Anomalous transport of magnetized electrons interacting with EC waves

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Abstract. We consider the nonlinear interaction of magnetized electrons with an oblique narrow-band electromagnetic wave-packet. The interaction is analyzed over and near the local threshold to chaos. The statistical character of the forcing that controls the trajectories of the particles is also studied. We focus our analysis on issues related to energy and spatial diffusion across the magnetic field by following the evolution of the ensemble mean squares $<(\gamma-\gamma_0)^2>$ and $<(\boldsymbol{r}_\perp-\boldsymbol{r}_{\perp 0})^2>$ for various values of the wave amplitude and angle of wave propagation. We study in particular the interaction of magnetized electrons with waves having strong and moderate amplitudes, near the transition to chaos where the dynamics is complex and a mixture of periodic and stochastic orbits co-exist. The electron diffusion in real and energy space is found to obey simple power law in time and the scaling exponents are indicative of sub-diffusion. This is a direct consequence of the effect of the resonant phase-space islands in the particle motion.

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1. Introduction

The nonlinear interaction of charged particles with electrostatic and electromagnetic waves is of great importance for the laboratory and astrophysical plasmas. This problem has been investigated in detail for the last thirty years. It was found that the coupling of non-relativistic magnetized ions with an ion-cyclotron electrostatic wave becomes extremely efficient when the wave amplitude and initial ion energy are above a threshold value, and the motion becomes chaotic leading to heating of particles in the stochastic region of the phase space [1, 2, 3]. In this case the chaotic region is bounded, and thus there is a limit in the maximum possible energy gain by the ions. This mechanism provided an explanation for the generation of energetic ion tails in lower-hybrid-heating experiments for the tokamak [4], as well as a possible explanation for observations of ion acceleration in the ionosphere [5].

The study of the nonlinear interaction of magnetized ions with many electrostatic waves showed that there can be non-resonant acceleration, and this may lead to unlimited energy gain regardless of the ion initial energy, depending on the parameters of the wave spectrum [6].

The importance of relativistic effects has also been recognized [7, 8]. The properties of the phase space are altered when the relativity theory is included and the energization appears more intense. The relativistic effects can be very important for electrons, but not as important as for the dynamics of heavier ions. Low energy electrons gain substantial energy only when the relativistic effects are included [8, 9].

Electromagnetic waves, due to their nature, have been proved more effective than electrostatic waves in accelerating and heating the electrons, under certain conditions. Early studies indicated that, in magnetized plasmas, resonance of the relativistic cyclotron motion with the Doppler-shifted wave frequency can lead to unlimited coherent acceleration, as long as the wave propagation is parallel to the magnetic field with the refraction index close to unity [10, 11]. These results were extended to the more realistic case of oblique monochromatic propagation [12, 13, 14]. Furthermore, when the parallel phase velocity of the wave is larger than c, the phase space is open, and thus unlimited electron acceleration is possible [13], contrary to the perpendicular propagation of electrostatic waves studied in [1, 2], but also to the non-relativistic approximation studied in [14], where the phase spaces are closed and the energy gain is limited.

The interaction with more than one electromagnetic waves was studied by means of the Hamiltonian formalism [15] or by using mapping approximations [16], where enhancement of the stochastic acceleration mechanism was predicted. Such applications are very important for the acceleration in the ionosphere, but also in modern fusion experiments; for example, single and multi-frequency electron-cyclotron resonance heating (ECRH) is an essential component for the fusion devices of current research, such as stellarators and tokamaks [17].

The rate of particle transport is crucial for the evolution of plasmas, especially for the laboratory plasma where controlling the particle diffusion is important for the fusion reactors. It has been pointed out in the past that in stochastic acceleration scenarios, as e.g. in [1, 13], the energy gain has a diffusive nature. Early studies of Hamiltonian chaos showed that, for a number of cases, the quasilinear approximation is fairly good, and the corresponding Fokker-Planck (FP) equation describes sufficiently well the evolution of the distribution function [18, 19].

Since then, a lot of work has been done on the applications of the quasilinear theory, but also on investigating its domain of validity. For electromagnetic waves interacting with electrons, theoretical predictions have been obtained on the basis of an FP equation with a local quasilinear diffusion coefficient using a monochromatic electron-cyclotron (EC) wave propagating perpendicular [20, 21] or obliquely to the magnetic field [22, 23], and also for narrow wave-packet propagation [15]. In [20] the analysis was performed in the globally stochastic regime, and the quasilinear predictions were in agreement with the numerical results from the equations of motion. On the contrary, in the case of a locally stochastic phase-space [21], appreciable deviations were found at long times. In [15, 22] the diffusion process was found to proceed in stages following different scalings. The time scales involved are much longer than those in [1], and the FP approximation becomes invalid for long times. In another, more recent work, ECRH simulations were performed using a full nonlinear treatment in contrast with the quasilinear theory [24]. The results show that the deviation from the quasilinear theory can be strong for present day fusion experiments.

In numerous publications [25, 26, 27, 28], examples from the interaction of electrostatic waves with ions have been studied and it was shown that the quasilinear theory breaks down due to the presence of resonant periodic orbits (islands) in the phase-space. These formations cause large time-space scaling of the particle kinetics, and thus non-Gaussian diffusion. The transition to normal diffusion occurs only for system parameters corresponding to strong chaos. In [27, 28], but also in a later review [29], anomalous transport was considered, with strong arguments (as described in [30]), to be a result of a Lèvy process [31]. In systems undergoing anomalous diffusion, the scaling of the ensemble mean square displacement of the particles was shown to be [29, 30]

$$< (\Delta \boldsymbol{R})^2 > \propto t^\alpha \tag{1}$$

where a is the transport exponent for the process. The type of anomalous diffusion is determined by the exponent ($a = 1$ obviously means normal diffusion): if $a < 1$ the evolution is called sub-diffusive, while for $a > 1$ we have super-diffusion. Furthermore, it was suggested that the Fractional Fokker-Planck (FFP) formalism is the appropriate tool for the study of the diffusion properties of such dynamical systems [32, 33, 34, 35]. All the above results suggest that the problem of particle transport is more complicated than one might have expected.

