Electrostatic solitary waves in superthermal plasmas:

nonlinearity off the Maxwellian frontier

Ioannis Kourakis

Centre for Plasma Physics, Queen's University Belfast
Northern Ireland, UK

In collaboration with: Naresh Pal Saini & Sharmin Sultana (QUB, Belfast, UK)
Manfred Hellberg & Thomas Baluku (UKZN, Durban, S Africa)

Special thanks to: Frank Verheest (U. Gent, Belgium)
A Sharma, A Danehkar (QUB)

Acknowledgments: UK EPSRC funding via CPP/QUB

Queen's University Belfast

Modern Challenges in Nonlinear Plasma Physics, June 15-19 2009, Sani, Greece

Layout

- 1. Motivation: Superthermal electrons – occurrence
- 2. Fundamental framework: κ distribution function, relation to electrostatic (ES) plasma modes
- 3. Question 1: Effect on ES solitary waves
- 4. Question 2: Nonlinear self-modulation of ES wavepackets
  - Modulational instability
  - Envelope solitons
- 5. Conclusions

1. Motivation: Ubiquitous superthermal plasma behavior

- Superthermal electrons are ubiquitously observed in Space: Montgomery *et al*, PRL (1965), Vasyliunas, JGR (1968), Fitzenreiter *et al,* GRL (1998)

- Saturn's Magnetosphere: Schippers *et al.* JGR (2008)

- Solar wind: Gaelzer – Yoon ApJ (2008)

- Plasma laboratory experiments: Kharchenko *et al,* Nucl. Fusion (1961), Kardfidov *et al,* Sov. Phys. JETP (1990), Yoon *et al*, PRL (2005), S. Magni *et al,* PRE (2005)

- Numerical simulations: Kawahara *et al* JPSJ (2006), Petkaki JGR (2003)

- Beam-plasma interactions - Yoon et al, PRL (2005)

- Intense laser-matter interactions: M. Nakatsutsumi *et al*, NJP (2008)

2. κ (kappa) distribution - basics

$$f_\kappa(v) = \frac{n_0}{(\pi\kappa\theta^2)^{3/2}} \frac{\Gamma(\kappa+1)}{\Gamma(\kappa-1/2)} \left(1 + \frac{v^2}{\kappa\theta^2}\right)^{-\kappa-1}$$

[Ref. Vasyliunas JGR (1968),

Baluku & Hellberg, PoP (2008)]

Effective thermal speed:

$$\theta^2 = \frac{\kappa-3/2}{\kappa}\left(\frac{2k_B T}{m}\right)$$

T : kinetic temperature

κ : spectral index

[Fig. from:

Summers & Thorne, PF (1991)]

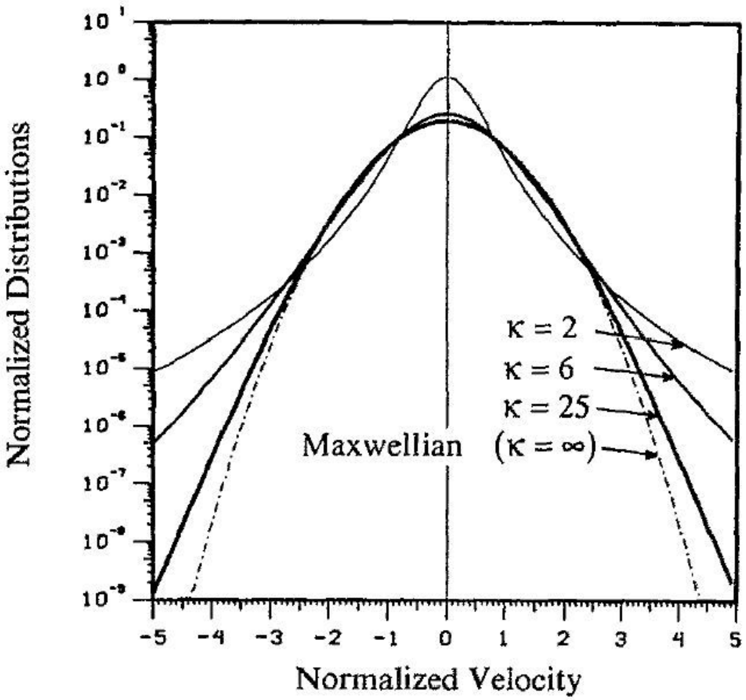


FIG. 1. Comparison of generalized Lorentzian distributions for the spectral index $\kappa = 2$, 6, and 25, with the corresponding Maxwellian distribution ($\kappa = \infty$).

