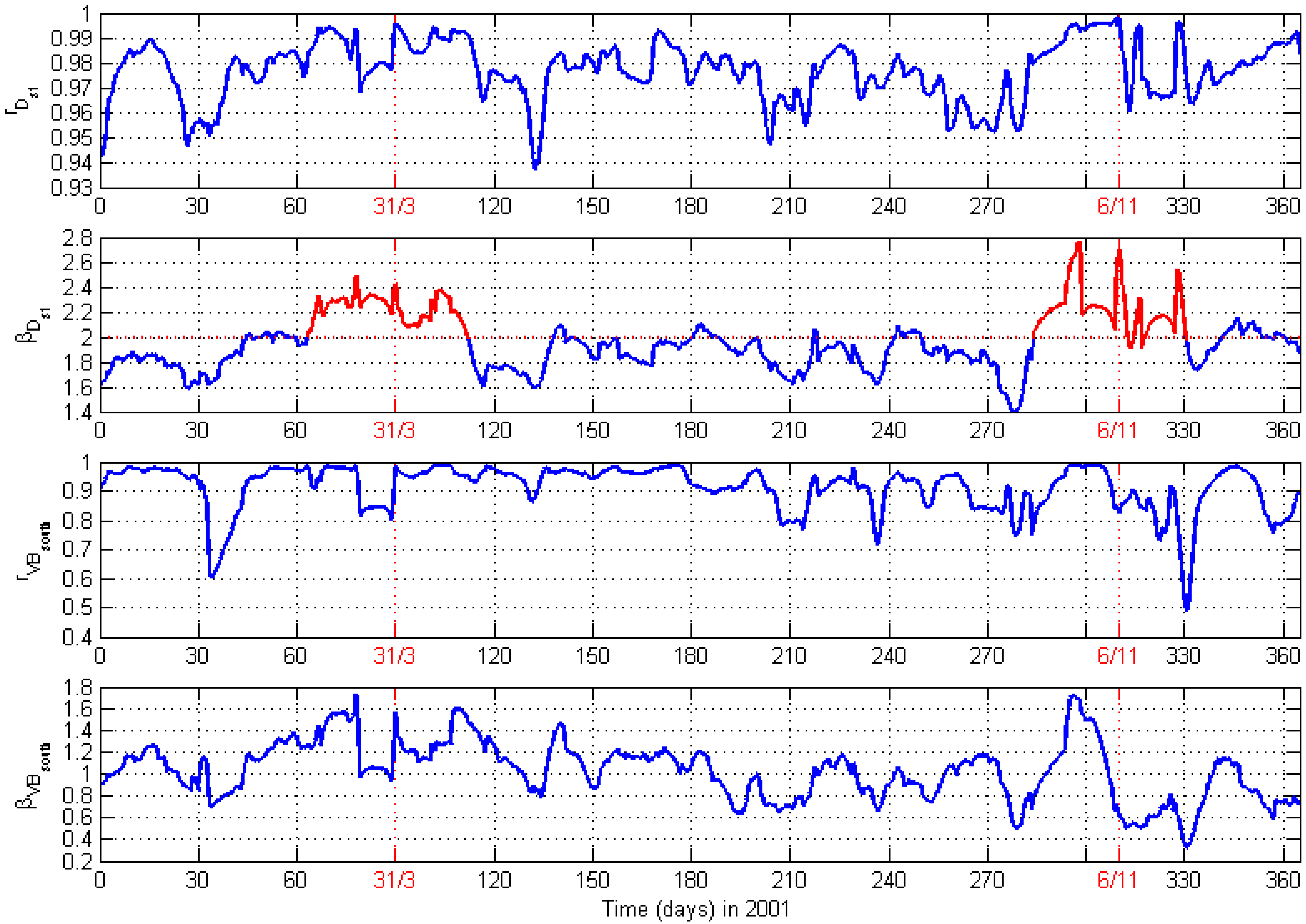
$\beta=2H+1$, where H is the Hurst exponent.