In this article, we focus our attention on the interaction of magnetized relativistic electrons with electromagnetic waves. This problem has been studied in the past (see references mentioned already), but there is still strong disagreement on the character of particle diffusion both in energy and real space. We analyze, for the first time, the diffusion of electrons perpendicular to the external magnetic field in the presence of electron-cyclotron waves, without using a quasilinear approximation for the phase space. We pose two questions: (1) Does the nonlinear interaction of EC waves with magnetized electrons follow a normal or anomalous diffusion? (2) Is the quasilinear approximation valid for the interaction of EC waves with plasma? The forcing that controls the trajectories of the particles is also examined, and its complicated character is emphasized. The EC waves used are right-handed circularly polarized and propagate in the (x, z) plane at an angle θ with respect to a uniform background magnetic field $B_0\boldsymbol{z}$. The ambient plasma is assumed to be cold.

The article is organized as follows: in section 2, the formulation of the problem is presented by the means of an autonomous relativistic Hamiltonian, and also our model parameters and characteristics are given, while in section 3 the character of the forcing that affects the particles is discussed. In section 4, the numerical results concerning the anomalous diffusion are presented and analyzed. Finally, in the last section our conclusions are summarized and discussed.

2. Our model

2.1. Hamiltonian formulation

In our model, the EC wave-packet consists of N discrete modes which propagate at frequencies $\omega_{\mathrm{i}} \in [\omega - \Delta\omega/2, \omega + \Delta\omega/2]$. The bandwidth $\Delta\omega$ of the wave-packet is a small fraction of the main frequency ω. The amplitudes of the subterminal modes A_{01}, A_{0N} are a fraction of the amplitude A_0 of the main mode ω, while the amplitudes of the remaining modes are distributed within these bands using linear interpolation. The vector potential describing the wave-particle interaction is [15]

$$\boldsymbol{A} = \sum_{j=1}^{N} A_{0\mathrm{j}}(\cos\theta \sin\phi_{\mathrm{j}}\boldsymbol{x} + \cos\phi_{\mathrm{j}}\boldsymbol{y} - \sin\theta\sin\phi_{\mathrm{j}}\boldsymbol{z}) + xB_0\boldsymbol{y} \tag{2}$$

with $A_{0\mathrm{j}} = (4\pi c S_j/\omega_j^2)^{1/2}$ the vector potential amplitudes, expressed in terms of the power flux, $\phi_j = \omega_j(n_{\mathrm{xj}}x + n_{\mathrm{zj}}z - ct)/c$ the wave phases and n_j the refraction index [36]

$$n_j^2 = 1 - \frac{\omega_{\mathrm{p}}^2}{\omega_j(\omega_j - \omega_{\mathrm{c}})} \tag{3}$$

corresponding to wave frequency ω_j, $\omega_{\mathrm{c}} = eB_0/m_{\mathrm{e}}c$ and $\omega_{\mathrm{p}} = (4\pi n_{\mathrm{e}}e^2/m_{\mathrm{e}})^{1/2}$ are the cyclotron and plasma frequencies, n_{e}, m_{e} are the electron plasma density and electron rest mass respectively, and $n_{\mathrm{xj}} = n_{\mathrm{j}}\sin\theta$, $n_{\mathrm{zj}} = n_{\mathrm{j}}\cos\theta$ are the perpendicular and parallel refraction indices with respect to the ambient magnetic field. We should note that the simple dispersion relation (3) is not the exact for obliquely propagating EC waves (for the exact relation see [36]), however it is a good approximation for the parameters used in this study [13].

An autonomous Hamiltonian can be obtained with the use of proper canonical transformations [15, 22]. As a first step, the generating functions $F_j = F_2 = x'p_x + y'p_y + (z' - ct/n_{\mathrm{zj}})p_{zj}$ $(j = 1, .., N)$, where the old variables are denoted as primed, are used to transform the time-dependent Hamiltonian of the motion

$$H = [(\mathbf{p} + \frac{e\mathbf{A}}{c})^2 + m^2c^2]^{1/2} \tag{4}$$

After the N transformations, the system becomes of $N+2$ degrees of freedom, namely the coordinates $(x, y, [z_j, j = 1, .., N])$. However, the degrees of freedom can be reduced to 4 (x, y, z_a, z_h) using the generating function $F_R = -F_3 = z_a \sum_{j=1}^{N}(1 - n_h/n_{zi})p_{zi} + z_h \sum_{j=1}^{N} n_h p_{zi}/n_{zi}$, and the resulting Hamiltonian is [15]

