Injection of ELF Magnetic Fields and Currents into the Equatorial E-Region Ionosphere

Modern Challenges in Nonlinear Plasma Physics

Halkidiki, Greece, 15–19 June 2009

Bengt Eliasson
Ruhr-University Bochum, Germany

in collaboration with

Dennis Papadopoulos
University of Maryland

Motivation and Outline

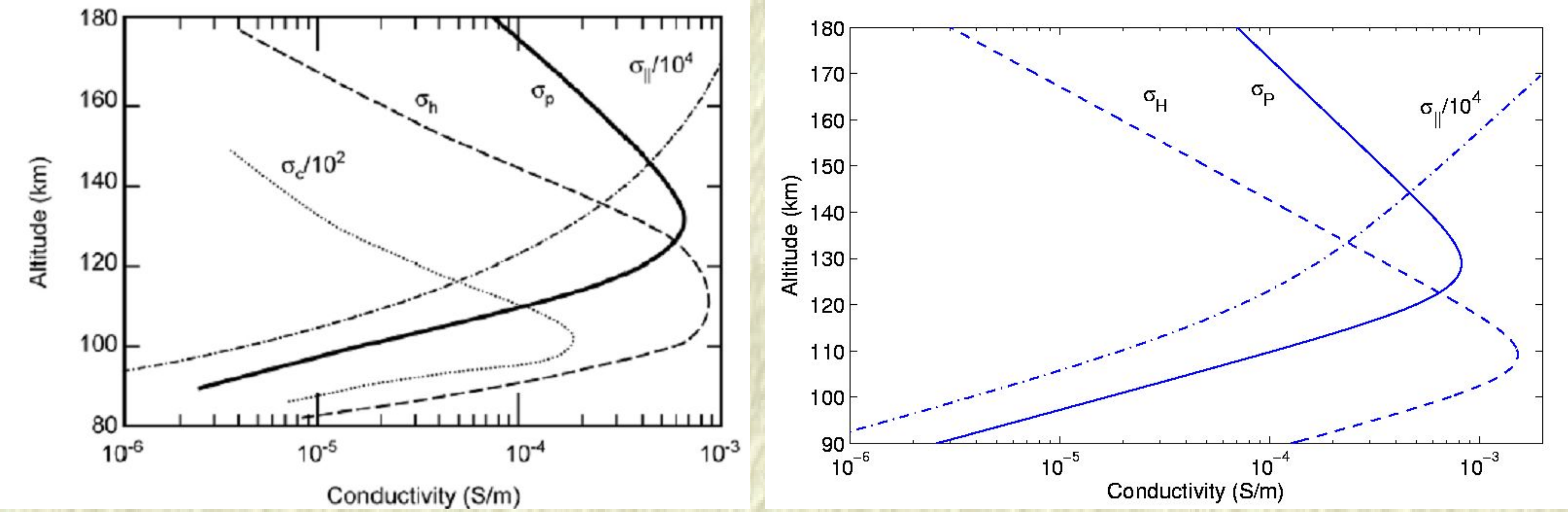
A. Motivation: The understanding of E-region transient in time ($\sim 10^{-3}$–1 s) and small scale size (50–100 km) field and current structures driven by lightning discharges, seismic events, antennas, etc.

B. Potential application: Conversion of Horizonal Electric Dipole (HED) to ionospheric Vertical Electric Dipole (VED) via Hall conductivity. VED radiates 10^5 times more efficient than HED (Field et al., 1989)

C. Numerical modeling results: Helicon/Whistler mode dynamics in 10 Hz range. Dependence on ionospheric conductivity profiles.

E. Future developments.

Equatorial E-region conductivities



Hall conductivity dominates in the E-region $90\text{–}120\,\mathrm{km}$ because electrons strongly magnetized ($\nu_{en} \ll \omega_{ce}$) while ions viscously coupled to neutrals ($\nu_{in} \gg \omega_{ci}$). (Forbes, 1976.) Right: Numerical fit used in simulations.

Model: Helicon waves

- Faraday's and Ampère's law

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j} = \mu_0 e n_0(z)(\mathbf{v}_i - \mathbf{v}_e)$$

- Generalized Ohm's law ($\mathbf{B}_0 = B_0 \widehat{\mathbf{x}}$)

$$\begin{bmatrix} j_x \\ j_y \\ j_z \end{bmatrix} = \begin{bmatrix} \sigma_{||} & 0 & 0 \\ 0 & \sigma_P & -\sigma_H \\ 0 & \sigma_H & \sigma_P \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix},$$

- Evolution equation

$$\frac{\partial \mathbf{B}}{\partial t} = -\frac{1}{\mu_0} \nabla \times [\bar{\bar{\sigma}}^{-1} \cdot (\nabla \times \mathbf{B})] \qquad (1)$$

Conductivities

Pedersen conductivity

$$\sigma_P = \varepsilon_0 \frac{\omega_{pe}^2}{\omega_{ce}} \left[\frac{\nu_{en}\omega_{ce}}{\omega_{ce}^2 + \nu_{en}^2} + \frac{\nu_{in}\omega_{ci}}{\omega_{ci}^2 + \nu_{in}^2} \right]$$

