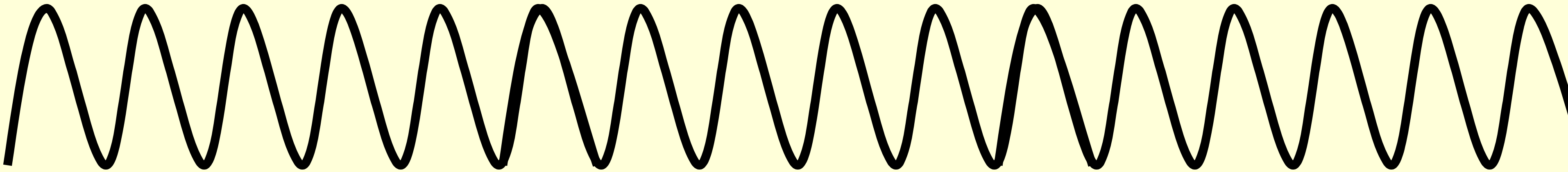
Modern Challenges in Nonlinear Plasma Physics
A Conference Honoring the Career of Dennis Papadopoulos
June 19, 2009

Validity of Plasma Resonance & Pulse-Particle Interaction

Kaz Akimoto
Teikyo University,
Utsunomiya, Japan

Landau Damping

- Damping mechanism of plasma es waves
- Effective for linear waves



Why Landau damping now？
→ getting popular outside plasma physics！

- **sound waves in tenuous neutral gases**
- **instabilities of interstellar gases & star systems （many body systems in gravitational field）, etc.**

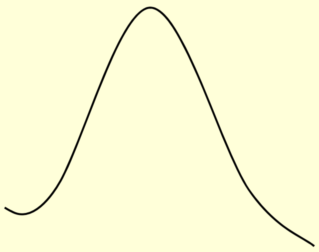
- Landau damping is based on sinusoidal waves that are ideal entities.
  But, all the waves are pulses!

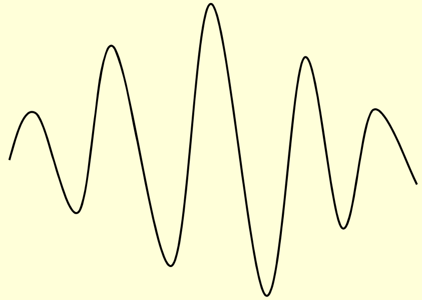
Is Landau damping applicable to pulses?

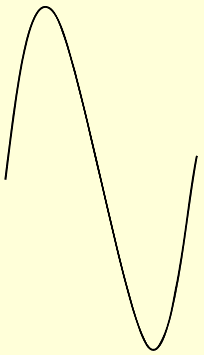
- Short pulses are dissipated via transit-time acceleration.

What is transit-time acceleration?

- **Damping mechanism of pulse waves**
- **Pulse waves emerge as dissipative structures in turbulence**
- **Applied to strong Langmuir turbulence etc.**







Motivation :

to clarify the relationship between Landau damping and transit-time acceleration

2 ways to derive Landau resonance

1. Mathematical one by Landau(1946), in which complex integrals are rigorously evaluated.

2. More physical one by Dawson(1961)

Physics of Landau resonance revisited (Stix)

Eq. of motion for a charged particle in an sinusoidal wave:

$$m\frac{dv}{dt} = qE\cos(k\,z - \omega\,t)$$

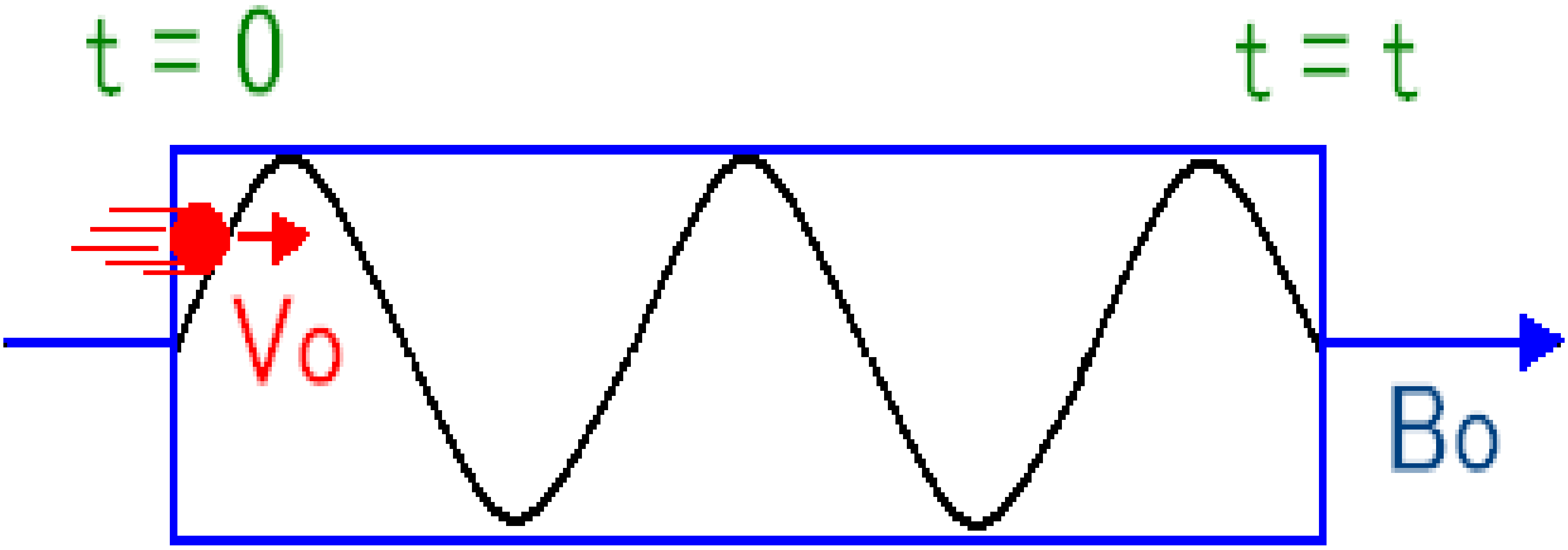
Let $z = v_o t + z_o$, **and solve above with** $v_1 = 0$ **at** $t=0$**, then the particle velocity at t equals**

$$v_1 = \frac{qE}{m\alpha}\left\{\sin(k\,z_0 - \alpha\,t) - \sin(k\,z_0)\right\}$$

$$(\alpha = kv_0 - \omega)$$

This equals the velocity shift due to transit-time acceleration of a particle that has penetrated a square pulse after *t* !

Transit-time acceleration of a particle and a square pulse



Power due to transit-time acceleration

Thus, position-averaged power becomes

$$\left\langle \frac{d}{dt}\left(\frac{1}{2}mv^2\right)\right\rangle_{z_0} = \frac{q^2E^2}{2m}\left[-\frac{\omega \sin \alpha t}{\alpha^2} + t\cos \alpha t - \frac{\omega t \sin \alpha t}{\alpha}\right]$$

We utilize f(v) to obtain the power P(t).

$$P(t) = \int_{-\infty}^{\infty} \left\langle \frac{d}{dt}\left(\frac{1}{2}mv^2\right)\right\rangle_{z_0} f(v)\,dv$$

$$= \frac{q^2E^2}{2m}\int_{-\infty}^{\infty}\left[-\frac{\omega \sin \alpha t}{\alpha^2} + t\cos \alpha t - \frac{\omega t \sin \alpha t}{\alpha}\right] f(v)\,dv$$

Stix had approximated each term, but

→ More accurate derivation becomes possible if one notices below.

$$P(t)=\frac{q^2E^2}{2m}\int_{-\infty}^{\infty} f(v)\frac{d}{dv}\left(v\frac{\sin(kv-\omega)t}{kv-\omega}\right)dv$$

$$=-\frac{q^2E^2}{2m}\int_{-\infty}^{\infty}\frac{df(v)}{dv}\left(v\frac{\sin(kv-\omega)t}{kv-\omega}\right)dv$$

This is the power due to transit-time acceleration by the square pulse of interaction time t.

In the limit $t\to\infty$, the following identity may be used.

$$\lim_{t\to\infty}\frac{\sin(kv-\omega)t}{kv-\omega}=\frac{\pi}{k}\delta\left(v-\frac{\omega}{k}\right)$$

Thus, the power in the **lilmit** $\boldsymbol{t \to \infty}$ is

$$P(\infty) = \frac{dW}{dt} = -2\gamma W$$

$$= -\frac{q^2 E^2}{2m} \int_{-\infty}^{\infty} \frac{df(v)}{dv} v \delta\left(v - \frac{\omega}{k} \right) dv$$

Hence, Landau's damping rate is obtained as an extreme case of transit-time acceleration.

$$\gamma = \frac{\pi}{2} \frac{\omega_e^2}{k^2} \frac{df(v)}{dv}\bigg|_{v=\frac{\omega}{k}}$$

More rigorous expression than Landau damping

・Approximation t → ∞ ignores nonlinearity.