$$H = \gamma - \frac{p_{\mathrm{h}}}{n_{\mathrm{h}}} \tag{5}$$

where $\gamma = [1 + (p_{\mathrm{x}} + \cos\theta \sum_{j=1}^{N} \varepsilon_{\mathrm{j}} \sin\phi_{\mathrm{j}})^2 + (p_{\mathrm{y}} + x + \sum_{j=1}^{N} \varepsilon_{\mathrm{j}} \cos\phi_{\mathrm{j}})^2 + (p_{\mathrm{a}} + p_{\mathrm{h}} - \sin\theta \sum_{j=1}^{N} \varepsilon_{\mathrm{j}} \sin\phi_{\mathrm{j}})^2]^{1/2}$ is the relativistic Lorentz factor, $\phi_{\mathrm{j}} = \omega_{\mathrm{j}}[n_{\mathrm{xj}}x + (n_{\mathrm{zj}} - n_{\mathrm{h}})z_{\mathrm{a}} + n_{\mathrm{h}}z_{\mathrm{h}}]$ are the dimensionless autonomous wave phases, $\varepsilon_{\mathrm{j}} = eA_{0\mathrm{j}}/m_e c^2$ are the normalized amplitudes, n_{h} is the harmonic median of the parallel refraction indices n_{zj} and $x, y, z_{\mathrm{a}}, z_{\mathrm{h}}, p_{\mathrm{x}}, p_{\mathrm{y}}, p_{\mathrm{a}}, p_{\mathrm{h}}$ stand for the new canonical variables of the system, arising from the dimension-reduction transformation. The Hamiltonian is normalized with $m_{\mathrm{e}}c^2$, the time with ω_{c}^{-1} and the wave frequency with ω_{c}. The dimensionless coordinates and canonical momenta are $\omega_{\mathrm{c}}x/c$, $\omega_{\mathrm{c}}y/c$, $\omega_{\mathrm{c}}z/c$ and $p_{\mathrm{x}}/m_{\mathrm{e}}c$, $p_{\mathrm{y}}/m_{\mathrm{e}}c$, $p_{\mathrm{z}}/m_{\mathrm{e}}c$. Note that the Hamiltonian does not depend on the coordinate y, and thus y is a cyclic variable and its conjugate momentum p_{y} is a constant of the motion.

From the above analysis, the Hamiltonian equations of motion can be easily obtained. In the special case of a monochromatic wave ($N = 1$), the system is simpler: the degrees of freedom of the autonomous system are 3 (with y cyclic) and no transformation for reducing the dimensions is needed. The Hamiltonian for this case can be obtained from (5) after setting $z_{\mathrm{h}} = z$, $p_{\mathrm{a}} = 0$, $p_{\mathrm{h}} = p_{\mathrm{z}}$ and $n_{\mathrm{h}} = n_{\mathrm{z}}$.

2.2. Our model parameters

Our choice for the plasma parameters is the following: the uniform magnetic field is $B_0 = 3.5 \cdot 10^{-5}$ T, corresponding to a cyclotron frequency $\omega_{\mathrm{c}} = 1.96\pi$ MHz, while the plasma density is $n_{\mathrm{e}} = 10^2$ cm^{-3}, and so the plasma frequency is $\omega_{\mathrm{p}} = 0.564$ MHz. Practically, these values correspond to the night-time ionosphere at an altitude ~ 130 km. The main wave frequency is $\omega = 6\pi$ MHz, in the range of radio waves, and the bandwidth of the wave-packet, which consists of N modes, is $\Delta\omega = 0.12\pi$ MHz (2% of the main frequency). The angle of propagation, unless differently stated, is $\theta = 40^{\mathrm{o}}$. Several wave amplitudes within $(0, 0.5]$ are used when following the system evolution. An amplitude value ε corresponds to total power flux $S = 30\sum_{j=1}^{N} n_j \omega_j^2 \varepsilon_j^2$ (W/cm^2). The equations of motion are integrated using a 4^{th} order Runge-Kutta algorithm. The accuracy of our scheme was checked using the error in the calculation of the motion constant H. This error is of the order 10^{-8}.

We briefly report below some of the characteristics of our model:

2.2.1. Local threshold to chaos For this system, significant chaos exists only for amplitudes larger than a critical value $\varepsilon_{\rm cr}$, which depends on the other parameters (e.g. wave frequency, propagation angle). A local estimate of $\varepsilon_{\rm cr}$ can be found by utilizing the fact that for $\varepsilon > \varepsilon_{\rm cr}$ the acceleration is possible, as seen in figure 1(a) where the mean energy of a monoenergetic ensemble of 10000 electrons after $T = 3000$ (0.48 ms) is given as a function of ε, for the interaction with a single wave ($N = 1$) as well as with a wave-packet of $N = 5$ modes. The initial energy of the ensemble, constituted by $N = 1000$ particles, is $\gamma_0 = 2.5$ (1.279 MeV in physical units). The energy remains almost constant for small ε, until a sudden increase appears near $\varepsilon_{cr} = 0.03$ for $N = 1$, a value $S = 246$ mW/cm^2 for the power flux, and for $N = 5$ near $\varepsilon_{cr} = 0.015$ (a lower value $S = 167$ mW/cm^2). The onset of chaos in the system is similar with respect to the angle of propagation [14]; for fixed values of the other parameters, there is a critical angle value over which chaos exists.

2.2.2. Poincarè surfaces of section The interaction of electrons with a monochromatic wave (case $N = 1$) is essentially of two degrees of freedom, and so the dynamics can be visualized using Poincarè surfaces of section (PSS). In [14] there is an extensive study in this direction; here we present only an indicative case in order to uncover the intrinsic complexity of the phase space. In figure 1(b), the PSS is shown for $\varepsilon = 0.1$. For forming the surfaces of section, 20-50 electron orbits are followed for $T = 2000 - 3000$, and the section points are taken "stroboscopically" every time when $\omega n_{\rm z} z$ is multiple to 2π with the same direction of crossing $\dot{z} < 0$. Note that the phase space is inhomogeneous, a highly-complex mixture of periodic and stochastic behavior. A large number of resonant islands of different scales exists in the stochastic sea. These islands serve as regions of trapping for the particle motions.