Kappa (k) parameter measures deviation from thermal equilibrium

Smaller kappa value → longer superthermal df tail, harder spectrum

Infinite kappa value → Maxwellian *df, no* superthermal particles

Kappa distribution function (continued)

- First introduced to fit early Space observations [Vasyliunas, PF 1968], suggesting superthermal electrons + power-law dependence in *v*
  [Montgomery *et al*, PRL 1965]

- Kappa distribution studied in linear regime: Z_κ dispersion function
  [Summers & Thorne, PF (1991), Mace & Hellberg PoP (1995)]

- Anomalous Landau damping of ES plasma modes
  [Podesta PoP (2005); Lee PoP (2007)]

- Satellite observations; Foreshock, Magnetotail, Plasma sheet; Solar Exosphere, Solar wind, …

- Solar Corona anomalous temperature variation explained via kappa theory
  [Scudder ApJ (1992), Maksimovic *et al*, A&A (1997)]

- Cassini data, Saturn: s/thermal c/h-*e* obs. [Schippers *et al* JGR (2008)]

JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 113, A07208, doi:10.1029/2008JA013098, 2008

Multi-instrument analysis of electron populations in Saturn's magnetosphere

P. Schippers,[1] M. Blanc,[1] N. André,[2] I. Dandouras,[1] G. R. Lewis,[3] L. K. Gilbert,[3] A. M. Persoon,[4] N. Krupp,[5] D. A. Gurnett,[4] A. J. Coates,[3] S. M. Krimigis,[6] D. T. Young,[7] and M. K. Dougherty[8]

Received 15 February 2008; revised 2 May 2008; accepted 7 May 2008; published 18 July 2008.

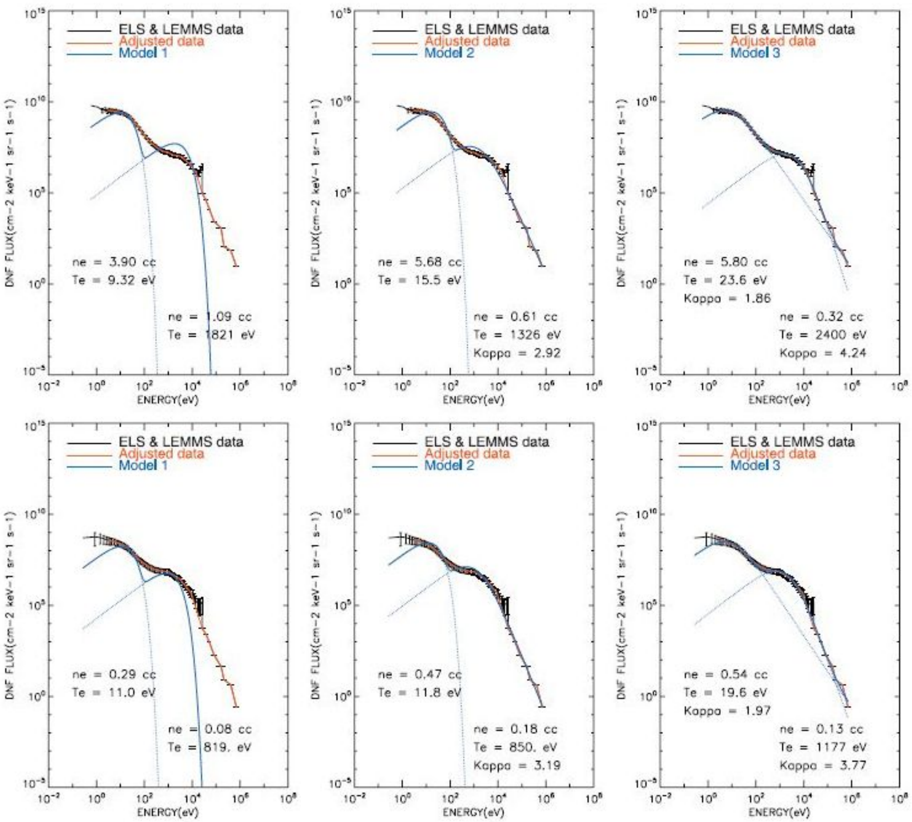


Figure 2. Composite CAPS/ELS and MIMI/LEMMS (energy channels C0-C7) spectral plots of electron intensities versus energy, observed at (top) 2200 UT (R = 9 R_s, local time 18.32 h, latitude 0.23 degrees) and at (bottom) 0727 UT (R = 12.8 R_s, local time 19.82 h, latitude 0.35 degrees) on days of year 142 and 143 of 2006 during Rev. 24, respectively. Original data are represented in black, our interpolated data are represented in red, and the results of our various models are represented in blue. (left) Model with 2 Maxwellian distributions. (middle) Model with one Maxwellian and one kappa distribution. (right) Model with two kappa distributions.

Cassini data from Saturn;

from:

Schippers *et al* JGR (2008)

Excellent 2-kappa df fit

over regions

$5.4\ R_S < R < 18\ R_S$

PRL 95, 215003 (2005) PHYSICAL REVIEW LETTERS week ending 18 NOVEMBER 2005

Self-Consistent Generation of Superthermal Electrons by Beam-Plasma Interaction

Peter H. Yoon
Institute for Physical Science and Technology, University of Maryland, College Park, Maryland 20742 , USA

Tongnyeol Rhee and Chang-Mo Ryu
Pohang University of Science and Technology (POSTECH), Pohang, Korea
(Received 11 July 2005; published 16 November 2005)

