Evolution of Whistler Turbulence in the Magnetosphere

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Sponsored by ONR and DARPA

Modern Challenges in Nonlinear Plasma Physics
Halkidiki, Greece, June 15 – 19, 2009

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Acknowledgment

Dennis Papadopoulos,
for numerous stimulating discussions

Introduction

- **Whistler waves are ubiquitous in the space plasma environment**
  - **Transports momentum and energy**
  - **Signatures of important space plasma processes; e.g., reconnection, lightning, plasmaspheric hiss, etc.**

- **Whistlers are also injected into the near-Earth space by man-made VLF transmitters**

- **Both natural and man-made whistlers affect the space plasma environment**
  - **Hence, important to “space weather”**

- **Need to understand the evolution of whistler turbulence in the space plasma environment accurately**
  - **Numerical simulations are necessary**
  - **Prerequisite for accurate simulation is knowledge of whistler wave properties**

Linear Whistler Wave Properties: Homogeneous Plasma

- **Whistler dispersion relation in cold plasma for $\Omega_e > \omega > \Omega_i$ obtained by;**

$$\vec{\nabla}\cdot\vec{j}/e=\vec{\nabla}\cdot n_0\underbrace{\left(\frac{c\vec{E}\times\vec{b}}{B_0}+\frac{i\omega}{\Omega_e}\left(1-\frac{\omega_{LH}^2}{\omega^2}\right)\frac{c\vec{E}_\perp}{B_0}+i\frac{eE_z\vec{b}_0}{m\omega}\right)}_{\text{Relative drift }(\mathrm{v}_i-\mathrm{v}_e)}=0$$

- **From Maxwell equations with Coulomb gauge it can be shown that**

$$\underbrace{E_x=-ik_x\varphi\frac{(\bar{k}_z^2+\bar{k}_\perp^2)}{\bar{k}_\perp^2},}_{\text{Electrostatic for } k_\perp>k_z}\qquad \underbrace{E_y=E_x\frac{i\omega}{\Omega_e\bar{k}^2},}_{\text{Electromagnetic}}\qquad \underbrace{E_z=-ik_z\varphi\frac{\bar{k}^2}{(1+\bar{k}^2)}}_{\text{Electrostatic for } k>1}$$

- **For $\omega_{pe} > \Omega_e$ the dispersion relation is,**

$$\omega^2=\left(\frac{\bar{k}_\parallel^2}{(1+\bar{k}_\perp^2)}+\frac{m_e}{m_i}\right)\frac{\bar{k}^2\Omega_e^2}{1+\bar{k}^2}$$

$$\bar{k}^2=\bar{k}_\perp^2+\bar{k}_\parallel^2$$

$$\bar{k}=kc/\omega_{pe}$$

- **Frequency in limiting cases:**
  - **LH limit:** $\bar{k}_\perp \gg 1,\quad k_\perp \gg k_\parallel,\quad k_\parallel/k_\perp \ll \sqrt{m_e/m_i} \rightarrow \omega^2=\Omega_e\Omega_i$
  - **Whistler limit:** $\bar{k} \ll 1,\quad \bar{k}_\parallel^2 \gg m_e/m_i \rightarrow \omega^2=\bar{k}_\parallel^2\bar{k}^2\Omega_e^2$
  - **Magnetosonic limit:** $\bar{k} \ll 1,\quad \bar{k}_\parallel^2 \ll m_e/m_i \rightarrow \omega^2=k^2V_A^2$

Whistlers and Lower Hybrid are the same wave at different propagation angle

Linear Whistler Wave Properties: Inhomogeneous Plasma

- **With density inhomogeneity the (E X B) drift gives a large term**
  - **Inhomogeneity could be external or self-consistent**

$$\vec{\nabla}\cdot\vec{j}/e = \vec{\nabla}\cdot\left(n_0+\delta n(x,y)\right)\left(\frac{c\vec{E}\times\vec{b}}{B_0}+\frac{i\omega}{\Omega_e}\left(1-\frac{\omega_{LH}^2}{\omega^2}\right)\frac{c\vec{E}_\perp}{B_0}+i\frac{eE_z\vec{b}_0}{m\omega}\right)=0$$

$$-n_0\frac{\omega}{\Omega_e}\left(\frac{1+\bar{k}^2}{\bar{k}^2}-\frac{\omega_{LH}^2}{\omega^2}-\frac{\Omega_e^2}{\omega^2}\frac{\bar{k}_z^2}{1+\bar{k}_x^2}\right)\frac{ck_xE_x}{B_0}-\frac{c}{B_0}(\vec{E}\times\vec{\nabla}\delta n)\cdot\vec{b}_0=0$$

- **Essentially 3 dimensional**
  - **Extends instability relaxation time**

- **Density fluctuations introduces new solution (sort of drift waves)**

$$\frac{\omega}{\Omega_e}=-\frac{\nabla_y\delta n}{2n_0k_x}\frac{\bar{k}^2}{1+\bar{k}^2}\pm\left(\left(\frac{\nabla_y\delta n}{2n_0k_x}\frac{\bar{k}^2}{1+\bar{k}^2}\right)^2+\frac{m}{M(1+\bar{k}_x^2)}+\frac{\bar{k}_z^2\bar{k}^2}{(1+\bar{k}_x^2)(1+\bar{k}^2)}\right)^{1/2}$$

$$\omega\rightarrow-\frac{\Omega_e\nabla_y\delta n}{n_0k_x}\frac{\bar{k}^2}{1+\bar{k}^2}$$

- **Nonlinear pondermotive force along B_0 can lead to second order density fluctuations**

$$\delta n(x,y)\propto k_{\|}\equiv\partial/\partial z$$

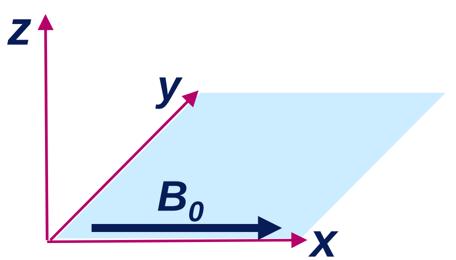
Small $\delta n/n$ (> max[$(m/M)^{1/2}$, k_z/k]) leads to big change in whistler mode character