2.2.3. Energy distribution function and electric current The distribution of the particle energies is an important concept, because the probability density function (PDF) can be described as the solution of FP/FFP equations, depending on the statistical characteristics of the motion. In figure 1(c), the distribution function is given for $\varepsilon = 0.1$ after $T = 5000$, for interaction with the single wave ($N = 1$) and with the wave-packet of $N = 5$ modes. The simulations were performed for initially mono-energetic ensembles $f(\gamma, t = 0) = \delta(\gamma - \gamma_0)$ of $N = 10000$ particles with $\gamma_0 = 2.5$. In both cases, it is certain that the evolution of the system is diffusive and associated with energy gain, and the particles are accelerated more efficiently as the number of modes increases. This behavior is also seen in figure 1(d), where the electric current $I = Ne < \upsilon >$ resulting from the particle motion is shown as a function of time for the same parameters. In this figure, the current values are normalized to the initial current $I_0 = Ne\upsilon_0$.

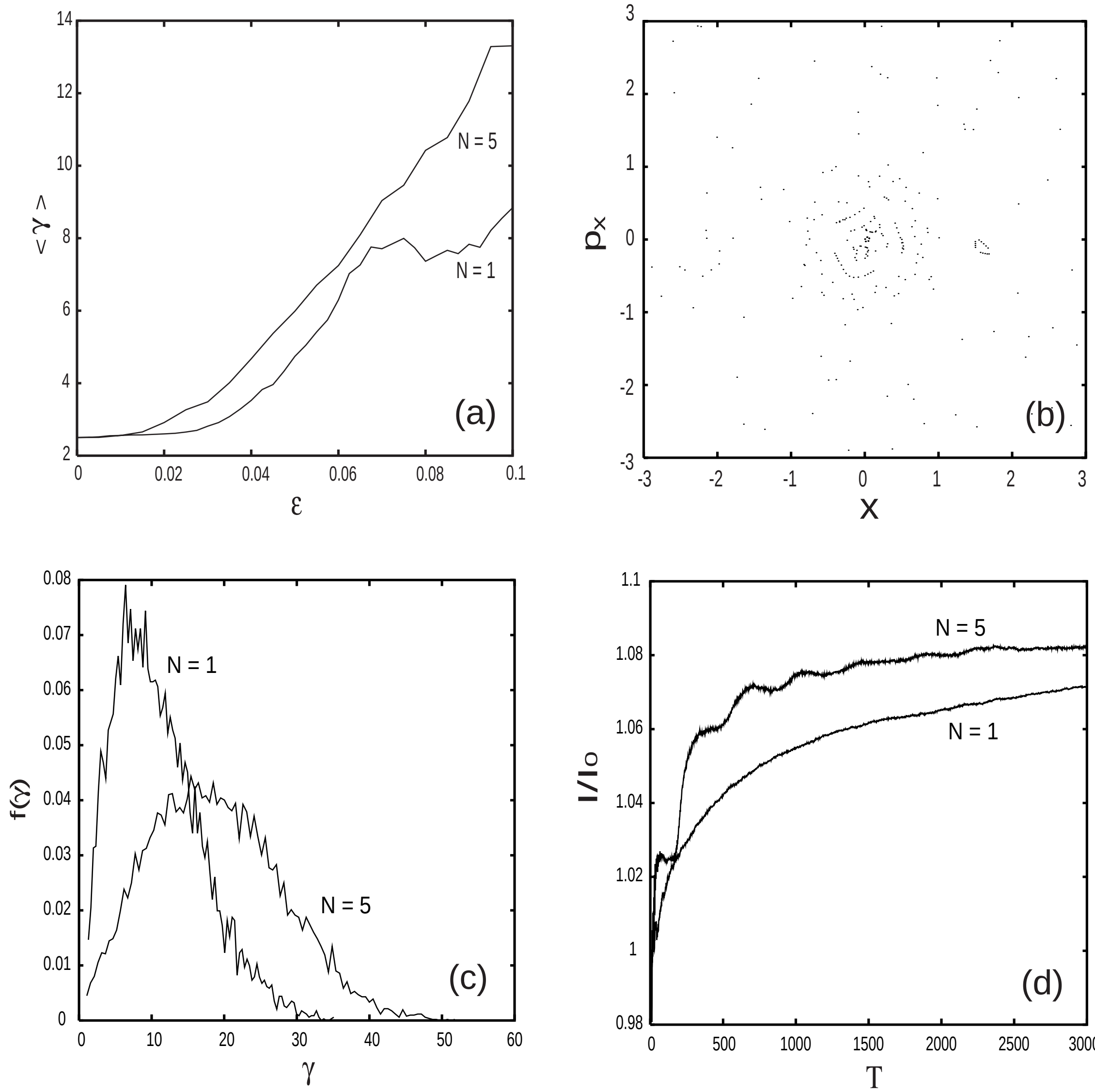

Figure 1. (a) Ensemble mean energy $<\gamma>$ as a function of ε, (b) Poincarè surface of section (x, p_{x}) for $\varepsilon = 0.1$, (c) Energy distribution function $f(\gamma)$ for $N = 1, 5$ ($\varepsilon = 0.1$), (d) Normalized electric current I/I_0 vs time for $N = 1, 5$ ($\varepsilon = 0.1$).

