The turbulent bremsstrahlung (nonlinear plasma-maser) effect

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Outline

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Background

- In a weakly turbulent plasma, ***resonant*** and ***nonresonant*** interactions of waves with particles occur; the energy is transferred among the participants
- First theories on the onset of turbulence (wave instability, growth, and saturation) were based on the lowest order effects and included ***wave-particle resonance*** of Landau type and/or ***wave-wave resonance*** involving exact matching of the wave frequencies and wave vectors
- Terms describing ***nonresonant*** effects were usually ignored (relatively small magnitudes). However, such higher order effects can become dominant when the resonant effects are prohibited or saturated early
- The ***turbulent bremsstrahlung***, or ***nonlinear plasma-maser***, effect was first discussed more than 30 years ago by Tsytovich, Stenflo and Wilhelmsson [Phys. Scripta 1975] and by Nambu [PRL 1975]. This involves the nonresonant interaction of plasma particles with a pair of plasma modes of large frequency difference; the wave energy is converted into particle energy and/or back

The physics

- In the lowest order: the wave-particle resonance is $\omega - \mathbf{k} \cdot \mathbf{v} = 0$, where ω is the wave frequency, $\mathbf{k}$ is the wave vector, and $\mathbf{v}$ is the particle velocity
- In the next order: there are nonlinear ***three-wave*** interactions, with the wave frequency and wave vector matching $\omega_1 + \omega_2 = \omega_3$, $\mathbf{k}_1 + \mathbf{k}_2 = \mathbf{k}_3$, with 1, 2, 3 denoting the three waves involved. There can also be resonance of two waves at the ***beat frequency*** with plasma particles; this nonlinear scattering wave-particle resonance satisfies $\omega_1 - \omega_2 - (\mathbf{k}_1 - \mathbf{k}_2) \cdot \mathbf{v} = 0$
- The ***nonlinear plasma-maser*** (or ***turbulent bremsstrahlung***) process involves one wave (ω, $\mathbf{k}$) that satisfies the linear wave-particle resonance condition, and another wave (Ω, $\mathbf{K}$) that satisfies neither the linear nor the nonlinear resonance condition: $\Omega - \mathbf{K} \cdot \mathbf{v} \neq 0$ and $\Omega - \omega - (\mathbf{K} - \mathbf{k}) \cdot \mathbf{v} \neq 0$
- In the plasma maser, the energy of the resonant mode (ω, $\mathbf{k}$) is transferred to the nonresonant mode (Ω, $\mathbf{K}$) and plasma particles; the frequency difference $|\Omega - \omega|$ can be quite large. The nonresonant wave-particle interactions are important for the global energy and momentum conservation, and can affect the overall evolution of the system

Perturbations of the distribution function

Assume that the perturbation function of plasma particles of sort α (= e,i for simplicity) can be decomposed into a regular and turbulent parts

$$f_\alpha(t,\mathbf{r};\mathbf{p}) = \Phi_\alpha + \delta f_\alpha(t,\mathbf{r};\mathbf{p}), \quad \Phi_\alpha = \left\langle f_\alpha(t,\mathbf{r};\mathbf{p})\right\rangle$$

The statistically averaged distribution Φ_α can evolve slowly in time:
$\Phi_\alpha = \Phi_\alpha(t;\mathbf{p})$
Perturbations of the distribution function because of (turbulent wave) fields $\mathbf{E}(t,\mathbf{r})$ are:

$$\delta f_\alpha(t,\mathbf{r};\mathbf{p}) = \sum_j f_\alpha^{(j)}(t,\mathbf{r};\mathbf{p}), \qquad f_\alpha^{(j)}(t,\mathbf{r};\mathbf{p}) \sim \mathbf{E}^j(t,\mathbf{r})$$

We have for the regular part

$$\frac{\partial \Phi_\alpha}{\partial t} + \mathbf{v}\cdot\frac{\partial \Phi_\alpha}{\partial \mathbf{r}_\alpha} = -\left\langle q_\alpha \mathbf{E}(t,\mathbf{r})\cdot\frac{\partial \delta f_\alpha}{\partial \mathbf{p}}\right\rangle$$

and in the lowest order for the turbulent part

$$\frac{\partial f_\alpha^{(1)}}{\partial t} + \mathbf{v}\cdot\frac{\partial f_\alpha^{(1)}}{\partial \mathbf{r}_\alpha} + q_\alpha \mathbf{E}(t,\mathbf{r})\cdot\frac{\partial \Phi_\alpha}{\partial \mathbf{p}} = 0$$