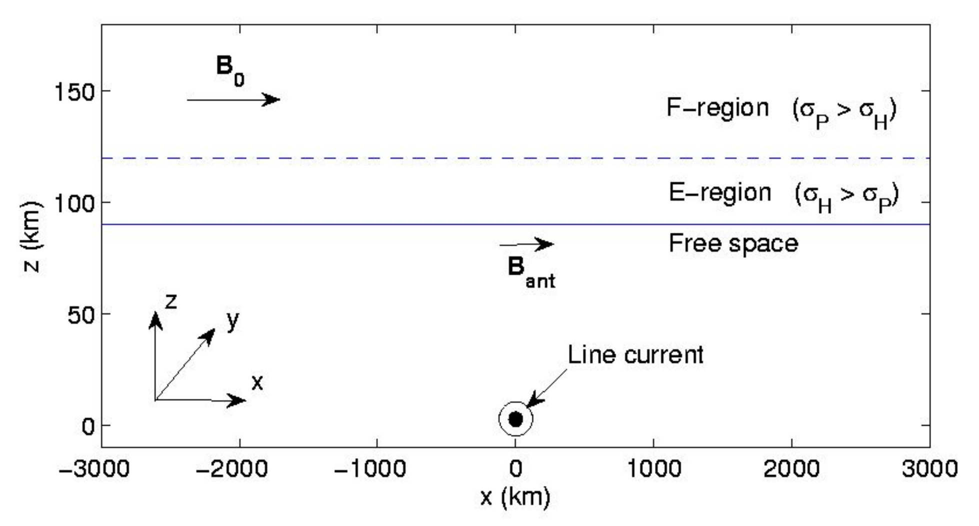
Hall conductivity

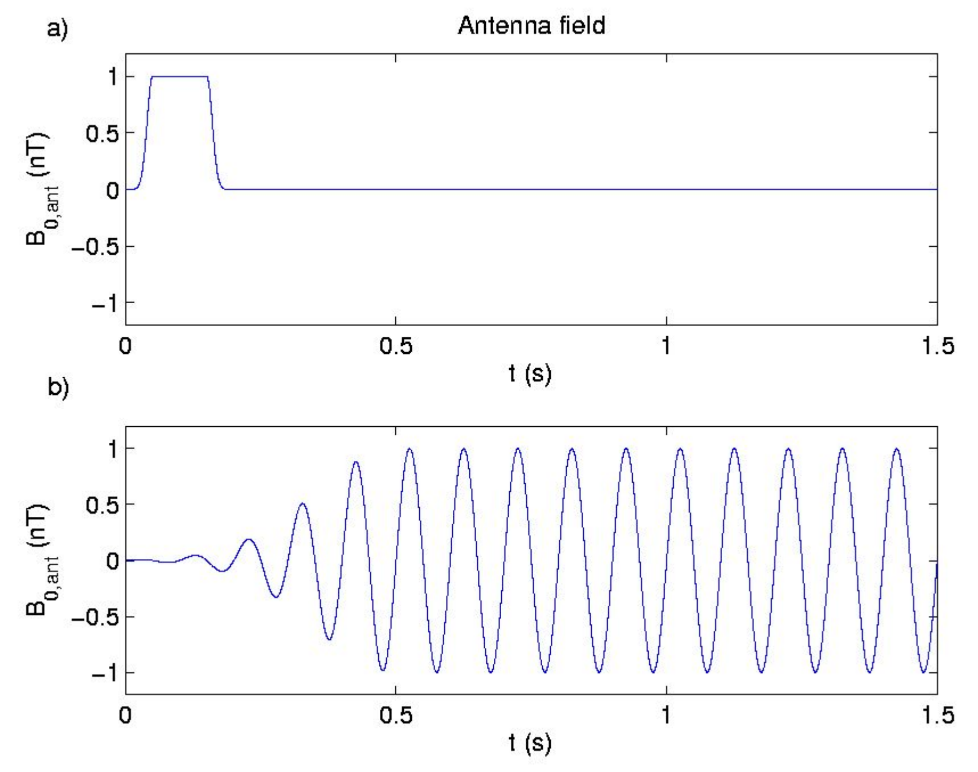
$$\sigma_H = \varepsilon_0 \frac{\omega_{pe}^2}{\omega_{ce}} \left[\frac{\omega_{ce}^2}{\omega_{ce}^2 + \nu_{en}^2} - \frac{\omega_{ci}^2}{\omega_{ci}^2 + \nu_{in}^2} \right]$$

Parallel conductivity

$$\sigma_{||} = \varepsilon_0 \frac{\omega_{pe}^2}{\nu_{en}}$$

Geometry of the model





What is special with Equatorial E-Region

- ❑ Ions viscously glued to the neutrals so that $\sigma_H \gg \sigma_P$, which gives rise to whistler-like helicon wave dynamics in the low-frequency range below ion cyclotron frequency.

- ❑ Horizontal magnetic field. Very small vertical Hall current j_z, so that

$$E_z = -\frac{\sigma_H}{\sigma_P} E_y \tag{2}$$

and

$$j_y = \underbrace{\frac{(\sigma_P^2 + \sigma_H^2)}{\sigma_P^2}}_{\sim 10^3 - 10^4} \sigma_P E_y \equiv \sigma_C E_y \tag{3}$$

are large (σ_C is the Cowling conductivity). Gives rise to the Equatorial Electrojet! $\sigma_C \gg \sigma_H \gg \sigma_P$!

Boundary conditions at plasma–free space boundary

Continuity of the normal component B_z and its normal derivative $\partial B_z/\partial z$.

Free space response: $\nabla^2 \mathbf{B}_{free} = 0$ gives evanescent field $\propto \exp(|k_x| z)$ generated by the plasma:

$$\mathbf{B}_{free} = \underbrace{(\mathbf{B} - \mathbf{B}_{ant})_{z=z_0} \exp[|k_x|(z - z_0)]}_{\text{Generated by the plasma}} + \mathbf{B}_{ant}.$$

Coupling to the antenna field at boundary $z = z_0$:

$$\frac{\partial B_z}{\partial z} - |k_x| B_z = \frac{\partial B_{z,ant}}{\partial z} - |k_x| B_{z,ant}.$$

Numerical fits of conductivities

$$\sigma_{||} = \frac{1}{a_{1,||}\exp(-z/L_{1,||}) + a_{2,||}\exp(-z/L_{2,||})},$$