Nonlinear Whistler Properties

- Nonlinear quasi-electrostatic Lower Hybrid (LH) waves extensively studied
  - Porkolab, 1974
  - Hasegawa and Chen, 1975
  - Shapiro, Shevchenko, Papadopoulos and Sagdeev, 1977-1993

- Simulations based on EMHD equation (no density perturbation)
  - D. Biskamp, E. Schwartz, and J. F. Drake, Phys. Rev. Lett. 76, 1264 (1996)
  - S. Dastgeer, A. Das, P. Kaw, and P. H. Diamond, Phys. Plasmas 7, 571 (2000)

- 2D PIC simulations $(\vec{k} \times \vec{\nabla} \delta n) \cdot \vec{B}_0 = 0$
  - D. Biskamp, E. Schwartz, and J. F. Drake, Phys. Rev. Lett. 76, 1264 (1996)
  - S. P. Gary, S. Saito, and H. Li, Geophys. Res. Lett., 35, L02104 (2008)
  - S. Saito, S. P. Gary, H. Li, and Y. Narita, Phys. Plasmas 15, 102305 (2008)

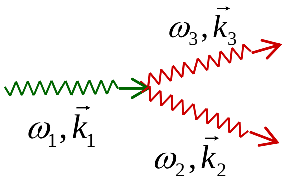


Plasma Weak and Structural Turbulence

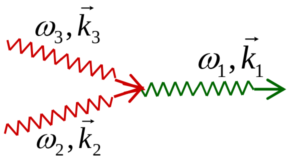
- **Induced whistler wave scattering while radiating low frequency wave**
  - **Waves energy and momentum are conserved**

$$\omega_1 = \omega_2 + \omega_3, \quad \vec{k}_1 = \vec{k}_2 + \vec{k}_3$$

$$\omega_1 >> \omega_3 \quad (= LH\ /\ MS\ , \vec{k} c_s\ , l\Omega_i\ )$$



Decay

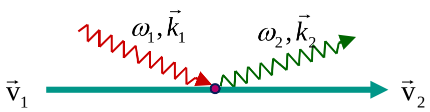


Coalescence

- **Induced wave scattering by plasma particles**
  - **Wave momentum need not be conserved if particles are magnetized (principal momentum conserved)**

$$\Delta\vec{P}_\perp = (ne/c)\Delta\vec{A}_\perp + \Delta n\vec{v}_\perp = \left(mnv_e(\Delta\vec{r}_\perp / \rho_e) + \sum_k (\Delta\vec{k}_\perp) N_k \right) = 0$$

Resonance Condition $\quad \mathrm{v}_{\|e} = \dfrac{\omega_1 - \omega_2}{k_{1\|} - k_{2\|}}$



W-P interactions are less restrictive than W-W interactions

Waves Spectra Nonlinear Evolution

- Calculate nonlinear conversion of W/LH waves (E_1) into LH/W waves (E_2)
  - Maxwell Equation,
  - Fluid equations for ion
  - Vlasov equation for electrons in drift approximation

- Whistler waves in a medium with slowly varying density perturbation induced by beat waves (ω_1 - ω_2):

$$\nabla \cdot \vec{j} = n_0 \frac{\omega}{\Omega_e}\left(\frac{1+\bar{k}^2}{\bar{k}^2} - \frac{\omega_{LH}^2}{\omega^2} - \frac{\Omega_e^2}{\omega^2}\frac{\bar{k}_z^2}{1+\bar{k}_x^2}\right)\frac{c}{B_0}\nabla_\perp \cdot \vec{E}_\perp^{(1)} + \frac{c}{B_0}(\vec{E}^{(1)} \times \vec{\nabla}\delta n_e{}^{(2)}) \cdot \vec{b}_0 = 0$$

- 2nd order density perturbation due to pondermotive force along B_0.
  - Maxwell electrons and unmagnetized ions

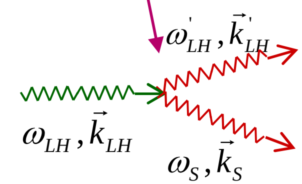
$$\frac{\delta n_e^{(2)}}{n_0} = \left(-\frac{c}{B_0}(\vec{E}_{k2} \times \vec{k}_1)_z \frac{\bar{k}_{1\perp}^2 e\varphi_{k1}}{\omega_{k1}(1+\bar{k}_{1\perp}^2)T_e}\left(1+\zeta Z(\zeta)\right)\right), \quad \zeta = \frac{\omega_{k1}-\omega_{k2}}{(k_{1z}-k_{2z})v_{te}}$$

- Subsonic ion condition: $(\omega_{k1}-\omega_{k2})^2 < (\bar{k}_{1\perp} - \vec{k}_{2\perp})^2 c_s^2$

Nonlinear Scattering : Short Wavelength

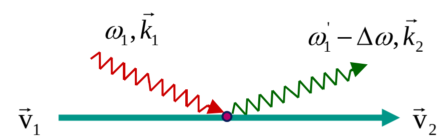
- For narrow frequency band ($\delta\omega < \gamma_{NL}$) Re Z leads to modulation instability, NLS equation and collapse of localized LH 3D wave packets (if $T_e >> T_i$)

$$i\frac{\partial E_{k2}}{\partial t} \sim -E_{k2}\omega_{LH}\frac{M}{m}\frac{W_{k2}}{n_0 T_e} + E_{k2}(\omega - \omega_{LH}) \qquad W_k \equiv \frac{\omega_{pe}^2}{\Omega_e^2}\frac{|E_k|^2}{8\pi}$$



