Mode coupling in non-axisymmetric solar dynamo models

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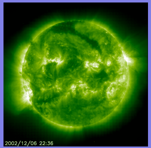
Chalkidiki, 27 Sept 2003

Basic Processes of Turbulent Plasmas
Chalkidiki, Greece.



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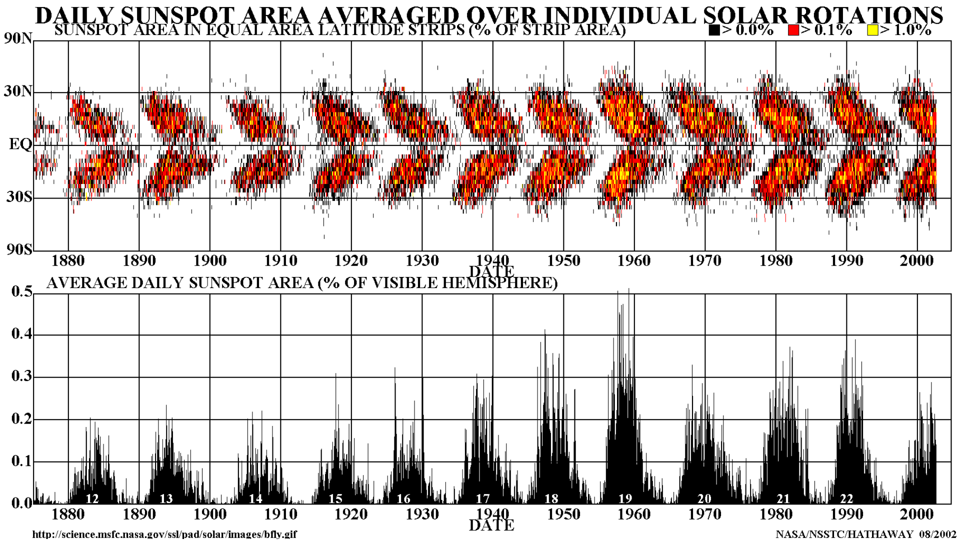
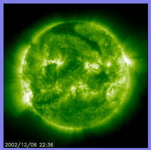
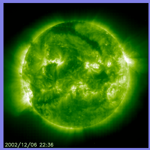
The Sun and the Sunspot Cycle



- At the beginning of any new cycle, sunspots appear at latitudes between 30 and 45 degrees. and subsequently migrate equatorwards, concentrating within the 30° latitude belt.

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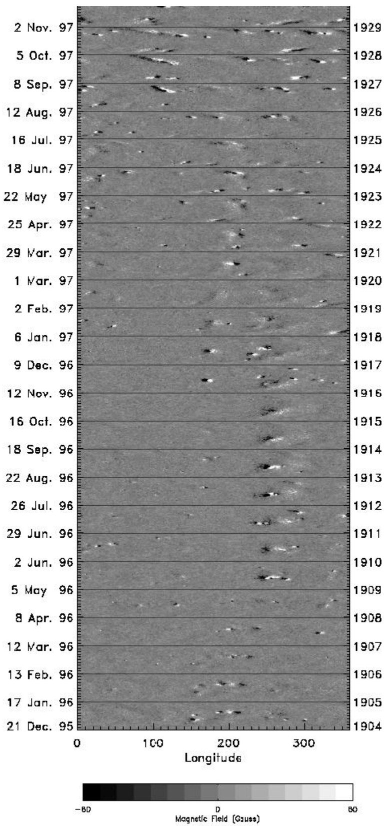
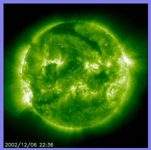


Longitudinal structure: preferred longitudes

- Magnetic features appear at particular longitudes.
- Cycle 22: sol min (1996)
- Five major active regions emerge all at the **same** Carrington longitude of about 250°. (mid April - late July). DeToma, White & Harvey, 2000
- Persistence of active longitudes has been calculated up to 120 years.

  Berdyugina & Usoskin, 2003
- Threshold mechanism in the presence of an underlying mean-field component ? Ruzmaikin 1999



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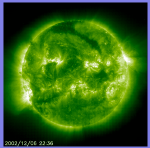
Preferred longitudes in the Solar Wind

- The poloidal field of the Sun opens into the interplanetary space carried by the solar wind.
- Non-axisymmetric components of the poloidal field appear as rotating patterns in the interplanetary field.
- In interplanetary field, magnetic field patterns have been found to have a period of 27.03 days (428nHz), through several solar cycles. Neugebauer, Smith, Feynman, Ruzmaikin, 2000.

