# Universal extreme statistical properties (of plasma edge transport)

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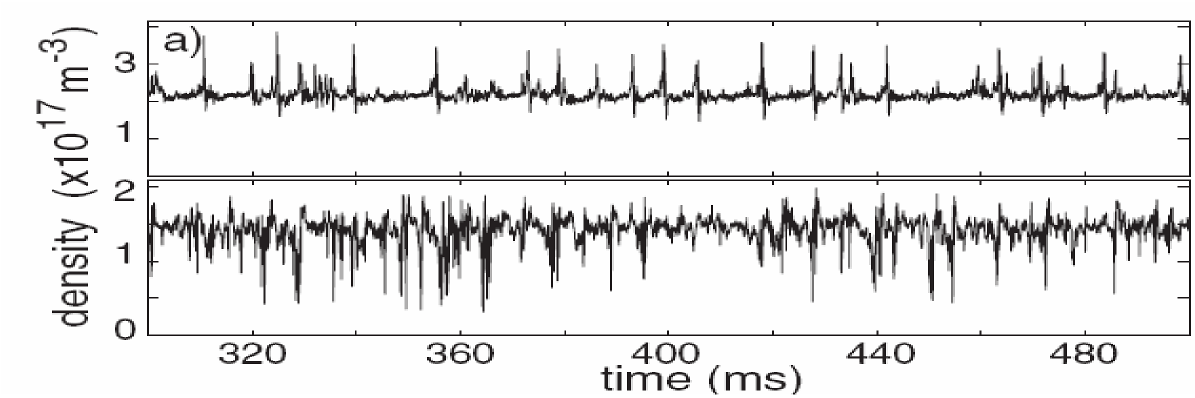
# Outline

- Motivation
- Introduction
- Experimental/observational results
- Introducing an *extreme* stochastic process $w(t)$
- Basic properties of $PDF(w)$
- Comparisons with observations
- Conclusions

# Motivation

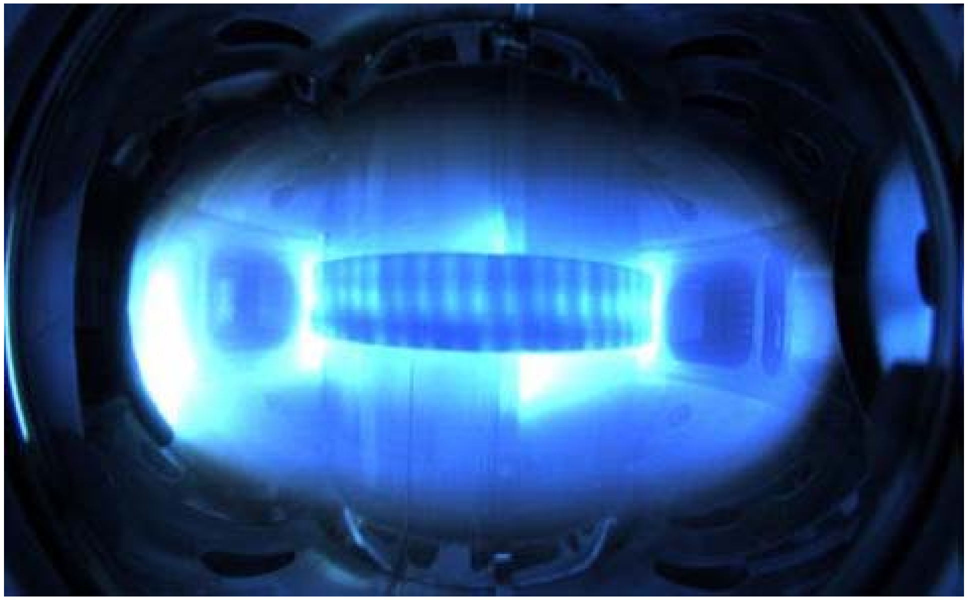



- Understand statistical properties of plasma transport in the edge of tokamaks, stellarators, spheromaks *etc.*
- Explain the observed universality
- Use generic properties of plasma turbulence

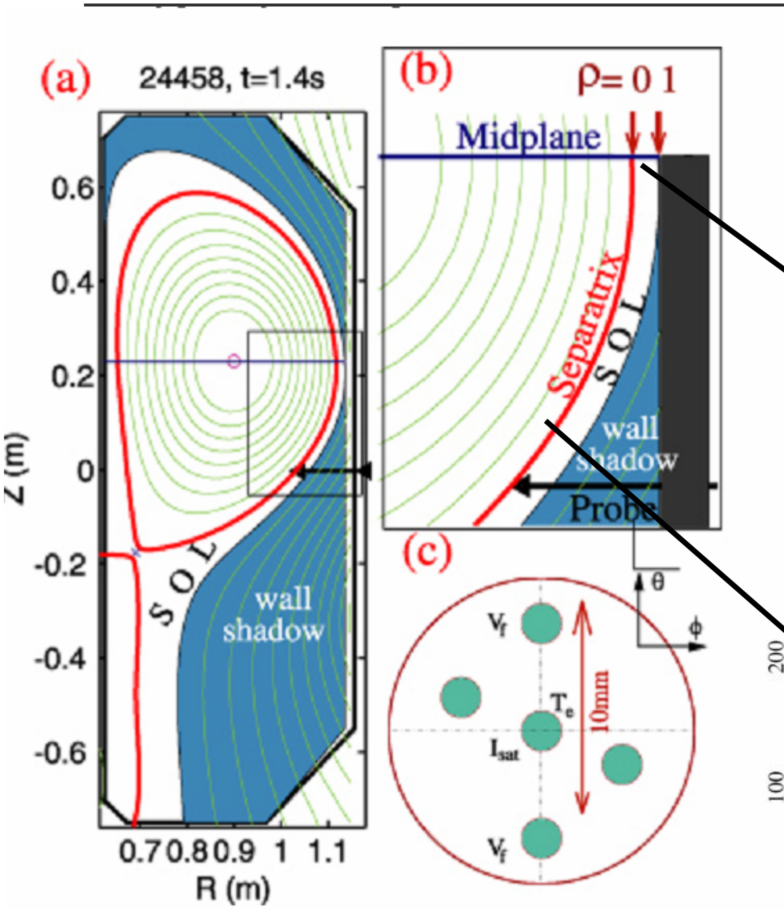
# Transport in plasma edge

In plasma edge:

- Cross-field transport is known to be dominated by large radial transport bursty events
- Turbulent *diffusion* is nearly absent: *bursty convective transport* accounts for the totality of the plasma that survived parallel transport