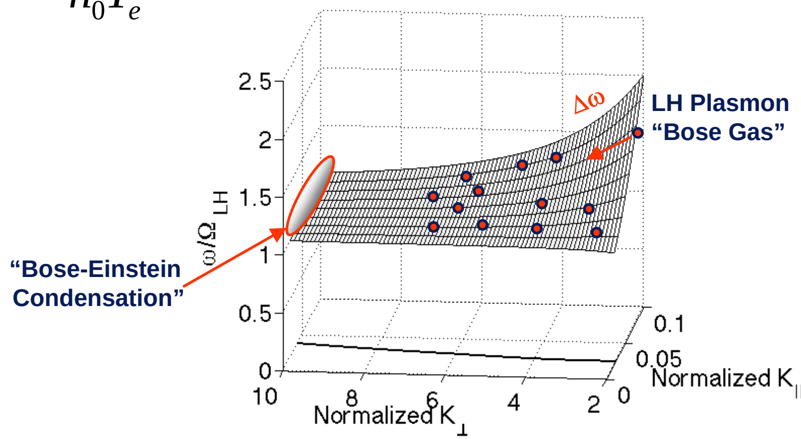
- For broad frequency band turbulence Im Z leads to nonlinear scattering by plasma electrons
  - Short wavelength electrostatic case discussed by Hasegawa and Chen, 1975

$$\gamma_{LH\to LH} \equiv \frac{\partial \ln W_{k2}}{\partial t} = \omega_{LH}\frac{M}{m}\sum_{k1}\frac{(\vec{k}_1\times\vec{k}_2)_z^2}{k_{1\perp}^2 k_{2\perp}^2}\zeta \,\mathrm{Im}\, Z(\zeta)\frac{W_{k1}}{n_0 T_e}$$



- Frequency decreases while wave scatters

$$\Delta\omega \sim \min\left\{ |k_{1z} - k_{2z}|\,\mathrm{v}_{te}, |\vec{k}_{1\perp} - \vec{k}_{2\perp}|\,c_s \right\}$$



Nonlinear Scattering : Long Wavelength Generalization

- **Generalization of Hasegawa and Chen**
  - **- Long wavelength electromagnetic regime**

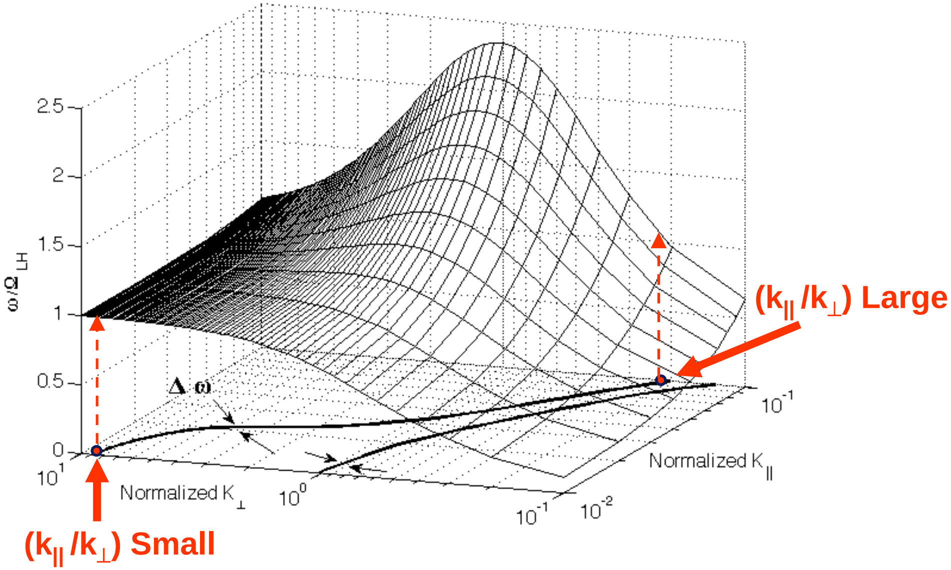
$$\gamma_{NL} = \frac{dN_{k2}}{N_{k2}dt} \sim \Omega_e^2 \frac{\bar{k}_{2\perp}^2}{1+\bar{k}_{2\perp}^2} \sum_{k1} \frac{(\vec{k}_1 \times \vec{k}_2)_z^2}{k_{1\perp}^2 k_{2\perp}^2} \frac{\bar{k}_{1\perp}^2}{1+\bar{k}_{1\perp}^2} \zeta \operatorname{Im} Z(\zeta) \frac{N_{k1}}{n_0 T_e} \qquad N_k = W_k / \omega_k$$

- **Scattering rate decreases frequency slightly and conserves "plasmons" N**

$$\Delta\omega / \omega_{LH} < |\vec{\bar{k}}_{1\perp} - \vec{\bar{k}}_{2\perp}| \beta_e^{1/2}$$

- **Wave-particle resonance can be easily met for any combinations of ($k_{||}$, $k_\perp$) in a thin slot in which ω ~ const.**

$$\omega^2 = \left(\frac{\bar{k}_{||}^2}{(1+\bar{k}_\perp^2)} + \frac{m_e}{m_i} \right) \frac{\bar{k}^2 \Omega_e^2}{1+\bar{k}^2}$$

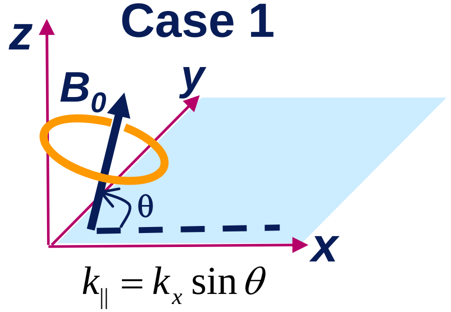


Short wavelength can scatter into long wavelength and vice-versa: γ_{NL} largest for $k_1 \perp k_2$

