KINETIC FORMULATION OF TRANSPORT OF CHARGED PARTICLES INTERACTING WITH ELECTROMAGNETIC WAVES IN MAGNETIZED PLASMAS

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Modern Challenges in Nonlinear Plasma Physics
Conference Honoring the Career of Dennis Papadopoulos
June 15 - 19, 2009
Halkidiki, Greece

GEOPHYSICAL RESEARCH LETTERS, VOL. 7, NO. 11, PAGES 1014-1016, NOVEMBER 1980

STOCHASTIC ACCELERATION OF LARGE M/Q IONS BY HYDROGEN CYCLOTRON WAVES IN THE MAGNETOSPHERE

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Abstract. It is shown that in hydrogen dominated multi-ion plasmas supporting coherent hydrogen cyclotron waves, the minority ion species with large M/Q are preferentially accelerated and the maximum energy achieved scales as $(M/M_{H^+})^{5/3}$. The importance of this scaling to O^+ acceleration in the auroral zones and to other high energy heavy ion observations in the earth's and Jupiter's magnetospheres is discussed.

STANDARD (CHIRIKOV-TAYLOR) MAP

Interaction of a charged particle with an infinite set of plane waves

$$\frac{dx}{dt} = v$$

$$\frac{dv}{dt} = \frac{qE}{m} \sum_{n=-\infty}^{\infty} \sin\left(kx - n\omega t\right)$$

STANDARD MAPPING EQUATIONS

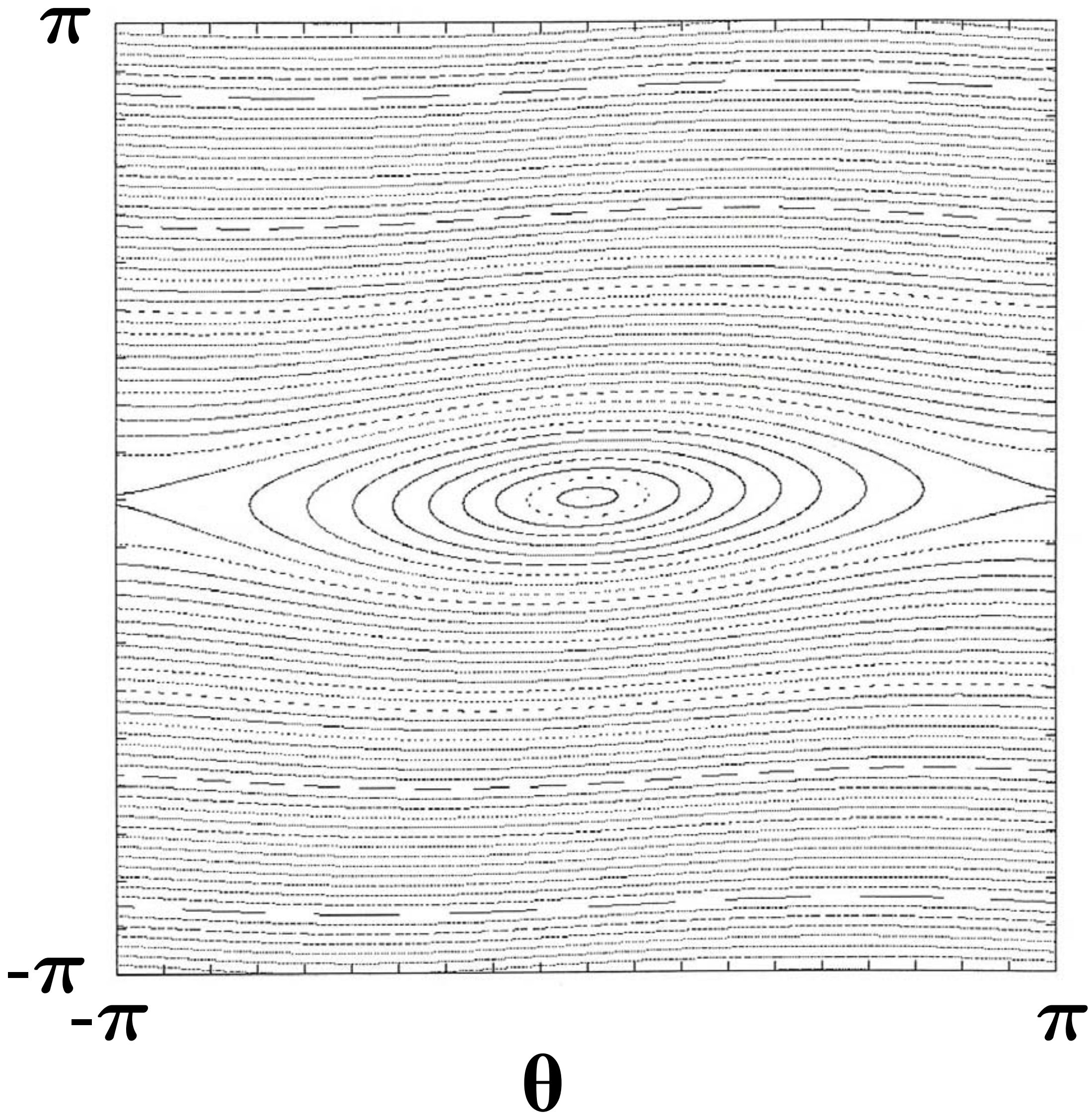
The change in particle velocity and wave phase after every time step T = 2π/ω is