[Landau approximation?]

Hence, the following equation is better at t＜∞.

$$P(t) = -\frac{q^2E^2}{2m}\int_{-\infty}^{\infty}\frac{df(v)}{dv}[v\frac{\sin(kv-\omega)t}{kv-\omega}]\,dv$$

・This general power is due to transit-time acceleration of particles with interaction time *t*.

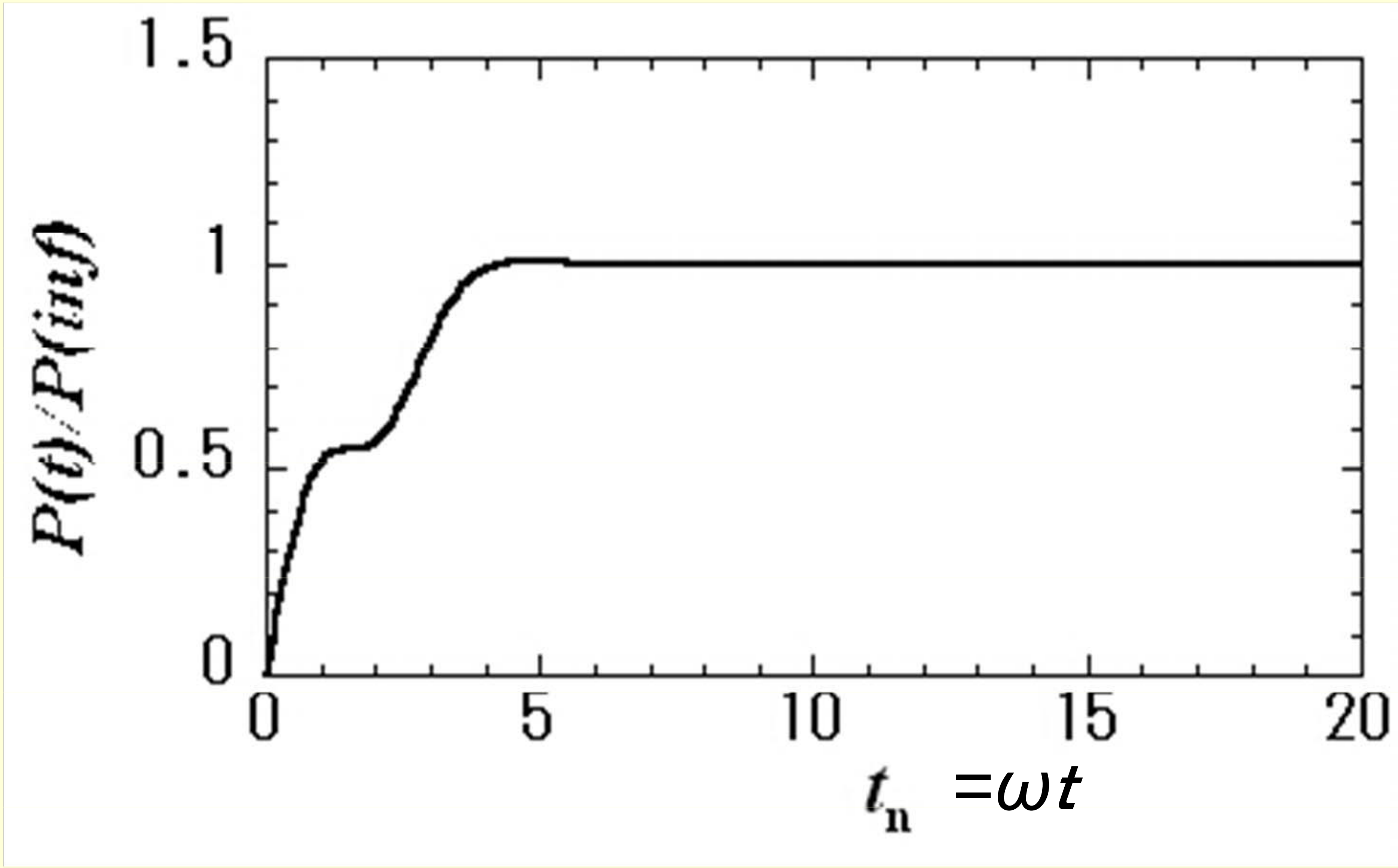
・This is more realistic than Landau's expression that is based on sinusoidal waves.