3. Forcing term

The equation of motion for a magnetized electron in the presence of an electromagnetic wave-packet is

$$\frac{d(\gamma m_e \boldsymbol{v})}{dt} = -e\boldsymbol{E}_1(\boldsymbol{r}, t) - \frac{e}{c}[\boldsymbol{v} \times (\boldsymbol{B}_0(\boldsymbol{r}, t) + \boldsymbol{B}_1(\boldsymbol{r}, t))] \tag{6}$$

where $\boldsymbol{E}_1(\boldsymbol{r}, t) = -1/c\partial \boldsymbol{A}(\boldsymbol{r}, t)/\partial t$ and $\boldsymbol{B}_1(\boldsymbol{r}, t) = \nabla \times \boldsymbol{A}(\boldsymbol{r}, t)$ are the electric and magnetic field of the wave-packet, which determine the forcing on the particle.

The statistical properties of the forcing term

$$\boldsymbol{F} = -e\boldsymbol{E}_1(\boldsymbol{r}, t) - \frac{e}{c}\boldsymbol{v} \times \boldsymbol{B}_1(\boldsymbol{r}, t) \tag{7}$$

are extremely important for the evolution of the gyrating particle and its transport properties. Several studies have discussed the properties of magnetized ions in the presence of pre-described forcing terms: in [35], a homogeneous and isotropic electric field with non-Gaussian, Lèvy-stable statistics is considered and the relevant FFP equation is solved. The stationary states are essentially non-Maxwellian and the characteristic displacement of the particles grows super-diffusively with time. In [37] a non-resonant, frictionless "impulsive" forcing was studied. The forcing term has random direction and acts on the particle at random times. The motion is a superposition of a Brownian motion and a gyration and its transport characteristics are described well with the FP description when the number of impulsive events is large. In [38], a turbulent electrostatic field was used to describe magnetized ion motion. It was shown that the transport properties are anomalous and an FFP equation was necessary to describe the evolution of the particle distribution function. A similar field was used in [39] where, apart from anomalous diffusion, it was found that a barrier for transport can be generated through the randomization of phases of the turbulent field.

The forcing term described in (7) depends on the spatial coordinates and it is highly inhomogeneous along the trajectory of the particle. In figure 2(a), the distribution of the perpendicular component of the forcing is given for $N = 1$, $\epsilon = 0.1$, while in figure 2(b) the same is shown for the same amplitude and $N = 5$. The initial conditions are taken in the chaotic part of the phase space, and the particle is followed for $T = 3000$. The deviation from Gaussian statistics is evident, being more intense in the second case, which implies that the forcing contains non-Gaussian characteristics. This is connected to the inhomogeneous character of the forcing mentioned above, and is bound to affect the chaotic motions with spatially-dependent statistics. The particles show a strong preference to lie on regions of small perpendicular forcing, especially in the second case. The situation may become different when different initial conditions are chosen, but the general picture remains the same.

The effect of the forcing term on the orbits can be uncovered by studying the motion of the gyro-center under the $\boldsymbol{E} \times \boldsymbol{B}$ drift force. In a "first order" approximation, for values of the wave amplitude where $B_1 < B_0$, one may neglect the contribution of the wave magnetic field and study a simplified equation for the perpendicular motion

$$\frac{d(\gamma m_e \boldsymbol{v}_\perp)}{dt} = -e\boldsymbol{E}_{1\perp} - \frac{e}{c}(\boldsymbol{v}_\perp \times \boldsymbol{B}_0) \tag{8}$$

Based on (8), an equivalent drift velocity for the gyro-center, which in this case is non-constant and relativistic, may be defined as

$$\boldsymbol{w}_{\mathrm{D}} = c\frac{\boldsymbol{E}_{1\perp} \times \boldsymbol{B}_0}{\gamma B_0^2} \tag{9}$$

In figure 2(c), (d), the orbit of the gyro-center after $T = 1000$ under the drift force $\boldsymbol{E}_{1\perp} \times \boldsymbol{B}_0$ is given for $N = 1$ and $N = 5$ respectively. In both cases, the value for the wave amplitude is $\epsilon = 0.1$, for which $B_1/B_0 \approx 0.3$. The initial conditions are taken in the chaotic part of the phase space. The complex character of the motion is evident