$$I_{n+1} = I_n + K \sin \theta_n \qquad \mathrm{mod}\ 2\pi$$

$$\theta_{n+1} = \theta_n + I_{n+1} \qquad \mathrm{mod}\ 2\pi$$

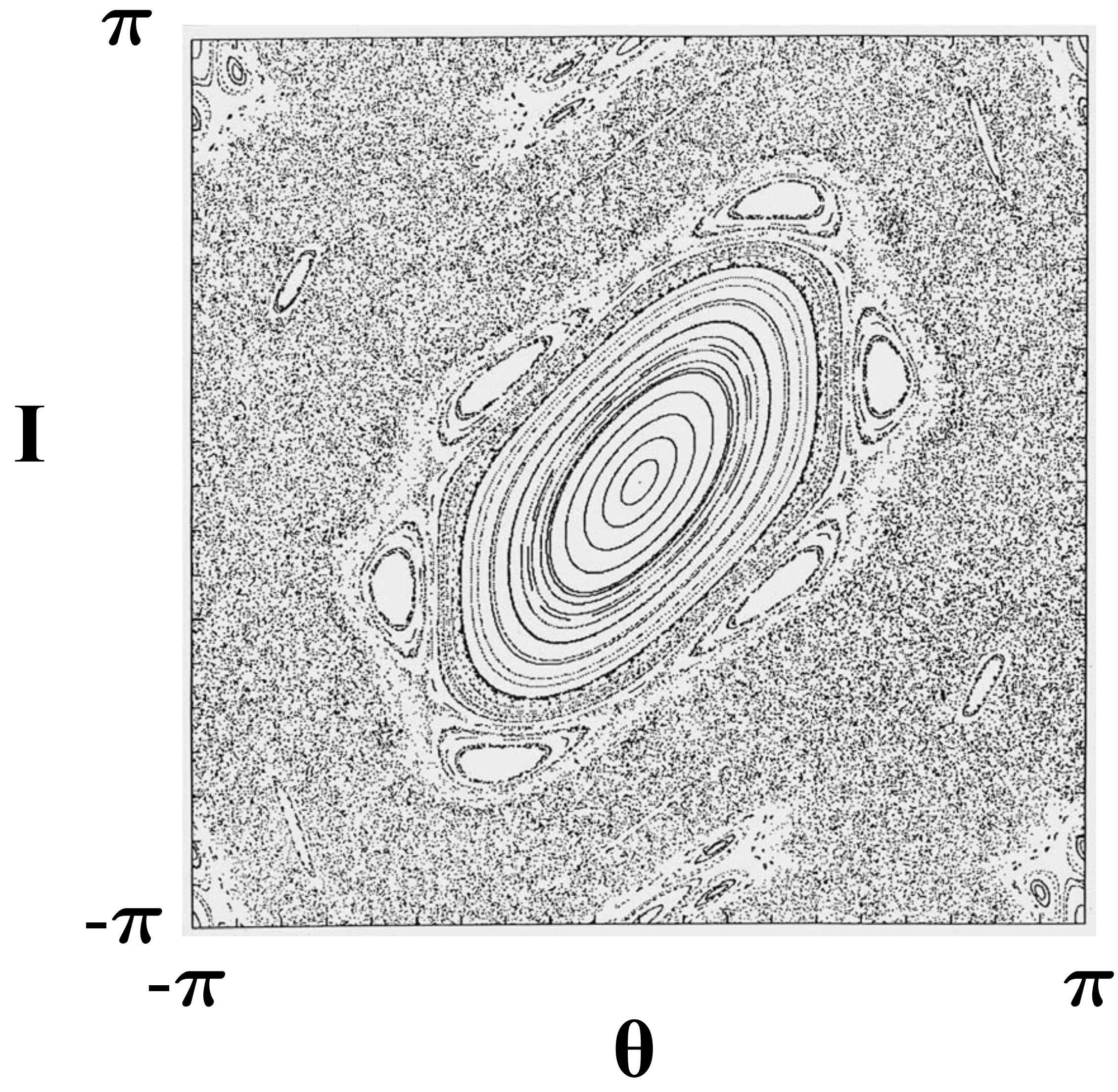
where $kx = \theta, \quad kTv = I, \quad \left(\frac{2\pi}{\omega}\right)^2 \frac{qkE}{m} = K.$