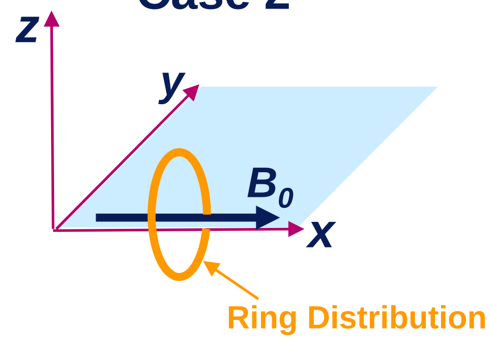
Electromagnetic 2D-3V PIC Simulation

- **Simulation box (X-Y) 512 x 256, equals 51.2 and 25.6 electron inertial lengths**
- **Magnetic field in (X-Z) plane with inclination $b_x = B_x/B_0$**

Case 1



Case 2



- **Simulation parameters**

$$m_i = 100m_e, \ \omega_{pe}^2 / \Omega_e^2 = 5, \ \mathrm{v}_{te} = 0.14c, \ \beta_e = 0.1, \ T_e = T_i$$

- **Whistlers self-consistently generated by “heavy ring electrons”**

$$n_r / n_e = 0.25;$$

$$V_r / c = 0.2$$

$$m_r / m_e = 3 \,\&\, 10$$

Instability Generation In Simulation

- **Hydro: Whistlers generated by ring beam for $\Omega_e > \omega > \omega_{LH}$ [Ganguli et al., JGR, 2007]**
  - **Large $k_\perp V_r / \Omega_r > 1$ necessary**

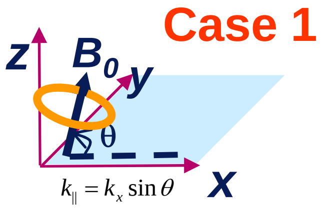
$$\omega = l\Omega_r$$

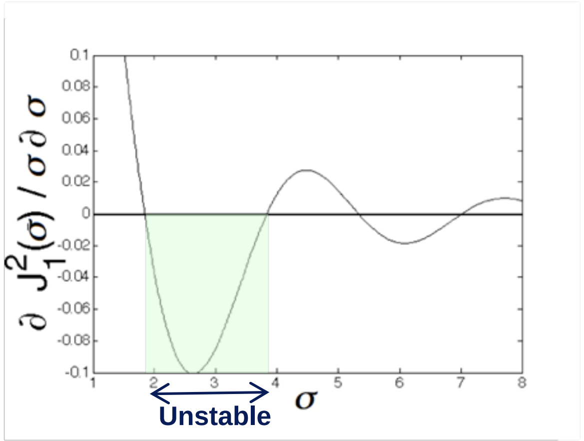
$$\frac{\gamma}{l\Omega_r} = \frac{1}{2}\sqrt{\frac{n_r m_r}{n_e m_e}\left|\frac{dJ_l^2(\sigma_r)}{\sigma_r d\sigma_r}\right|\frac{b_e}{\Gamma_l(b_e)}\left(\frac{\Omega_e^2 - l^2\Omega_r^2}{\Omega_e^2}\right)^2\frac{\bar{k}^2}{1+\bar{k}^2}}$$

  - **For the simulation parameters and for $l = 1$**

$$\sigma_r = k_\perp V_r/\Omega_r = 0.45\bar{k}_\perp (m_r/m_e)$$

$$b_e = (k_\perp \rho_e)^2/2 << 1 \Rightarrow b_e/\Gamma_1(b_e) \sim 2$$



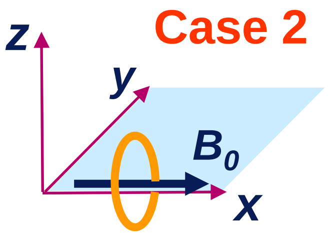


- **Kinetic: Whistlers generated by temperature anisotropy [Kennel and Petschek, JGR, 1966]**
  - **Small $k_\perp V_r / \Omega_r < 1$ necessary**

$$\frac{\omega}{\Omega_r} = 1 - \frac{1}{\kappa^2}$$

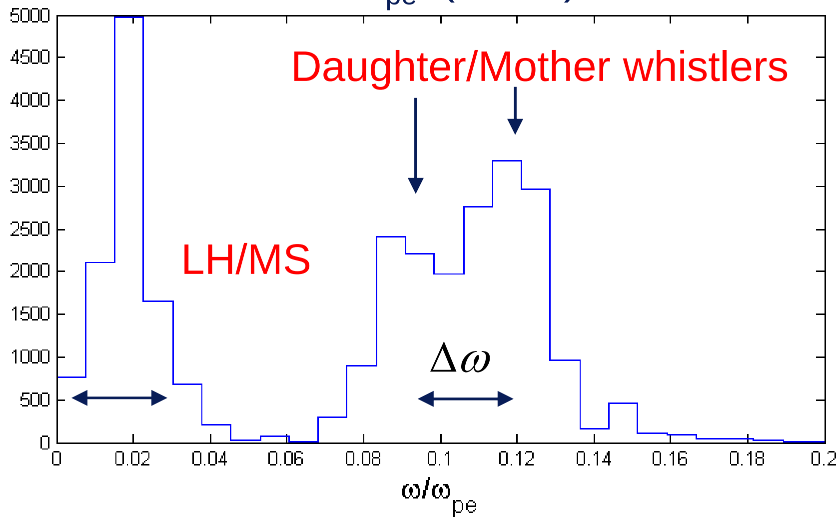
$$\frac{\gamma}{\Omega_r} = \sqrt{\pi}\frac{(\theta - \kappa^2)}{\theta^{-1/2}\beta_\perp^{1/2}|\kappa|^7}\exp\left(-\frac{1}{\theta^{-1}\beta_\perp\kappa^6}\right)$$

$$\kappa = k_\parallel c/\omega_{pr} \qquad \theta = m_r V_r^2/2T_{\parallel r} \qquad \beta_\perp = \frac{4\pi n_r m_r V_r^2}{B_0^2} \qquad \frac{\gamma_{max}}{\Omega_r} \sim \sqrt{\beta_\perp}\theta$$