Comparison between P(t) and P(∞)

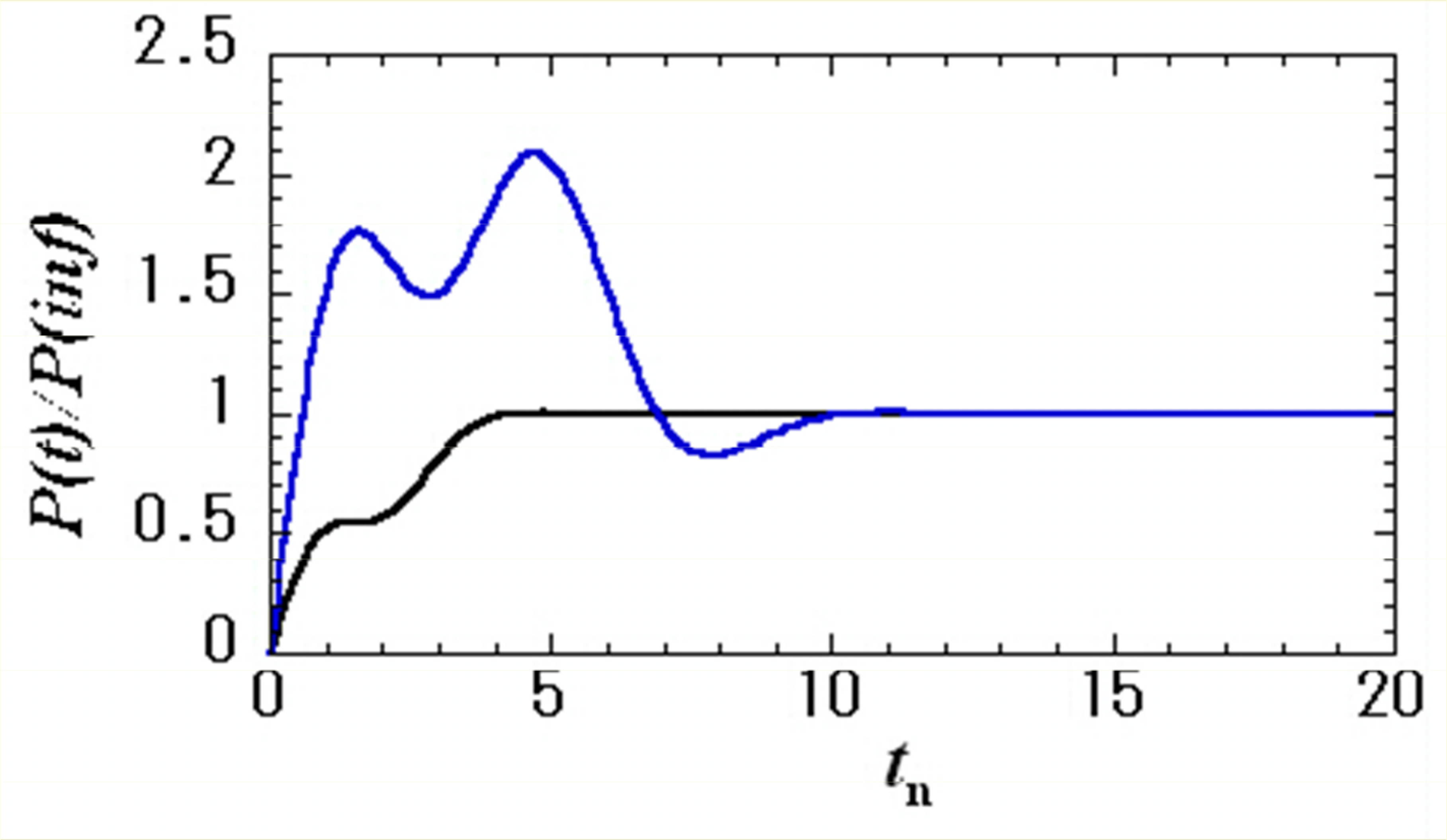
$$\frac{P(t)}{P(\infty)} = \frac{\int_{-\infty}^{\infty} \frac{df(v)}{dv} [v \frac{\sin(kv-\omega)t}{kv-\omega}]\, dv}{\frac{\omega}{k} \frac{df(v)}{dv}\Big|_{v=\frac{\omega}{k}}}$$

→ Let f(v) be Maxwellian distribution.