Higher-order perturbations

For Fourier-components we have in the lowest order

$$f^{(1)}_{\alpha,\omega\mathbf{k}} = \frac{-\mathrm{i}q_\alpha}{\omega - \mathbf{k}\cdot\mathbf{v}}\left(\mathbf{E}_{\omega\mathbf{k}}\cdot\frac{\partial\Phi_\alpha}{\partial\mathbf{p}}\right)$$

For higher-order perturbations

$$f^{(j+1)}_{\alpha,\omega\mathbf{k}} = \frac{-\mathrm{i}q_\alpha}{\omega - \mathbf{k}\cdot\mathbf{v}}\int d_{12}\left(\mathbf{E}_{\omega_1\mathbf{k}_1}\cdot\frac{\partial f^{(j)}_{\alpha,\omega_2\mathbf{k}_2}}{\partial\mathbf{p}} - \left\langle\mathbf{E}_{\omega_1\mathbf{k}_1}\cdot\frac{\partial f^{(j)}_{\alpha,\omega_2\mathbf{k}_2}}{\partial\mathbf{p}}\right\rangle\right)$$

Here, $d_{12} = d\omega_1 d\omega_2\delta(\omega - \omega_1 - \omega_2)\, d\mathbf{k}_1 d\mathbf{k}_2\delta(\mathbf{k} - \mathbf{k}_1 - \mathbf{k}_2)$. For example,

$$f^{(3)}_{\alpha,\omega\mathbf{k}} = \frac{-\mathrm{i}q_\alpha}{\omega - \mathbf{k}\cdot\mathbf{v}}\int d_{12}\left(\mathbf{E}_{\omega_1\mathbf{k}_1}\cdot\frac{\partial f^{(2)}_{\alpha,\omega_2\mathbf{k}_2}}{\partial\mathbf{p}} - \left\langle\mathbf{E}_{\omega_1\mathbf{k}_1}\cdot\frac{\partial f^{(2)}_{\alpha,\omega_2\mathbf{k}_2}}{\partial\mathbf{p}}\right\rangle\right)$$

$$= \frac{-\mathrm{i}q_\alpha}{\omega - \mathbf{k}\cdot\mathbf{v}}\int d_{12}\left[\left(\mathbf{E}_{\omega_1\mathbf{k}_1}\cdot\frac{\partial}{\partial\mathbf{p}}\right)\frac{-\mathrm{i}q_\alpha}{\omega_2 - \mathbf{k}_2\cdot\mathbf{v}}\int d_{34}\mathbf{E}_{\omega_3\mathbf{k}_3}\cdot\frac{\partial f^{(1)}_{\alpha,\omega_4\mathbf{k}_4}}{\partial\mathbf{p}} - \langle\ldots\rangle\right]$$

Linear and nonlinear plasma responses

From Poisson's equation for longitudinal turbulent waves $\mathbf{E}_{\Omega\mathbf{K}}=\mathbf{K}E_{\Omega\mathbf{K}}/K$ and $\mathbf{E}_{\omega\mathbf{k}}=\mathbf{k}E_{\omega\mathbf{k}}/k$, with

$$\left\langle \mathbf{E}_{\Omega\mathbf{K}}\mathbf{E}_{\Omega_1\mathbf{K}_1}\right\rangle = -\left|E\right|^2_{\Omega\mathbf{K}}\,\delta\left(\Omega+\Omega_1\right)\delta\left(\mathbf{K}+\mathbf{K}_1\right),\quad \left\langle \mathbf{E}_{\omega\mathbf{k}}\mathbf{E}_{\omega_1\mathbf{k}_1}\right\rangle = -\left|E\right|^2_{\omega\mathbf{k}}\,\delta\left(\omega+\omega_1\right)\delta\left(\mathbf{k}+\mathbf{k}_1\right)$$

we obtain

$$\left(\varepsilon^{l}_{\Omega\mathbf{K}}+\varepsilon^{nl(3)}_{\Omega\mathbf{K}}\right)\left|E\right|^2_{\Omega\mathbf{K}}=0$$