- KEYWORDS:
  - Persistence
  - Clustering



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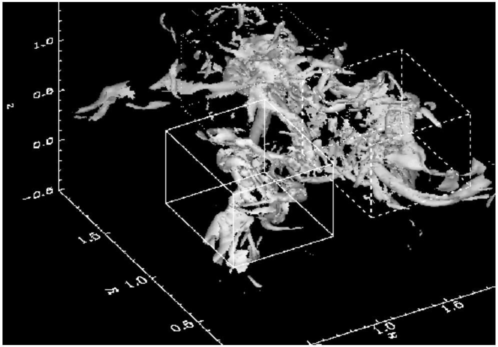
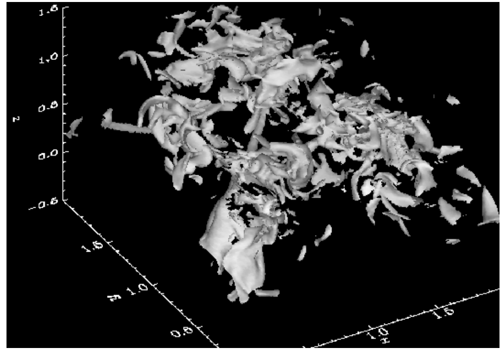
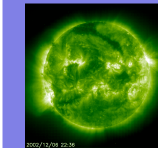




Coherent Structures in MHD turbulence.

- Analysis of turbulence at moderate Reynolds number
  Brandenburg,Jennings,Nordlund,Rieutord,Stein,Tuominen 1995
- Structures in vorticity and in magnetic field do not coincide.

Bigazzi,Brandenburg,Moss , Phys.Plasmas **6** 72-80 (99)



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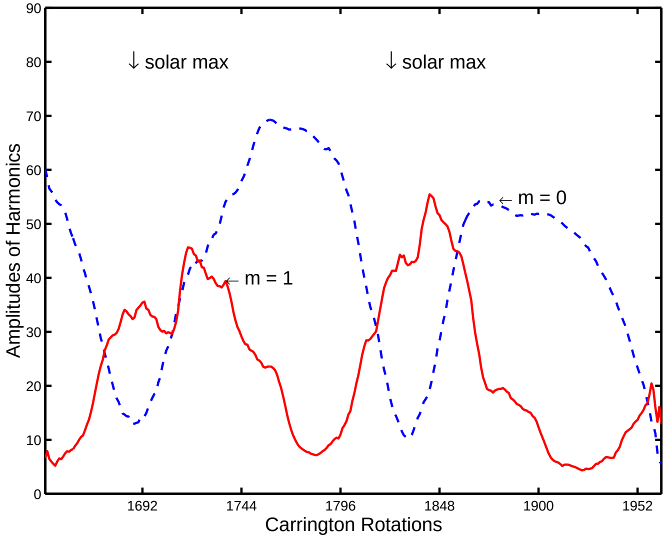
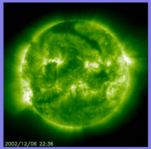
Coupling of the dynamo azimuthal m-modes



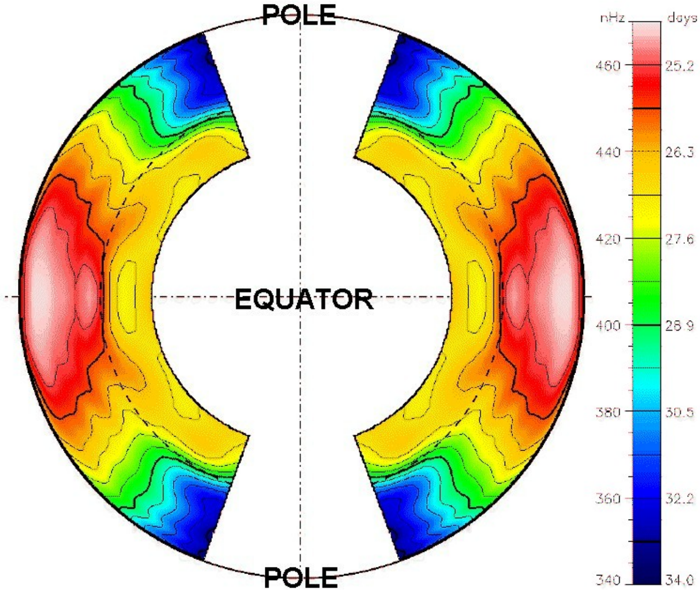
- Observations suggest, that modes are coupled.