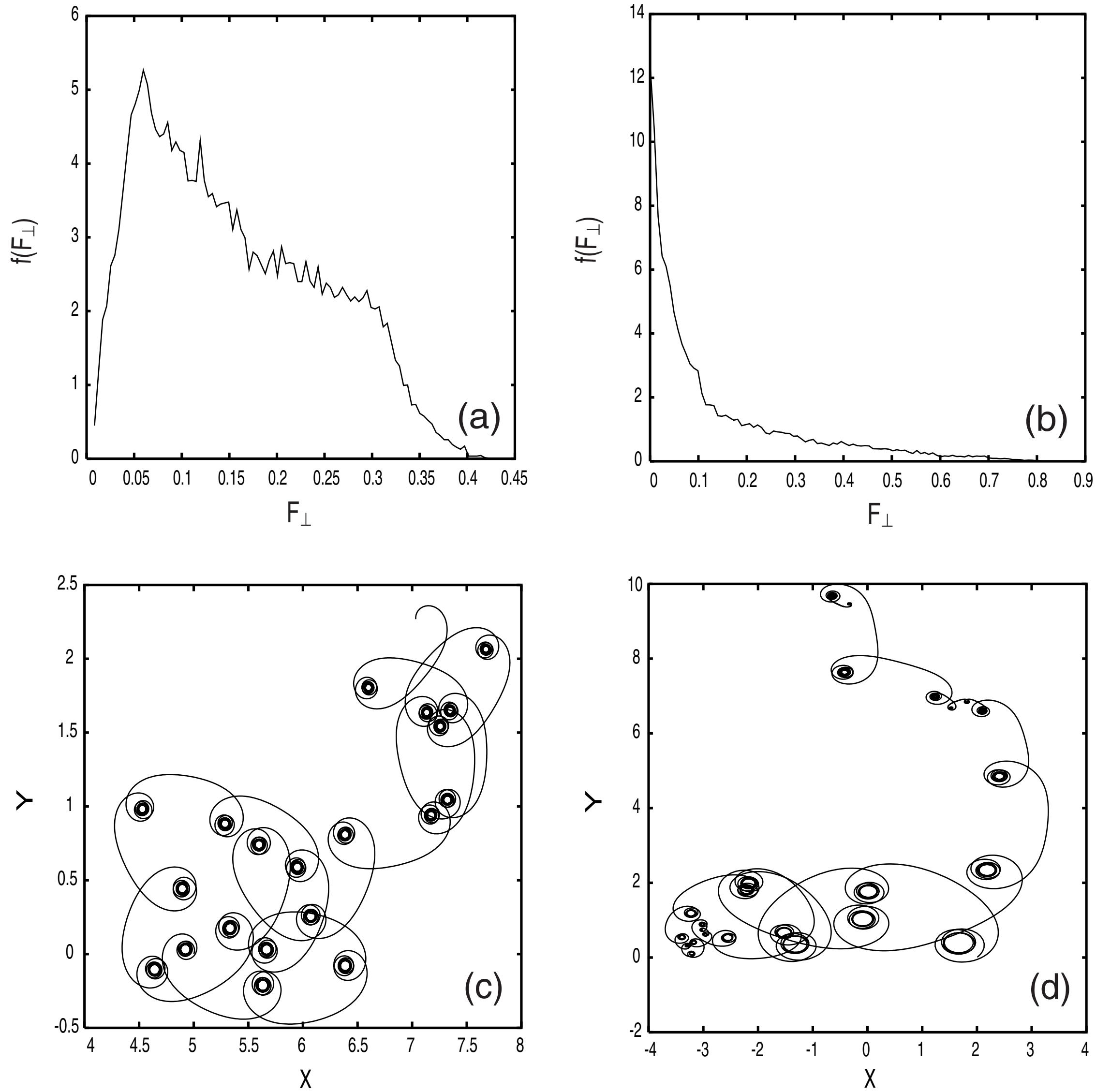

Figure 2. Distribution function $f(F_\perp)$ of the perpendicular forcing $F_\perp$ for (a) $N = 1$, (b) $N = 5$ ($\epsilon = 0.1$), and the orbit of the gyro-center under the $\boldsymbol{E}_{1\perp} \times \boldsymbol{B}_0$ drift force for (c) $N = 1$, (d) $N = 5$ ($\epsilon = 0.1$).

from these figures. The gyro-center motion consists of smooth jumps among positions where it remains trapped for large times. The trapping is indicative of the non-Gaussian characteristics mentioned above. The results are qualitatively the same even when the wave magnetic field $\mathbf{B_1}$ is included in the definition of (9).

4. Anomalous diffusion

We estimate the transport properties of the magnetized electrons interacting with the EC waves by following the orbits of 10000 electrons with initial energy $\gamma_0 = 2.5$. The initial conditions of the particles are chosen randomly. We construct the mean square displacements $< (\gamma - \gamma_0)^2 >$ and $< (\boldsymbol{r}_\perp - \boldsymbol{r}_{\perp 0})^2 >$ (see [29, 30]).

In figure 3(a) we plot $\ln < (\gamma - \gamma_0)^2 >$ as a function of $\ln t$ for $N = 1$ and $\varepsilon = 0.5, 0.1$, and for $N = 5, \varepsilon = 0.5$. In all cases, the log-log curves become linear after some characteristic time, which is a function of the wave amplitude. Thus, the power law dependence for the mean values is established after a characteristic time interval. For the smaller wave amplitudes (e.g. $\varepsilon = 0.1$), the time needed for the diffusion rates to settle to a characteristic power law is longer because the phase space becomes a very complex mixture of periodic and stochastic orbits. This complexity has been actually visualized in figure 1(b). During their motion in the chaotic regions of the phase-space the particles are trapped for some time around resonant islands, and this makes the motion a mixture of diffusion and organized motions. Nevertheless, the linear scaling is still present, at least after the sufficient time has elapsed.

Increasing the number of wave modes causes, in general, enhancement in the acceleration of the particles. However, the time needed for the scaling variations to diminish is not always smaller, as seen for $N = 5$. One reason for this is that the presence of many modes may bring up significant alterations in the phase space, new stability islands may arise or existing ones may disappear. It is not easy to make a prediction whether or where this might happen. The evolution of $< (\boldsymbol{r}_\perp - \boldsymbol{r}_{\perp 0})^2 >$ is similar to $< (\gamma - \gamma_0)^2 >$; the corresponding diagrams are given for the same values of N, ε in figure 3(b).