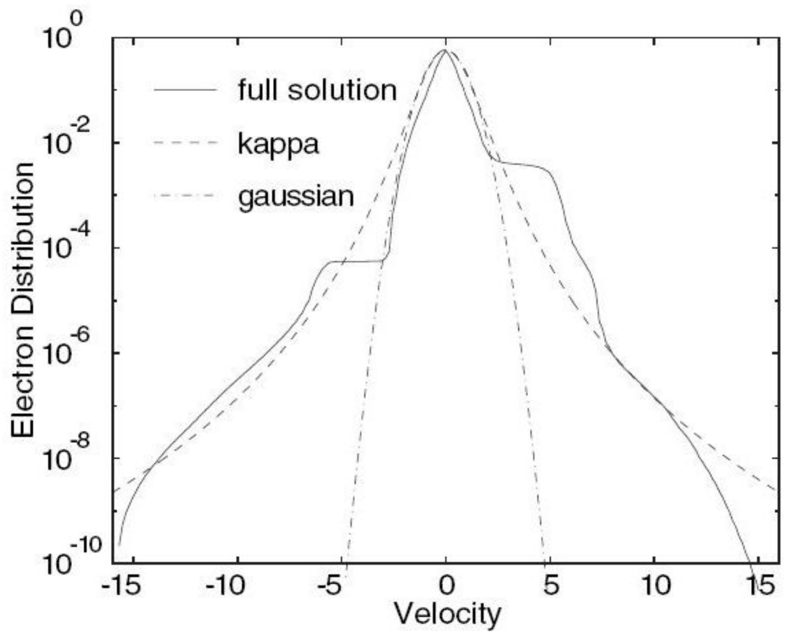


FIG. 4. Comparison of $F(u)$ at $\omega_{pe}t = 2 \times 10^4$ computed for $g = 5 \times 10^{-3}$ with κ distribution with index $\kappa = 3.5$ and the Gaussian.

Beam-plasma interactions; from:
Yoon *et al* PRL (2005)

3. (dust-) ion-acoustic excitations (fluid description) (+ superthermal *e* background)

Continuity:

$$\frac{\partial n}{\partial t} + \frac{\partial (n\,u)}{\partial x} = 0$$

Momentum:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = -\frac{\partial \phi}{\partial x}$$

Poisson Eq.:

$$\frac{\partial^2 \phi}{\partial x^2} = -n + n_e \mp Z_d n_d$$

dust - defects (stationary)

S/thermal *e* density:

$$n_e = n_{e,0}\left(1 - \frac{\phi}{\kappa - 3/2}\right)^{-\kappa + 1/2}$$

Scaling:

$$n = \frac{n_i}{n_{i0}}, \quad u = \frac{u_i}{c_s}, \quad x = \frac{x}{\lambda_D}, \quad \phi = \frac{e\phi}{k_B T_e}, \quad t = \omega_{pi} t$$

$$c_s = \left(\frac{k_B T_e}{m_i}\right)^{1/2} \qquad \omega_{pi} = \left(4\pi n_{i0} e^2 / m\right)^{1/2} \qquad \lambda_D = \left(k_B T_e / 4\pi n_{i0} e^2\right)^{1/2}$$

Stationary profile solitary waves – pseudopotential formalism

[Vedenov & Sagdeev 1961, Sagdeev 1966]

$$\frac{1}{2}\left(\frac{d\phi}{d\xi}\right)^2 + V(\phi) = 0 \qquad \xi = x - Mt$$

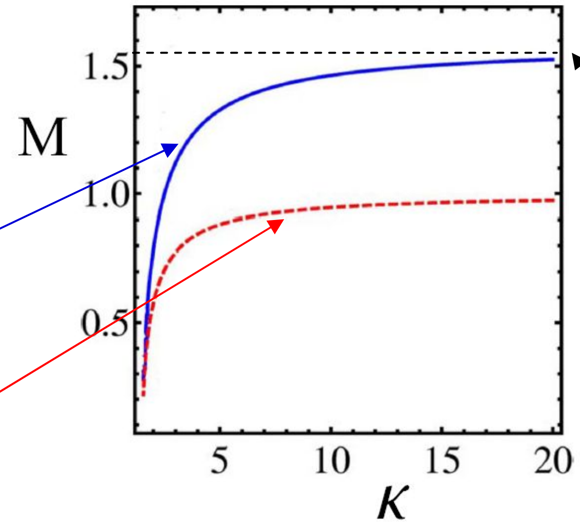
$$V(\phi) = M^2\left(1 - \sqrt{1 - \frac{2\phi}{M^2}}\right) + 1 - \left(1 - \frac{\phi}{\kappa - 3/2}\right)^{-\kappa+3/2}$$

Slower "supersonic" solitons

for smaller kappa values:

M_2: infinite compression point (choked flow)

M_1: κ-dependent sound speed



Known limit 1.58 for infinite κ

FIG. 1. (Color online) IA soliton existence domain in the parameter space of κ and Mach number, M. Solitons may be supported in the region between the two curves. The lower, dashed curve represents the minimum (soliton) condition, M_1, and the upper, solid curve the infinite compression limit, M_2.