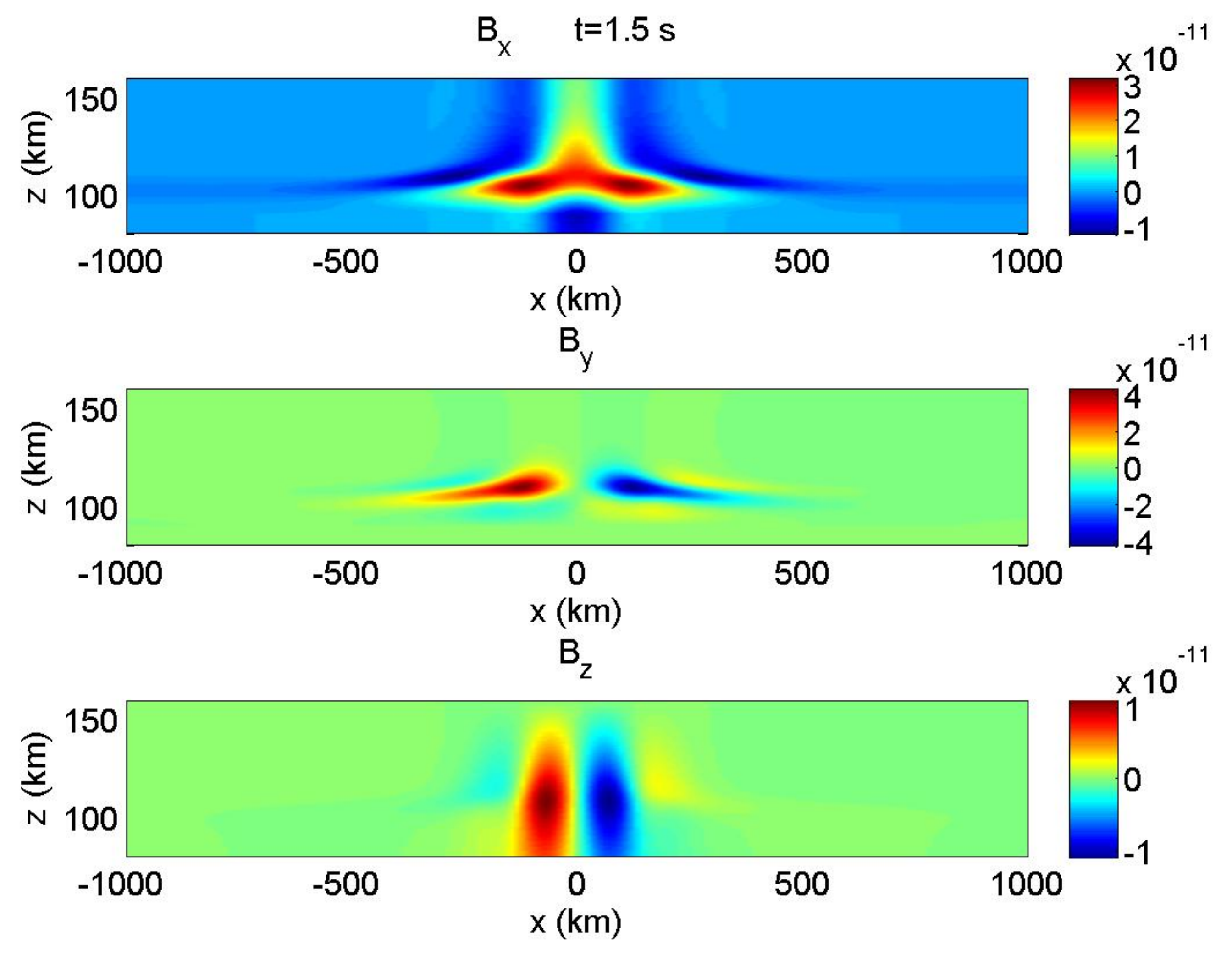
$$\sigma_P = \frac{1}{a_{1,P}\exp(-z/L_{1,P}) + a_{2,P}\exp(z/L_{2,P})},$$

$$\sigma_H = \frac{1}{a_{1,H}\exp(-z/L_{1,H}) + a_{2,H}\exp(z/L_{2,H})},$$

Parameter values				
Day	$a_{1,\|\|} = 3.1 \times 10^9\,\Omega\mathrm{m}$	$L_{1,\|\|} = 5.4\,\mathrm{km}$	$a_{2,\|\|} = 0.63 \times 10^3\,\Omega\mathrm{m}$	$L_{2,\|\|} = 18\,\mathrm{km}$
	$a_{1,P} = 7.7 \times 10^{12}\,\Omega\mathrm{m}$	$L_{1,P} = 5.4\,\mathrm{km}$	$a_{2,P} = 0.99\,\Omega\mathrm{m}$	$L_{2,P} = 19\,\mathrm{km}$
	$a_{1,H} = 1.5 \times 10^{11}\,\Omega\mathrm{m}$	$L_{1,H} = 5.4\,\mathrm{km}$	$a_{2,H} = 0.0157\,\Omega\mathrm{m}$	$L_{2,H} = 11\,\mathrm{km}$
Night	$a_{1,\|\|} = 1.5 \times 10^{10}\,\Omega\mathrm{m}$	$L_{1,\|\|} = 5.4\,\mathrm{km}$	$a_{2,\|\|} = 3.2 \times 10^3\,\Omega\mathrm{m}$	$L_{2,\|\|} = 18\,\mathrm{km}$
	$a_{1,P} = 3.8 \times 10^{13}\,\Omega\mathrm{m}$	$L_{1,P} = 5.4\,\mathrm{km}$	$a_{2,P} = 5.0\,\Omega\mathrm{m}$	$L_{2,P} = 19\,\mathrm{km}$
	$a_{1,H} = 7.7 \times 10^{11}\,\Omega\mathrm{m}$	$L_{1,H} = 5.4\,\mathrm{km}$	$a_{2,H} = 0.078\,\Omega\mathrm{m}$	$L_{2,H} = 11\,\mathrm{km}$

