Alfvén waves propagation in an X point magnetic configuration

S. Landi (1), M. Velli (1) and G. Einaudi (2)

(1) Università di Firenze, (2) Università di Pisa

The dynamics of the development of extremely small scales in magnetic field is crucial to understand the heating and the energetic manifestations of the high temperature plasma of the solar corona. Here we illustrate what could be an essential aspect of the cascade of magnetic energy to small scales via numerical simulations of the propagation of (shear) Alfvén waves in a magnetic field with an X-point geometry in 2.5 D. The coupling of the waves with the background field leads to the development of fast mode shocks whose number depends on the Alfvén wave frequency. Though the X-point is essential to shock wave formation, dissipation occurs within the shocks which sweep the whole plasma volume. The shocks might also play an important role in modifying particle acceleration around the X-point.

Introduction

MHD waves can play an important role in the heating of the solar corona, a primary role if they are the agent by the which the photospheric energy is transmitted into the corona or a secondary role if they are one of the channel into which energy is brought by other heating mechanism.

In this work we address the question on how energy associated with these waves may dissipate once it is in the corona.

Given the smallness of the Reynolds number in the solar corona, $R_\eta = 10^{11}$, the only way of dissipating is to create small scale structures. Given the lack of homogeneity of the coronal magnetic field small scales can be created by the linear and non linear interaction between the waves and the magnetic field (Einaudi and Velli, 1994).

In this poster we present the study of the propagation of Alfvén waves in a potential magnetic field configuration with X-points for large and small values of the plasma β.

The main result is that the coupling of shear Alfvén waves with the X-point magnetic configuration leads to the formation of fast shocks. Fast shocks are able to dissipate independently of the Reynolds number and, given their motion, the dissipation is not focalized around the X-point but occurs in the whole volume.

The numerical model

We make use of resistive and viscous MHD for a compressible fluid.

In the numerical resolution we assume that one coordinate (say z) can be ignored

We consider a rectangular box of dimension $Lx = Ly = 2\pi$.

Along the y direction we assume periodicity; the spatial integration is performed by spectral methods.

In the x direction no assumption is made about periodicity. The spatial integration is performed by the use of compact difference schemes (Lele, 1992).

The initial conditions are chosen with structurally stable magnetic field configuration (Bulanov et al., 1999).

The drawback of periodicity is that it is impossible to treat an isolated X-point.

We make us of a current free magnetic field configuration:

$$\phi = \frac{1}{2\sinh(\pi)}\sinh(x-\pi)\sin(y)$$

$$B_x = \frac{\partial\phi}{\partial y} + B_{0x}$$

$$B_y = -\frac{\partial\phi}{\partial x} + B_{0y}$$

$$B_z = B_{0z}$$

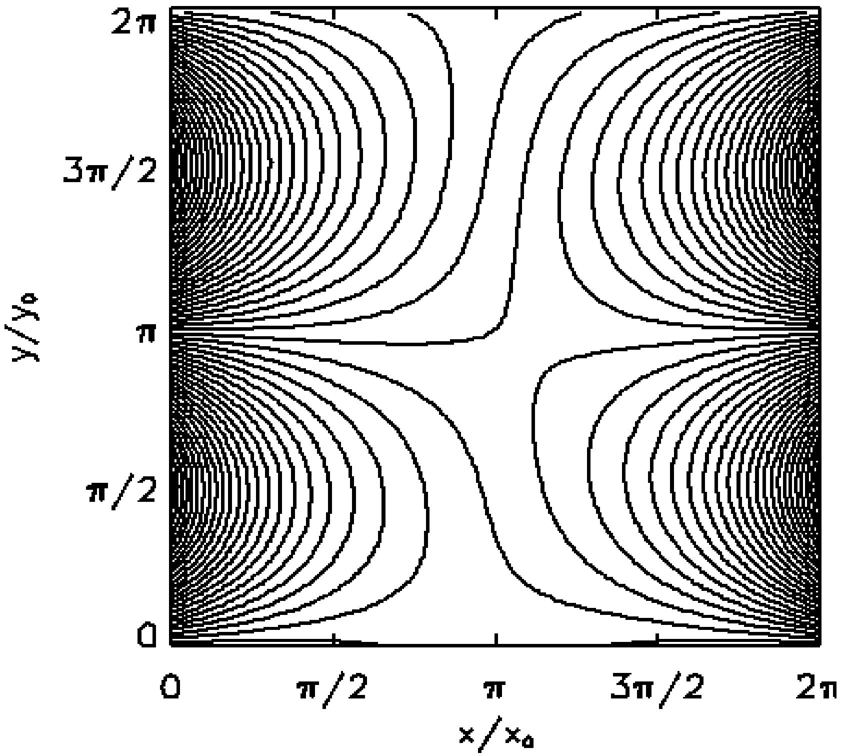


Fig.1 Field lines of the structurally stable magnetic configuration chosen as initial condition in our simulations.

The magnitude of the constant field is chosen so that the three components have similar rms:

$$B_{0x} = B_{0y} = 0.01 \qquad B_{0z} = 0.5$$

In the x direction boundary conditions are assigned via projected characteristic of the MHD equations allowing us essentially perfect non reflecting boundary conditions and full control over the type of fluctuations input into the numerical domain (Roe and Balsara, 1996).

We introduce Alfvén waves from both the boundaries in x = 0 and x = 2π. We have chosen to force inward only in over the central half of the domain, i.e. over only one X-point.

$$L_a = \alpha \sin(y)\cos^2(y)\left(\frac{t^2}{t^2+1}\right)\cos(\omega t)$$

The maximum amplitude α of the Alfvén waves is chosen to be comparable to that of the equilibrium magnetic field so that linear and nonlinear coupling with the X-point are also comparable.

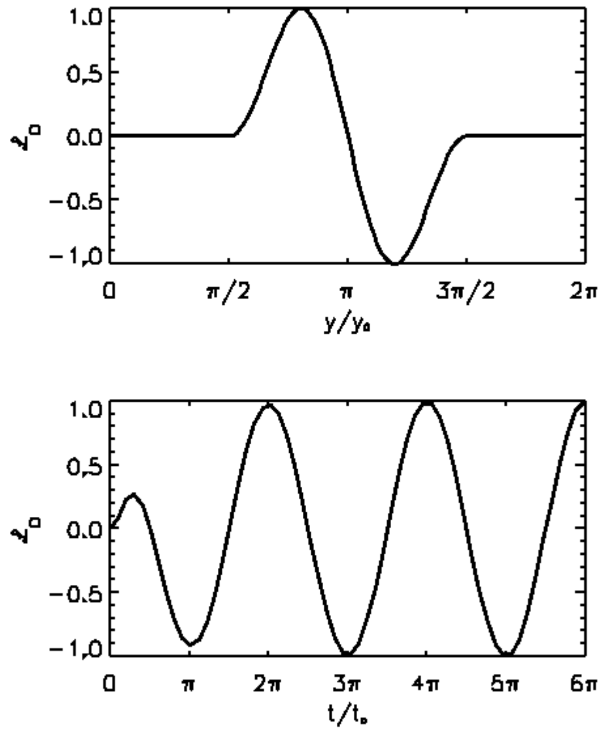


Fig.2 Spatial (top) and temporal (bottom) profiles of the Alfvén waves introduced at the boundaries in x = 0 and x = 2π, via the projected characteristic method.

High β simulations

In this set of simulation we have adopted an uniform temperature $T_0 = 0.75$; β varies between $2/3$ at the boundaries and 6 near the X-point.