Increased soliton amplitude for higher speed M (for given κ):

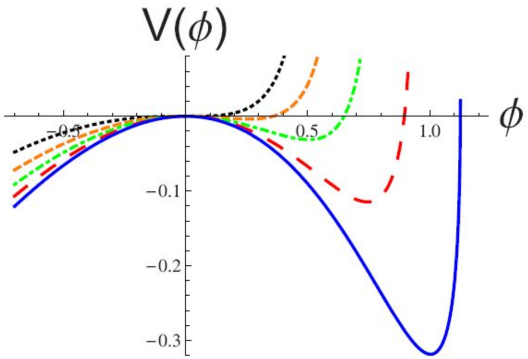


FIG. 2. (Color online) Variation of $V(\phi)$ for $\kappa=16$ and different values of Mach number, M. From top to bottom: Dotted curve: $M=0.97$; dashed curve: $M=1.10$; dotted-dashed curve: $M=1.23$; long-dashed curve: $M=1.36$; and solid curve: $M=1.50$.

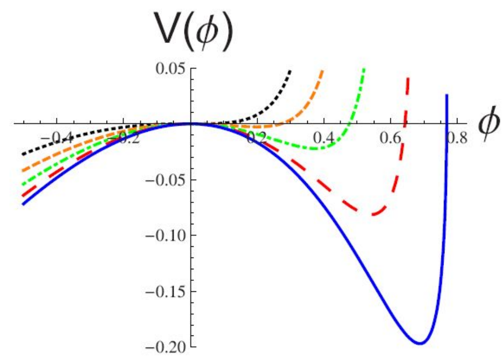


FIG. 3. (Color online) Variation of $V(\phi)$ for $\kappa=4$ and different values of Mach number, M. From top to bottom: Dotted curve: $M=0.85$; dashed curve: $M=0.95$; dotted-dashed curve: $M=1.05$; long-dashed curve: $M=1.15$; and solid curve: $M=1.24$.

and…

increased soliton amplitude for smaller kappa values (for fixed M) by a factor ~ 1.1 – 5: see bottom left

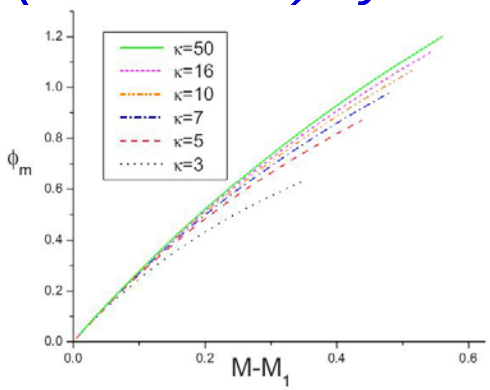


FIG. 4. (Color online) Variation of ϕ_m with $M-M_1$ for different values of κ. The dotted curve corresponds to $\kappa=3$, the dashed curve to $\kappa=5$, the dotted-dashed curve to $\kappa=7$, the dotted-dotted dashed curve to $\kappa=10$, the short-dashed curve to $\kappa=16$, and the solid curve to $\kappa=50$.

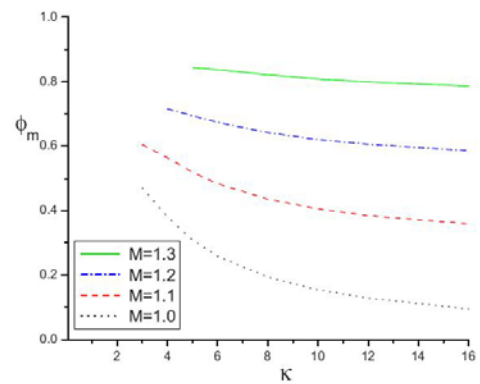


FIG. 8. (Color online) Variation of ϕ_m with κ for different values of the Mach number, M. The dotted curve corresponds to $M=1.0$; the dashed curve to $M=1.1$; the dotted-dashed curve to $M=1.2$; and the solid curve to $M=1.3$.

From: N S Saini, I Kourakis and MA Hellberg, Phys. Plasmas **16** 062903 (2009)

4. Nonlinear self-modulation of ES/EM plasma modes

- Nonlinear self-modulation of ES plasma wavepackets: generic nonlinear mechanism, involving *harmonic generation*, *modulational instability*, *envelope soliton* generation, …

- A Maxwellian background was generally considered for:
  - *Ion-acoustic waves, e-i* plasmas, cold model [Kakutani & Sugimoto, PF (1974)]
  - IAWs, warm model [Durrani et al., PF (1979)]
  - Multi-ion plasma [Chabra & Sharma, PF (1986)]
  - Electron acoustic waves [Kourakis & Shukla, PRE (2004)]
  - Dusty plasmas

    [Kourakis & Shukla, PoP (2003); JPA (2003); Phys. Scr. (2004); NPG (2005)]

- Recent study of ES soliton modes in kappa-distributed plasmas:
  - Dust-acoustic mode in dusty plasmas [Saini & Kourakis, PoP (2008)]
  - IAWs [Saini *et al*, PoP (2009); Sultana *et al* (in preparation)]

κ–dependent charge balance: expansion near equilibrium

Normalized electron density:

$$\left(1-\frac{\phi}{\kappa-3/2}\right)^{-\kappa+1/2} \cong 1+c_1\phi+c_2\phi^2+c_3\phi^3+\ldots$$