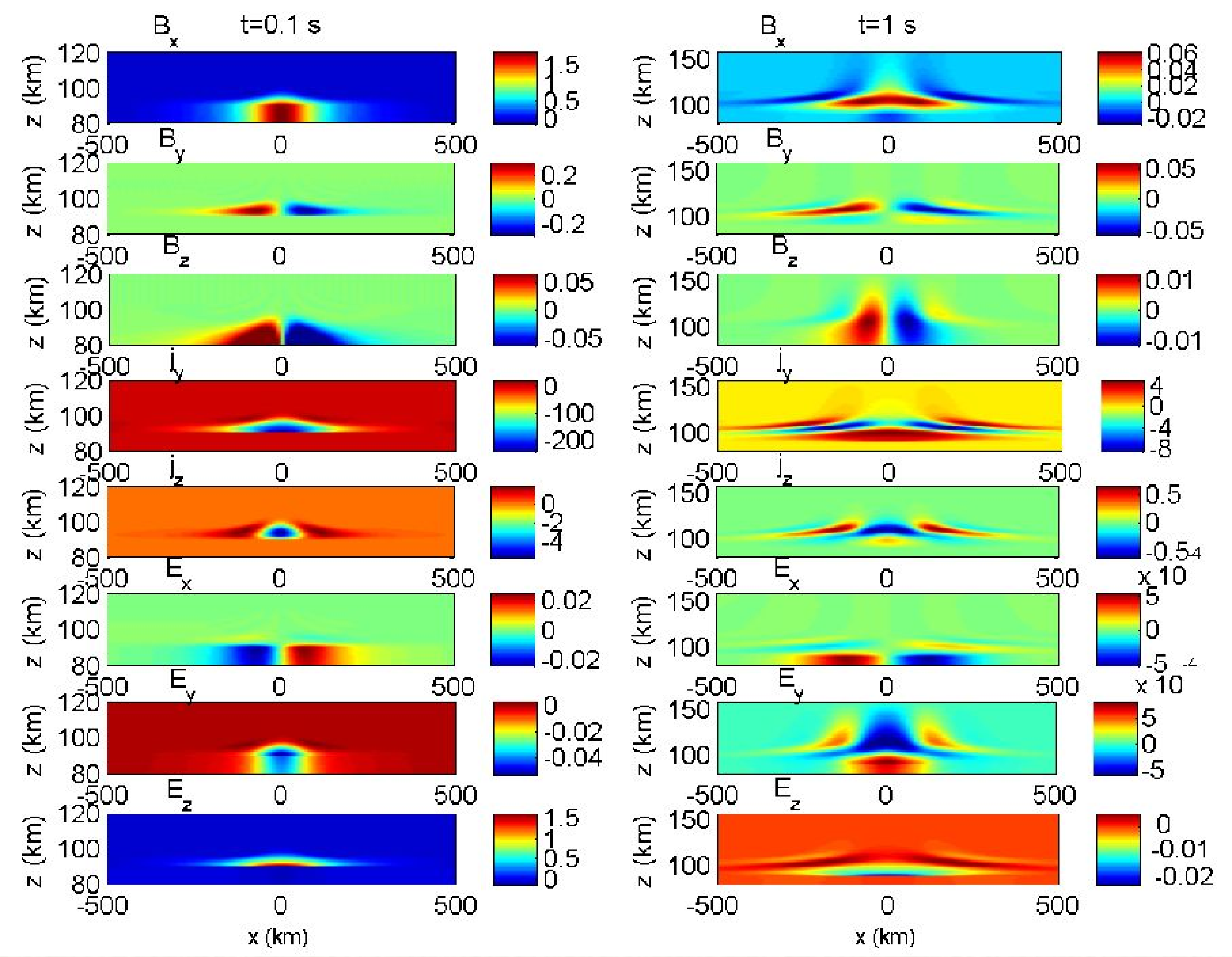
(Conductivities 5 times smaller for nighttime compared to daytime conditions.)

Pulsed antenna field, daytime conditions

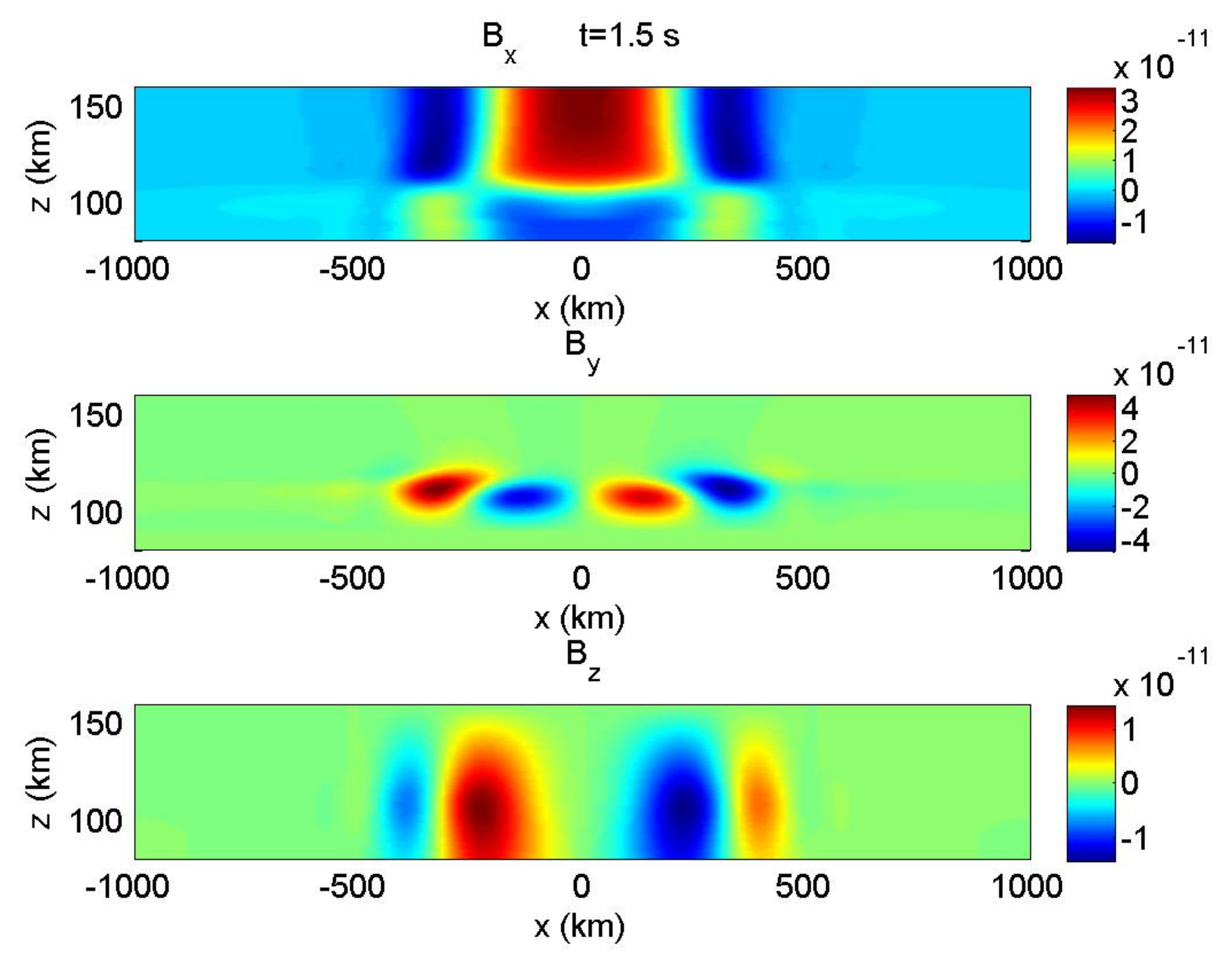


Pulsed antenna field, daytime conditions

Profiles of $\mathbf{B}$ (nT), $\mathbf{j}$ $(\mathrm{nA/m^2})$ and $\mathbf{E}$ $(\mathrm{mV/m})$