P(t) for $v_p = v_e$



P(t) for $v_p=2v_e$



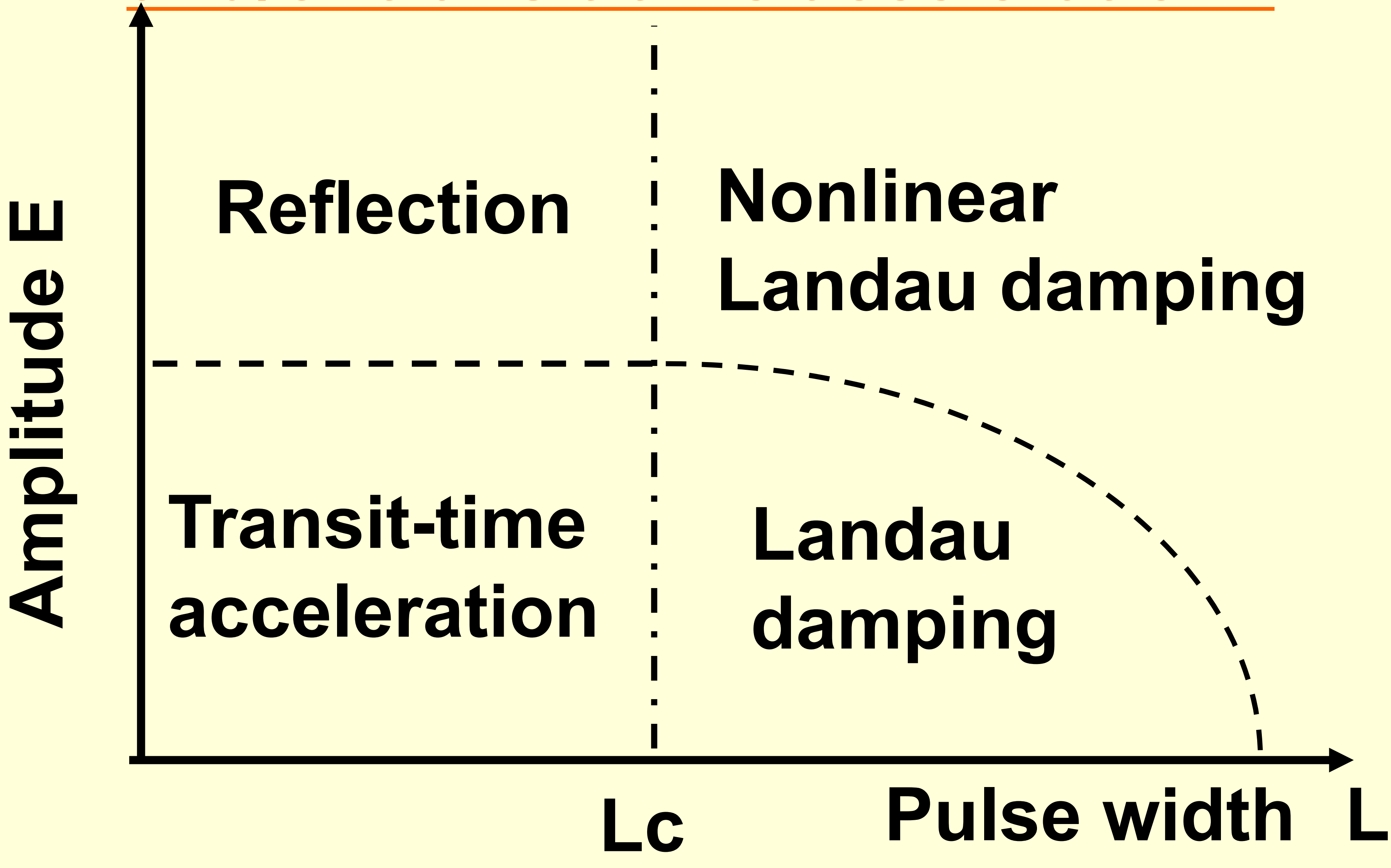
Comparison between P(t)&P(∞)

$$\frac{P(t)}{P(\infty)} = \frac{\int_{-\infty}^{\infty} \frac{df(v)}{dv}[v\frac{\sin(kv-\omega)t}{kv-\omega}]\,dv}{\frac{\omega}{k}\frac{df(v)}{dv}\Big|_{v=\frac{\omega}{k}}}$$

How close the sinc fn. is to the δ fun is important.
→$t_n=\omega t \gg 1$ is necessary for Landau approximation.
(depends on vp and vg)

However, when E is large, nonlinearity becomes important at that time.→Landau approximation fails!

Where can we use Landau damping and/or transit-time acceleration?