Superthermality traced via the κ–dependent coefficients:

$$c_1=\mu\frac{2\kappa-1}{2\kappa-3},\quad c_2=\mu\frac{4\kappa^2-1}{2(2\kappa-3)^2},\quad c_3=\mu\frac{(4\kappa^2-1)(2\kappa+3)}{6(2\kappa-3)^3}$$

Maxwellian *e-i* plasma limit (infinite κ): $c_n=1/n! \quad (n=1,2,3\ldots)$

The parameter μ measures the dust concentration:

$$\mu=1+s\frac{Z_d\, n_d}{Z_i\, n_{i,0}},\quad s=\pm1 \quad \text{(for +/- dust charge sign)}$$

Multiple scales perturbation technique

State variables $S = (n, u, \phi)$ expanded near $S^{(0)} = (1,0,0)$

$$S = S^{(0)} + \sum_{m=1}^{\infty} \varepsilon^m S^{(m)}; \quad m = 1, 2, 3.......$$

Harmonic expansion

$$S^{(m)} = \sum_{l=-m}^{m} S_l^{(m)}(X_m, T_m) \ \exp[il(kx - \omega t)]$$

Space/time stretching: $X_m = \varepsilon^m x, \quad T_m = \varepsilon^m t$

Solution obtained to 2nd order (0th, 1st, 2nd harmonics):

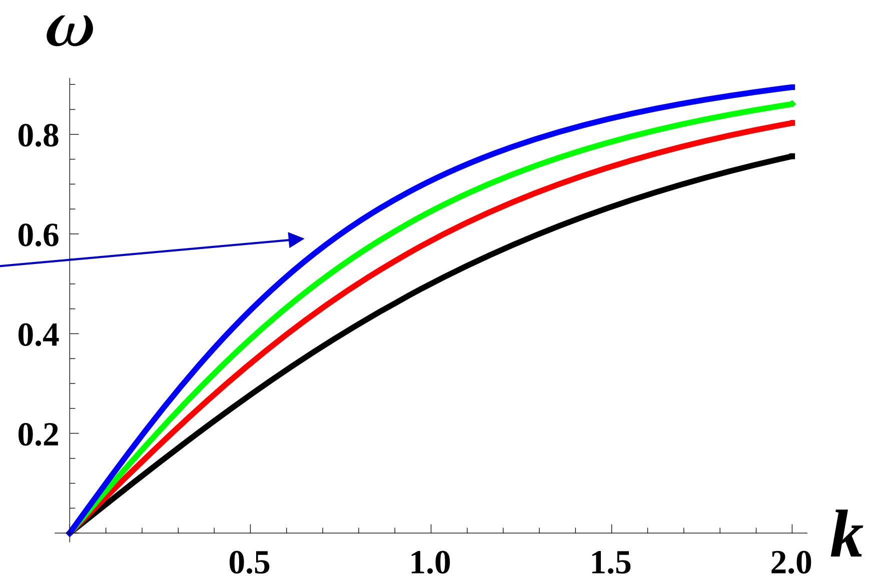
$$S \cong \varepsilon\, S_1^{(1)} e^{i(kx-\omega t)} + \varepsilon^2 \left[S_2^{(0)} + S_2^{(2)} \ e^{2i(kx-\omega t)} \right] + O(\varepsilon^3)$$

Linear regime (*l=m=1*): Dispersion relation

$$n_1^{(1)} = \left(k^2 + c_1\right)\phi_1^{(1)} \qquad u_1^{(1)} = \frac{k}{\omega}\,\phi_1^{(1)}$$

$$\omega^2 = \frac{k^2}{k^2 + c_1} \qquad c_1 = \frac{2\kappa - 1}{2\kappa - 3}$$

κ–modified Debye screening



Blue: κ=∞ (Maxwellian)
Green: κ=4
Red: κ=2.6
Black: κ=2

[Perfect agreement with Bryant JPP (1996)]

Non-linear Schrödinger Equation (NLSE)

Solution obtained to ~ε^3 :

$$\phi \cong \varepsilon\, \psi\, e^{i(kx-\omega t)} + \varepsilon^2 \left[\phi_2^{(0)} + \phi_2^{(2)} e^{2i(kx-\omega t)}\right] + O(\varepsilon^3), \qquad \psi = \phi_1^1$$

The potential amplitude $\phi_1^{(1)} \equiv \psi(\zeta, \tau)$ satisfies:

$$i\frac{\partial \psi}{\partial \tau} + P\frac{\partial^2 \psi}{\partial \zeta^2} + Q\,|\psi|^2 \psi = 0$$

Slow envelope variables: $\zeta = \varepsilon(x - v_g t)$, $\tau = \varepsilon^2 t$

Dispersion coefficient P: $P = -\frac{3c_1}{2}\frac{\omega^5}{k^4} = \frac{\omega''(k)}{2}$