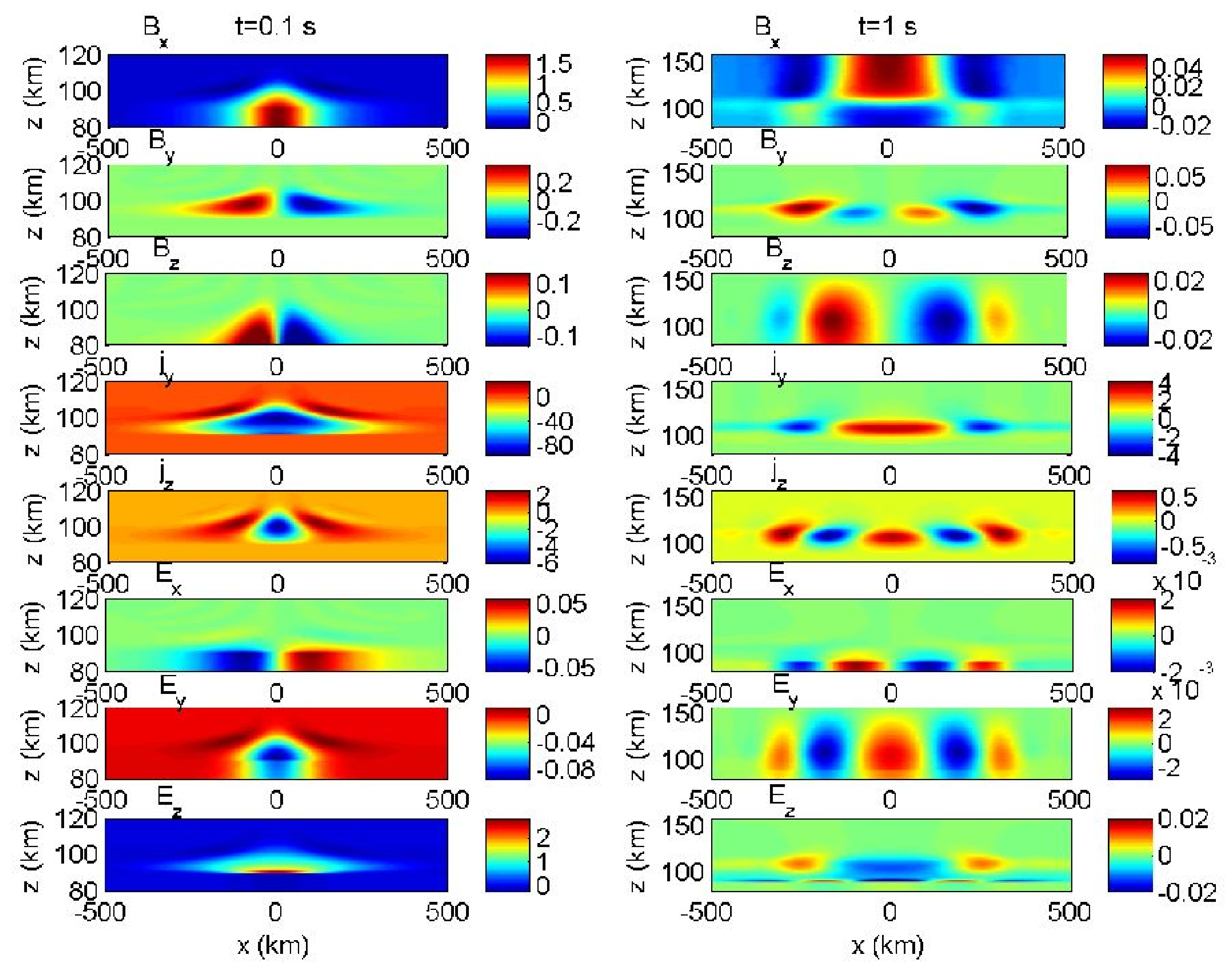


Pulsed antenna field, nighttime conditions

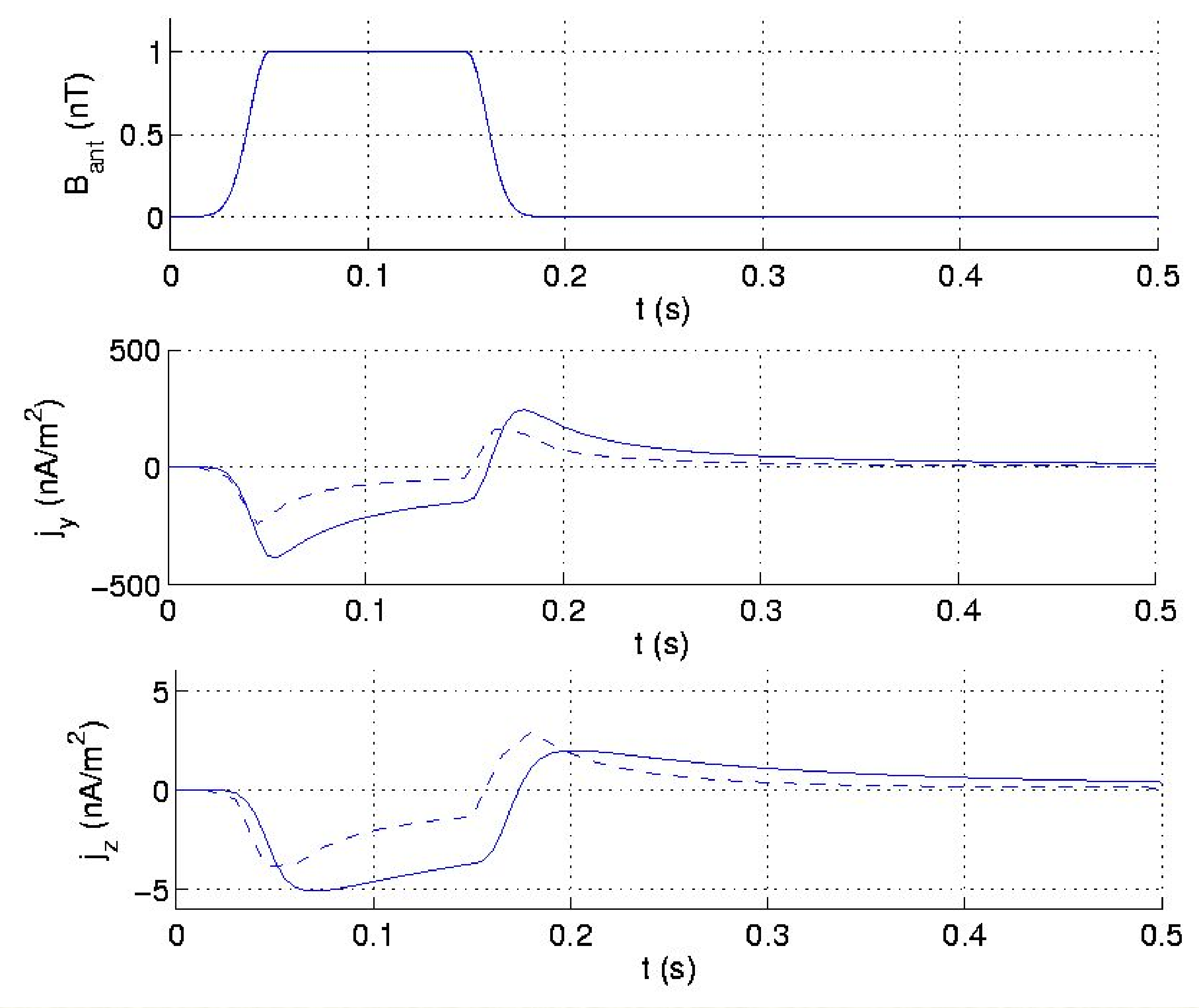


Pulsed antenna field, nighttime conditions