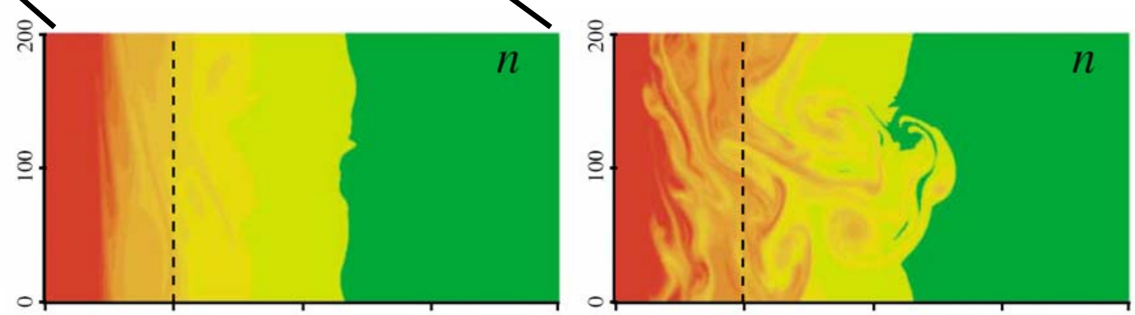


# Blobs, avaloids, filaments..



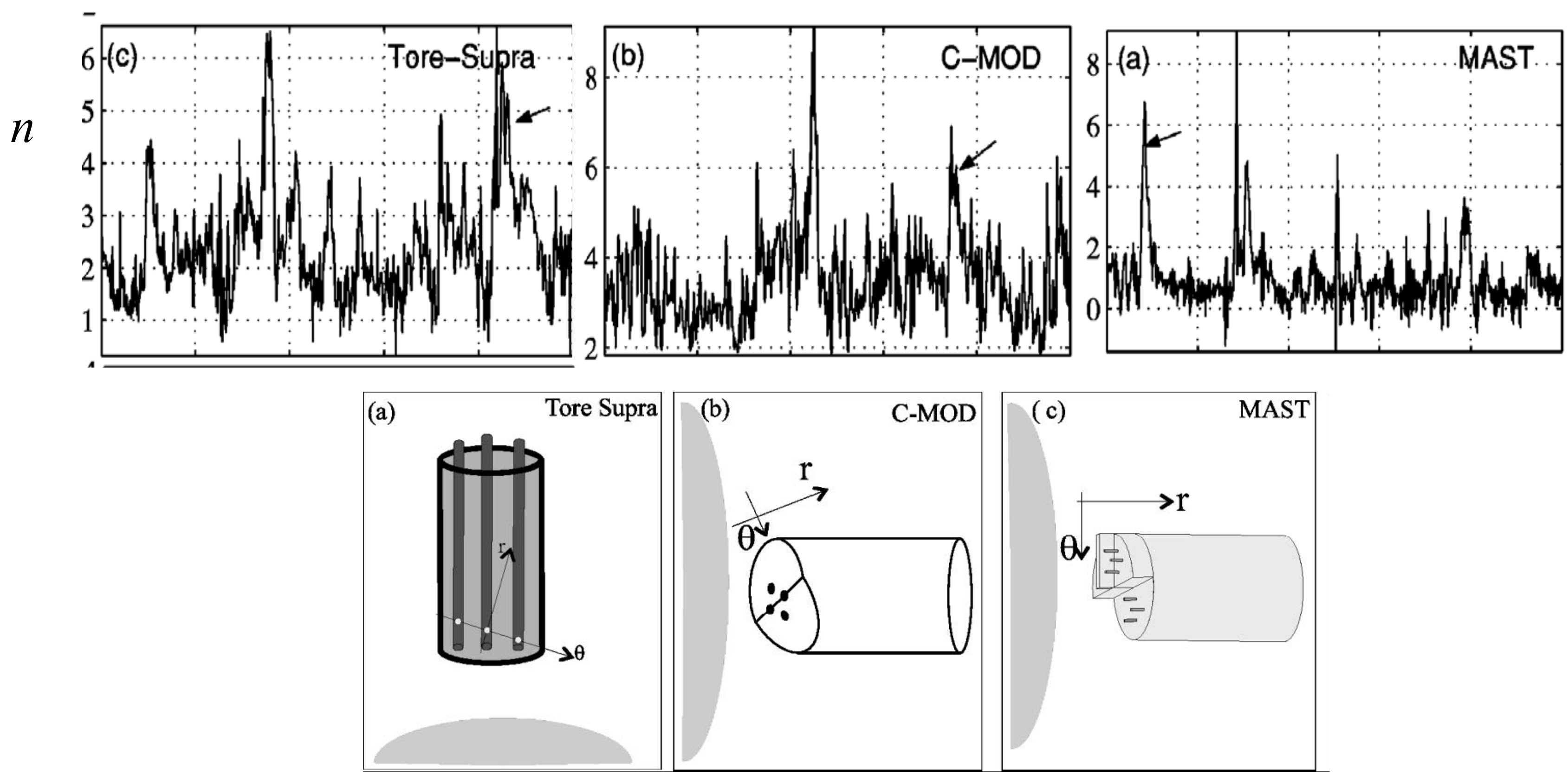

Graves, *et al* PPCF **47** L1-L9 (2005)

- Interchange instability
- Electron motion cannot cancel the space-charge separation
- Blobs are convected outwards



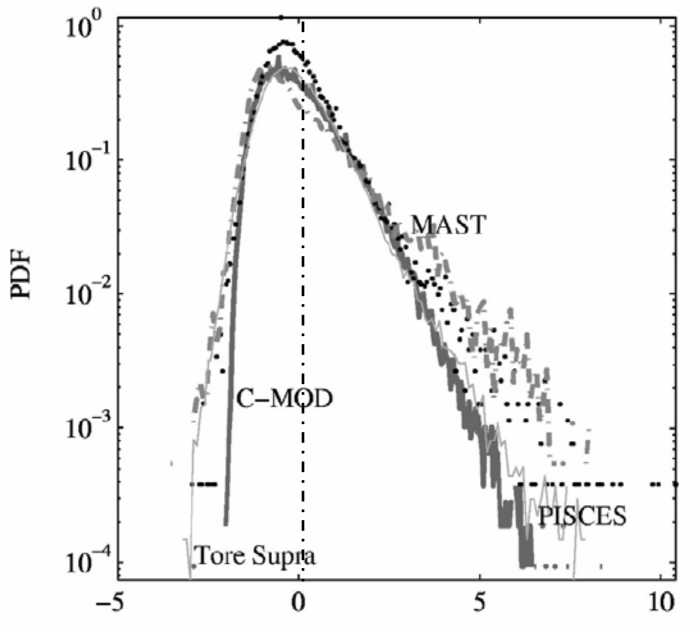

Garcia *et al* PRL **92** 165003 (2004)

# Plasma edge fluctuations

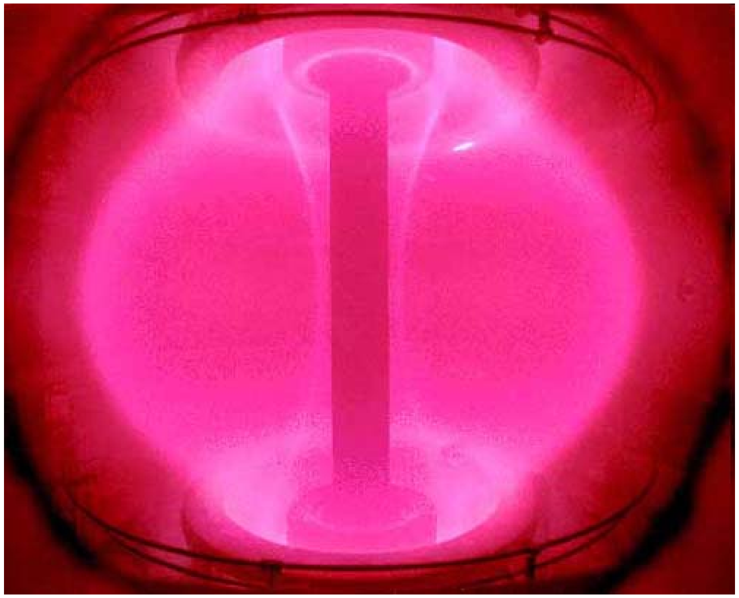



Antar *et al* PoP **10** (2) 419 (2003)