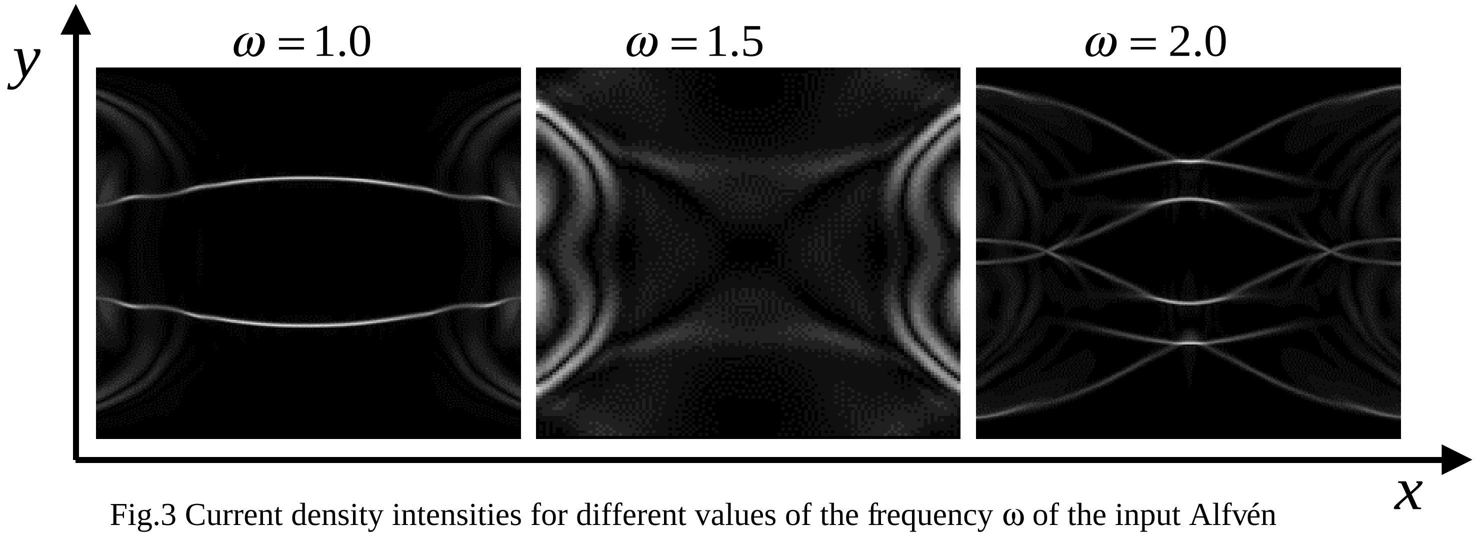


Fig.3 Current density intensities for different values of the frequency ω of the input Alfvén wave.

Once the stationary regime has been reached on average, several current sheet appear in the domain depending on the frequency ω of the input Alfvén wave.

These current sheet appear to extend along the x direction and propagate along the y direction in a quasi one-dimensional fashion.

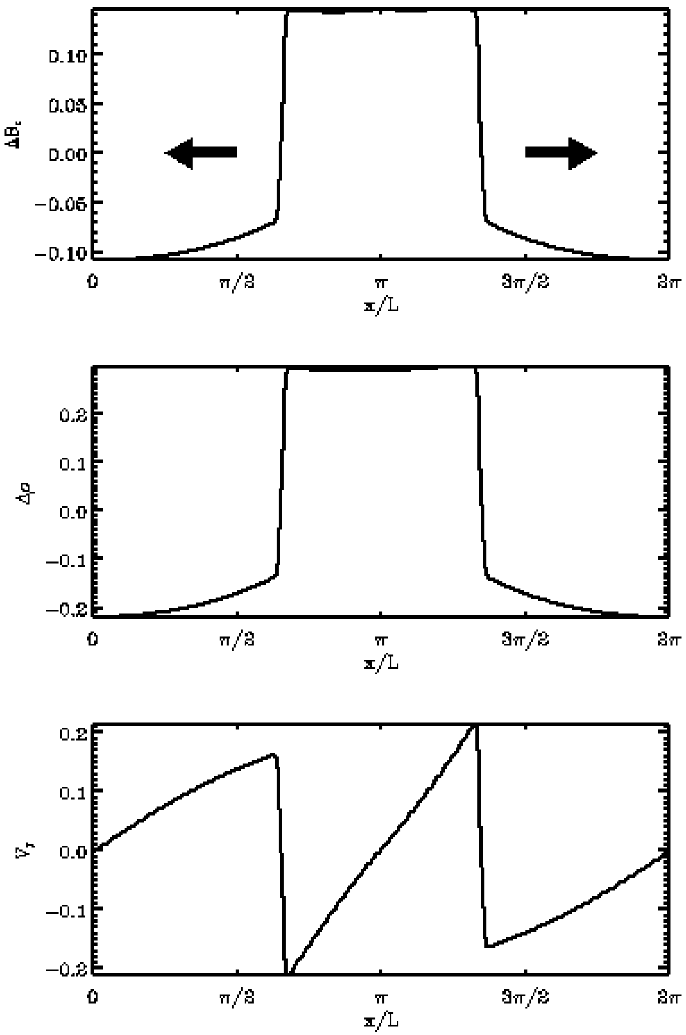


Fig.4 For the ω = 1 simulation, from top to bottom: z-component magnetic fluctuations, density fluctuations and y-directed fluid velocity as a function of y in x = π. The arrows mark the direction of propagation of the two shock waves.