- The exponent H characterizes the ***persistent/anti-persistent*** properties of the signal. The range $0<H<0.5$ $(1<\beta<2)$ during the normal period indicates ***anti-persistency***, reflecting that if the fluctuations increase in a period, they are likely to decrease in the interval immediately following and vice versa.

Transition from anti-persistent to persistent behavior

- We pay attention to the fact that the time series exhibits ***persistent*** properties ($0.5<\boldsymbol{H}<1,\ 2<\boldsymbol{\beta}<3$) close to the two MS, meaning that if the amplitude of fluctuations increases in a time interval it is likely to continue increasing in the interval immediately following.

Scaling parameters of the D_{st} index



Transition from anti-persistent to persistent behavior

- ***H*=*0.5 (β=2)*** suggests no correlation between the repeated increments. Consequently, this particular value has a special physical meaning:

  It marks the transition between persistent and anti-persistent behavior in the time series.

Summary

- We show that distinctive alterations in scaling parameters of D_{st} index time series occur as an intense magnetic storm approaches.

- The transition from anti-persistent to persistent behavior may indicate that the onset of an intense magnetic storm is imminent.

Outline

- **From pre-storm activity to magnetic storms: a transition described in terms of fractal dynamics**

- **Dynamical complexity in D_{st} time series using non-extensive Tsallis entropy**

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Introduction

- The uncertainty of an open system state can be quantified by the ***Boltzmann-Gibbs (B-G) entropy***, which is the widest known uncertainty measure in statistical mechanics.
- ***B-G entropy** (S_{B-G}) cannot, however, describe non-equilibrium physical systems with large variability and multi-fractal structure such as the solar wind [Burlaga et al., 2007].*
- Inspired by multi-fractal concepts, ***Tsallis*** [1988, 1998] proposed a generalization of the B-G statistics.

Extensivity

- One of the crucial properties of the S_{B-G} in the context of classical thermodynamics is ***extensivity***, namely proportionality to the number of elements of the system.

- The S_{B-G} satisfies this prescription if the subsystems are statistically (quasi-) independent, or typically if the correlations within the system are essentially local. In such cases the system is called ***extensive***.

Tsallis entropy

- In general the situation is not of this type and correlations may be far from negligible at all scales. In such cases the S_{B-G} is ***non-extensive***.
- ***Tsallis*** [1988, 1998] introduced an entropic expression characterized by an index ***q*** which leads to non-extensive statistics

$$S_q = k\frac{1}{q-1}\left(1-\sum_{i=1}^{W} p_i^q\right)$$

where p_i^q are the probabilities associated with the microscopic configurations, W is their total number, q is a real number, and k is Boltzmann's constant.

Tsallis entropy

- The value of q is a measure of the **non-extensivity** of the system: $q = 1$ corresponds to the standard, extensive, B-G statistics.
- This is the basis of the so called non-extensive statistical mechanics, which generalizes the B-G theory.

Tsallis entropy and complexity

- Time variations of Tsallis entropy for a given q (S_q) quantify the dynamic changes of the complexity of the system.
- *Lower S_q values characterize the portions of the signal with lower complexity.*
- Herein, we estimate S_q based on the concept of ***symbolic dynamics***: from the initial measurements we generate a sequence of symbols, where the dynamics of the original system has been projected [Bailin, 1989].

Tsallis entropy in terms of symbolic dynamics

- ***Symbolic dynamics*** is based on a coarse-graining of the measurements, i.e., the original D_{st} time series of length N, $(X_1, X_2, \ldots, X_N)$, is projected to a symbolic time series $(A_1, A_2, \ldots, A_N)$ with An from a finite alphabet of λ letters $(0, \ldots, \lambda - 1)$.
- After symbolization, the next step in identification of temporal patterns is the construction of symbol sequences with size L. We use the technique of lumping. Thus, we stipulate that the symbolic sequence is to be read in terms of distinct successive “blocks” of length L,

$$A_1, A_2, \ldots, A_L \,/\, A_{L+1}, \ldots, A_{2L} \,/\, A_{jL+1}, \ldots, A_{(j+1)L}\ .$$