# Anomalous transport in laboratory

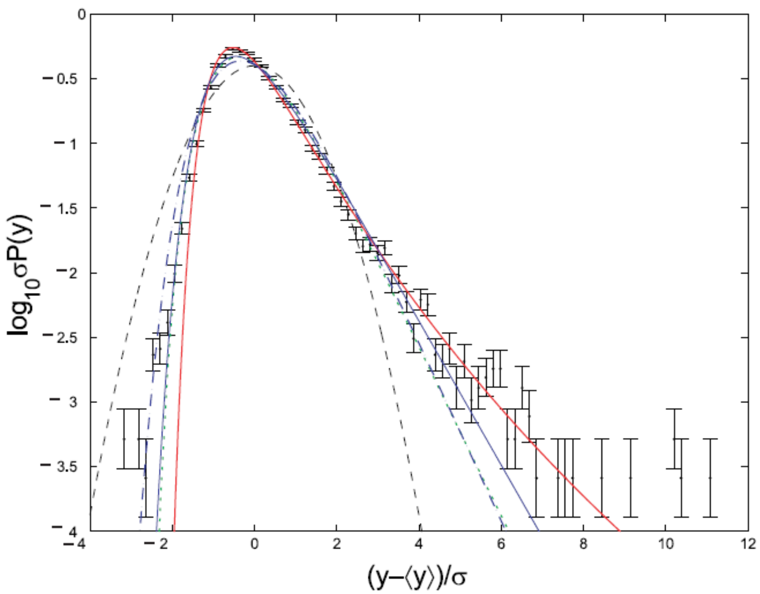


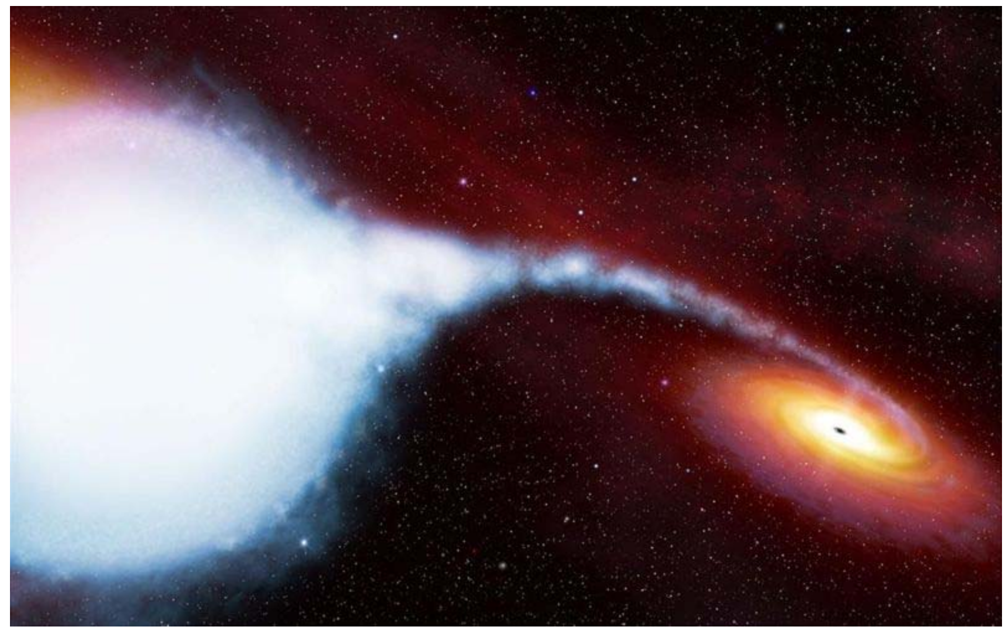

$n$

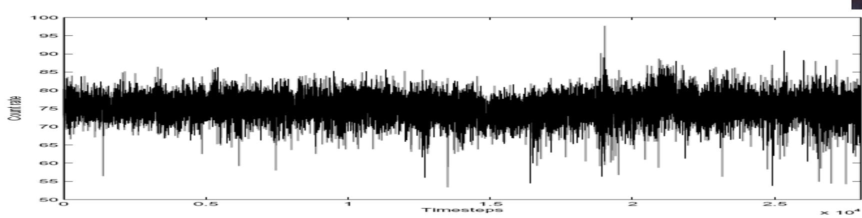


Antar *et al* PoP **10** (2) 419 (2003)

# Anomalous transport on Cygnus-X1 accretion disk



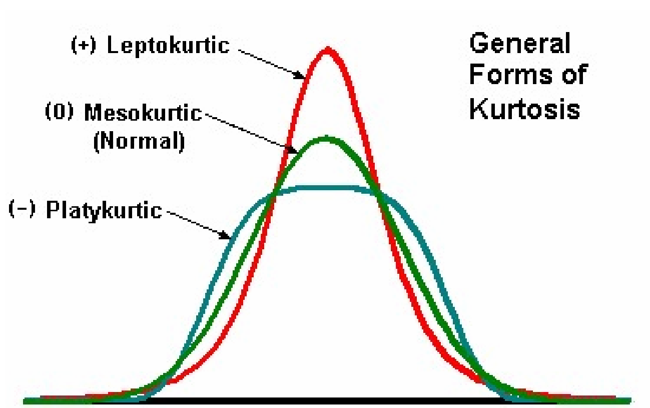


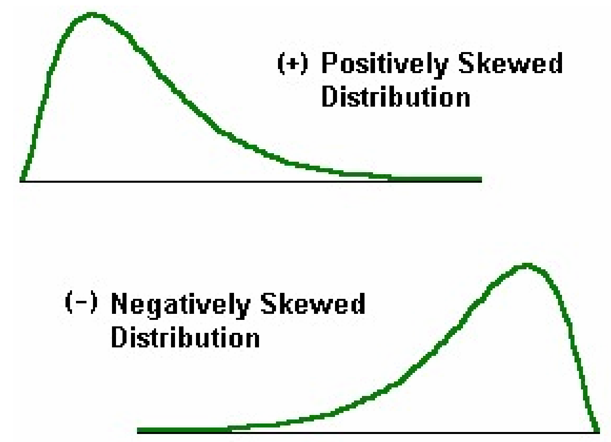

R. Dendy, this conference, Monday

# High order moments

$$K = \frac{\mu_3}{\sigma^3} - 3 = \frac{\left\langle \left( X - \langle X \rangle \right)^4 \right\rangle}{\sigma^4} \text{-3}$$

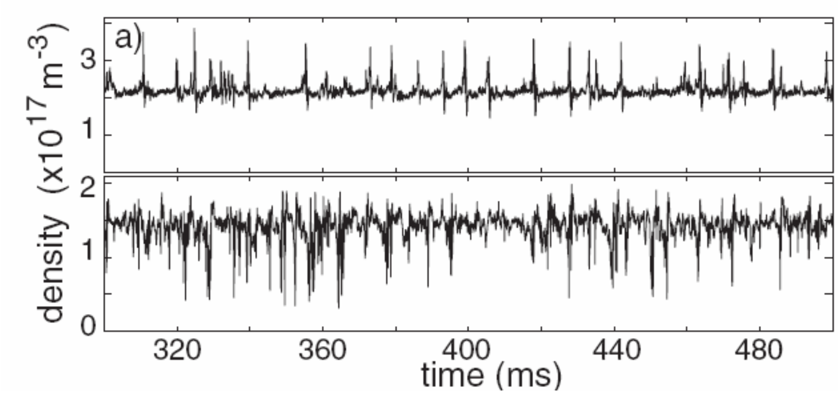
$$S = \frac{\mu_3}{\sigma^3} = \frac{\left\langle \left( X - \langle X \rangle \right)^3 \right\rangle}{\sigma^3}$$