Linear dielectric plasma permittivity

$$\varepsilon^{l}_{\Omega\mathbf{K}}=1+\frac{4\pi e^2}{K^2}\int\frac{1}{\Omega-\mathbf{K}\cdot\mathbf{v}}\left(\mathbf{K}\cdot\frac{\partial\Phi}{\partial\mathbf{p}}\right)d\mathbf{p}$$

Nonlinear third-order response

$$\varepsilon^{nl(3)}_{\omega\mathbf{k}}=-\frac{4\pi e^4}{K^2k^2}\int d\mathbf{p}\left|E\right|^2_{\omega\mathbf{k}}\,d\omega d\mathbf{k}\,\frac{1}{\Omega-\mathbf{K}\cdot\mathbf{v}}\left(\mathbf{k}\cdot\frac{\partial}{\partial\mathbf{p}}\right)\frac{1}{\Omega-\omega-\left(\mathbf{K}-\mathbf{k}\right)\cdot\mathbf{v}}$$
$$\times\left[\left(\mathbf{K}\cdot\frac{\partial}{\partial\mathbf{p}}\right)\frac{1}{\omega-\mathbf{k}\cdot\mathbf{v}-i0}\left(\mathbf{k}\cdot\frac{\partial\Phi}{\partial\mathbf{p}}\right)+\left(\mathbf{k}\cdot\frac{\partial}{\partial\mathbf{p}}\right)\frac{1}{\Omega-\mathbf{K}\cdot\mathbf{v}}\left(\mathbf{K}\cdot\frac{\partial\Phi}{\partial\mathbf{p}}\right)\right]$$

Imaginary part of the nonlinear response

The imaginary part of the third-order response is

$$\mathrm{Im}\,\varepsilon^{nl(3)}_{\omega\mathbf{k}} = \frac{12\pi^2 e^4}{m_e^2}\int d\mathbf{p}\,\frac{\mathbf{K}\cdot\mathbf{k}}{k^2}\left|E\right|^2_{\omega\mathbf{k}}\,d\omega d\mathbf{k}\,\frac{\delta\left(\omega-\mathbf{k}\cdot\mathbf{v}\right)}{\left(\Omega-\mathbf{K}\cdot\mathbf{v}\right)^4}\left(\mathbf{k}\cdot\frac{\partial\Phi}{\partial\mathbf{p}}\right)$$

It is closely associated to resonant, or quasilinear, particle heating:

$$\mathrm{Im}\,\varepsilon^{nl(3)}_{\omega\mathbf{k}} = -\frac{4\pi e^2}{m_e}\int d\mathbf{p}\,\frac{1}{\left(\Omega-\mathbf{K}\cdot\mathbf{v}\right)^3}\left(\mathbf{k}\cdot\frac{\partial}{\partial\mathbf{p}}\right)\frac{\pi e^2}{k^2}\delta\left(\omega-\mathbf{k}\cdot\mathbf{v}\right)\left(\mathbf{k}\cdot\frac{\partial\Phi}{\partial\mathbf{p}}\right)\left|E\right|^2_{\omega\mathbf{k}}\,d\omega d\mathbf{k}$$

$$= -\frac{4\pi e^2}{m_e}\int d\mathbf{p}\,\frac{1}{\left(\Omega-\mathbf{K}\cdot\mathbf{v}\right)^3}\frac{\partial}{\partial p_i}D^{ql}_{ij}\frac{\partial\Phi}{\partial p_i}$$

The nonresonant fields can affect the degree of the resonant particle heating or cooling. Since energy is continuously flowing between the resonant waves and the resonant particles, the turbulent bremsstrahlung mechanism provides a channel by which a part of the resonant energy is distributed to the nonresonant waves and particles

Nonstationary plasma systems

Quasilinear evolution: the system is (weakly) nonstationary

$$\frac{d\Phi}{dt} = \frac{\partial}{\partial p_i} D_{ij}^{ql} \frac{\partial \Phi}{\partial p_i} \equiv \hat{I}^R \Phi$$