SURFACE-OF-SECTION PLOT



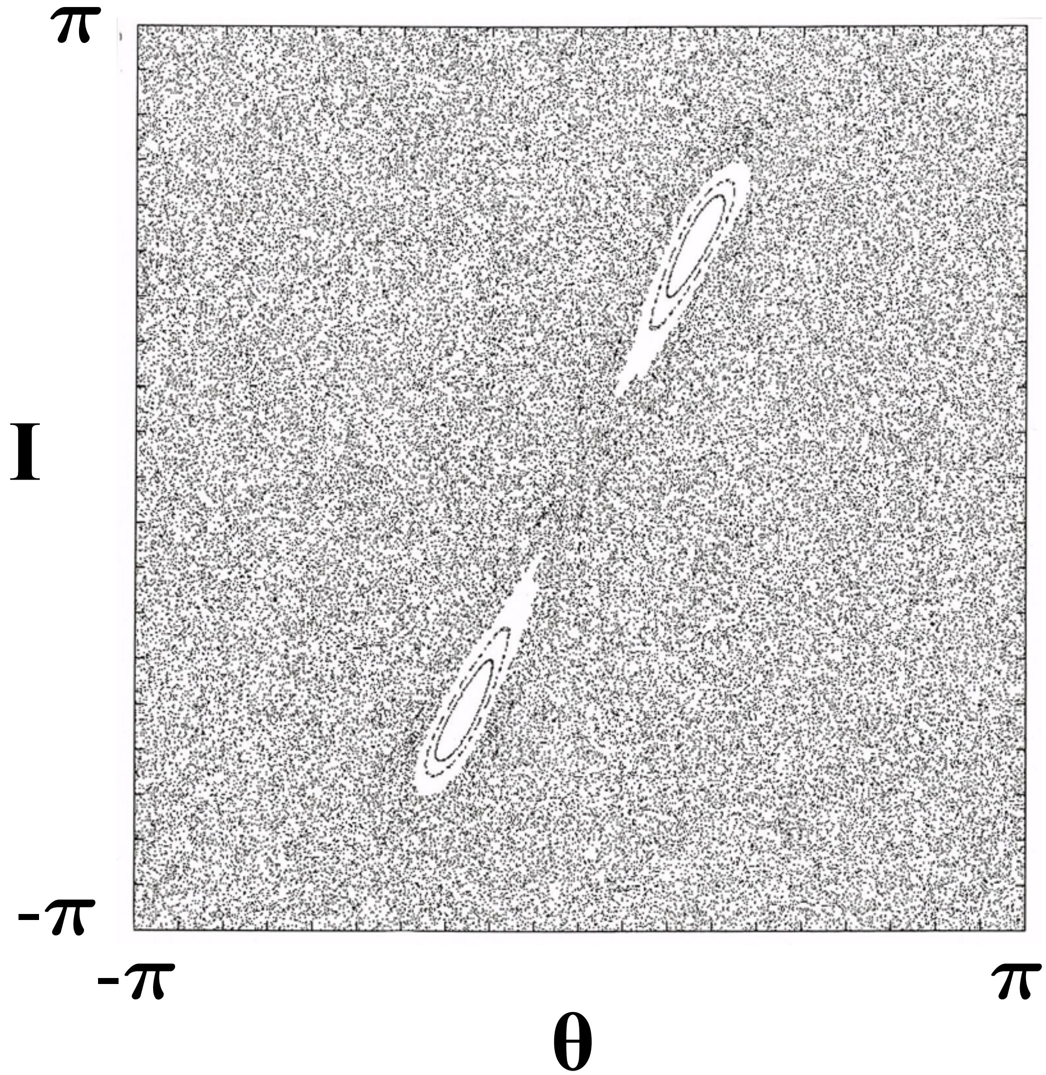
K = 0.1

SURFACE-OF-SECTION PLOT



K = 1.5

SURFACE-OF-SECTION PLOT



K = 4.5

EVOLUTION OF A DISTRIBUTION FUNCTION

Fick's second law, or Fokker-Planck equation

$$\frac{\partial f(I)}{\partial n} = \frac{\partial}{\partial I}\left(D(I)\ \frac{\partial}{\partial I}\ f(I) \right)$$

f(I) is the distribution function

D(I) is the diffusion coefficient

What is to be substituted for ***n*** and ***D(I)*** ?

DIFFUSION IN VELOCITY SPACE

Evaluation of the diffusion coefficient

- Single step jump in velocity ($I_0 \rightarrow I_1$)
  - Markovian assumption;
  - random walk (or Brownian motion).
- Multiple step jump in velocity ($I_0 \rightarrow I_n$)
  - $n \gg n_c$
  - n_c is the number of steps for phase randomization.

DIFFUSION IN VELOCITY SPACE

Quasilinear diffusion coefficient **(n=1)**

$$D_{QL} = \frac{<(\Delta I_1)^2>_{\theta_0}}{2}$$

$$= \frac{1}{4\pi}\int_0^{2\pi} d\theta_0 (I_1 - I_0)^2 = \frac{K^2}{4}$$

- Independent of I

Define the correlation function:

$$C_n = <(I_n^p - I_{n-1}^p)\,(I_1^p - I_0^p)>_p$$

where $<\ldots>_p$ is an ensemble average for a set of randomly distributed particles.

The correlation "time" n_c is such that

for $n > n_c$, $C_n \approx 0$.

DIFFUSION IN VELOCITY SPACE

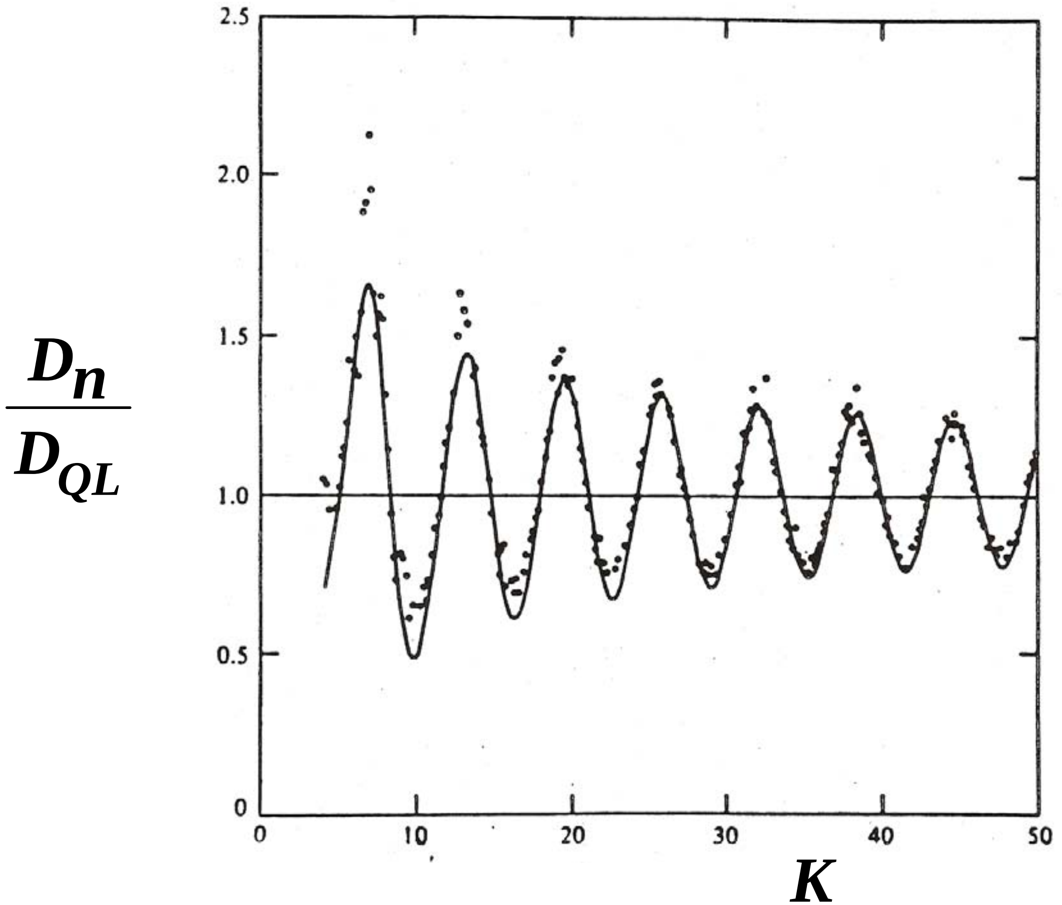
Diffusion coefficient for **$n > n_c$**:

$$D_n = \lim_{n > n_c} \frac{< (I_n - I_0)^2 >}{2n}$$

< .. > is the ensemble average.

$$\frac{\partial f(I)}{\partial n} = \frac{\partial}{\partial I}\left(D_n(I) \frac{\partial}{\partial I} f(I) \right)$$

DIFFUSION COEFFICIENT FOR THE STANDARD MAP



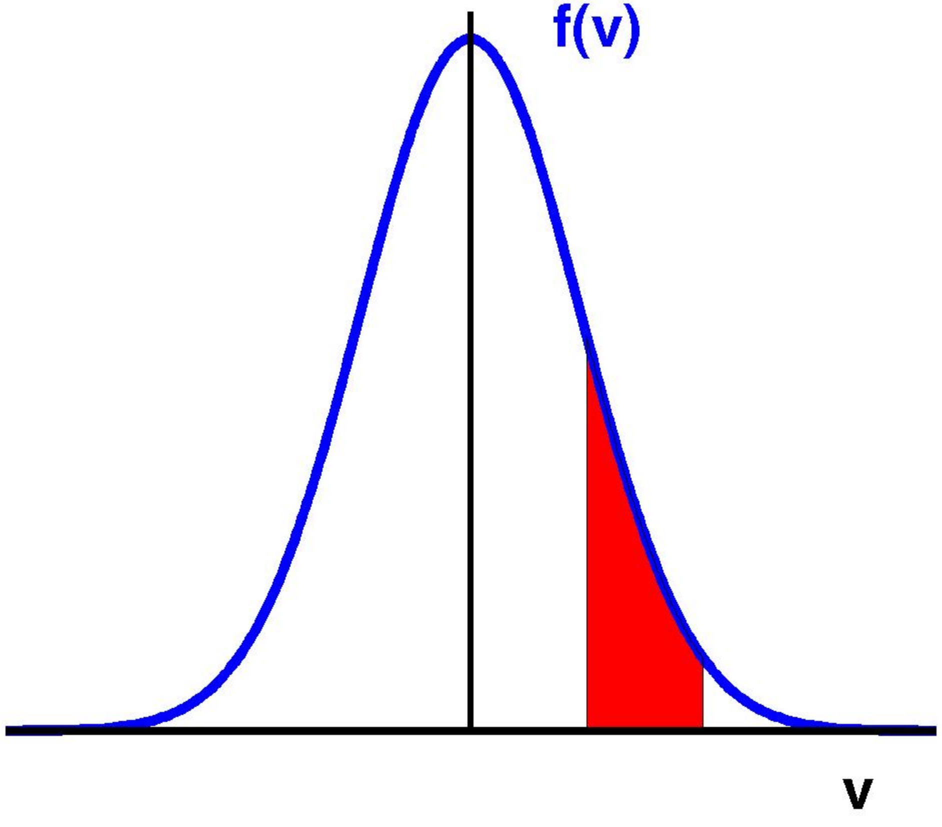
n = 50

➢ **D is independent of I**

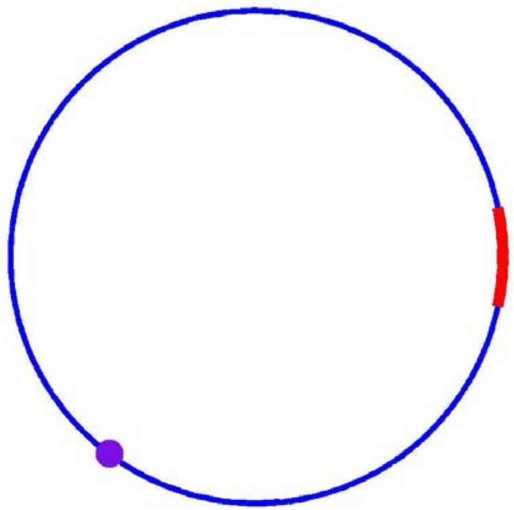
Rechester & White, *Phys. Rev. Lett*. **44**, 1586 (1980)

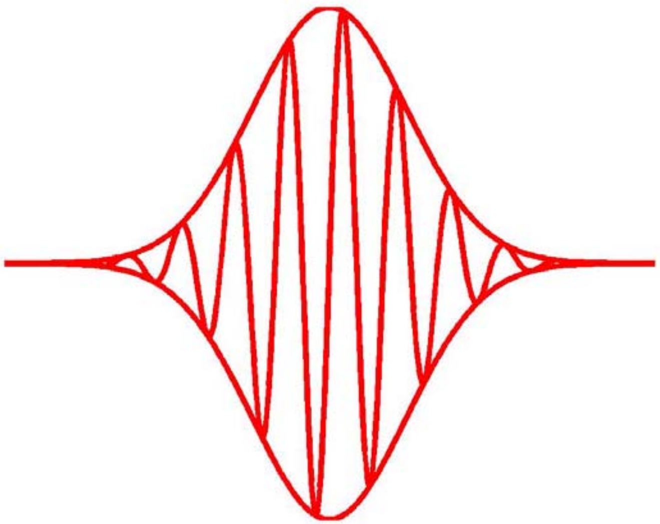
MODIFICATION TO THE DISTRIBUTION FUNCTION

- In the standard map, the entire particle distribution function is affected.