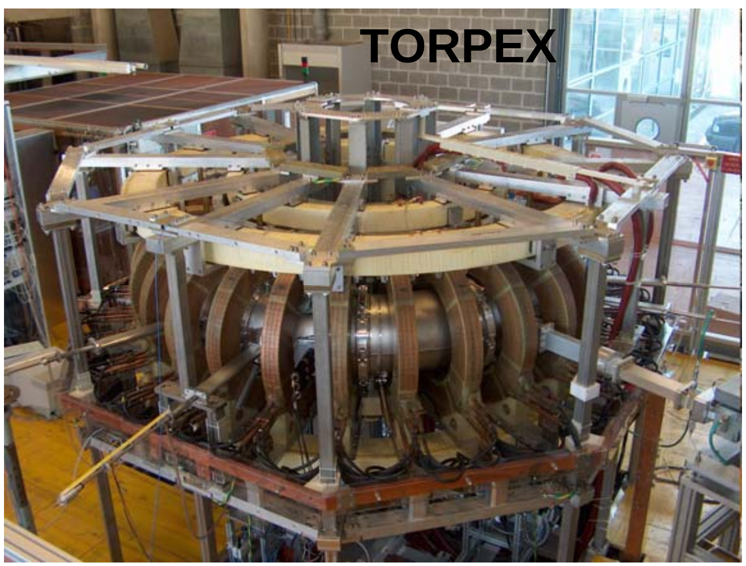


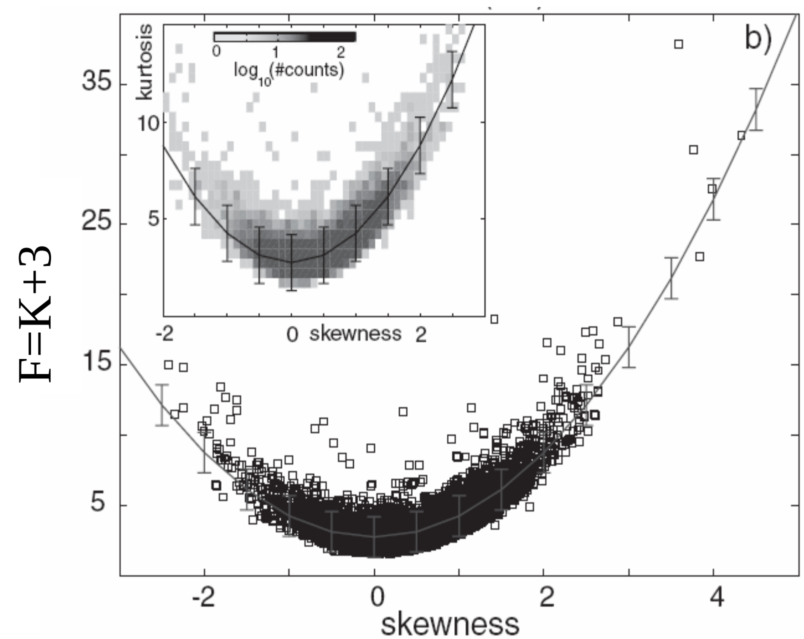




$$\mu_k = \left\langle \left( X - \langle X \rangle \right)^k \right\rangle, \qquad \sigma^2 = \mu_2 = \left\langle \left( X - \langle X \rangle \right)^2 \right\rangle$$