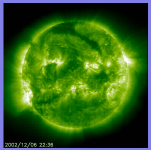
Ruzmaikin, Feynman, Neugebauer, & Smith, 2001



Where is the solar dynamo?

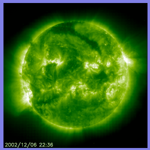
- Solar rotation curve. Helioseismic data. (M.J.Thompson)





What is this telling us about the dynamo

- Presence of a non-axisymmetric mean-field which
  1. Is concentrated at low latitudes
  2. (possibly) maximum close to the tachocline
  3. Rotates with a frequency close to 27 days
  4. Is modulated with the solar cycle period.



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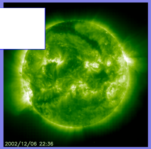
Turbulence and the α-effect: mean field dynamo.



- Take the induction equation

$$\frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{\nabla} \times (\boldsymbol{u} \times \boldsymbol{B}) + \boldsymbol{\nabla} \times \eta \boldsymbol{\nabla} \times \boldsymbol{B}$$

- and separate out the mean from the fluctuating part.

$$\boldsymbol{B} = \overline{\boldsymbol{B}} + \boldsymbol{B}' \qquad\qquad \boldsymbol{u} = \overline{\boldsymbol{u}} + \boldsymbol{u}'$$

- You get an equation for the mean field.

$$\frac{\partial}{\partial t}\overline{\boldsymbol{B}} = \boldsymbol{\nabla} \times (\overline{\boldsymbol{u}} \times \overline{\boldsymbol{B}}) + \boldsymbol{\nabla} \times \overline{\boldsymbol{u}' \times \boldsymbol{B}'} + \eta \nabla^2 \overline{\boldsymbol{B}}$$

- An electric field proportional to the mean fiald and its derivative results.

$$\boldsymbol{\mathcal{E}}_{\ i} = \overline{\boldsymbol{u}' \times \boldsymbol{B}'}_i = \alpha \delta_{ij} \overline{\boldsymbol{B}}_j + \beta \epsilon_{ijk} \overline{\boldsymbol{B}}_{j,k}$$

- You have a source and a diffusion term coming from your underlying turbulence.

$$\frac{\partial}{\partial t}\overline{\boldsymbol{B}} = \underbrace{\boldsymbol{\nabla} \times (\overline{\boldsymbol{u}} \times \overline{\boldsymbol{B}})}_{\Omega-\text{effect}+\text{merid circ}} + \underbrace{\boldsymbol{\nabla} \times \alpha \overline{\boldsymbol{B}}}_{\alpha-\text{effect}} + \underbrace{(\eta + \beta)}_{\text{Turb diff}} \nabla^2 \overline{\boldsymbol{B}}$$

Toroidal and poloidal potentials

- Two variables: T, P:

$$\boldsymbol{B}_T = \boldsymbol{\nabla} \times \mathbf{r}T$$

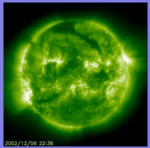
$$\boldsymbol{B}_P = \boldsymbol{\nabla} \times \boldsymbol{\nabla} \times \mathbf{r}P$$

- Two coupled equations:

$$\begin{aligned}
\partial_t T &= R_\Omega V_\Omega + R_\alpha V_\alpha + R_M V_M \\
&+ \eta \nabla^2 T + \partial_r \eta \cdot \frac{1}{r} \partial_r (rT), \\
\partial_t S &= R_\Omega U_\Omega + R_\alpha U_\alpha + R_M U_M \\
&+ \eta \nabla^2 S
\end{aligned}$$

- Non-dmensional numbers:

$$R_\Omega = \frac{\Omega_0 R_\odot^2}{\eta_0}, \quad R_\alpha = \frac{\alpha_0 R_\odot}{\eta_0}, \quad R_M = \frac{u_M R_\odot}{\eta_0}.$$



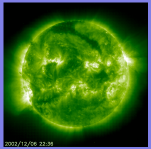
Toroidal and poloidal potentials

$$
\begin{aligned}
\partial_t T &= R_\Omega V_\Omega + R_\alpha V_\alpha + R_M V_M \\
&+ \eta \nabla^2 T + \partial_r \eta \cdot \frac{1}{r} \partial_r (rT), \\
\partial_t S &= R_\Omega U_\Omega + R_\alpha U_\alpha + R_M U_M \\
&+ \eta \nabla^2 S
\end{aligned}
$$