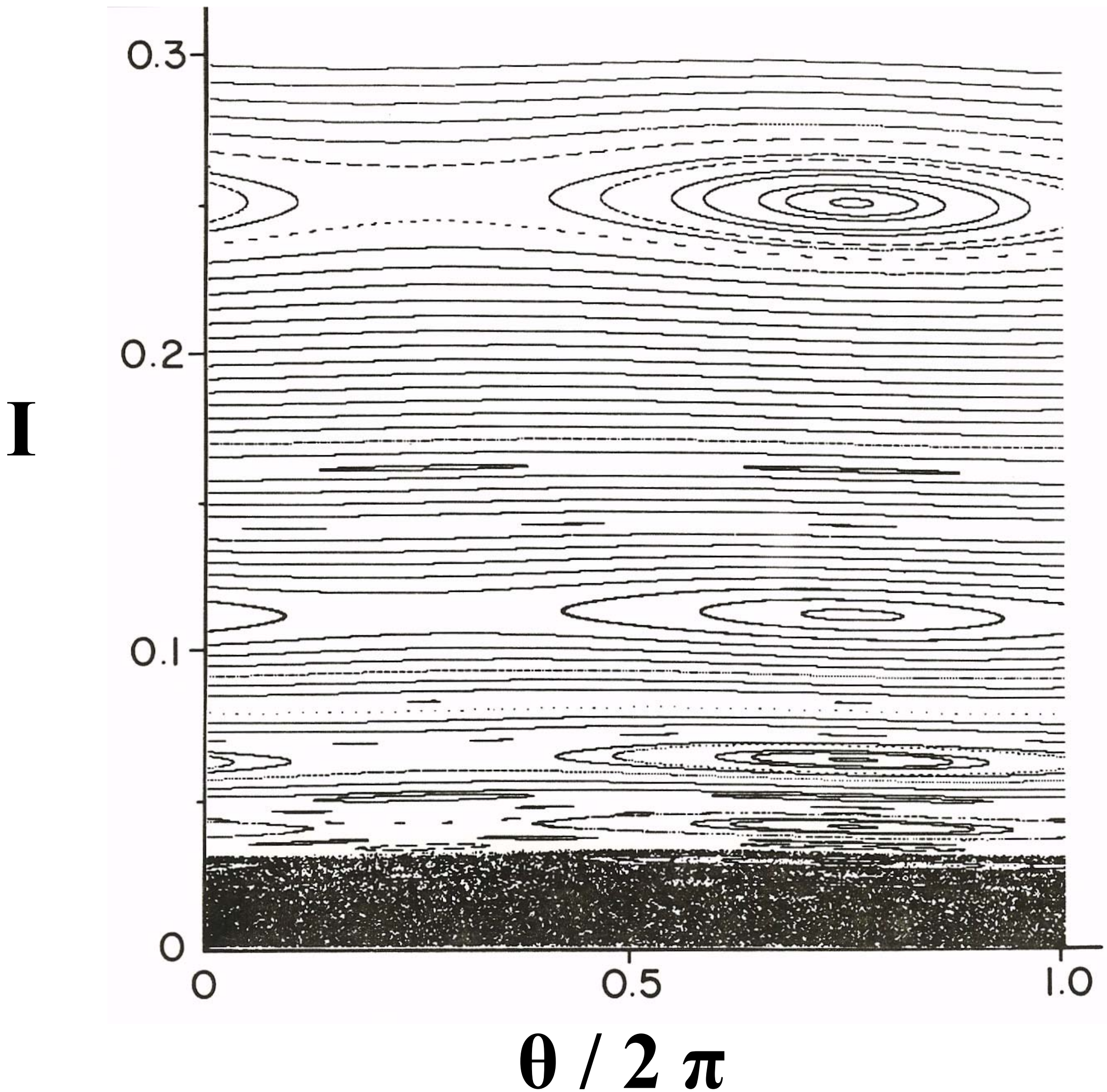


PARTICLE INTERACTION WITH A SPATIALLY LOCALIZED FIELD

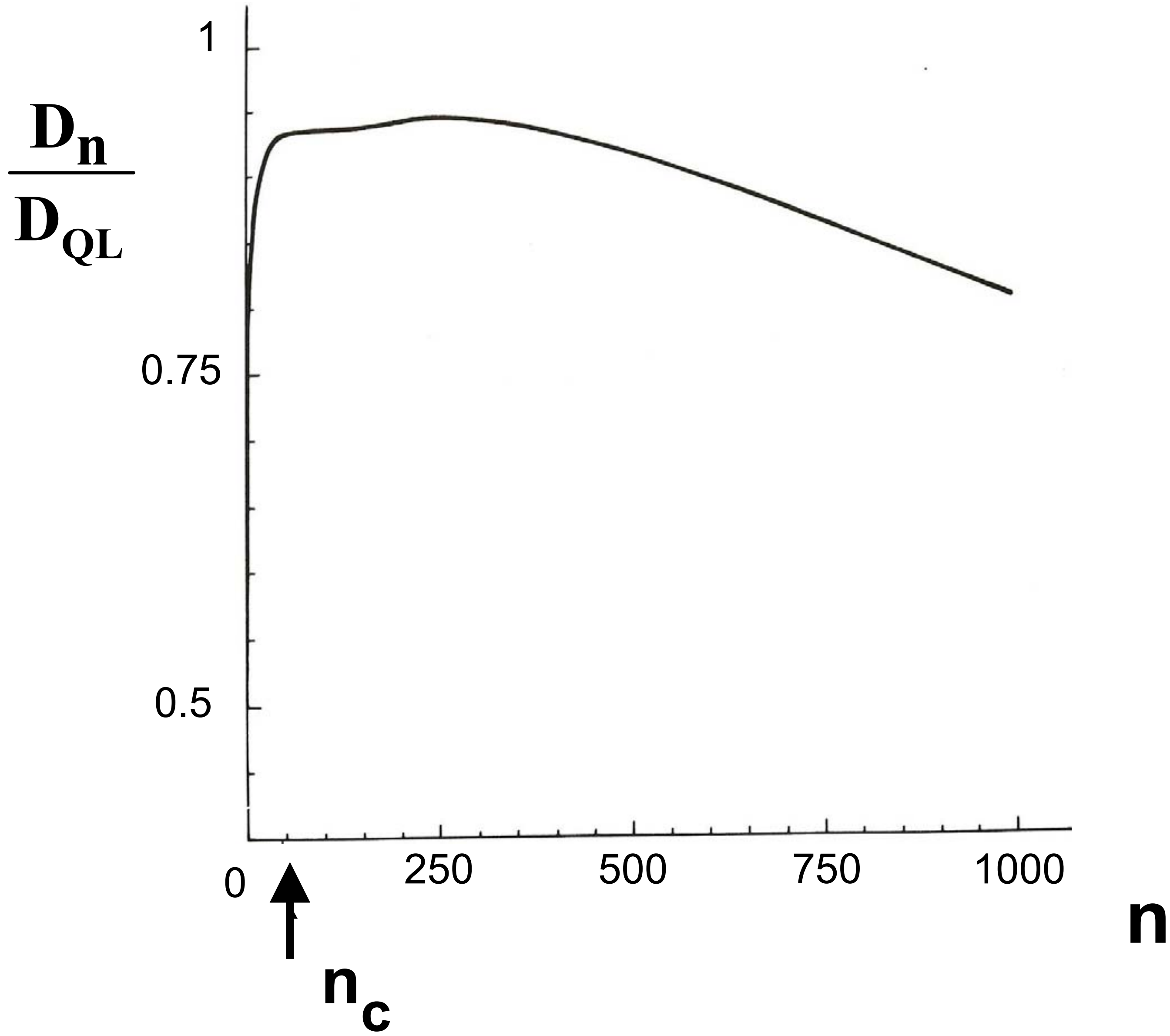




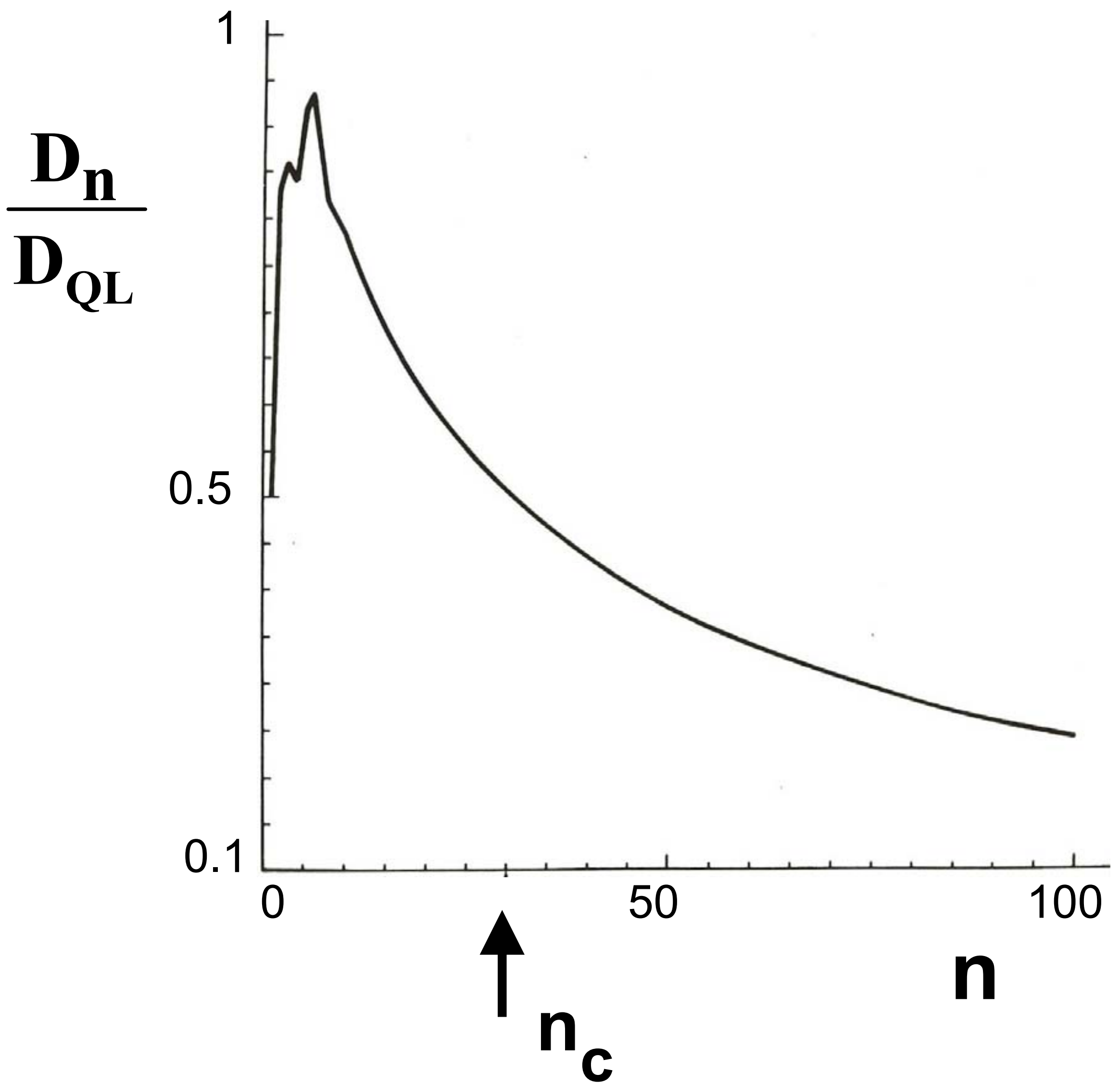
SURFACE-OF-SECTION PLOT



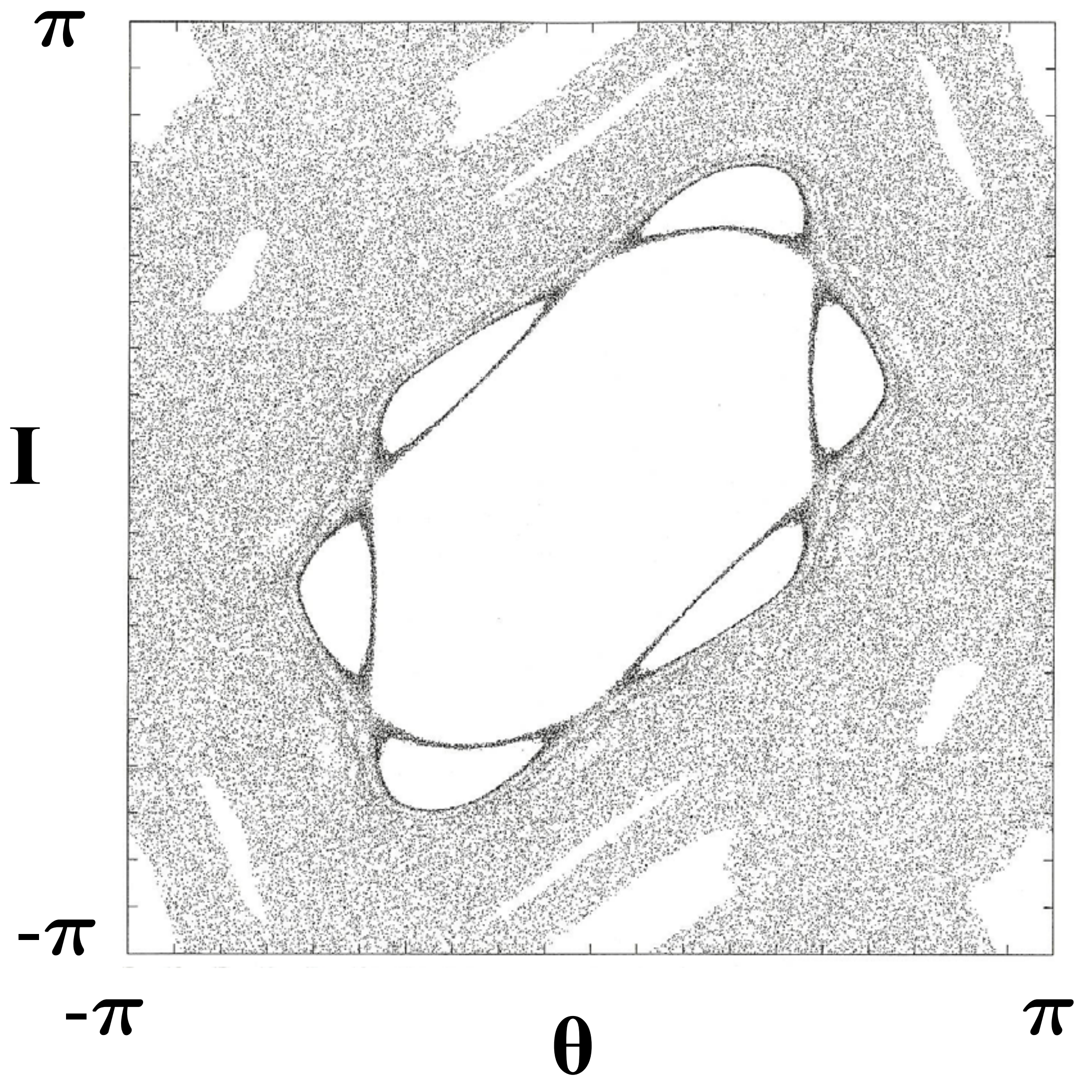
DIFFUSION COEFFICIENT FOR LOCALIZED CHAOS



DIFFUSION COEFFICIENT FOR LOCALIZED CHAOS



STICKINESS OF ORBITS



K = 1.5

PREVIOUS APPROACHES TO QUASILINEAR DIFFUSION EQUATION

- Linearize the Vlasov equation and obtain an equation for the perturbed distribution function.

- Assume that the underlying particle dynamics is chaotic
  - Brownian motion (random walk);
  - no structure to phase space.

- Long time evolution is the same as for short times
  - allows the limit ($t \rightarrow \infty$) in evaluating D.

- Obtain time-independent, singular diffusion operator

$$\delta(\omega - n\omega_c - k_{\|} v_t)$$

A different approach is needed to describe the evolution of a distribution function of particles interacting with plasma waves.

LIE PERTURBATION SERIES METHOD

Hamiltonian approach to particle dynamics and wave particle interactions:

$$H(\mathbf{J},\boldsymbol{\theta}) = H_0(\mathbf{J}) + \epsilon\, H_1(\mathbf{J},\boldsymbol{\theta},t)$$

$H_0(\mathbf{J})$ describes the motion of the particle in the absence of plasma waves.