Cyclotron resonance of circularly polarized EM wave

- Power due to linear cyclotron resonance

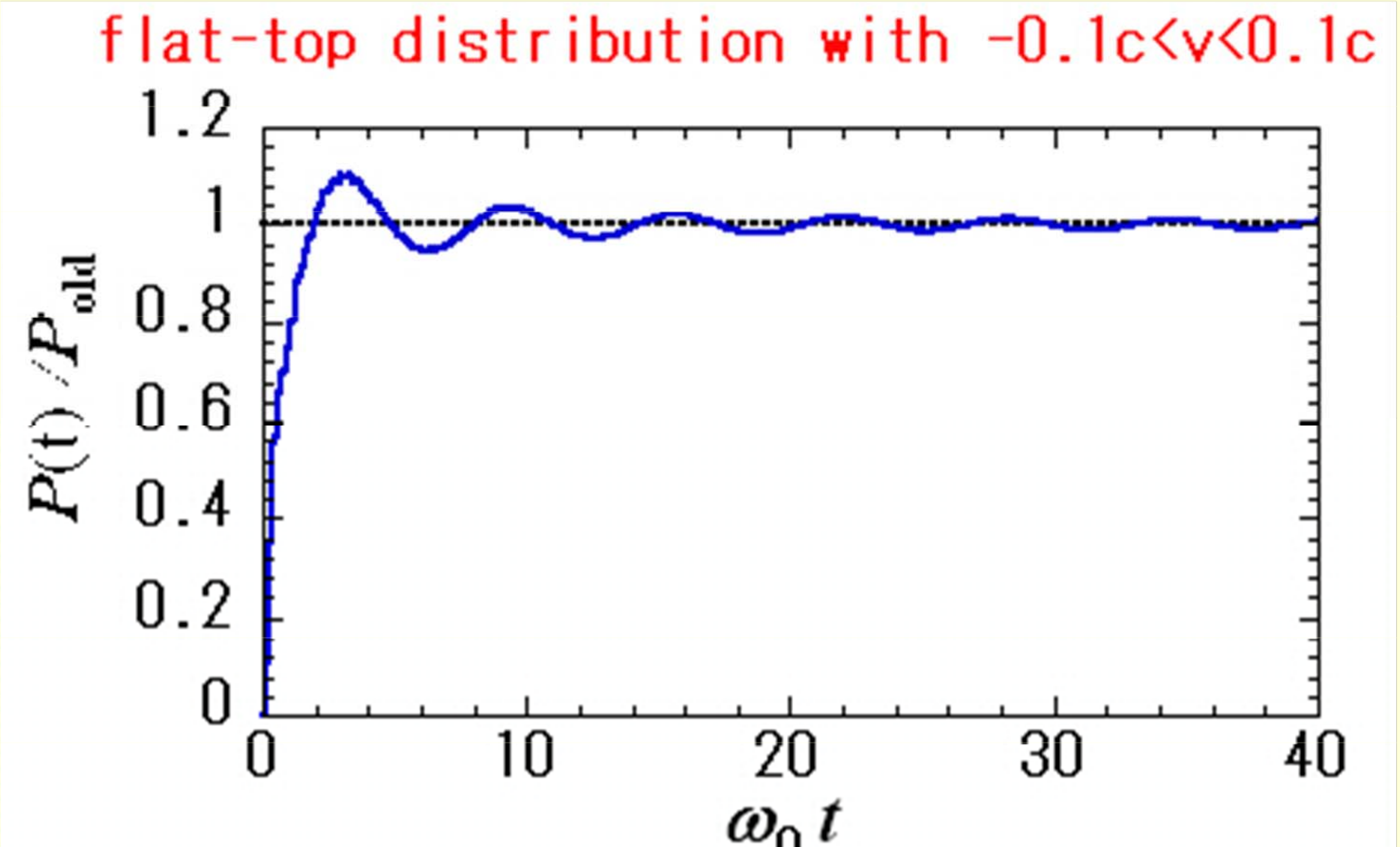
$$P_{old} = \int f(v_{z0}) \left\{ \frac{d}{dt}\left(\frac{1}{2} m v^2\right) \right\} dv_{z0} = \frac{\pi e^2 E^2}{m k} \left(-\frac{\varepsilon \Omega}{\omega} \right) f\left(\frac{\omega + \varepsilon \Omega}{k} \right)$$

- General power due to transit-time acceleration

$$P(t) = \int f(v_0) \left\{ \frac{d}{dt}\left(\frac{1}{2} m v^2\right) \right\} dv_0$$

$$= \frac{e^2}{m} E_0^2 \int f(v_0) \left(1 - k v_0 / \omega\right) \frac{\sin\left\{\varepsilon(k v_{z0} - \omega) + \Omega_p\right\} t}{\varepsilon(k v_{z0} - \omega) + \Omega_p} dv_0$$

How about EM waves?
EM Cyclotron Resonance vs.
Transit-Time Acceleraion



Conclusions

- Damping/resonance mechanisms of sinusoidal wave with a square envelope was evaluated.