- Relation between the scalars U, V and the sources.

$$
\begin{aligned}
((\boldsymbol{\Omega} \times \mathbf{r}) \times \boldsymbol{B})_T &= -\mathbf{r} \times \boldsymbol{\nabla} U_\Omega, \\
\boldsymbol{\nabla} \times ((\boldsymbol{\Omega} \times \mathbf{r}) \times \boldsymbol{B})_P &= -\mathbf{r} \times \boldsymbol{\nabla} V_\Omega, \\
(\boldsymbol{u}_M \times \boldsymbol{B})_T &= -\mathbf{r} \times \boldsymbol{\nabla} U_M, \\
\boldsymbol{\nabla} \times (\boldsymbol{u}_M \times \boldsymbol{B})_P &= -\mathbf{r} \times \boldsymbol{\nabla} V_M, \\
(\alpha \boldsymbol{B})_T &= -\mathbf{r} \times \boldsymbol{\nabla} U_\alpha, \\
\boldsymbol{\nabla} \times (\alpha \boldsymbol{B})_P &= -\mathbf{r} \times \boldsymbol{\nabla} V_\alpha.
\end{aligned}
$$

- Those relations can be solved numerically



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Longitudinal m-modes.

- Expansion in longitudinal m-modes as:

$$T(r,\theta,\phi) = \sum_{m=0}^{N} T^m(r,\theta)e^{im\phi} + \text{cc}, \ldots$$

- Theorem: when $\overline{\boldsymbol{u}}$, α and η are axisymmetric, the equation decompose into an independent set per each m-mode.

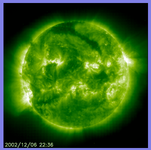
$$\begin{aligned}\partial_t T^m &= L^m(T^m, S^m) \\ \partial_t S^m &= G^m(T^m, S^m)\end{aligned}$$

- Modes are decoupled.
- Non-axisymmetric α naturally couples the modes.

$$(\alpha\boldsymbol{B})^m = \sum_{j=-N}^{N} \alpha^j(r,\theta)\boldsymbol{B}^{m-j}(r,\theta)$$

Consider the lowest modes $m = 0$, $m = 1$.

$$(\alpha\boldsymbol{B})^1 = \alpha^0\boldsymbol{B}^1 + \epsilon\alpha^1\boldsymbol{B}^0$$



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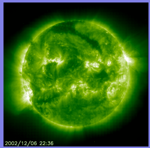
Our dynamo model: numerical setup.

- Non-spectral
  - Legendre transform is numerically expensive
  - No fast algorithm like FFT exists
  - Ease of parallelization
- Regular grid in r, θ, typically 80 x 160 grid points.
- Solves for $m = 0$ and the first non-axisymmetric mode $m = 1$
- Outer boundary conditions: potential.



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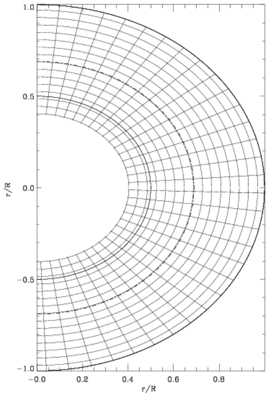
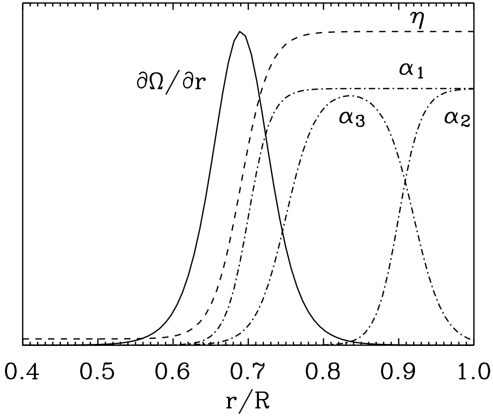
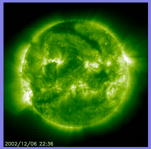
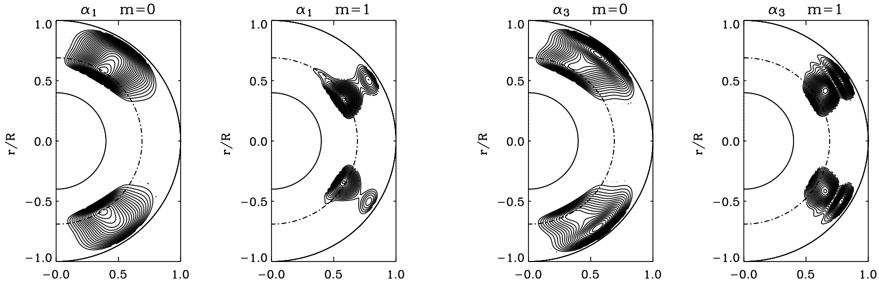