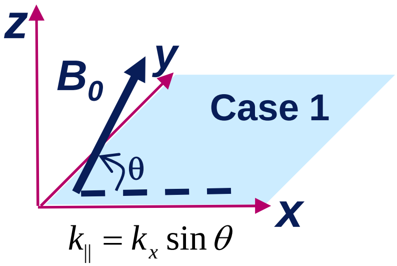


Simulation Results 1: Evidence Of Wave-Wave Interaction

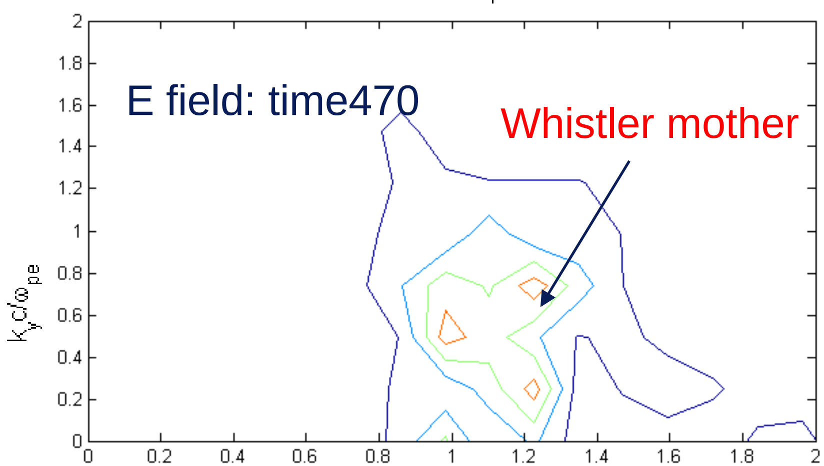
B field: $\omega_{pe}t$ (0-830)



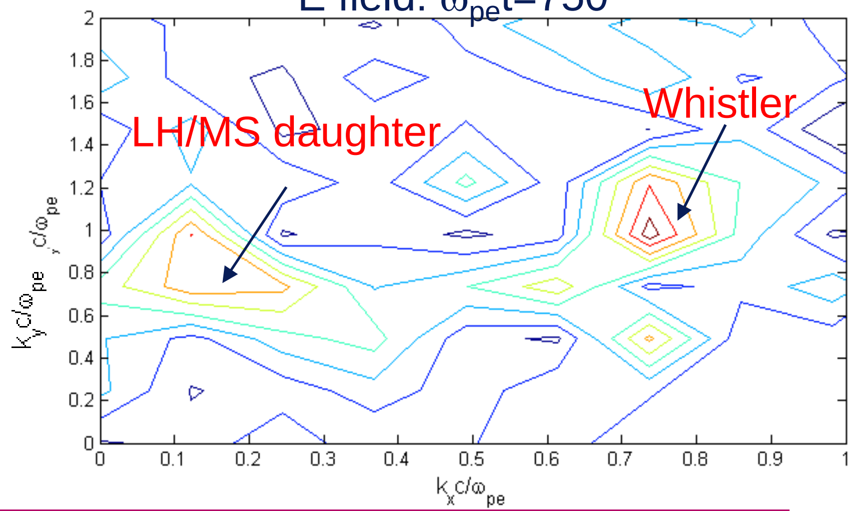
m_r = 3m_e b_x=1/2 (θ = 60^0)



$$\omega \sim 0.15\omega_{pe} \sim 3.3\omega_{LH}$$



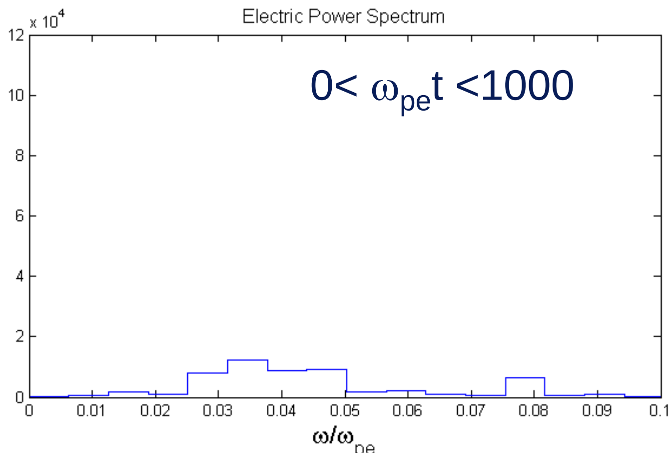
E field: $\omega_{pe}t$=750

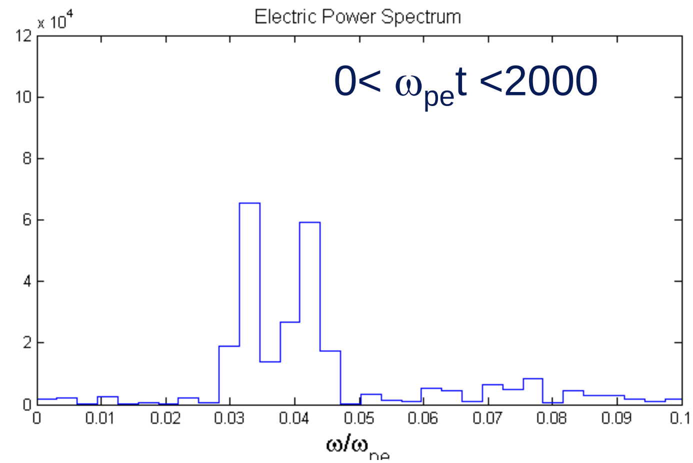


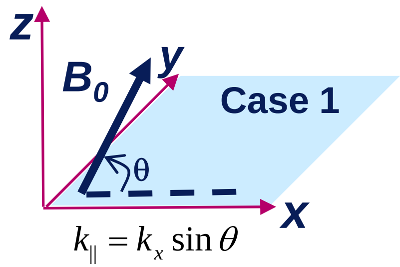
Whistler ω_M = 3.3ω_{LH} scatters radiating LH/MS wave $\omega_D \cong 0.5\omega_{LH}$