These current sheets are consequence of fast magnetoacustic shock waves propagating along the *y* direction.

We note the positive correlation between the z component of the magnetic field and the density fluctuations. The plasma is compressed crossing the discontinuity.

These waves are propagating with a velocity corresponding to the fast magnetoacustic mode.

In the shock front reference frame, the fluid velocity is superfast before the shock.

The Rankine-Hugoniot jump conditions for an isothermal gas are verified.

Simulations have been performed with different Reynolds number, i.e. $R_\eta = 200$ and $R_\eta = 500$. For the higher Reynolds number the resolution is $N_x = N_y = 256$ (128 for $R_\eta = 200$).

When shock waves are fully developed the ohmic dissipation rate become independent of the Reynolds number.

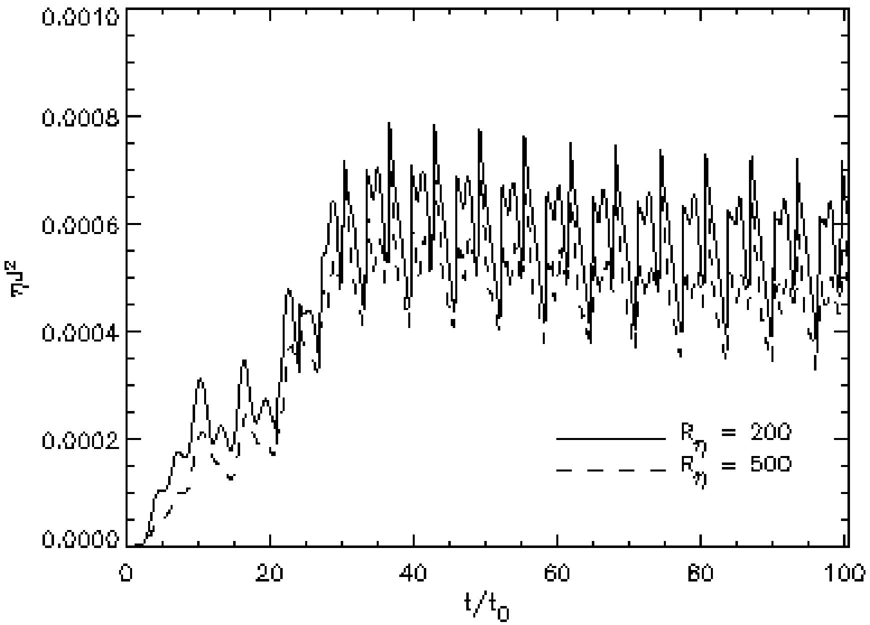


Fig 5. We compare the ohmic dissipation rate as function of time for $\omega = 1$ simulations which differ in the Reynolds number.