Nonlinearity coefficient Q: $Q = \ldots = Q(k; \kappa; \ldots)$

Modulational (in)stability analysis

Perturbing around a harmonic amplitude solution, we obtain the *nonlinear (amplitude) dispersion relation*:

$$\hat{\omega}^2 = P\hat{k}^2 \left(P\hat{k}^2 - 2Q \left| \hat{\psi}_{1,0} \right|^2 \right)$$

P/Q < 0, plane wave is modulationally stable

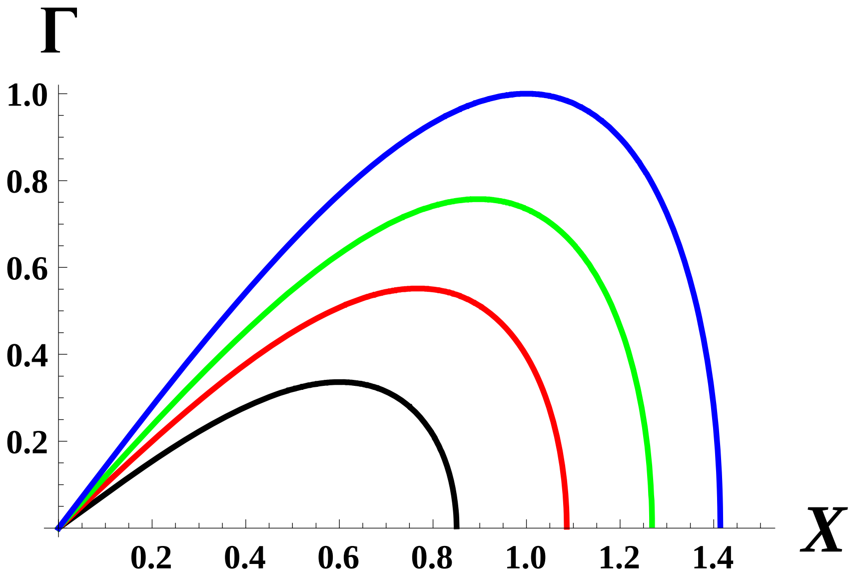
P/Q>0, unstable, modulational instability threshold:

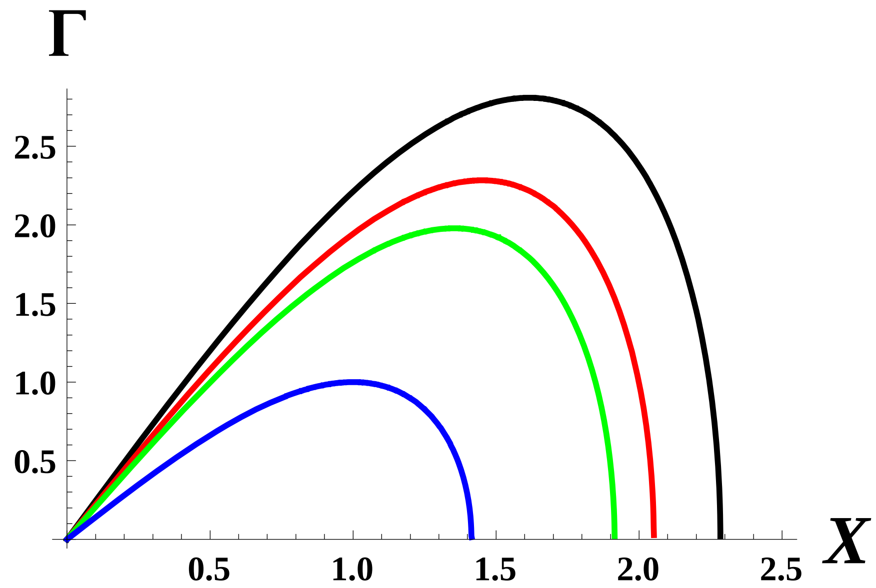
$$\hat{k} < k_{cr} \equiv \sqrt{\tfrac{2Q}{P}} \left| \hat{\psi}_{1,0} \right|$$

Maximum instability growth rate: $\Gamma_{\max} = Q \left| \hat{\psi}_{1,0} \right|^2$

(*k* dependent; see next slide)