In each one of the two spaces, energy and position, the time-scaling of the diffusion is determined by the exponent a_γ and a_{r} of the respective power law. In logarithmic scales, they are linearly related to the mean square displacements: $\ln < (\gamma - \gamma_0)^2 > \propto \alpha_\gamma \ln t$ and $\ln < (\boldsymbol{r}_\perp - \boldsymbol{r}_{\perp 0})^2 > \propto \alpha_{\mathrm{r}} \ln t$. In figure 4(a),(b) a_γ and a_{r} are plotted as a function of ε for (a) $N = 1$ and (b) $N = 5$. We observe that for $\varepsilon > \varepsilon_{\mathrm{cr}}$, both exponents a_γ and a_{r} take values less than 1. This means that the evolution is sub-diffusive. For $N = 1$, for stronger amplitudes $0.2 < \varepsilon < 0.5$ the exponents are found within narrow bands of values, especially for $N = 1$. For amplitudes less than 0.1, the exponents start to decrease. This was expected, as the phase-space for such ε presents resonant islands. These formations correspond to periodic motions and particle trapping, which suppresses the diffusive behavior. The decrease of the exponents becomes more radical as ε approaches $\varepsilon_{\mathrm{cr}}$.

The situation is similar for the interaction with the wave-packet using $N = 5$ modes. The evolution of a_γ and a_{r} is presented in figure 4(b). Each scaling exponent takes slightly larger values than the ones of the previous case, at least within the error boundaries, but apart from that no significant enhancement of the diffusion scaling is observed. Of course, such an enhancement may appear in the values of the properly defined diffusion coefficient, but such a calculation goes beyond this work. The exponents decrease as ε approaches the threshold, due to the enhancement of phase-space characteristics like organized motions.

The behavior of a_γ and a_{r} with respect to the angle of wave propagation does not show significant differences with the behavior as a function of the wave amplitude. In

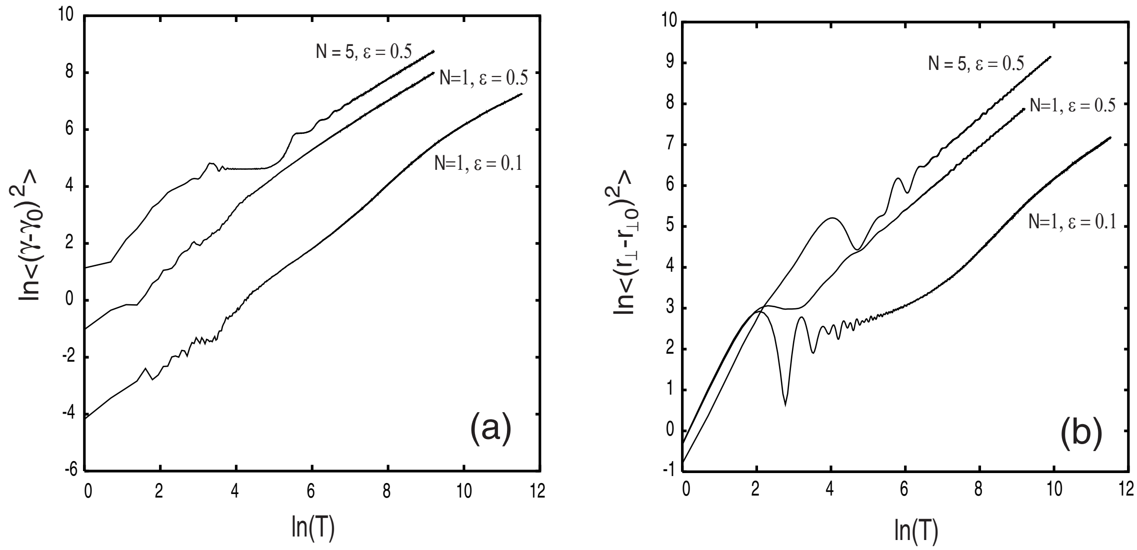

Figure 3. (a) $\ln < (\gamma-\gamma_0)^2 >$ as a function of $\ln t$ for $N=1$ ($\varepsilon=0.5, 0.1$) and $N=5$ ($\varepsilon=0.5$), (b) The same for $< (\boldsymbol{r}_\perp - \boldsymbol{r}_{\perp 0})^2 >$.

figure 4(c),(d) the scaling exponents are plotted as a function of the propagation angle θ for (c) $N=1$ and (d) $N=5$. For all the angles θ, the exponents a_γ and $a_{\rm r}$ take values corresponding to sub-diffusive scaling. For smaller values of the propagation angle the exponents start to decrease for the same reasons as in the above description.

5. Summary and Discussion

In this paper, the particle diffusion in the nonlinear interaction of magnetized electrons with an oblique electromagnetic wave-packet is studied. Electromagnetic waves are widely used for accelerating and heating charged particles in modern fusion reactors, but also play an important role in the ionosphere.

In numerous studies in the past, the interaction of charged particles with **electrostatic waves** was shown to be dominated by anomalous particle diffusion in real space [25, 27, 28, 39]. We have shown for the first time that, for **electromagnetic waves** interacting with electrons, the evolution in real space does not follow normal diffusion. The diffusion is found to obey simple power law in time and, for wave amplitudes near and over the threshold to chaos, the scaling exponents are indicative of sub-diffusion. The electromagnetic wave, as it was the case for the electrostatic wave [39], acts as a barrier. The physical reason for the anomalous scaling of diffusion is the inhomogeneous character of the system phase space, a mixture of periodic and stochastic orbits even for relatively large values of the wave amplitude. The exponents decrease as the wave power decreases, due to the enhancement of the periodic orbits in the phase space. The behavior of the scaling exponents with respect to the angle of wave propagation does not introduce significant differences to the described picture.