Introduce the number of quanta of nonresonant waves with the energy W:

$$W = \int d\mathbf{K} \Omega_{\mathbf{K}}(t) N_{\mathbf{K}}(t)$$

Temporal evolution of $N_{\mathbf{K}}(t)$~ $|\mathrm{E}|^2$ obeys the equation

$$\frac{dN_{\mathbf{K}}(t)}{dt} = 2\Gamma_{\mathbf{K}} N_{\mathbf{K}}(t)$$

In the nonstationary system

$$\Gamma_{\mathbf{K}} = -\left(\frac{\partial \varepsilon_{\Omega\mathbf{K}}}{\partial \Omega}\right)^{-1} \left[\mathrm{Im}\varepsilon_{\Omega\mathbf{K}}^{(3)} + \frac{1}{2}\frac{\partial^2 \varepsilon_{\Omega\mathbf{K}}}{\partial \Omega \partial t}\right]_{\Omega=\Omega_{\mathbf{K}}}$$

Closed and open plasma systems

When the nonstationarity is due to quasilinear evolution

$$\frac{1}{2}\frac{\partial^2 \varepsilon_{\Omega\mathbf{K}}}{\partial\Omega\partial t} = -\frac{2\pi e^2}{K^2}\int d\mathbf{p}\frac{1}{(\Omega - \mathbf{K}\cdot\mathbf{v})^2}\left(\mathbf{K}\cdot\frac{\partial}{\partial\mathbf{p}}\right)\frac{\partial\Phi}{\partial t}$$

$$= \frac{4\pi e^2}{m_e}\int d\mathbf{p}\frac{1}{(\Omega - \mathbf{K}\cdot\mathbf{v})^3}\frac{\partial}{\partial p_i}D_{ij}\frac{\partial}{\partial p_j} = -\mathrm{Im}\,\varepsilon_{\Omega\mathbf{K}}^{nl(3)}$$

- Thus, in a ***closed plasma system***, when particle distribution evolves quasilinearly due to interactions with resonant waves, the ***number of nonresonant quanta is conserved*** and there is no conversion of the resonant energy to the nonresonant waves
- This is the result of high degree of system symmetry and the absence of external sources and sinks. A violation can trigger the onset of energy flow in the plasma-maser process such as in open and/or nonstationary plasmas
- Furthermore, this energy conversion process can be strongly modified by anisotropy even in a closed system

The effect on particle distributions

The particle distribution function can be renormalized, so that the contributions of the linear and nonlinear wave-particle interactions as well as possible nonstationarity are taken into consideration

$$\frac{d\tilde{\Phi}}{dt} = \left(1 - \gamma_{\mathbf{p}} \hat{\tilde{I}}^{N} \gamma_{\mathbf{p}}^{-1}\right) \hat{\tilde{I}}^{R} \left(1 + \gamma_{\mathbf{p}} \hat{\tilde{I}}^{N} \gamma_{\mathbf{p}}^{-1}\right) \tilde{\Phi}; \quad \gamma_{\mathbf{p}} = \int \frac{d\mathbf{k}}{k} \left(\mathbf{k} \cdot \frac{\partial}{\partial p}\right)$$

The renormalized distribution is $\tilde{\Phi} = \left(1 - \hat{I}^{N}\right) \Phi$

The nonresonant operator is

$$\hat{I}^{N} = \frac{e^2}{2K^2} \int d\Omega d\mathbf{K} \left(\mathbf{K} \cdot \frac{\partial}{\partial \mathbf{p}}\right) \frac{|E|^2_{\Omega\mathbf{K}}}{(\Omega - \mathbf{K} \cdot \mathbf{v})^2} \left(\mathbf{K} \cdot \frac{\partial}{\partial \mathbf{p}}\right)$$