MI growth rate *Γ* vs perturbation wavenumber *X* *(normalized)*

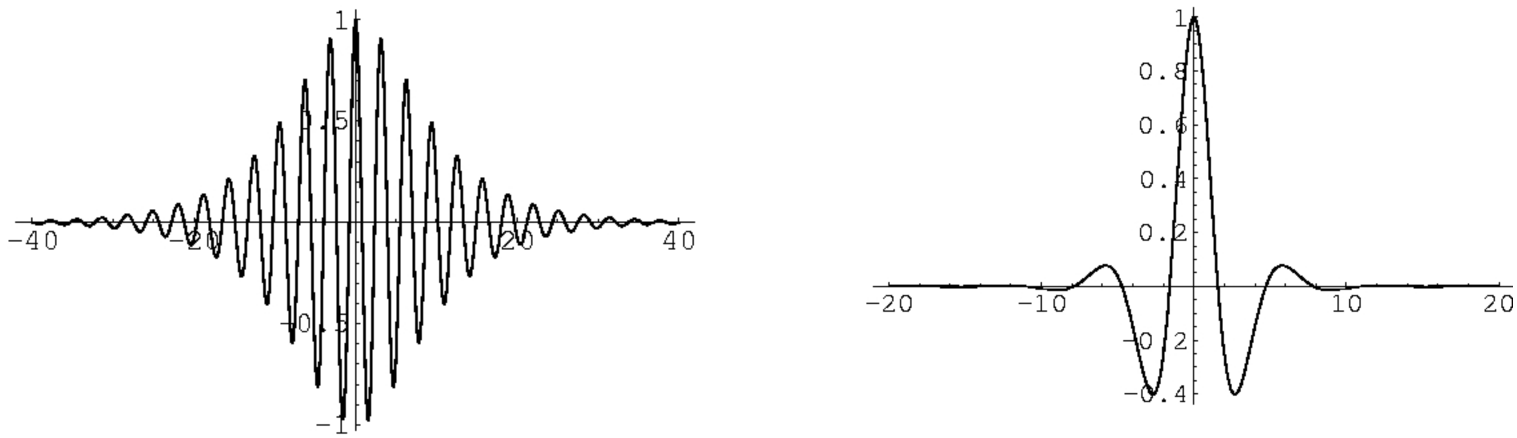




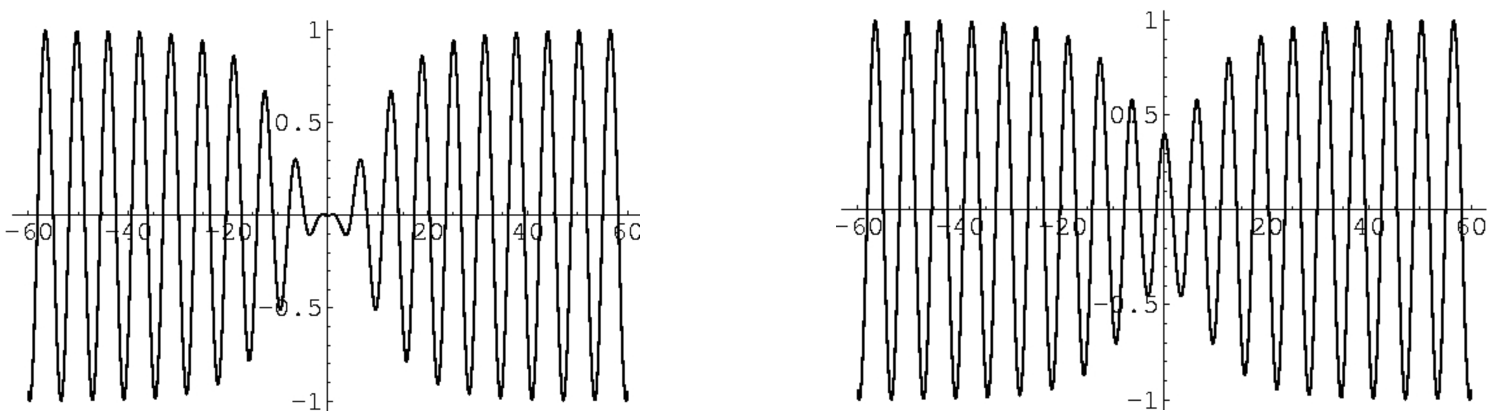
Blue: κ=∞ (Maxwellian)
Green: κ=1.602
Red: κ=1.601
Black: κ=1.6

Black: κ=2.5
Red: κ=3
Green: κ=3.5
Blue: κ=∞ (Maxwellian)

Envelope solitons



Bright-type envelope solitons (for P/Q>0)