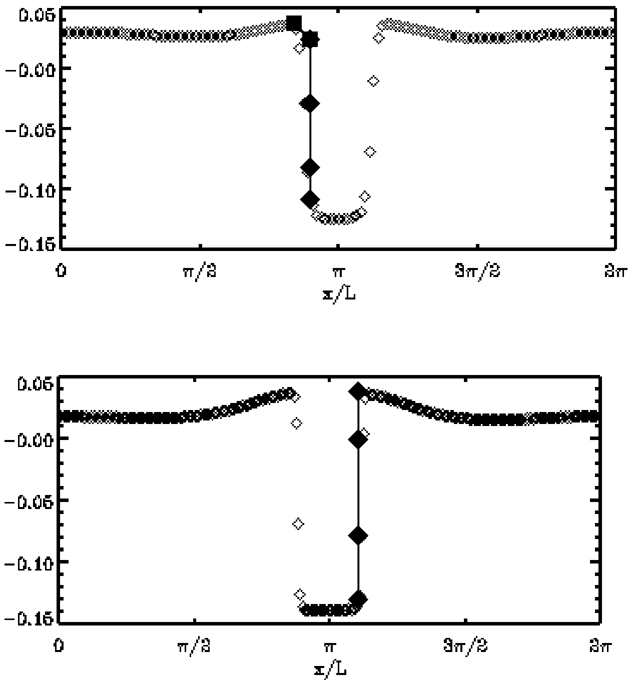


Fig 6. Comparison of the width l of the current sheet for 2 simulations which differs in the Reynolds number and resolution.

We can then found a relationship between the width l of the current sheet and the Reynolds number:

$$l \propto \frac{1}{R_\eta} \qquad\qquad l_{500} = \frac{2}{5} l_{200}$$

Taking into account the different resolution adopted this means that the number of point needed to resolve the discontinuity must scale as $4/5$.

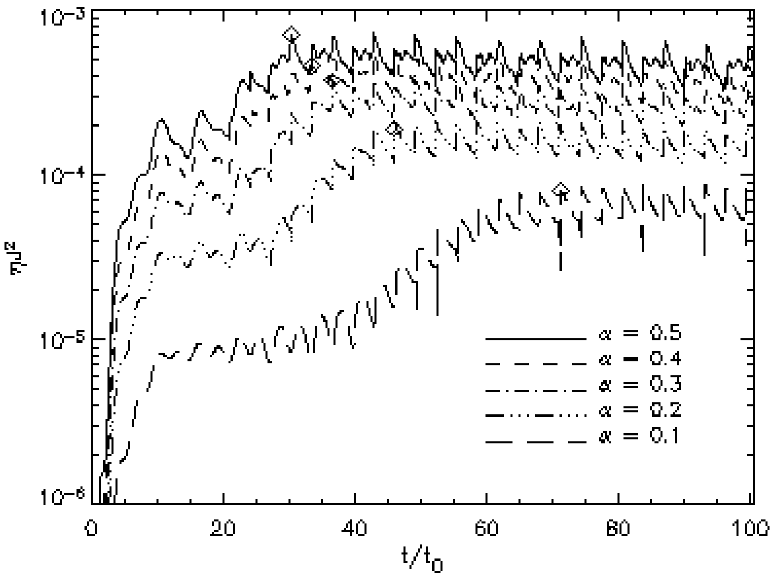


Fig. 7 Ohmic dissipation rate for simulations performed with different values of the amplitude α of the Alfvén wave pumped at the boundaries. The diamonds mark when a stationary condition has been reach.

The simulations has been performed also varying the amplitude of the injected Alfvén wave at the boundaries.

The dissipation rate show that the time when a stationary condition occurs depends on the amplitude. This means that only when a sufficient amount of energy is converted in fast mode shocks the energy equilibrium is reach.

The formation time, i.e. the time needed to reach a stationary regime, show an inverse square root dependency on the amplitude of the Alfvén wave:

$$\tau_s = \alpha^{1/2}$$

It is not clear the reason of such a dependence.The intensity of the dissipation scales as:

$$\tau_s = \alpha^{3/2}$$

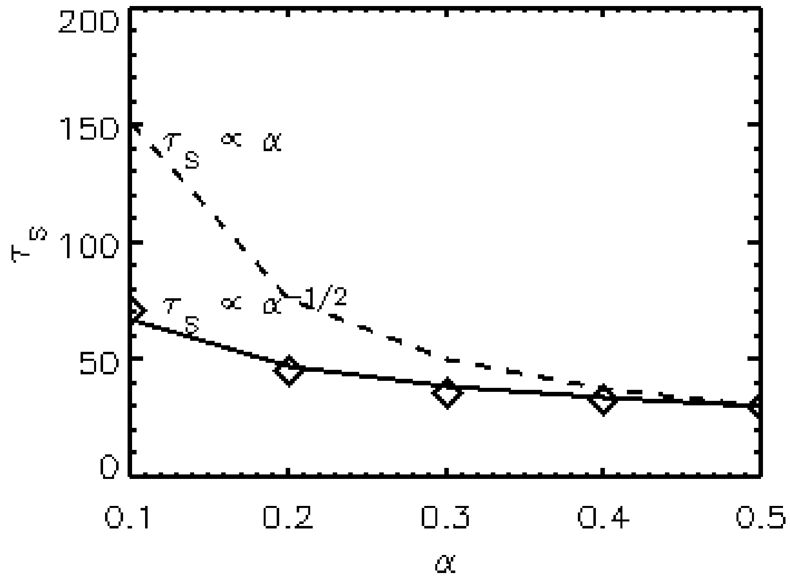


Fig. 8 We plot the formation time τ_s as function of α showing that the best fit is an inverse square root dependence on α. The $\alpha \propto \tau_s$ has been drawn as reference.

The low β simulations

We have adopted an uniform temperature one order of magnitude smaller then the previous case. With such a condition β varies from *0.15* at the boundaries up to *0.3* near the X-point.

These values are typical for an active-region corona with density $10^{10}\ \mathrm{cm}^{-3}$ and Alfvén velocity of $300\ \mathrm{Km\ s}^{-1}$.

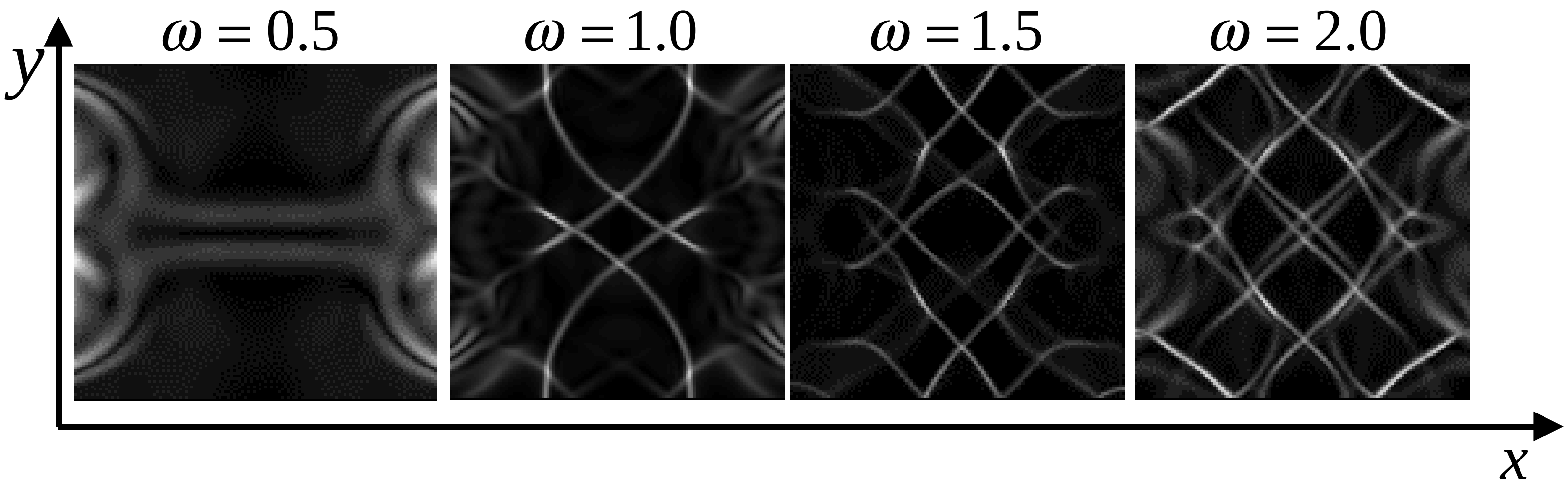
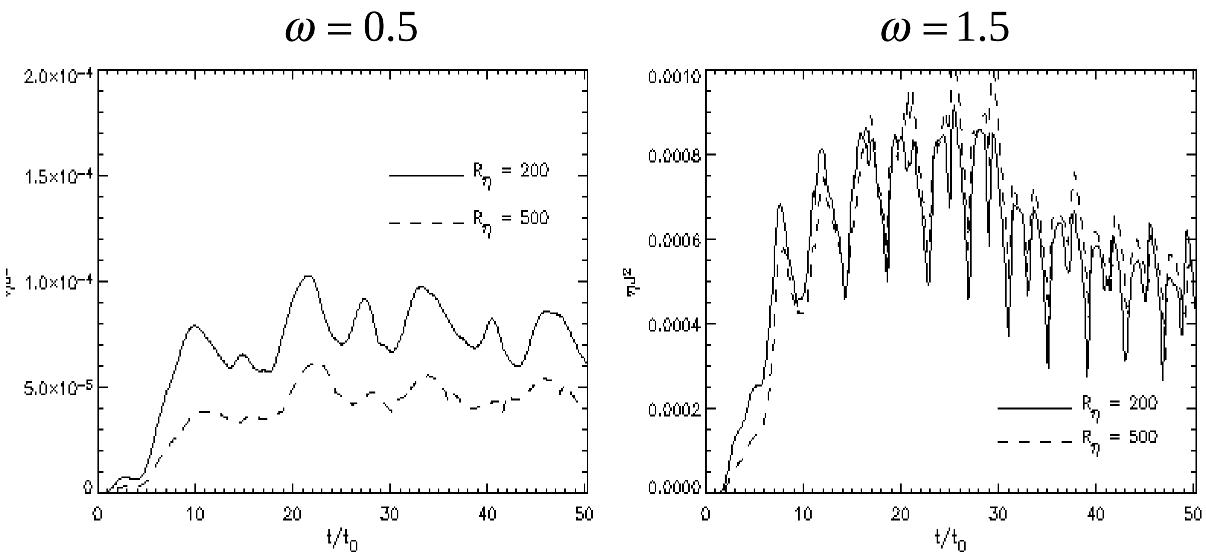