The renormalized operators are

$$\hat{\tilde{I}}^{N} = \hat{I}^{N} - \Gamma_{\mathbf{p}} \hat{I}^{N} \Gamma_{\mathbf{p}}^{-1} = -\frac{e^2}{m_e} \int d\Omega d\mathbf{K} \left(\mathbf{K} \cdot \frac{\partial}{\partial \mathbf{p}}\right) \frac{|E|^2_{\Omega\mathbf{K}}}{(\Omega - \mathbf{K} \cdot \mathbf{v})^3}; \quad \Gamma_{\mathbf{p}} = \int \frac{d\mathbf{K}}{K} \left(\mathbf{K} \cdot \frac{\partial}{\partial p}\right)$$

$$\hat{\tilde{I}}^{R} = \hat{I}^{R} + \frac{\pi e^2}{m_e^2} \int d\omega d\mathbf{k} \left(\mathbf{k} \cdot \frac{\partial}{\partial \mathbf{p}}\right) \int d\Omega d\mathbf{K} \frac{\left|\mathbf{E}_{\omega\mathbf{k}} \cdot \mathbf{E}_{\Omega\mathbf{K}}\right|^2}{(\Omega - \mathbf{K} \cdot \mathbf{v})^4} \delta(\omega - \mathbf{k} \cdot \mathbf{v}) \left(\mathbf{k} \cdot \frac{\partial}{\partial \mathbf{p}}\right)$$

A qualitative picture: quasilinear interaction

- Assume randomly distributed localized regions (of average size l and separation L) of electric fields with constant magnitude E_0 but randomly distributed signs (the average field of the entire system is zero)
- A particle moving through such a system will encounter the positive and negative field regions with the same probability
- Assume a positive particle charge (q>0) and positive initial particle velocity (v(0)>0). In a positive field region, it gains on average an amount of energy qE_0l, and in a negative field region it loses the same energy. That is, when the particle passes a positive field region, its velocity is increased by qE_0l/mv, and it will need a shorter time (t_+) to reach the next region containing electric field. In the opposite case, its traveling time (t_-) will be increased. The average change of the particle energy is

$$\frac{d\langle\mathcal{E}\rangle}{dt} = \left\langle \frac{qE_0l}{t_+} - \frac{qE_0l}{t_-} \right\rangle = \left\langle \frac{qE_0l}{L}\left(v + \frac{qE_0l}{mv}\right) - \frac{qE_0l}{L}\left(v - \frac{qE_0l}{mv}\right) \right\rangle$$

$$= \left\langle \frac{2q^2E_0^2l^2}{mvL} \right\rangle = \frac{2q^2L}{mv}\overline{E}_0^2$$

A qualitative picture: plasma-maser interaction

• In addition to the resonant field regions, include in the system also a nonresonant (higher frequency) oscillating electric field. For simplicity examine the case when there is an integer multiple of the wavelength of the nonresonant field, so that over the length l the particle will have an odd number of oscillations in the latter field

• When the particle velocity is increased it may not have the same number of oscillations (over the length l). When the particle leaves the resonant field region its energy will be slightly more (or less, depending on the sign of the resonant field) than $qE_0 l$. In a sufficiently weak nonresonant wave field this difference is proportional to the field strength E. That is, $\Delta\mathcal{E}_{\pm} = qE_0 l(1 \pm \alpha E)$, where α is a constant. The average energy change is

$$\frac{d\left\langle \mathcal{E} \right\rangle}{dt} = \left\langle \frac{\Delta\mathcal{E}_+}{L}\left(v + \frac{\Delta\mathcal{E}_+}{mv} \right) - \frac{\Delta\mathcal{E}_-}{L}\left(v - \frac{\Delta\mathcal{E}_-}{mv} \right) \right\rangle$$

$$= \frac{2q^2 L}{mv}\bar{E}_0^2\left(1 + \alpha^2\left\langle E^2 \right\rangle \right)$$

The Fermi-like pinball model

• Fermi [Phys. Rev. 1949] proposed that the motion of a pinball bouncing between ***a fixed and an oscillating wall*** can be used as a model for the acceleration of cosmic rays to ultra-high energies

• In the corresponding mechanical model the particle receives kicks not from the distributed electric fields but from its bouncing (mirror reflecting) between two walls

• In the classical Fermi-Ulam problem only ***one wall moves*** with the amplitude A and frequency ω

• The corresponding Fermi-like mechanical model for the plasma-maser process consists of many noninteracting particles bouncing elastically between ***two oscillating walls*** with the amplitude $A_{1,2}$ and frequency $\omega_{1,2}$