Simulation Results 2:
Evidence Of Wave – Particle Interaction

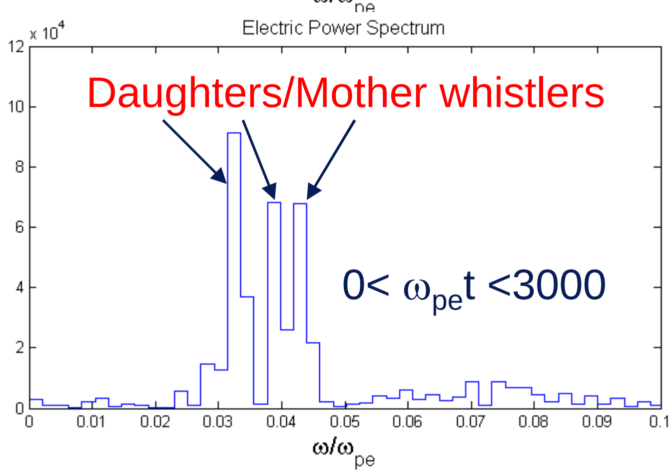
$m_r = 10m_e$, b_x=1/5 (θ = 78^0)

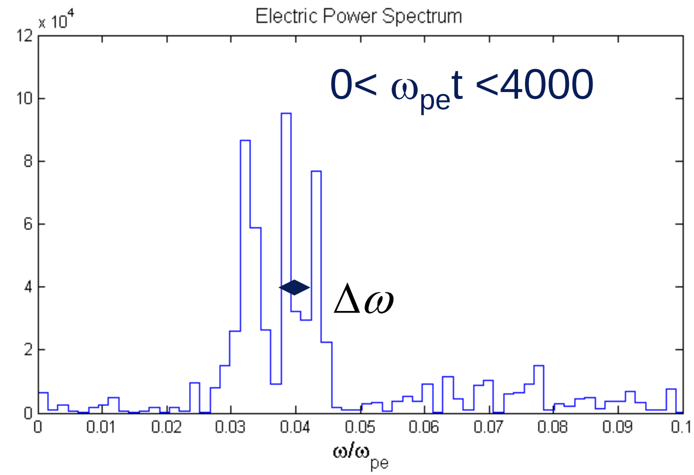






$\omega \sim 0.044\omega_{pe} \sim \omega_{LH}$

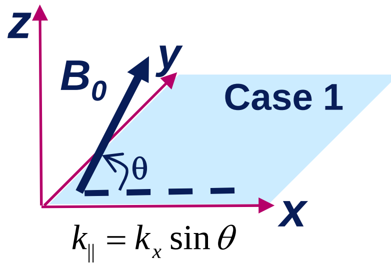


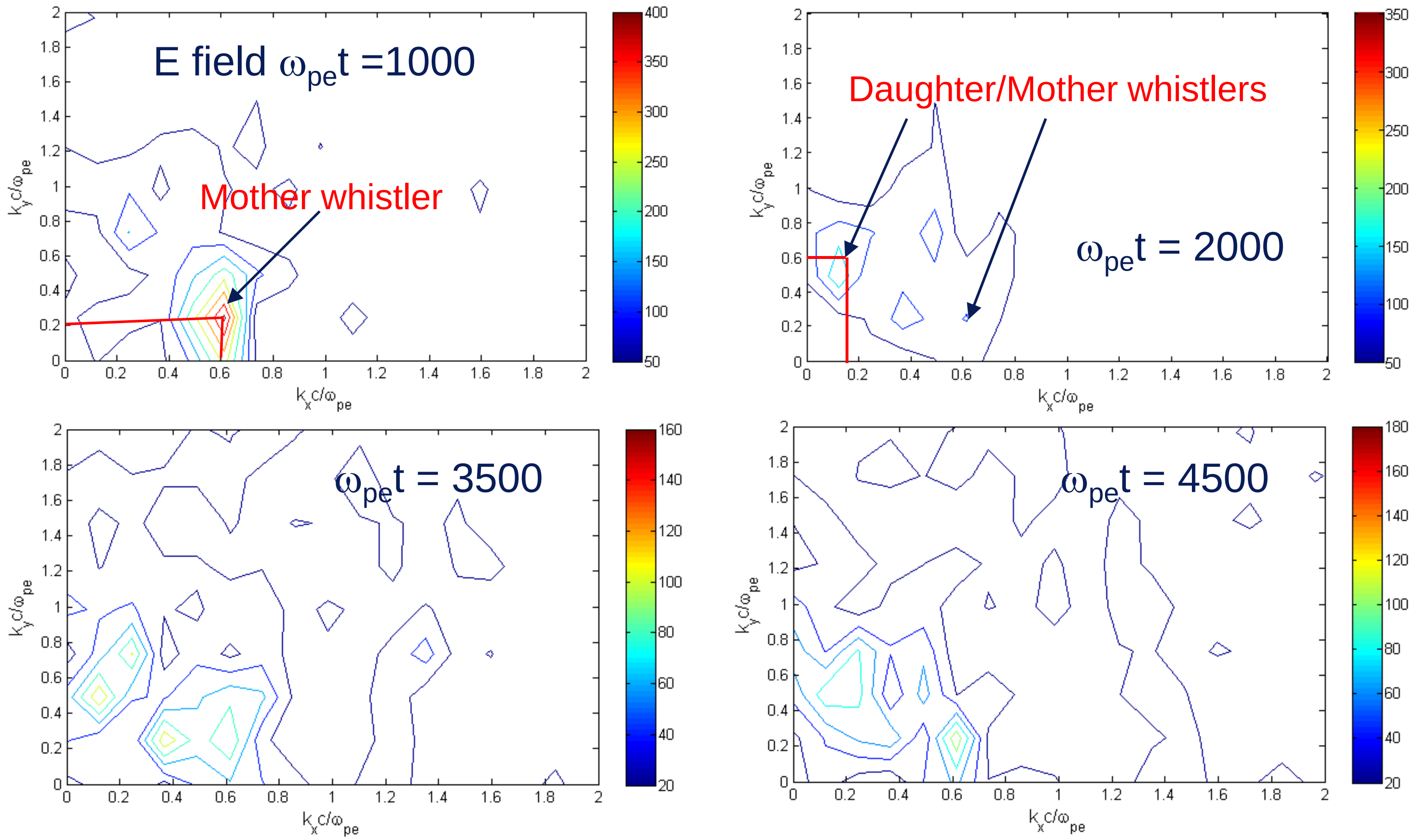


Whistler scatters radiating daughter waves $\Delta\omega/\omega_{LH} < \Delta k_{\perp} c/\omega_{pe}\, \beta^{1/2} < 0.2$.
No third low frequency wave to satisfy 3 wave decay condition.