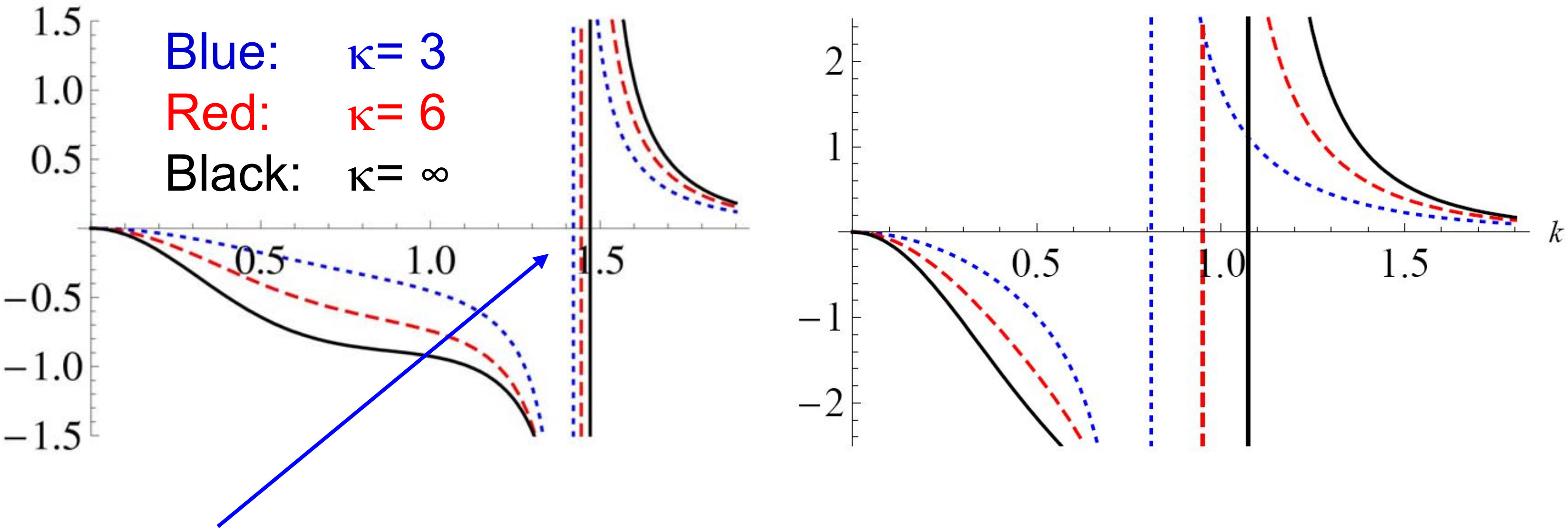


Dark (black/grey) type envelope solitons (for P/Q<0)

Parametric investigation of soliton characteristics 1

$$L\psi_0 = (P/Q)^{1/2}$$

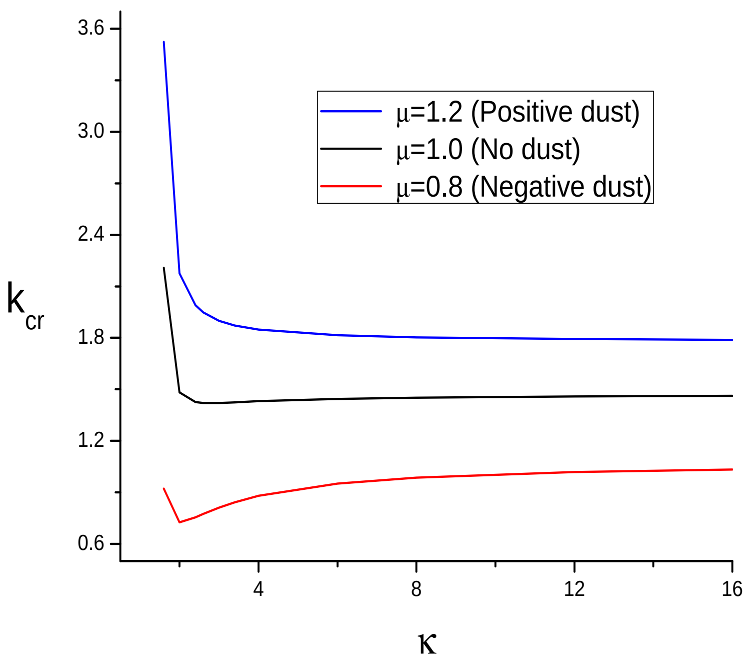
- Superthermality leads to a decrease in envelope width *L* (for given amplitude ψ): *enhanced envelope localization!*
- Lower instability threshold k_{cr} with smaller kappa
- Both effects increased with negative dust



- *Agreement (1.47) with Kakutani & Sugimoto PF,1974 (Maxwellian e-i plasma)*

Parametric investigation of soliton characteristics 2

- Modified instability threshold k_{cr} with kappa *and* with dust
- Modulational instability (MI) occurs at *longer wavelengths, in the presence of negative dust*
- MI less relevant with *positive dust*: stable wavepackets
- Remark: Landau damping omitted (yet less relevant for +d)



Conclusions

- Accelerated electrons are present in most plasmas

- Superthermal plasmas are efficiently modelled by a kappa *df*

- Increased superthermality (smaller κ) leads to:
  - A modification in the characteristics of ES solitary waves
  - Enhanced modulational instability of wavepackets
  - Stronger localization of energy stored in envelope solitons

- For infinite kappa, the Maxwellian limit is recovered.

- Results to be confirmed by Space observations or/and in experiments (+ M Borghesi, laser-plasma interactions).

- Minus: *Landau damping* neglected (fluid description): to be included.

Thank You!

Local team @QUB: Naresh Pal Saini, Ashutosh Sharma, Sharmin Sultana
Ashkbiz Danehkar

In collaboration with: Manfred Hellberg & Thomas Baluku (UKZN, Durban, S Africa)

Special thanks to: Frank Verheest (U. Gent, Belgium)

Acknowledgments: UK EPSRC funding via CPP/QUB

Material from:

N S Saini, I Kourakis and MA Hellberg, *Phys. Plasmas* **16** 062903 (2009)
M. A. Hellberg, T.K. Baluku, R.L. Mace, N. S. Saini and I. Kourakis,
Comment, submitted to *Phys. Plasmas* (2009)
N S Saini, I Kourakis and MA Hellberg, in preparation (2009)
S Sultana, I Kourakis, MA Hellberg & M Borghesi, in preparation (2009)
N S Saini, I Kourakis and MA Hellberg, *Phys. Plasmas* **15** 123701 (2008)

Slides at `www.kourakis.eu`