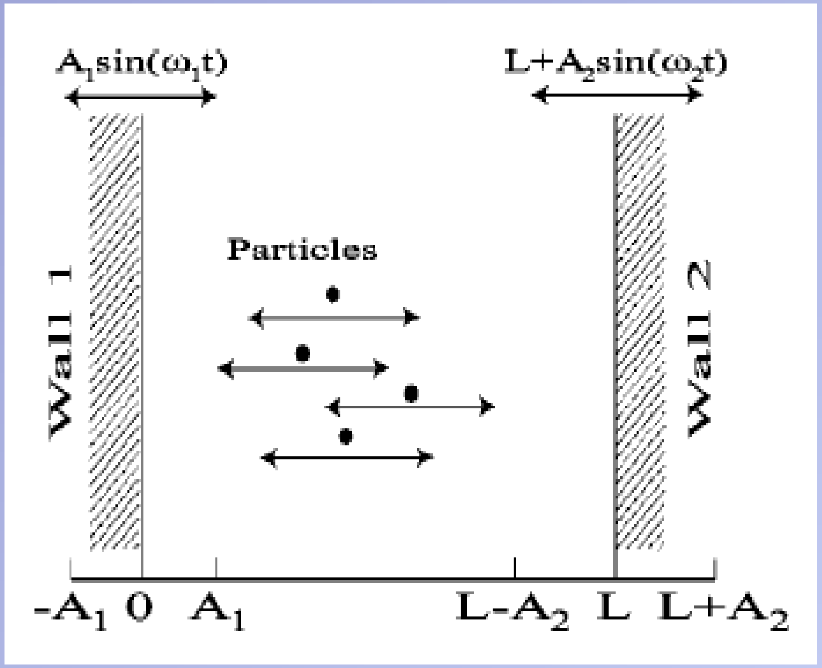
• The walls act as energy and momentum sources and sinks for the particles, analogous to the wave fields in a weakly turbulent plasma. The oscillations, in particular the amplitude and frequency characteristics, of the walls then determine the dynamics and distribution of the particles

The Fermi-like pinball model

• The mapping for the two-wall problem: a particle located at X_n at t_n and moving towards the right wall 2 will have its velocity mapped to

$$u_{n+1} = u_n - AF_2'\left(\Omega\Theta_n^{c2}\right)$$

$$\Theta_{n+1} = \Theta_n + \frac{2\pi M - x_n + AF_2'\left(\Omega\Theta_n^{c2}\right)}{2u_n} + \frac{2\pi M - x_n + AF_2'\left(\Omega\Theta_n^{c2}\right)}{2u_{n+1}}$$



Here $A = A_2/A_1$, $\Omega = \omega_2/\omega_1$, the prime denotes derivative with respect to the argument. The other notations follow that used in the Fermi-Ulam problem: $\Theta_n = \omega_1 t_n$, $u_n = v_n/2A_1\omega_1$, $M = L/2\pi A_1$, $x_n = X_n/A_1$, $F_2(\Theta) = \sin(\Theta)$, and $\Theta_n^{c2} = \Theta_n + [2\pi M - x_n + AF_2(\Omega\Theta_n^{c2})]/2u_n$ is the time when the particle collides with wall 2. After the subsequent collision with the left wall 1:

$$u_{n+2} = u_{n+1} - F_1'\left(\Theta_n^{c2}\right); \quad \Theta_{n+2} = \Theta_{n+1} + \frac{x_{n+1} + F_1'\left(\Theta_{n+1}^{c1}\right)}{2u_{n+1}} + \frac{x_{n+1} + F_1'\left(\Theta_{n+1}^{c1}\right)}{2u_{n+2}}$$

Here $\Theta_{n+1}^{c1} = \Theta_{n+1} + [\, x_n + F_1(\Theta_{n+1}^{c1})]/2u_{n+1}$ and $F_1(\Theta) = \sin(\Theta)$

The pinball model: numerical simulation

• Start with a given distribution of particles and velocities (a statistical description of a generalized Fermi system). The equations of motion are solved using variable time steps to obtain the time evolution of the energy of each particle as well as the velocity distribution function

• For the collision of particles with the walls consider several cases, including the possibility of multiple collisions, e.g., when the particle velocity is small compared with the velocity of the wall. The parameters used: the length of the system is L=1, the number of particles is N = 1000–10 000, and 50 is the number of output distributions. That is, if t_{tot} (typically t_{tot} =10^4–10^5) is the total calculation time, the distributions are sampled in 50 time steps t_i (i = 1 – 50) and averaged over $\Delta t = t_{tot}/50$. Note that Δt is not the same as the time step for the integration of the equations of motion; it is the time period for averaging the particle distribution function