Simulation Results 3: Evidence Of Large Angle Scattering

$m_r = 10m_e$, $b_x=1/5$ ($\theta = 78^0$)

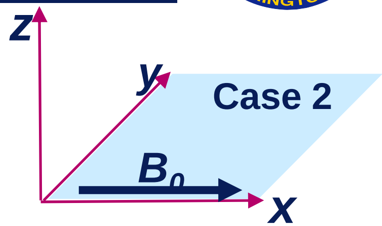




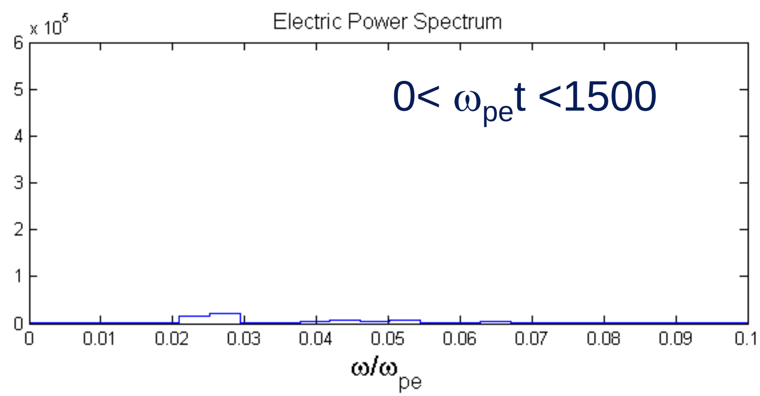
$$\gamma_{NL} \propto (\vec{k}_1 \times \vec{k}_2)_z^2$$

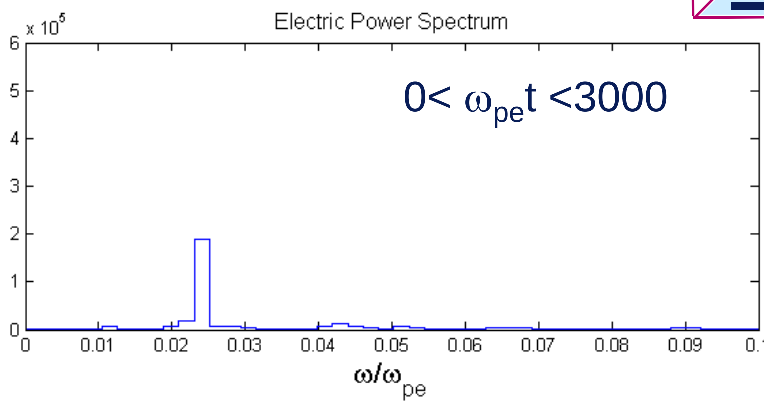
Whistler radiates daughter waves with large angle rotation for which γ_{NL} is large

Simulation Results 4: No Evidence Of Nonlinear Scattering

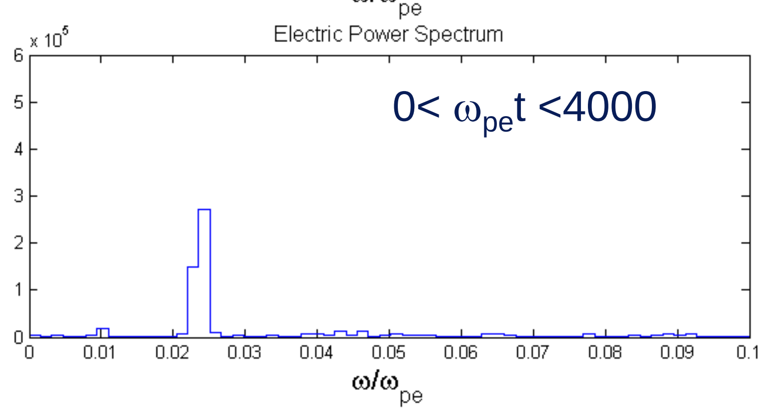


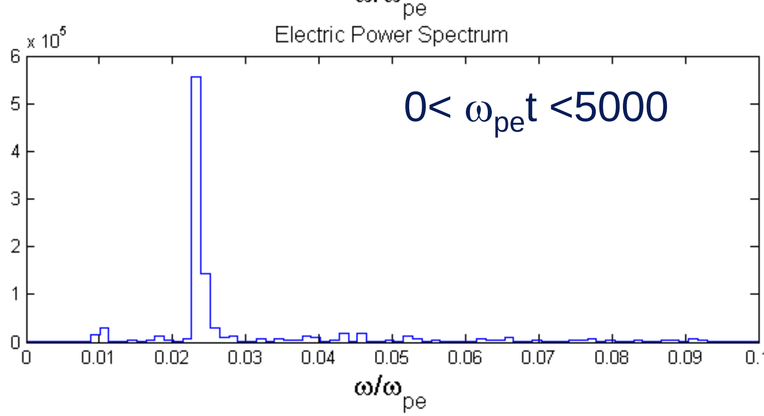
$m_r = 10m_e$, $b_x=1$ ($\theta = 0^0$)