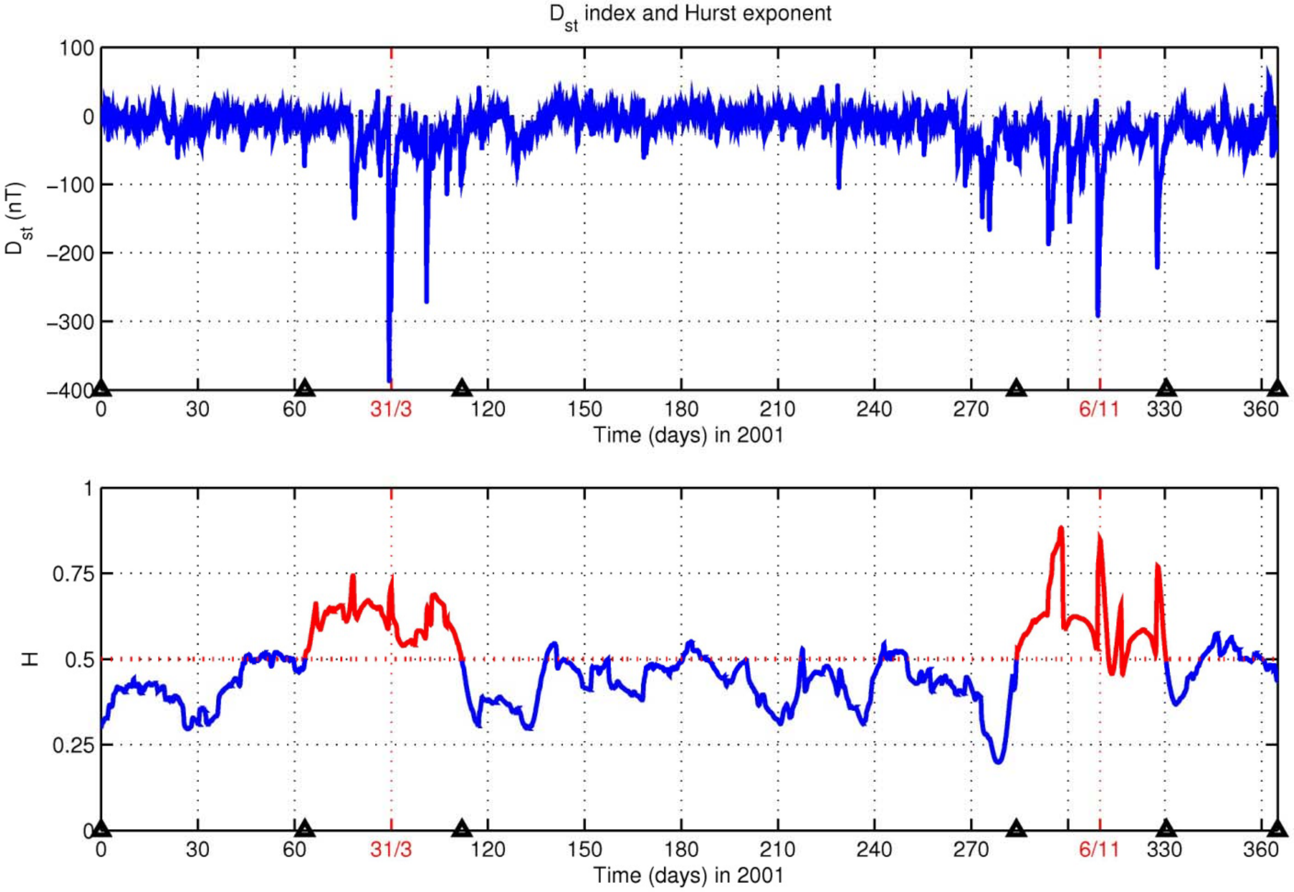
Tsallis entropy in terms of symbolic dynamics

- The simplest possible coarse graining of the D_{st} index is given by choosing a threshold C (usually the mean value of the data) and assigning the symbols “1” and “0” to the signal, depending on whether it is above or below the threshold (binary partition).
- Thus, we generate a symbolic time series from a 2-letter ($\lambda = 2$) alphabet ($0,1$), e.g. *0110100110010110…* .