Profiles of $\mathbf{B}$ (nT), $\mathbf{j}$ $(\mathrm{nA/m^2})$ and $\mathbf{E}$ $(\mathrm{mV/m})$



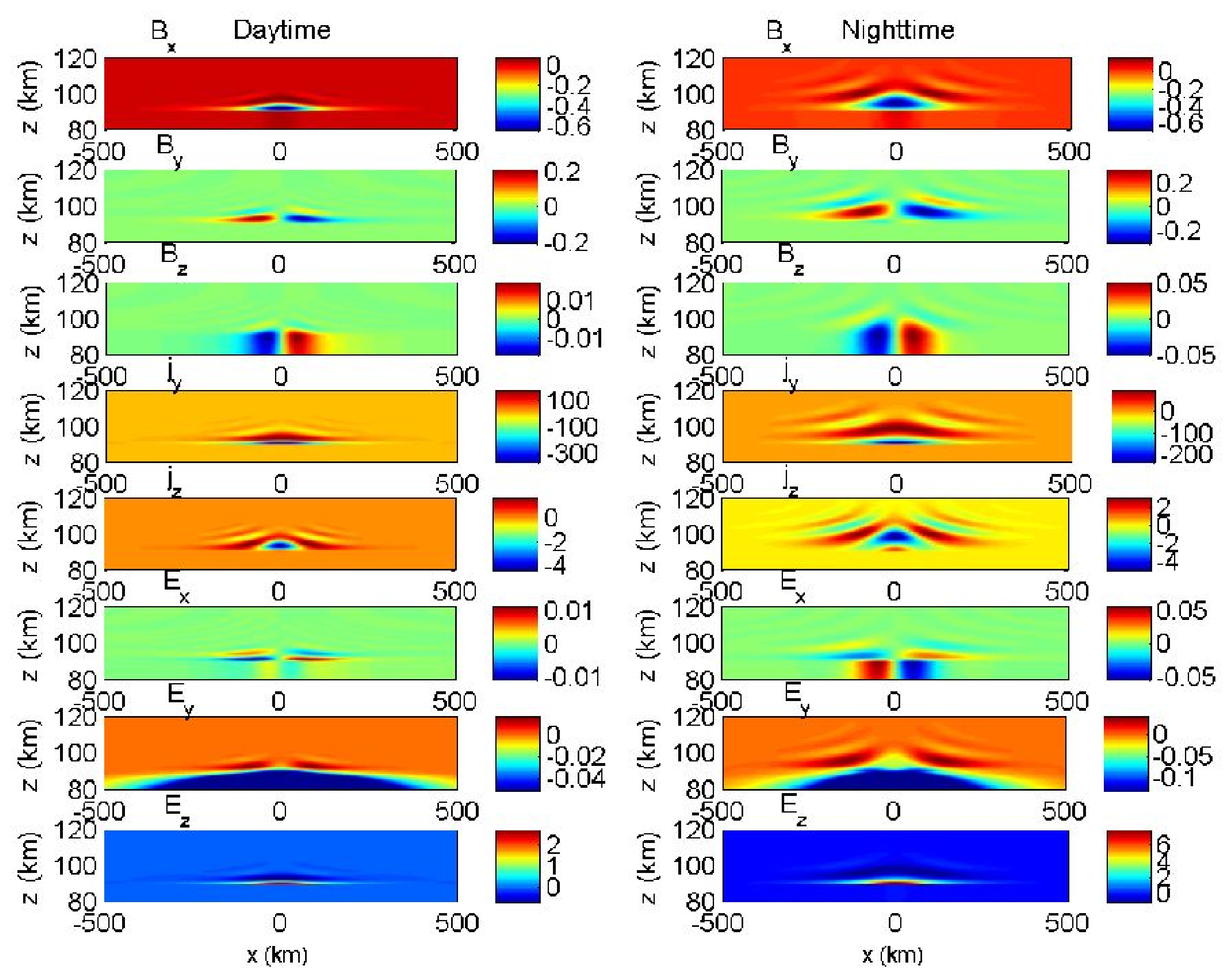
Currents at $z = 92$ km, pulsed antenna field