- Transit-time acceleration is the elementary process of Landau damping.

→ They agree with each other in the limit $\omega t \gg 1$.

→ Same is true for EM cyclotron resonance!

- Transit-time acceleration is applicable for plasma heating.

Next Step : Gaussian pulse

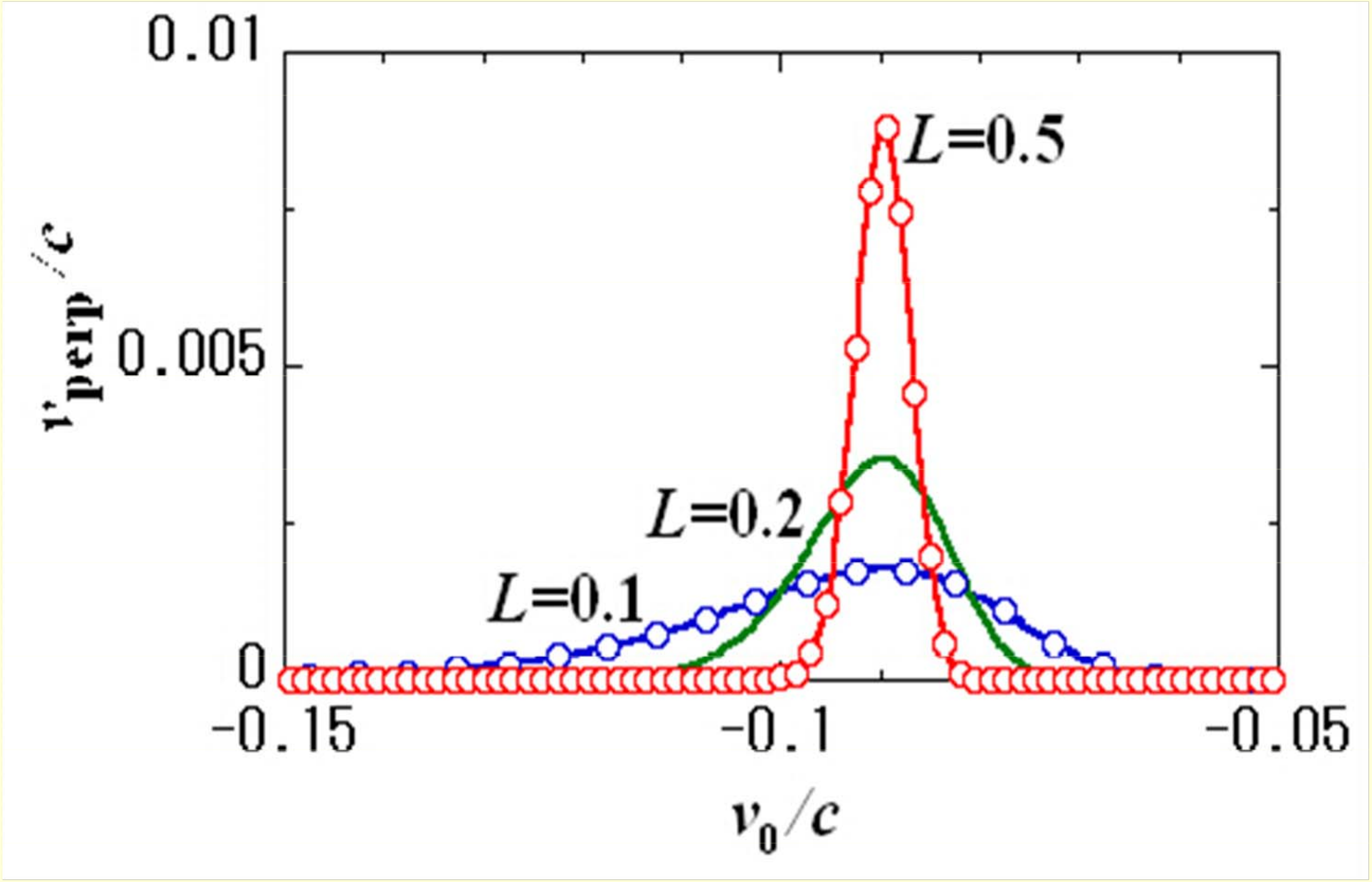
$$\Delta v = v_1 = \frac{\sqrt{\pi}\, q E_0 \cos\theta\, \Delta t}{m\gamma_0 |\alpha|} e^{-\frac{1}{4}\omega_0^2 \Delta t^2}$$

$$\alpha = \left| 1 - \frac{v_p}{v_0} \right|, \quad \omega_0\, \Delta t = \frac{\omega_0 (v_p - v_0)}{v_p (v_g - v_0)}$$