In this article, we find that the evolution of the electrons is sub-diffusive also in the

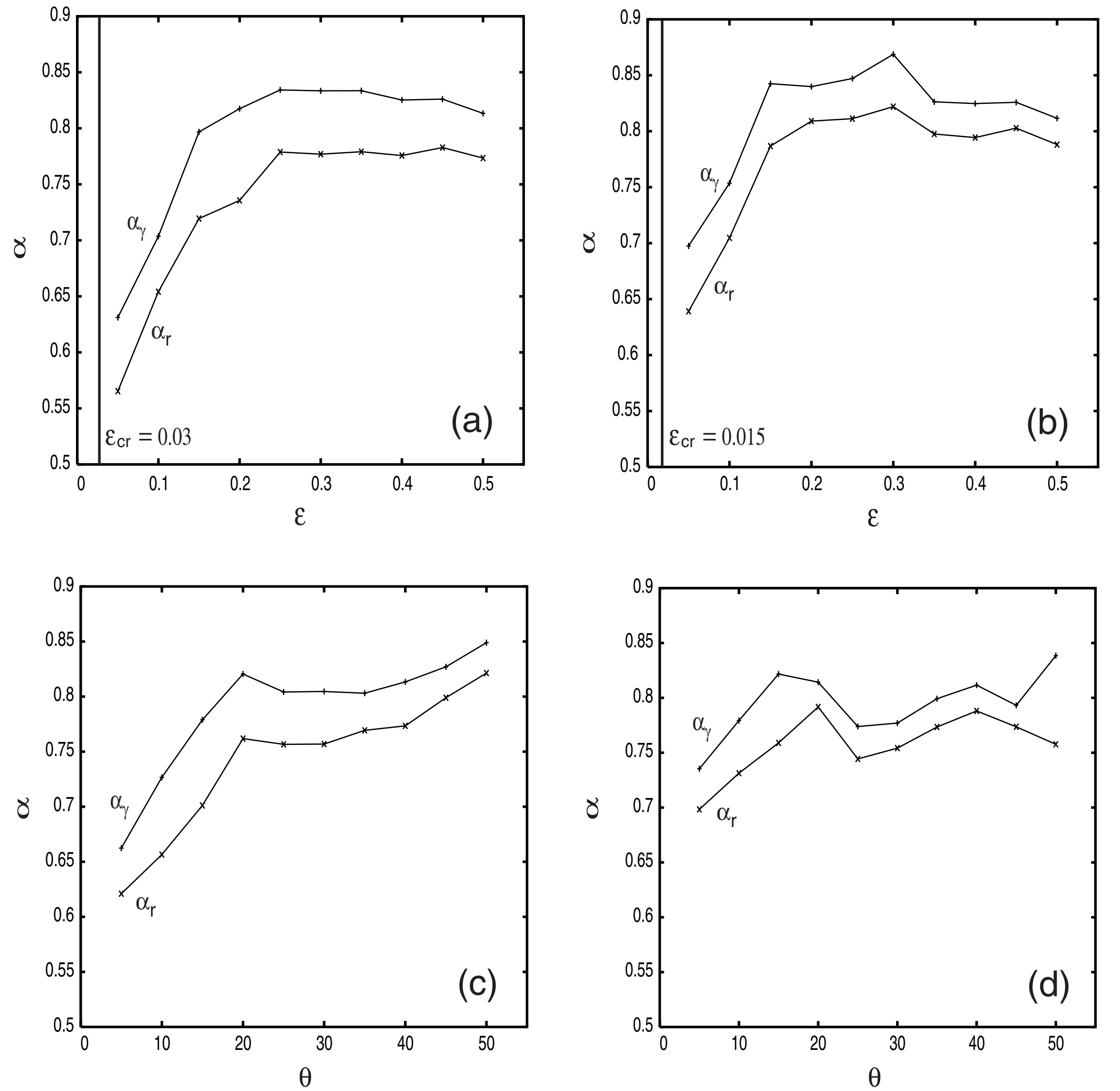

Figure 4. Exponents of diffusion a_γ and a_r as a function of the wave amplitude ε for (a) a single wave ($N = 1$) and (b) a wave-packet of $N = 5$ modes, and as a function of the propagation angle θ for (c) $N = 1$ and (d) $N = 5$.

energy space. The scaling exponents of the energy diffusion have the same properties as the ones in the real space described above. In this sense, the use of a quasilinear theory and the FP equation becomes questionable. In the existing literature, there is still a contradiction on this issue. The quasilinear theory is widely used to describe the particle diffusion in energy space [20, 21, 23]. In the interaction of ions with electrostatic waves, there are results showing that the use of the quasilinear is limited and cannot always describe sufficiently the velocity diffusion [26, 32]. Similar results exist for electromagnetic waves interacting with electrons [15, 22]. The quasilinear theory is also used in the theory and simulations of ECRH (see [24] and refs. therein). However, the results suggest that the quasilinear theory breaks down and the particle distribution function cannot be described sufficiently without including nonlinear effects [24].

We have also examined for the first time the forcing that controls the trajectories of the magnetized electrons in the presence of EC waves. In the past, several studies have discussed the properties of magnetized ions in the presence of pre-described forcing terms [35, 37, 38]. We have shown that the forcing is highly inhomogeneous along the trajectory of the particle. We find that the forces contain non-Gaussian characteristics. These aspects appear also in the orbit of the gyro-center under the drift force $\mathbf{E}_{1\perp} \times \mathbf{B}_0$, which consists of smooth jumps between locations of trapping. This result supports the findings reported above on the anomalous diffusion.

Our main contributions in this article are: (1) We demonstrated clearly that the quasilinear theory breaks down when chaos is not complete, which might be the case for present day experiments, and creates the need for different approaches to be used, such as the inclusion of nonlinear effects and the fractional kinetics, in order to obtain more consistent results. (2) We have also shown that the EC waves slow down the radial transport of the electrons, acting as a barrier, and this may have important consequences for the overall particle transport.

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