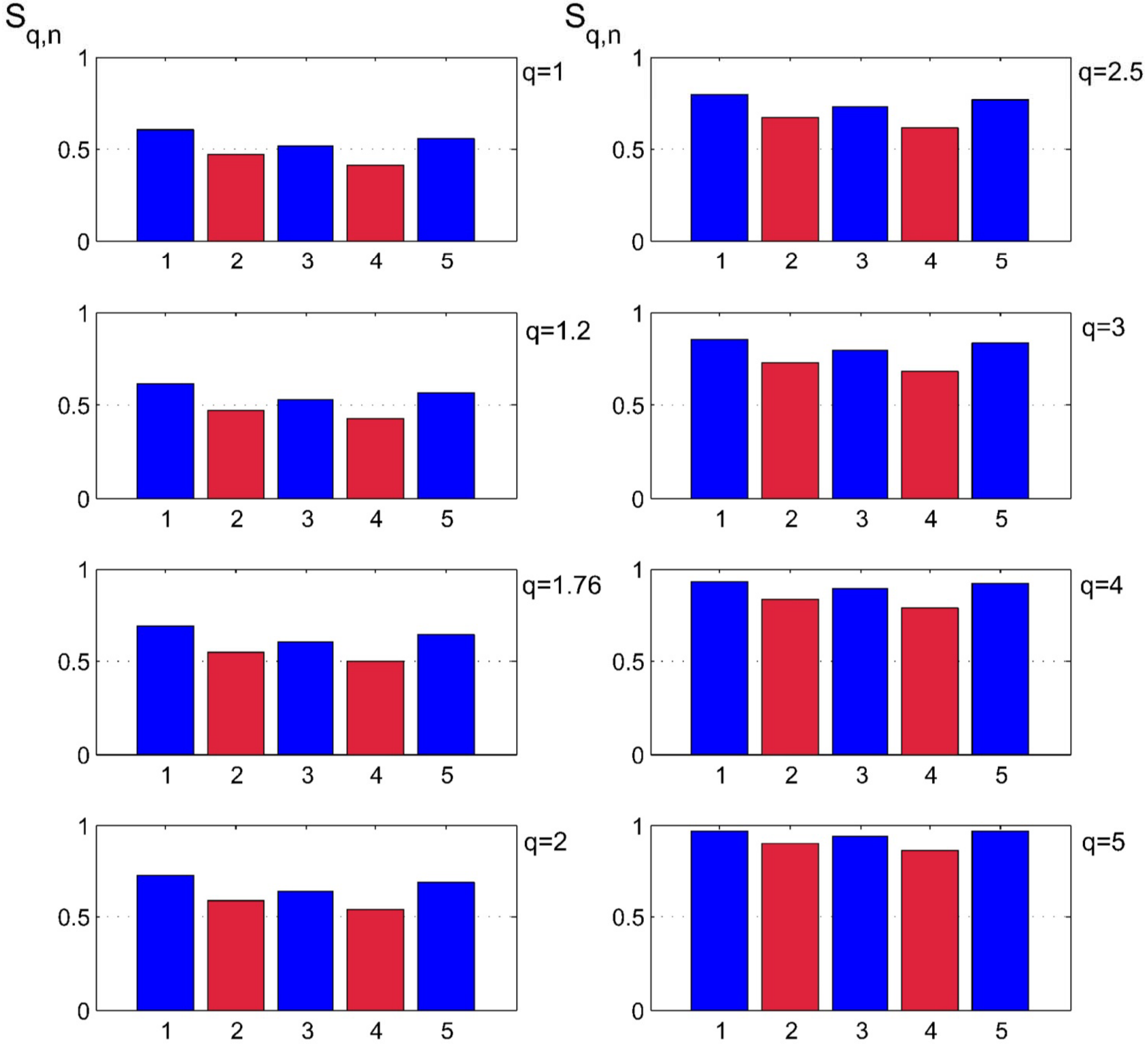
Tsallis entropy in terms of symbolic dynamics

- Reading the sequence by lumping of length *L=2,* the number of all possible kinds of blocks is $\lambda^{L} = 2^{2} = 4$, namely *00, 01, 10, 11*.
- Thus, the required probabilities for the estimation of the Tsallis entropy p_{00}, p_{01}, p_{10}, p_{11} are the fractions of the blocks *00, 01, 10, 11* in the symbolic time series.