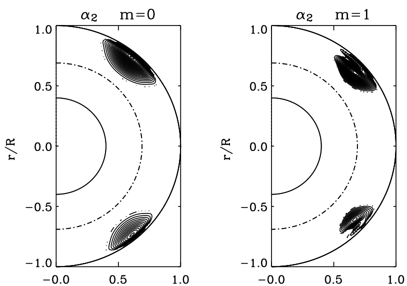
- We include the rotation curve of the Sun
- Coupling is introduced through the non-axisymmetric α
- A variable profile of turbulent diffusivity $\eta(r)$ defines the core boundary.
- We consider three different models for the distributions of α, see figure above.

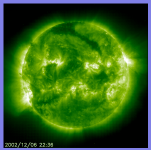


Localization of the field.

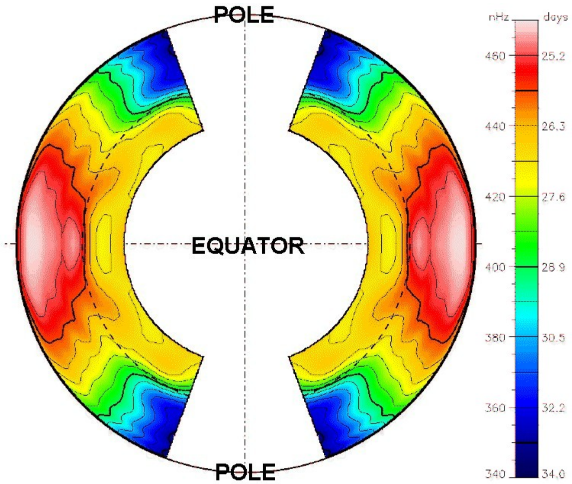


- In latitude: the non-axisymmetric mode concentrates around 30°
- In radius: the field maximises close to the Tachocline
- Surface α: No field at the tachocline.

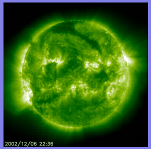




Localization: solar differential rotation

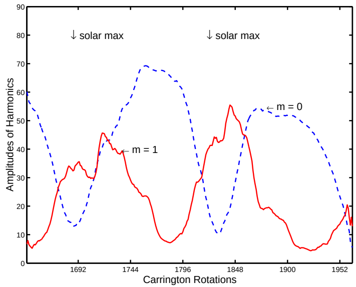


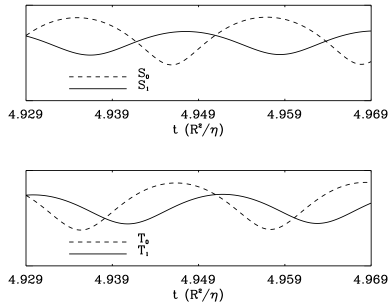
- The radial gradient of angular velocity is close to null at 30°
- That is where the non-axisymmetric (toroidal) field concentrates (when α overlaps with the shear layer, tachocline, at $0.6R_{\odot}$).
- The angular velocity distribution is reconstructed from helioseismic data Thompson, M.J. 2000



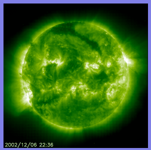
Cycle period and phase relations

- The $m = 1$ mode has the same cycle period as the $m = 0$ mode.

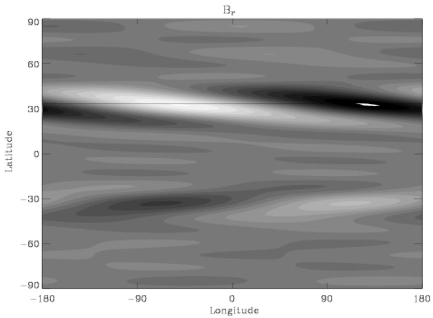


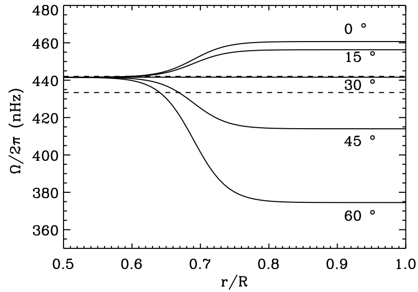


- The phases between S and T potentials, modes $m = 0$ and $m = 1$, are consistent with observations (case α_1 is displayed):
  - S_1 is max at T_0 min.
  - S_1 is max after S_0 min.