# Scaling between K and S of density fluctuations



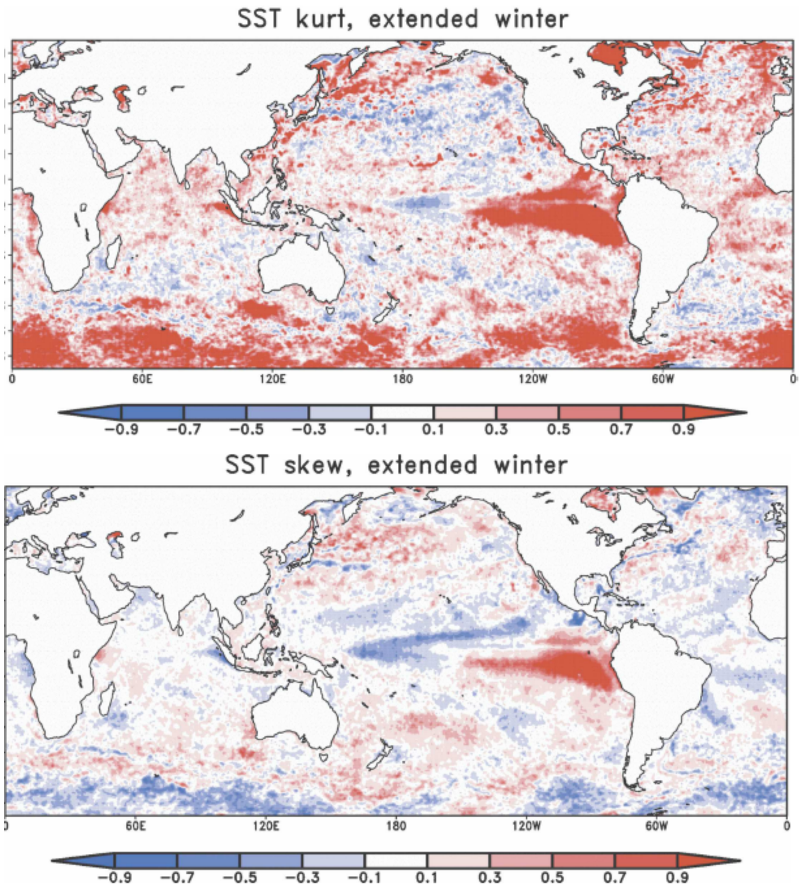



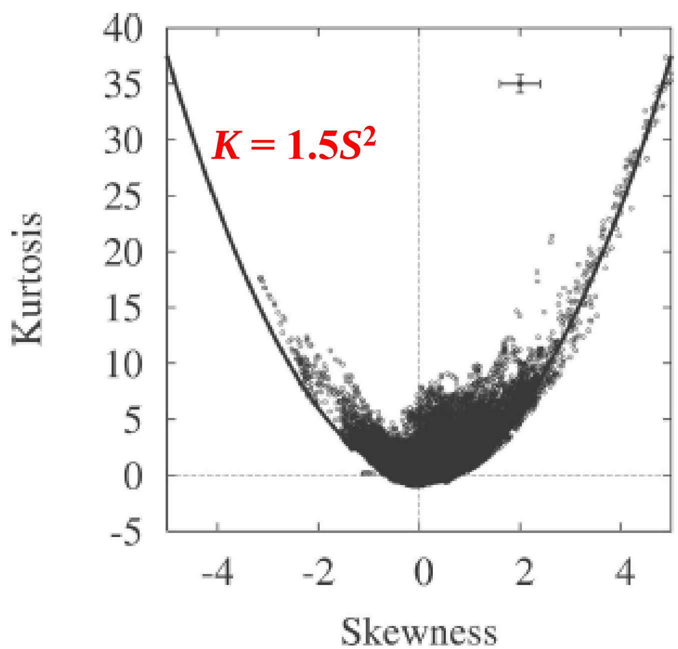




$$K = (1.502 \pm 0.015)S^2 - (0.226 \pm 0.019)$$

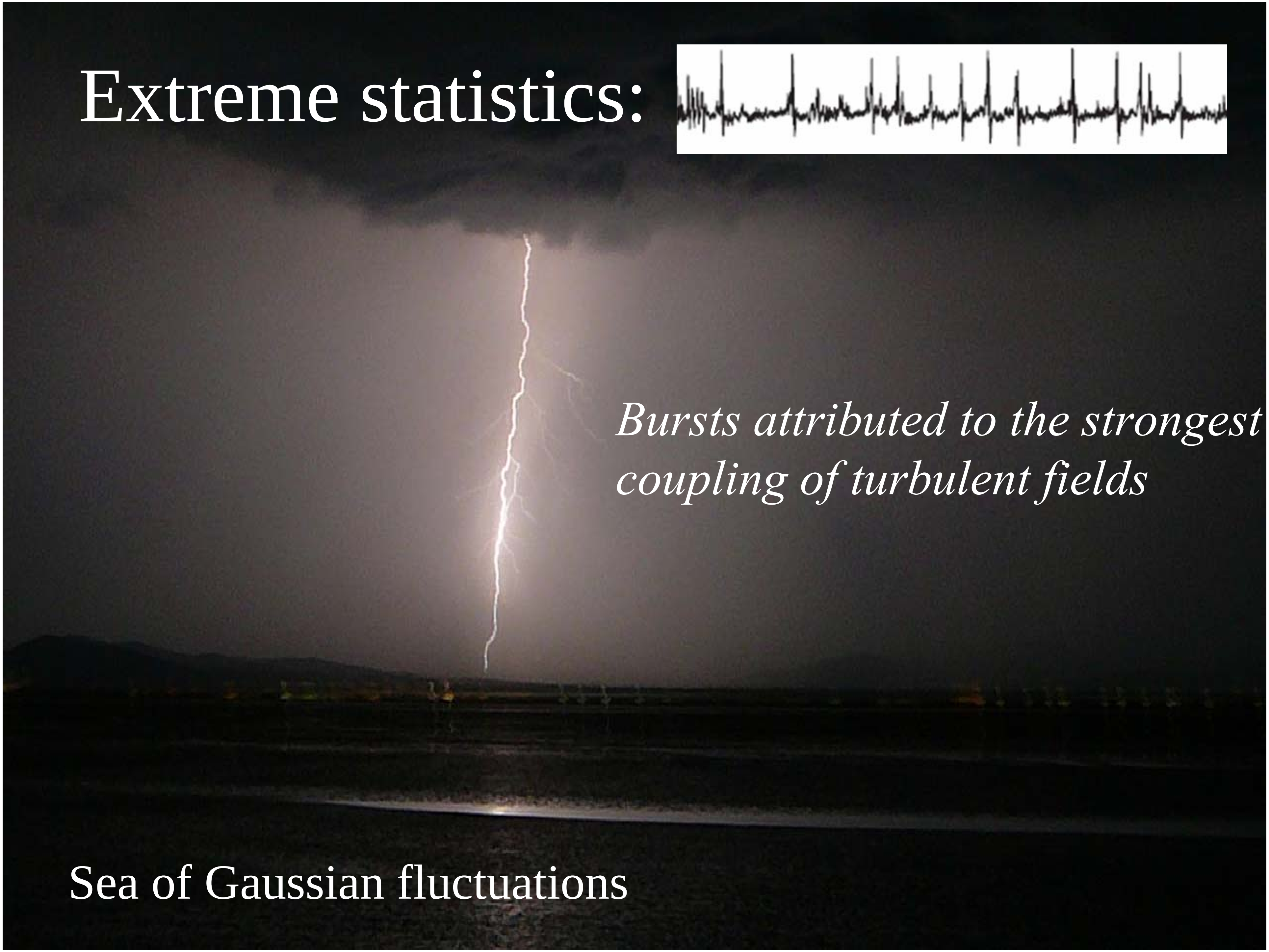
Labit *et al* PRL **98,** 255002 (2007)

# Scaling between K and S of SST variability




Sura and Sardeshmukh, J. Ph. Ocean. 639 (2008)


Extreme statistics:
Bursts attributed to the strongest coupling of turbulent fields
Sea of Gaussian fluctuations

# Stochastic process

Let us construct the simplest stochastic process that accounts strongly coupled turbulent fields:

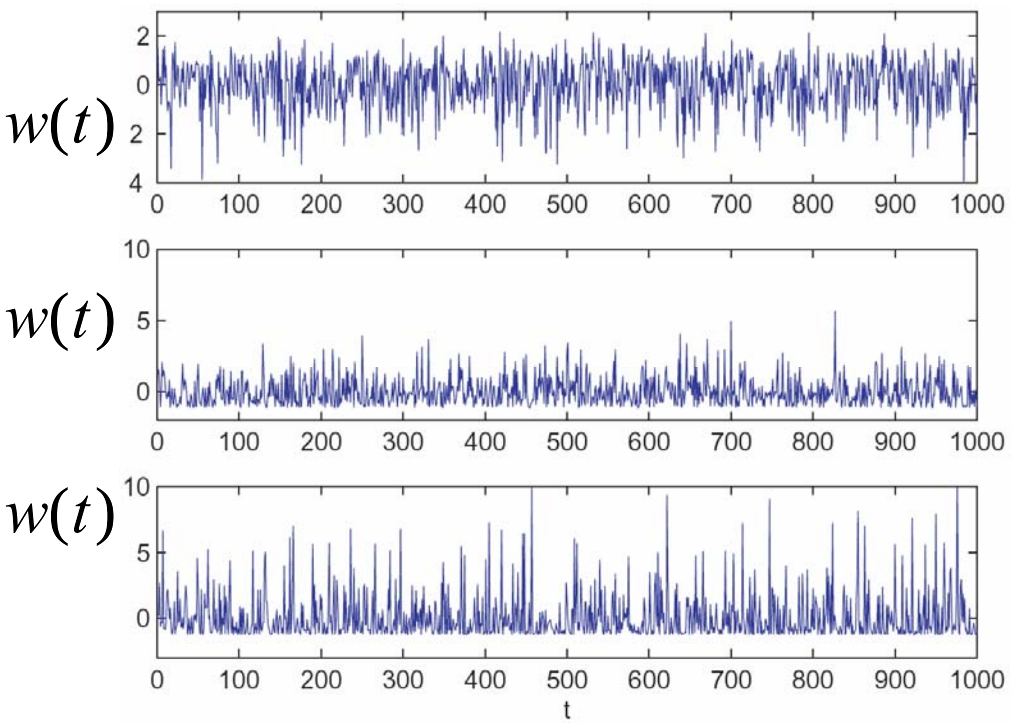
- given by the sum of a Gaussian and a non-Gaussian:

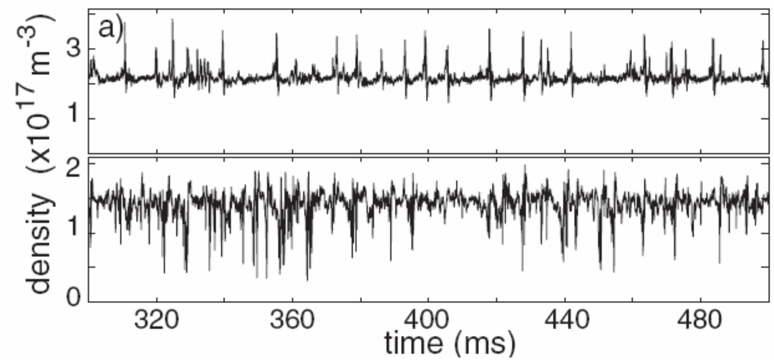
$$w(t) = z_G(\mathrm{t}) + \gamma z_{nG}(\mathrm{t})$$