$H_1(\mathbf{J},\boldsymbol{\theta},t)$ includes the interaction with waves.

LIE PERTURBATION SERIES METHOD

There exists an operator O_L (Lie operator) such that

$$\mathbf{O}_\mathbf{L} : \ (\mathbf{J}, \boldsymbol{\theta})_t \ \rightarrow \ (\mathbf{J}, \boldsymbol{\theta})_{t+\Delta t}$$

An advantage of the Lie operator is that

$$\mathbf{O}_\mathbf{L}^{-1} f(\mathbf{J},\boldsymbol{\theta}) = f(\mathbf{O}_\mathbf{L}\{\mathbf{J},\boldsymbol{\theta}\})$$

EVOLUTION EQUATION FOR THE DISTRIBUTION FUNCTION

$$f(\mathbf{J},\boldsymbol{\theta})_{t+\Delta t} - f(\mathbf{J},\boldsymbol{\theta})_t = (\mathbf{O}_{\mathbf{L}}^{-1} - \mathbf{I}) \bullet f(\mathbf{J},\boldsymbol{\theta})_t$$

Dividing by Δt and taking the limit $\Delta t \rightarrow 0$

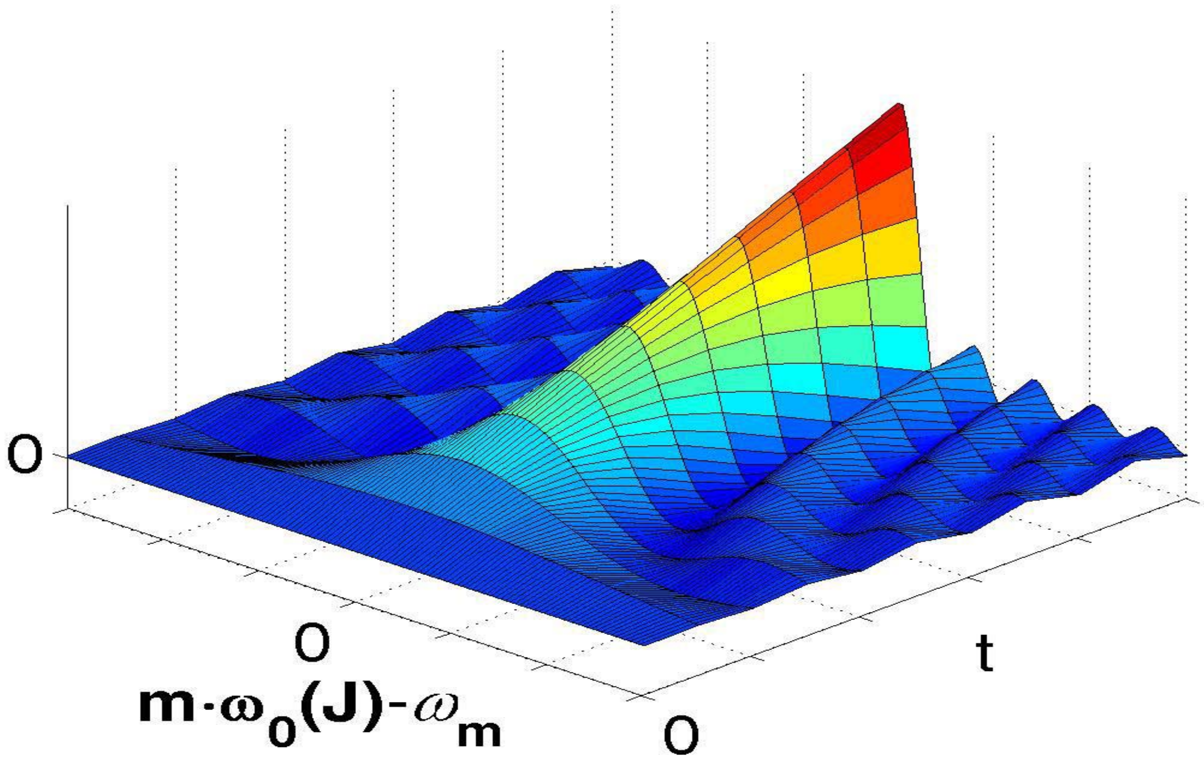
$$\frac{\partial}{\partial t} f(\mathbf{J},\boldsymbol{\theta},t) = \left[\frac{\partial}{\partial t}(\mathbf{O}_{\mathbf{L}}^{-1} - \mathbf{I})\right] \bullet f(\mathbf{J},\boldsymbol{\theta},t)$$

EVOLUTION EQUATION FOR THE DISTRIBUTION FUNCTION

By averaging over the angles **θ**

$$\frac{\partial}{\partial t} f(\mathbf{J},t) = \left[\frac{\partial}{\partial \mathbf{J}} \cdot \mathbf{D}(\mathbf{J},t) \cdot \frac{\partial}{\partial \mathbf{J}} \right] f(\mathbf{J},t)$$

EVOLUTION OF THE DIFFUSION COEFFICIENT



$$\lim_{t\to\infty} \mathbf{D}(\mathbf{J},t) \;\to\; \delta(\mathbf{m}\cdot\boldsymbol{\omega}_0 - \boldsymbol{\omega}_m)$$

CONCLUSIONS

- Dynamical studies of wave-particle interactions show a mixed phase space.
- The evolution of a distribution function requires proper accounting of this phase space.
- The Markovian assumption for evaluating the diffusion coefficient is invalid.
- Recent studies provide a detailed description for the evolution of the distribution function due to wave-particle interactions.

Y. Kominis, A.K. Ram, & K. Hizanidis, Phys. Plasmas **15**, 122501 (2008).