Fig.9 Current density intensities for different values of the frequency ω of the injected Alfvén waves at the boundaries, for the low β simulations.

As seen in the high β simulations, the appearance and the number of the current sheets depend on the frequency of the Alfvénic pump. In this case, however, their propagation yields an increasingly intricate pattern as they intersect and fragment and dissipate.

This more intricate structure can be explained recalling that, for a low β plasma, the fast magnetoacustic mode depends strongly on the magnetic field which varies of about a factor 2 in the whole domain.



As in the simulations with β>1 previously shown, once the shock waves formation is obtained, the current dissipation in the whole domain is almost insensitive of the resistivity. This is clearly shown in Fig.10, where we compare the dependence of the dissipation on the Reynolds number when current sheets are formed (ω=0.5) or not (ω=1.5).

Fig.10 We compare the current dissipation obtained with different Reynolds number when the Alfvén wave frequency is ω=1.5 (on top) and ω=0.5(bottom panel). When shock waves are generated, the dissipation is almost insensitive of the Reynolds number.

Conclusion

• We have presented simulations of the propagation and mode conversion of large amplitude Alfvén waves into fast mode shocks in equilibrium magnetic field with X-points.

• Dissipation via shock waves does not depend on the Reynolds number as this tend to infinity and therefore the whole problem related to the time scale of dissipation in the question of coronal heating is by-passed in this process.

• Though the X-point is essential to shock wave formation, dissipation occurs within the shocks which sweep the whole plasma.

• The presence of intricate patterns of shock waves could result in strong acceleration of particle (Vlahos, 1994).

• This simulations should be viewed as a preliminary result. One question is the extent to which our periodic boundary conditions are important in the generation of the waves.

• Another important question concern the more realistic case of a time-chaotic driver. It is important to know if this process survive or if the interaction between different fast mode waves created by the chaotic Alfvénic pump, become destructive.

References

Bulanov, S. V., Echkina, E. Y. , Inovenkon, I. N., Pegoraro, F. and Pichushkin, V. V., *Phys. Plasmas*, **6**, 802 (1999)

Einaudi, G. and Velli, M., *Space Science Reviews*, **68**, 97 (1994)

Lele, S. K., *J. Comp. Phys.*, **103**, 16 (1992)

Roe, P. L. and Balsara, D. S., *J. Appl. Math.*, **56**, 57 (1996)

Vlahos, L., *Space Science Reviews*, **68**, 39 (1994)