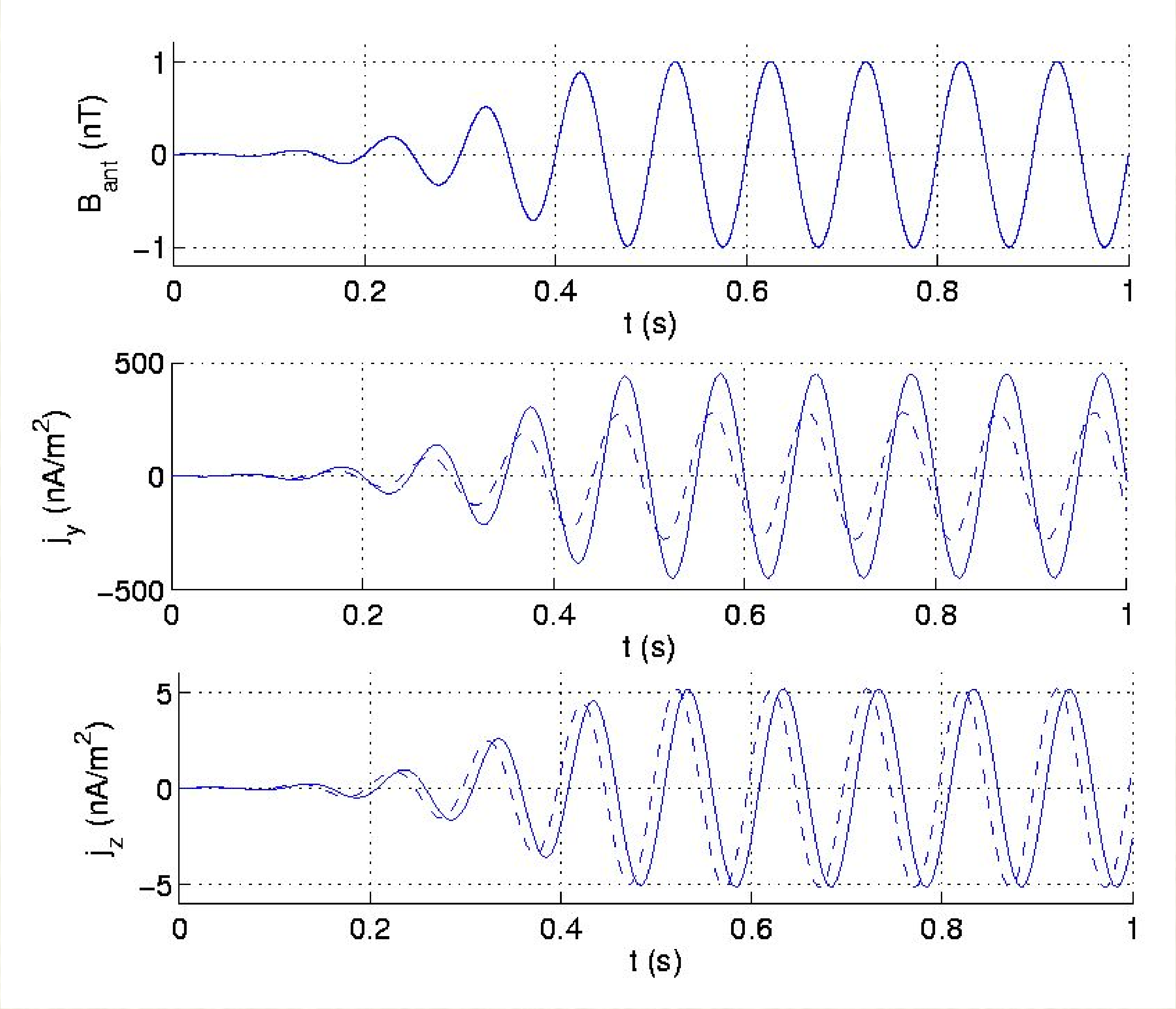
—— Daytime, - - - Nighttime

CW antenna field, day and nighttime conditions

Profiles of $\mathbf{B}$ (nT), $\mathbf{j}$ $(\mathrm{nA/m^2})$ and $\mathbf{E}$ $(\mathrm{mV/m})$

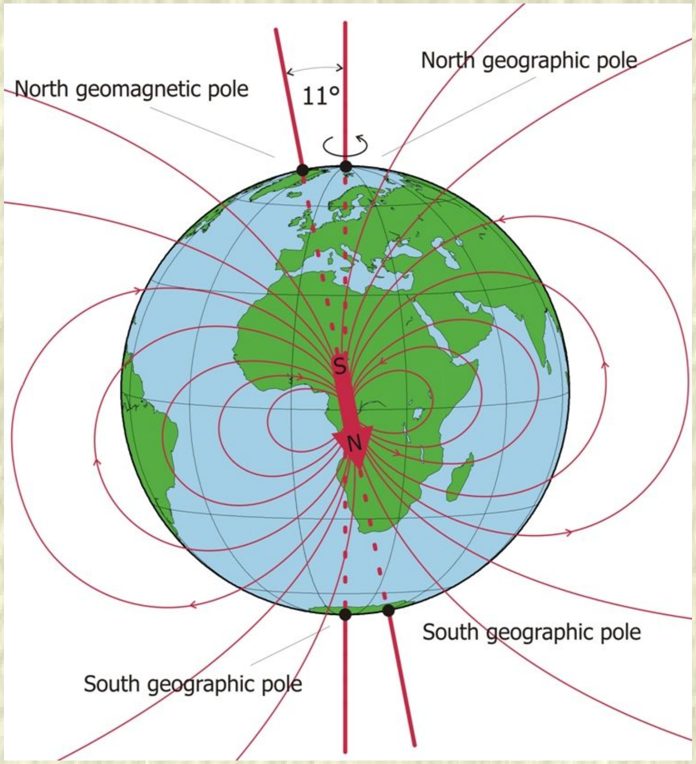


Currents at $z = 92$ km, CW antenna field



—— Daytime, - - - Nighttime

Work in progress



- ❑ Realistic geometry with oblique/curved magnetic field lines
- ❑ Coupling to Alfvén waves in the magnetosphere.