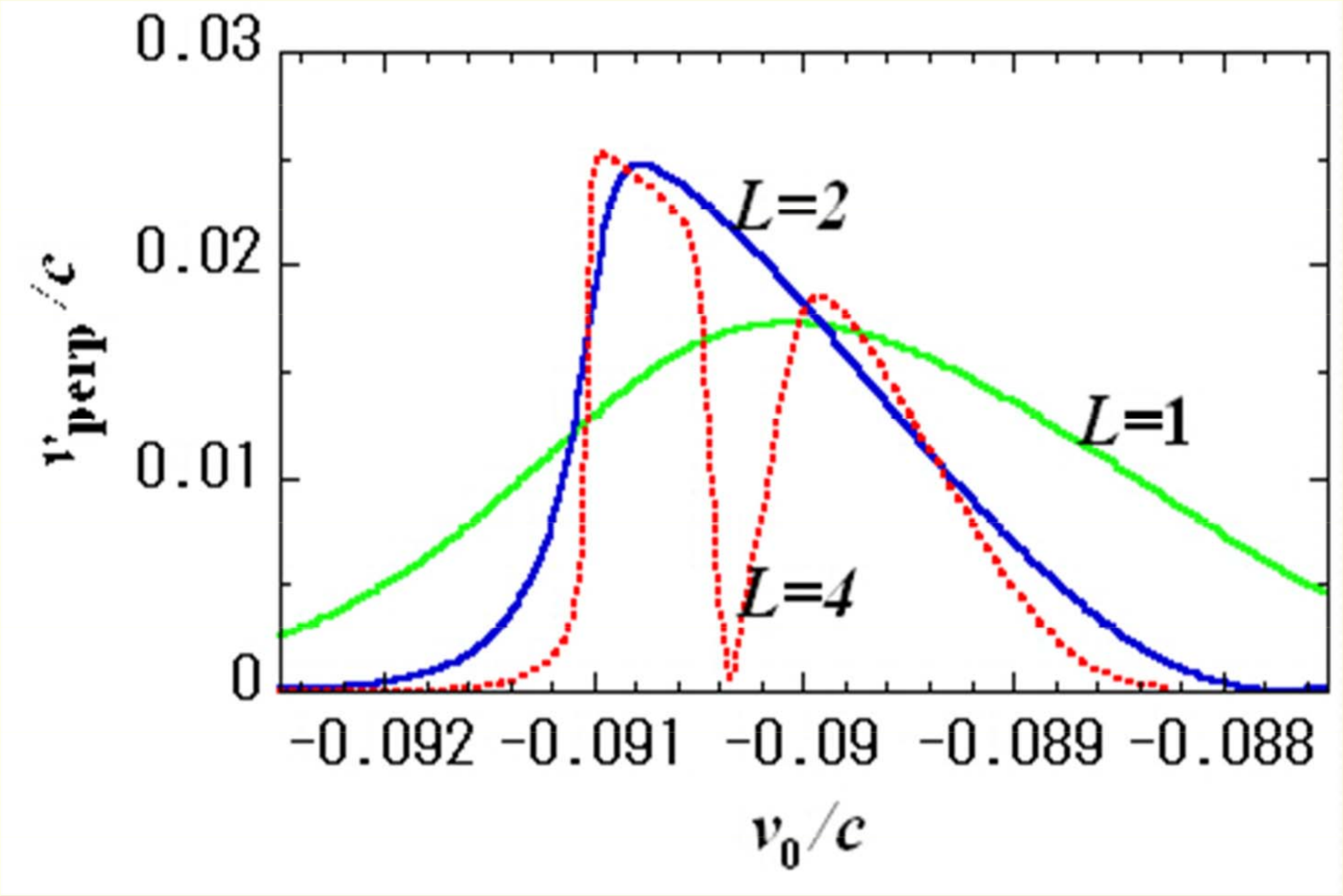
Increase in kinetic energy:

$$\Delta W = \pi \left(\frac{e E_0\, \Delta t}{2 m \gamma_0 |\alpha|} \right)^2 e^{-\frac{1}{2}\omega_0^2 \Delta t^2}$$

最終垂直速度（共鳴）のパルス幅L依存性



サイクロトロン共鳴のパルス長依存性



L (t) →∞でも共鳴は非デルタ関数的