Five time windows according to H=0.5



Tsallis entropies for the 5 windows and for various values of index q



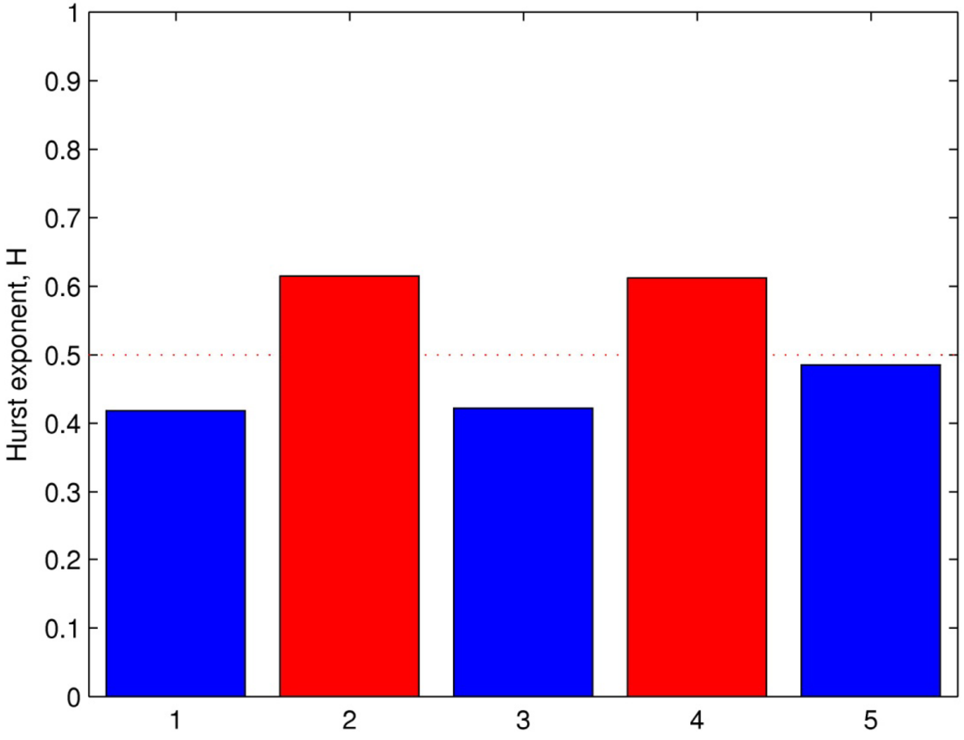
Conclusions

- The Tsallis entropy sensitively shows the complexity dissimilarity among different ***"physiological" (non-storm)*** and ***"pathological" states (magnetic storms)***. The Tsallis entropy implies the emergence of two distinct patterns:
- (i) a pattern associated with the intense magnetic storms, which is characterized by a higher degree of organization (lower Sq).
- (ii) a pattern associated with non-storm periods, which is characterized by a lower degree of organization.

Conclusions

- Results depend on Tsallis q value. Values in the range $1<q<2$ magnify differences of S_q and therefore of complexity as MS approaches.

Hurst index for same time windows



Conclusions

- The wavelet spectral analysis in terms of Hurst exponent, *H*, also shows the existence of two different patterns:
- (i) a pattern associated with the intense magnetic storms, which is characterized by a fractional Brownian persistent behavior
- (ii) a pattern associated with non-storm periods, which is characterized by a fractional Brownian anti-persistent behavior.

Conclusions

- We stress that the **anti-persistent time windows** correspond to the time windows of **high Tsallis entropies**, while the **persistent time windows** correspond to the time windows of **low Tsallis entropies**.
- In summary, a combination of the Tsallis entropy with the Hurst exponent can evolve into a powerful diagnostic tool for the prediction of intense magnetic storm development.

References

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