$$\frac{\omega}{\omega_{pe}} \sim 0.017 - 0.03$$

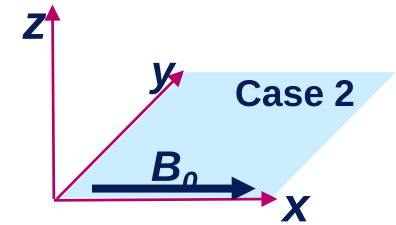


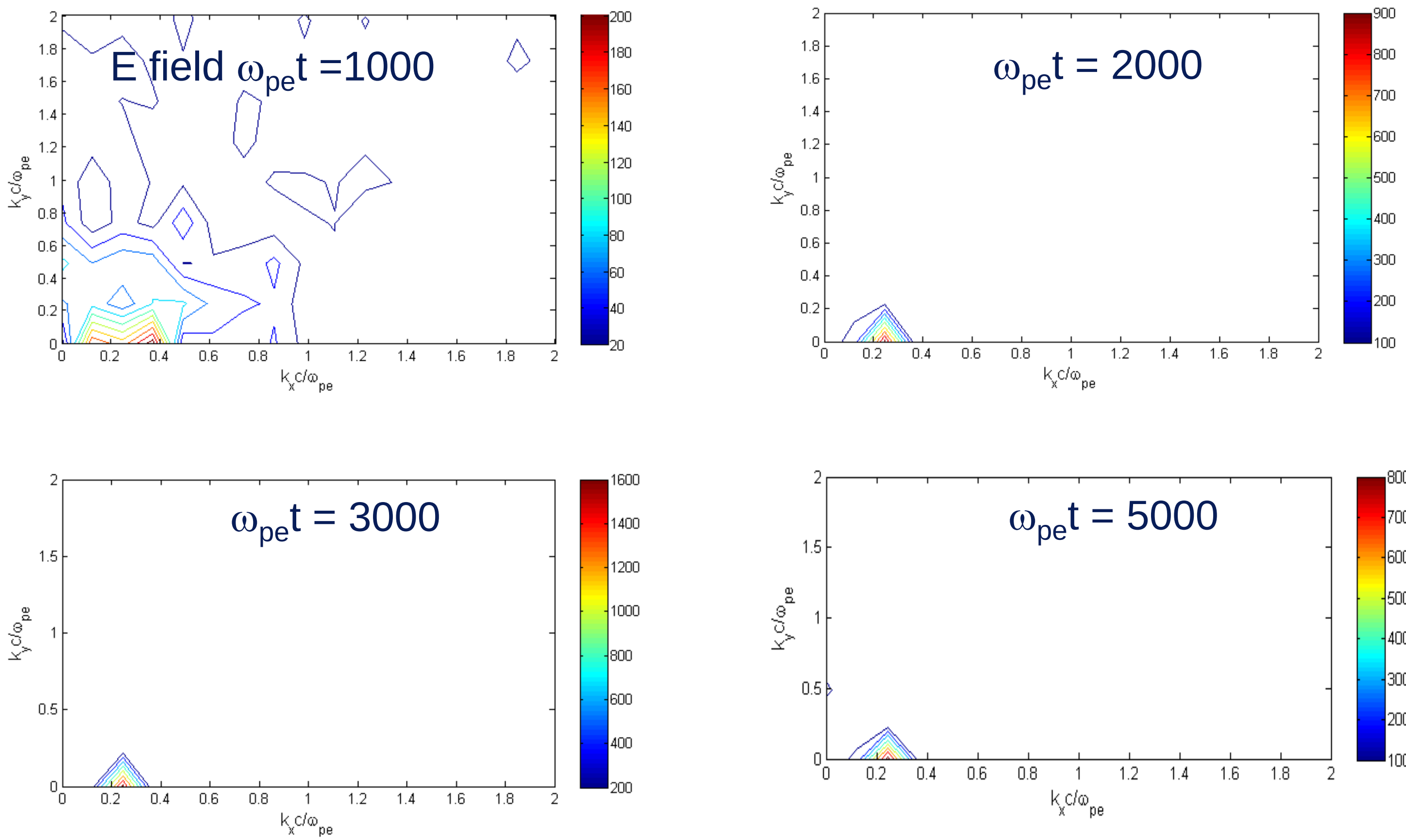


No nonlinear scattering on this time scale contrary to the b_x= 0.2 case

Simulation Results 5:
Whistlers Born With & Maintain Large $k_{||}/k_{\perp}$

$m_r = 10m_e$, b_x=1 ($\theta = 0^0$)





Only whistler with small $k_{\perp}$ / $k_{||}$ arise. No nonlinear scattering.

Conclusion

- Electromagnetic PIC simulations show that evolution of whistler turbulence is dominated by nonlinear ponderomotive force
- The ponderomotive force leads to higher (second) order density perturbation
- The density perturbation significantly changes the whistler evolution
  - Extends the instability relaxation time by orders of magnitude
  - Introduces an essentially 3 dimensional character
  - Nonlinear scattering (wave-wave and wave-particle) dominate the nonlinear phase
- Wave-particle interactions convert short wavelength quasi-em waves into long wavelength em waves and vice-versa
  - Large changes in wavelength possible because wave momentum need not be conserved