- where the non-Gaussian is attributed to quadratic non-linearity:

$$w_{nG}(\mathrm{t}) = \frac{z_G^2(t) - \left\langle z_G^2(t) \right\rangle}{\sigma_0^2}$$

# Bursty behavior: recovered!




Labit, PRL **98,** 255002 (2007)

# Calculation of *PDF(w)*

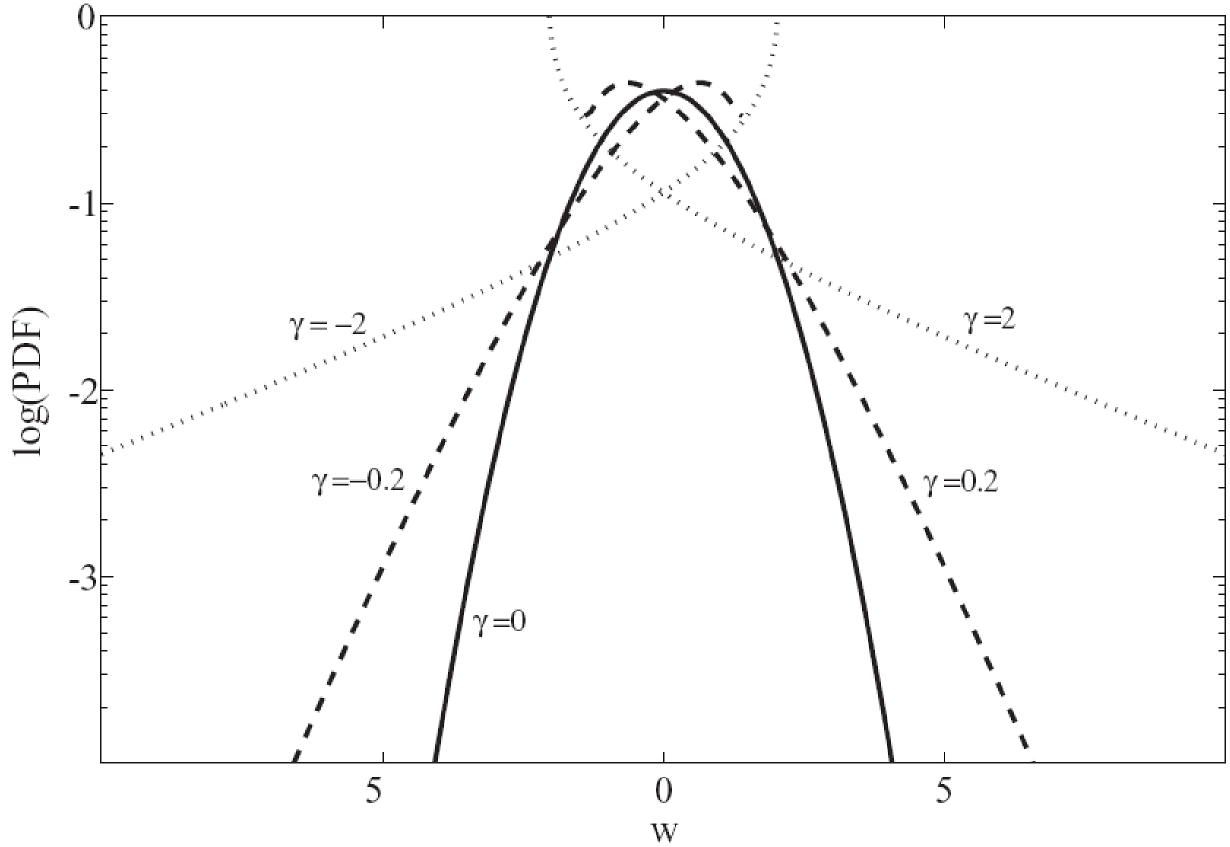
$$w_0 = -(4\gamma)^{-1} - \gamma \quad \longleftarrow$$ Cut off!

$$PDF\left(w; |w| > |w_0|\right) = \frac{2}{\pi\sqrt{\lambda(w,\gamma)}} \cosh\left(\frac{\sqrt{\lambda(w,\gamma)}}{4\gamma^2}\right) \exp\left(-\frac{1+\lambda(w,\gamma)}{8\gamma^2}\right)$$

$$\lambda = 1 + 4\gamma(w+\gamma) \quad \longleftarrow$$ Asymmetry induced

# PDF(w)

$w_0 = -(4\gamma)^{-1} - \gamma$



# Scaling of Kurtosis and Skewness

$$w(t) = z_G(\mathrm{t}) + \gamma \frac{z_G^2(t) - \langle z_G^2(t) \rangle}{\sigma_0^2} \longrightarrow S = 2\gamma \frac{3+4\gamma^2}{(1+2\gamma^2)^{3/2}} \text{ and } K = 48\gamma^4 \frac{1+\gamma^2}{(1+2\gamma^2)^2}$$