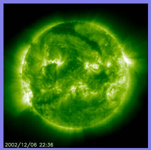


Rotation rate of the $m = 1$ mode





- The radial (poloidal) field at surface rotates with a rate of 442nHz (core rotation), M1 and M3, and 433nHz, M2.
- In interplanetary field a rotation of 27.03 days (428nHz) has been found. Neugebauer, Smith, Feynman, Ruzmaikin, 2000.
- Fast Ulysses scan at solar max, 2000-2001 (CR1970-CR1981): 432 $\div$ 437nHz rotation rate of the $m = 1$ mode (tachocline rate). Jones, Balogh & Smith, 2003



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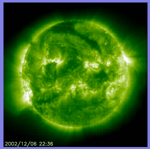
Conclusions

- The coupling of dynamo modes due to a non-axisymmetric α-effect, is responsible for
  - The latitudinal localization around 30° of the non-axisymmetric mode due to the solar rotation curve.
  - The 11 yr cycle for both the $m = 0$ and $m = 1$ components.
  - Preferred Longitudes:
    - Longitudinal localization of the fields due to the m-modes of the dynamo-generated fields.
    - Rate of rotation of surface fields determined by the global evolution of magnetic fields rather than from pure surface phenomena.
- How a non-axisymmetric α is produced?
  - Magnetic and/or hydro instabilities
  - Other mechanisms?

Bigazzi & Ruzmaikin 2003, ApJ, submitted



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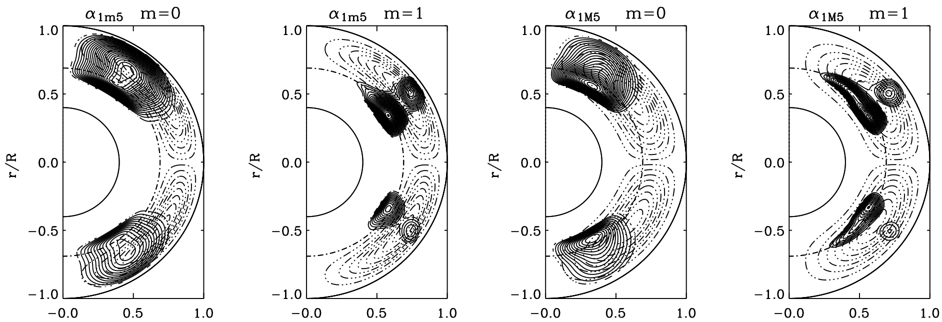
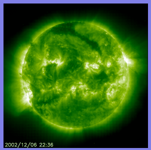
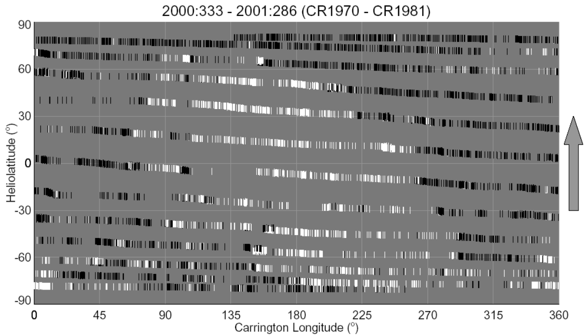
Meridional circulation



- Shallow and deep meridional circulation:
  - Diffusivity decreased, below the tachocline, to 1/200 the convection zone value.
  - Velocity close to the surface of order 20m/s
  - Velocity at the bottom 1/10 of surface velocity.
- Distribution is not radically changed.
  - $m = 1$ mode still concentrated at 30° latitude.
- Cycle period and symmetry are more sensitive.



Rotation rate of the $m = 1$ mode - Ulysses.



- Rotation rate calcutated for the fast Ulysses scan 2000-2001 (CR1970-CR1981). Jones, Balogh & Smith, 2003
- Ulysses data support a $432 \div 437$nHz rotation rate of the $m = 1$ mode, which correlates with the rotation rate of the tachocline.