• The particles are assumed to be initially located at $X^j_{init} = A_1 + (L - A_1 - A_1)(j - 0.5)/N$, where j (=1, . . . ,N) denotes the j^{th} particle and $v^j_{init} = (-1)^j\, u_{in}(V_1+V_2)/2$ is its initial velocity, where $V_{1,2} = \omega_{1,2}A_{1,2}$ are the velocity amplitudes of the walls 1 and 2, and an "initial velocity" parameter u_{in} has been introduced for convenience. Typically, for a time span of t_{tot} = 10000, corresponding to the output averaging time Δt = 200, a particle with an initial velocity of u_{in} =3 will have bounced off the walls a few thousand times

The simulations: small amplitude of the second wall

• Very small oscillation amplitude of the second wall: $A_2 = 10^{-5}$ and $A_1 = 0.1$. The effect of the second wall is negligible, and an evolution is similar to that for the classical Fermi problem. The energy of a particle oscillates with time but after an initial or transient quasilinear growth it becomes constant on the average

• For relatively small initial particle velocities, a plateau eventually appears in the particle energy (momentum) distribution, Fig. 2(a)

• For larger initial velocities, a change of the character of the distribution function occurs associated with the formation of stochastic islands and a qualitative change of the particle orbits in the phase space

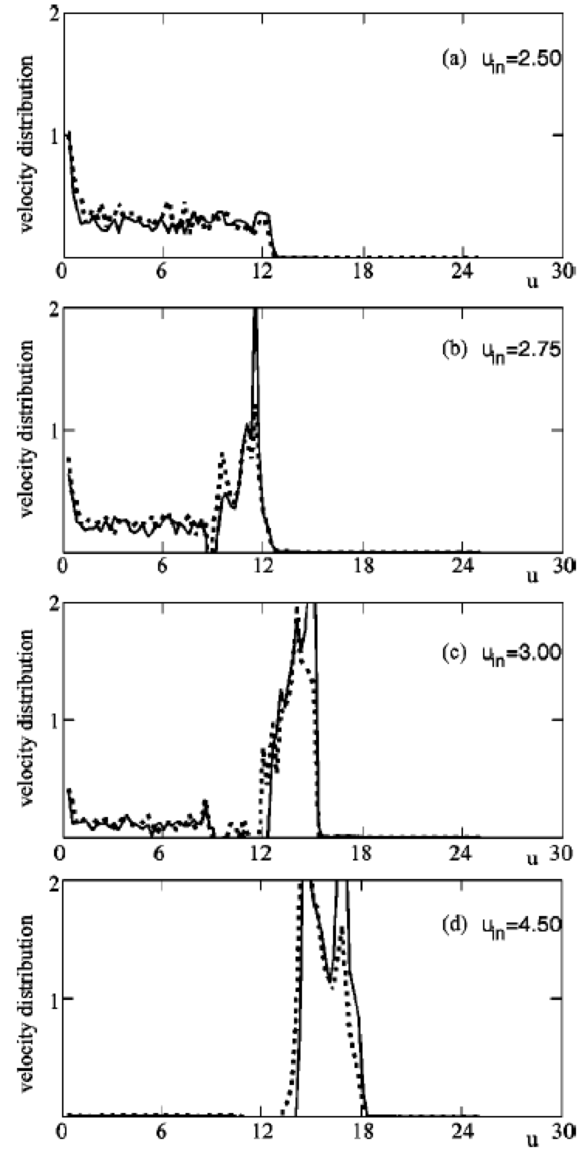


FIG. 2. The particle distribution function averaged at the 4th (dots) and 50th (solid lines) averaging intervals for different initial particle velocities and very small amplitudes of second-wall vibration: $A_1 = 0.1$ and $A_2 = 10^{-5}$.