Work in progress: General oblique magnetic field

The helicon evolution equation becomes

$$\frac{\partial \mathbf{B}}{\partial t} = -\frac{1}{\mu_0} \nabla \times [\bar{\bar{R}} \bar{\bar{\rho}} \bar{\bar{R}}^T \cdot (\nabla \times \mathbf{B})]. \tag{4}$$

where the rotation matrix and its inverse are

$$\bar{\bar{R}} = \begin{bmatrix} \cos(\theta) & 0 & -\sin(\theta) \\ 0 & 1 & 0 \\ \sin(\theta) & 0 & \cos(\theta) \end{bmatrix}, \qquad \bar{\bar{R}}^T = \begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) \end{bmatrix}. \tag{5}$$

with the inclination angle $\theta(\mathbf{x})$ and the ”impedance tensor”

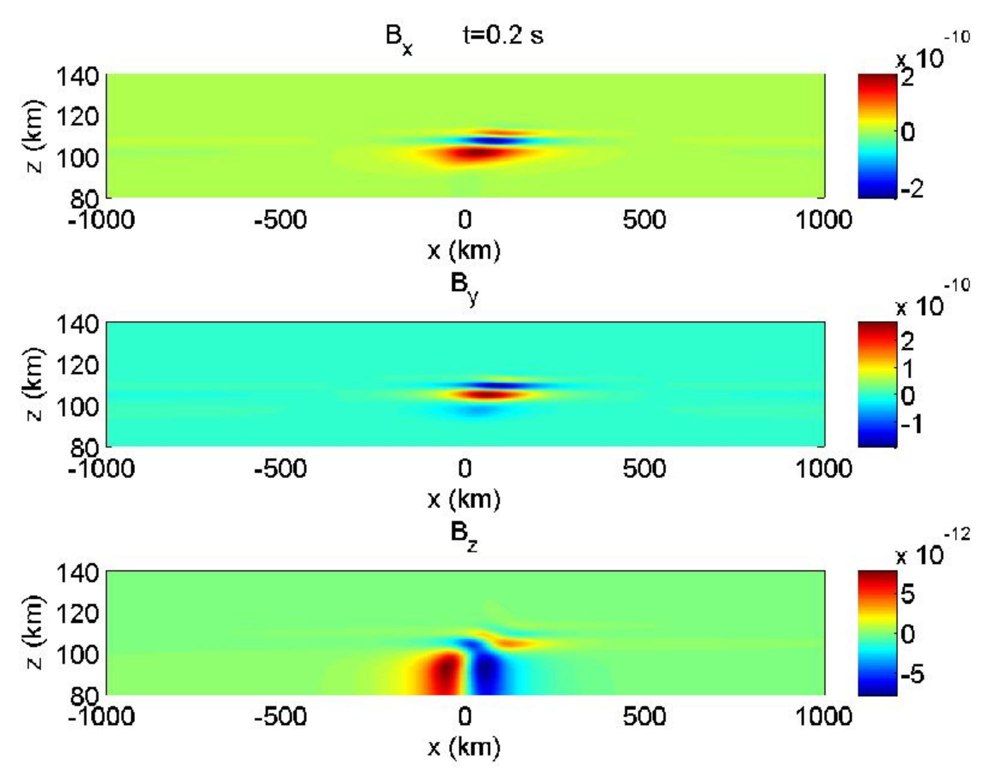
$$\bar{\bar{\rho}} = \bar{\bar{\sigma}}^{-1} = \begin{bmatrix} \rho_{||} & 0 & 0 \\ 0 & \rho_P & \rho_H \\ 0 & -\rho_H & \rho_P \end{bmatrix} \tag{6}$$

where $\rho_{||} = 1/\sigma_{||}$, $\rho_P = \sigma_P/(\sigma_P^2 + \sigma_H^2)$ and $\rho_H = \sigma_H/(\sigma_P^2 + \sigma_H^2)$.

Work in progress: General oblique magnetic field

We have

$$\bar{\bar{R}}\bar{\bar{\rho}}\bar{\bar{R}}^T = \begin{bmatrix} \rho_{||}\cos^2(\theta) + \rho_P\sin^2(\theta) & \rho_H\sin(\theta) & (\rho_{||} - \rho_P)\sin(\theta)\cos(\theta) \\ -\rho_H\sin(\theta) & \rho_P & \rho_H\cos(\theta) \\ (\rho_{||} - \rho_P)\sin(\theta)\cos(\theta) & -\rho_H\cos(\theta) & \rho_{||}\sin^2(\theta) + \rho_P\cos^2(\theta) \end{bmatrix} \qquad (7)$$



Pulse for $\theta = 0.1$ rad.

Work in progress: Alfvén wave dynamics

Evolution system of equations

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \left[\mathbf{E}_\perp + \frac{[\mathbf{B}_0 \cdot (\nabla \times \mathbf{B})]\mathbf{B}_0}{\mu_0 B_0^2 \sigma_{||}} \right] \tag{8}$$

$$\frac{\partial \mathbf{E}_\perp}{\partial t} = V_A^2 \left[(\nabla \times \mathbf{B})_\perp - \mu_0 \sigma_H \frac{\mathbf{B}_0 \times \mathbf{E}_\perp}{B_0} - \mu_0 \sigma_P \mathbf{E}_\perp \right], \tag{9}$$

Modified Alfvén speed $V_A^2 = \frac{\omega_{ci}^2 + \nu_{in}^2}{\omega_{pi}^2} c^2$, Perpendicular current density

$$(\nabla \times \mathbf{B})_\perp = \nabla \times \mathbf{B} - \frac{[\mathbf{B}_0 \cdot (\nabla \times \mathbf{B})]\mathbf{B}_0}{B_0^2} \tag{10}$$

Space-dependent profiles of $\mathbf{B}_0$, V_A, σ_P, σ_H, $\sigma_{||}$.

Summary

A. Investigated E-region transient field and current structures driven by transient time ($\sim 10^{-3}$–1 s) and small scale size (50–100 km) pulses.

B. Motivated by lightning discharges, Seismic events, Horizonal Electric Dipole (HED) antenna experiments.

C. Coupling of plasma dynamics to free space and antenna fields.

D. Helicon/Whistler mode dynamics and currents for different E-region plasma parameters.

E. Under investigation: Realistic geometry. Alfvén wave dynamics. Radiation into Earth-Ionosphere Waveguide.