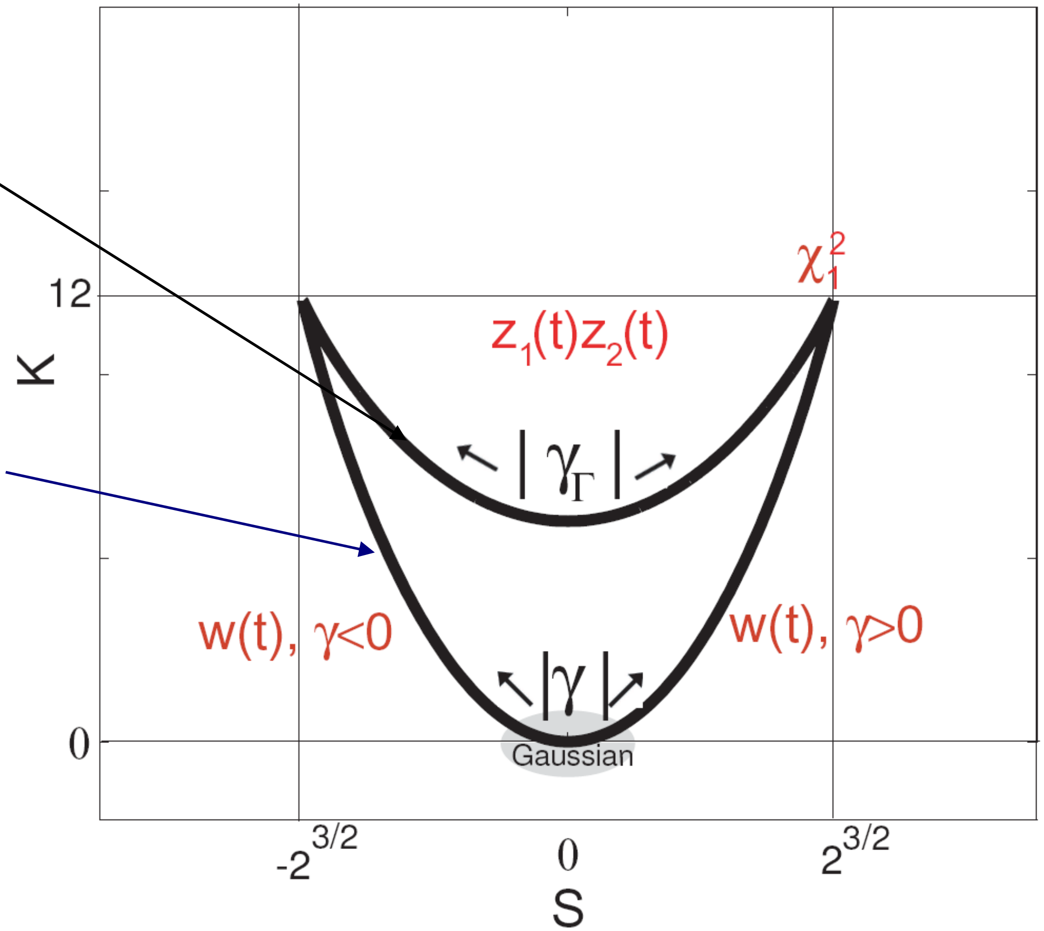
$$K = aS^2 + b$$

$$a = 12 \frac{(1+\gamma^2)(1+2\gamma^2)}{(3+4\gamma^2)^2} \longrightarrow \boxed{a = 3/2}$$

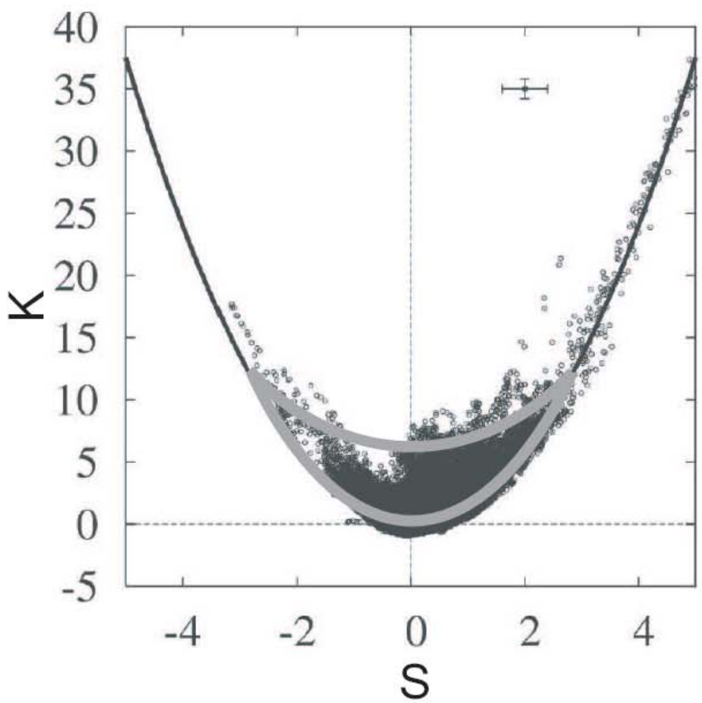
# K-S scaling

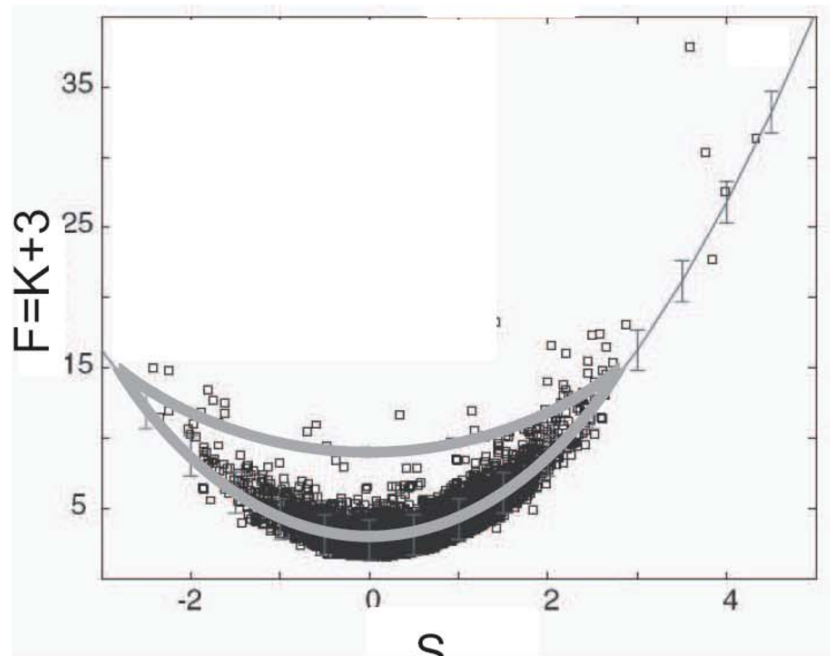
$$w_{\Gamma}(t) = \gamma_{\Gamma} z_{1G}(t) z_{2G}(t)$$

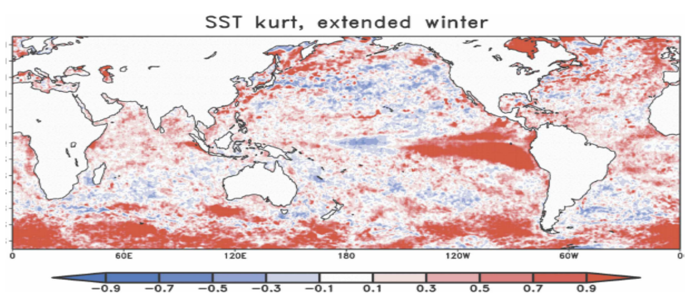
$$w(t) = z_G(t) + \gamma \frac{z_G^2(t) - \langle z_G^2(t) \rangle}{\sigma_0^2}$$

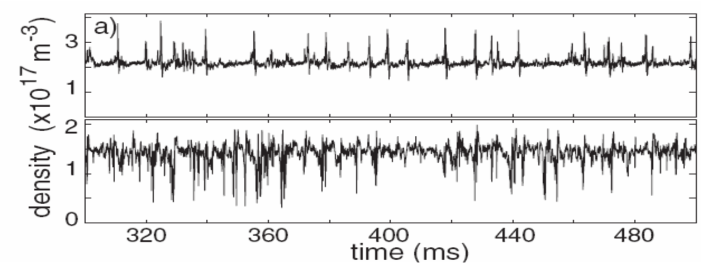


# K-S scaling: recovered

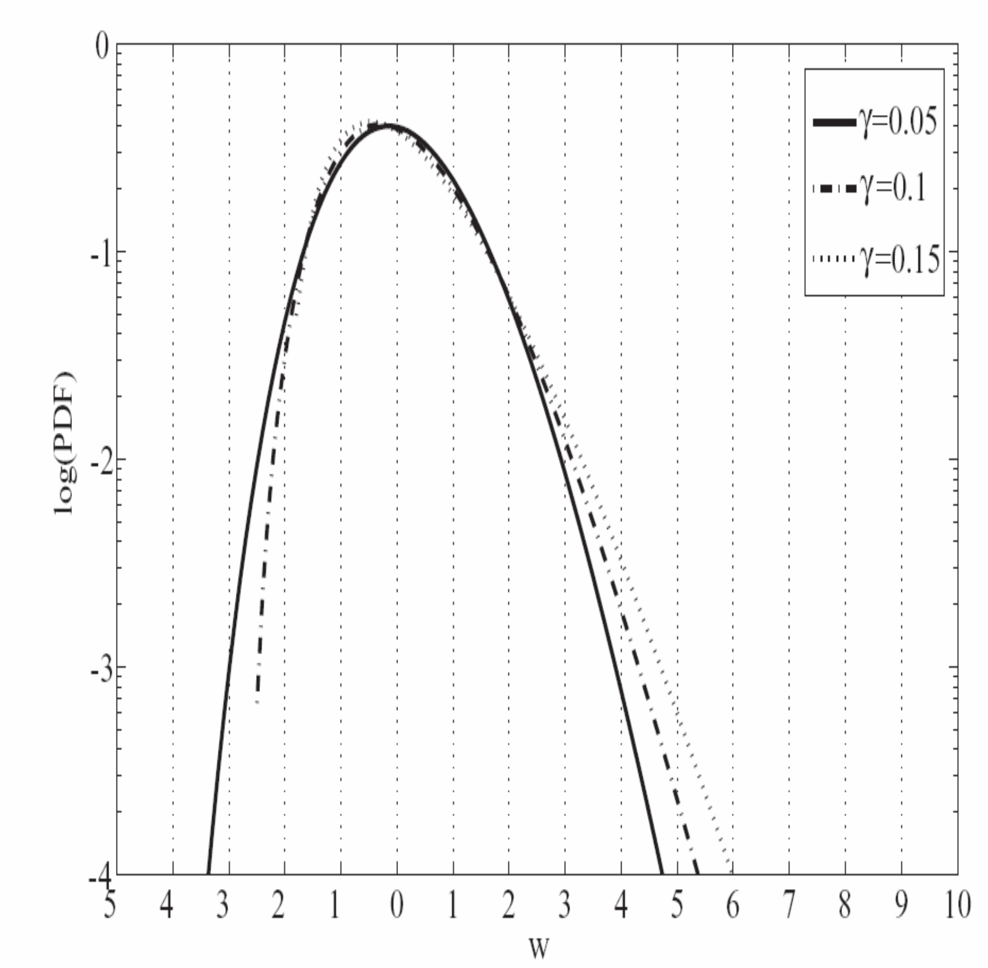


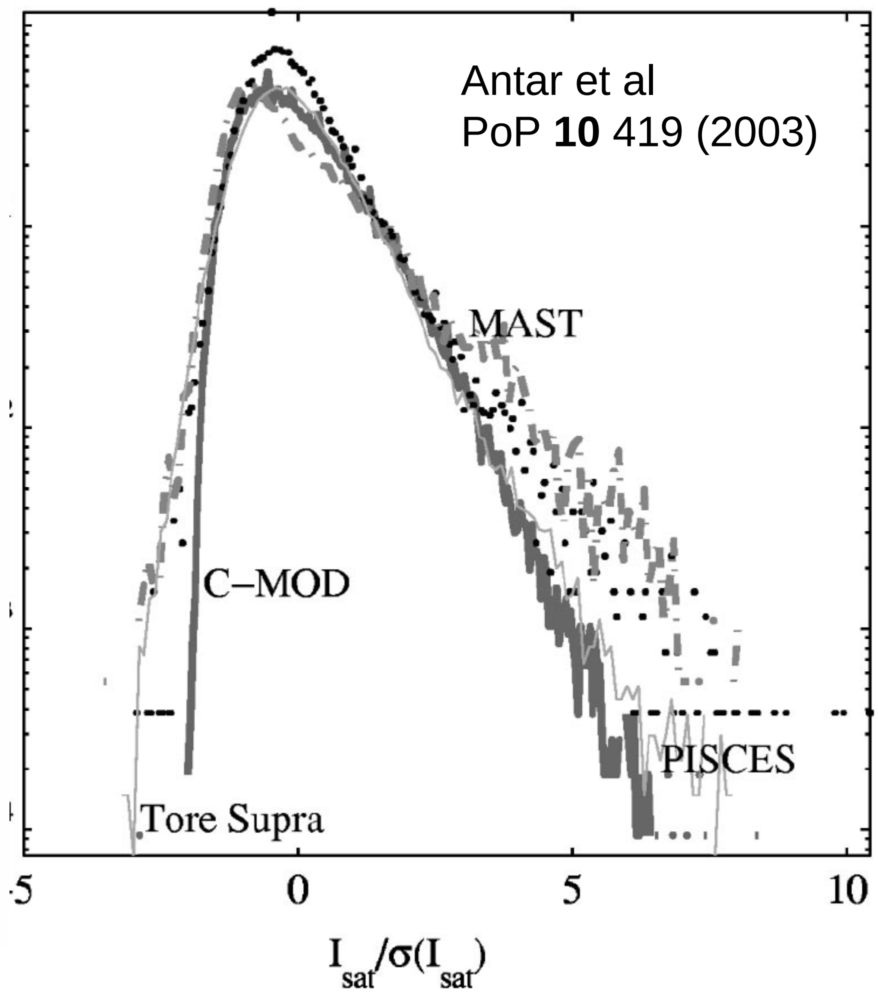



# Universal PDF: recovered



# Conclusions

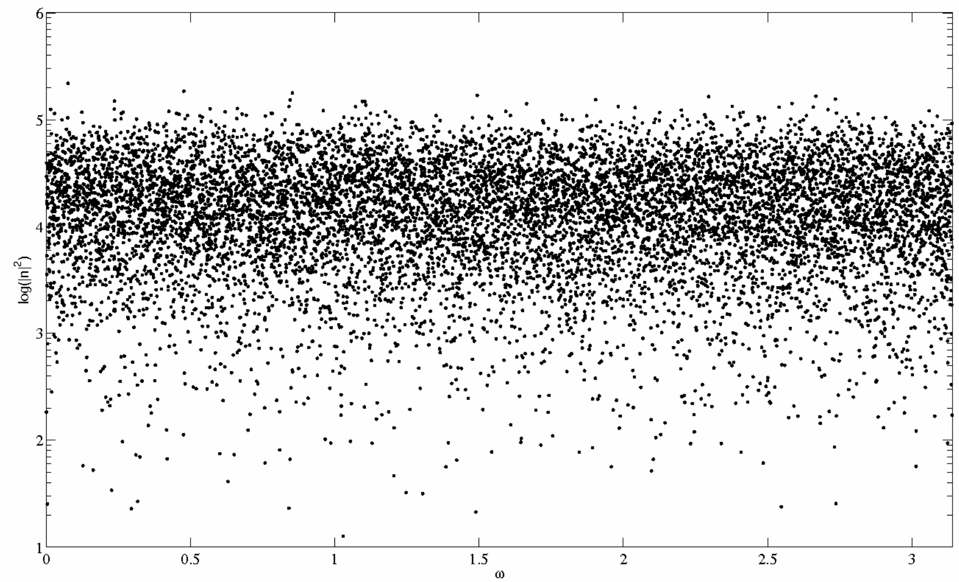
- A stochastic process *w(t)* for the description of bursty fluctuations based on generic properties of quadratic non-linearities was introduced
- The associated *PDF(w)* recovers observed features of extreme processes
- The analytic form of *PDF(w)* can be used as a diagnostic tool
- Do relaxation processes in convective systems lead to a state of extreme "intermittency"?


Thank you!

# White noise spectra

$$n(t) = n_G(\mathrm{t}) + \gamma n_{nG}(\mathrm{t})$$

$$n_{nG}(\mathrm{t}) = \frac{n_G^2(t) - \langle n_G^2(t) \rangle}{\sigma_0^2}$$




- White noise (<u>non-Gaussian</u>) type
- Signal does not change in time, H=0
- Absence of correlations
- Extreme bursty behavior