The simulations: distributions for increased amplitude of the second wall

- For $u_{\text{init}} = 3$ and $A_2 = 10^{-3}$–10^{-2}: no end of the quasilinear-like evolution regime. The particles are accelerated to high velocities without entering any strong-correlation regime in the phase space. There is also no stochastic island nor invariant curves in the phase space
- The presence of the second oscillating wall leads to a destruction of correlations for a much wider parameter range than in the case of a single oscillating wall
- Note the lack of other qualitative changes beside the appearance of high-energy tails in the distribution function for the cases in Figs. 3(b)–3(d), as opposed to that of Fig. 3(a)

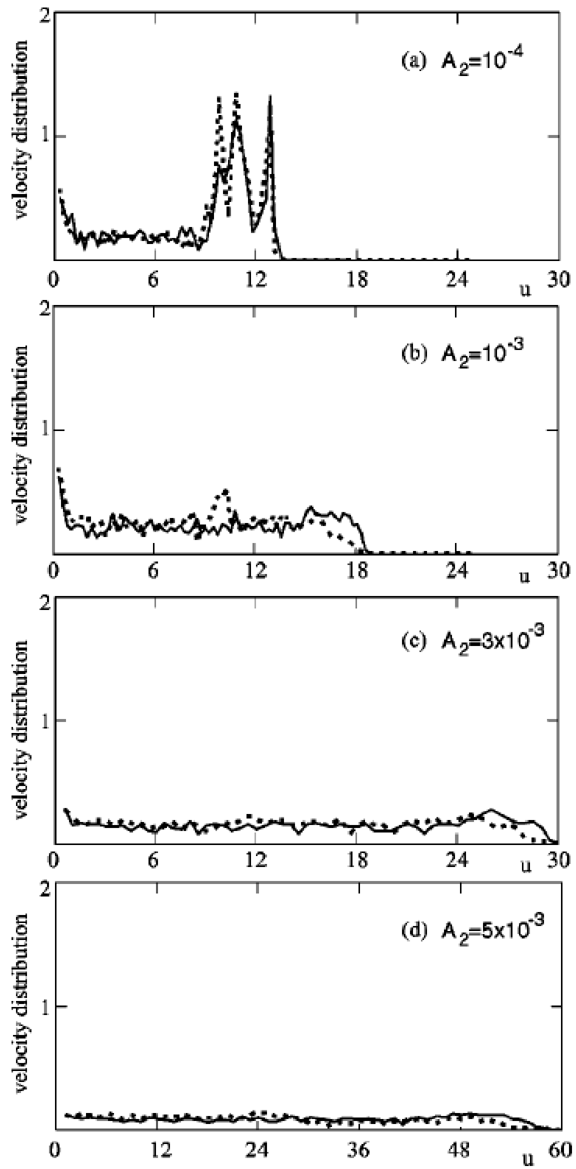


FIG. 3. The particle distribution function averaged at the 4th (dots) and 50th (solid lines) averaging intervals for higher amplitudes of second-wall vibration, with $A_1 = 0.1$ and $u_{\text{init}} = 3$.

The simulations: discussion

• High energy tails in the distribution function are formed because of modification of quasilinear diffusion due to the effect of the second oscillating wall. The initial stage of the quasilinear evolution is also affected by the nonlinear modulation, as exhibited by the change in the length of the transient period of stochastic diffusion when the second oscillating wall is added. The nonlinear terms proportional to $< V_1^2 >< V_2^2 >$ are responsible for the change of character in the quasilinear evolution

• Since the motion of the walls is fixed, the self-consistent nature of the plasma waves and their evolution are in general not covered by the model. That is, the simulation concentrates on the direct wave-particle interactions

• The result that the second oscillating wall can efficiently destroy correlations in the particle dynamics and thereby significantly change the character of the particle distribution at higher energies could not be predicted by a theory based on perturbation methods and the random-phase approximation

• Interaction among the particles, or generation and loss due to particle-particle collisions, are precluded. In dense low-temperature plasmas such collisions can dominate the purely dynamical phase randomization process. However, inclusion of these processes would change the physical nature of the problem, in particular with respect to the analogy to the classical Fermi process

Conclusions

- The plasma-maser process can affect weakly turbulent plasmas by converting the energy of resonant waves to nonresonant ones and plasma particles. As a result of this flow in the energy spectrum, the plasma system becomes nonstationary or/and inhomogeneous

- If the plasma system is closed, the quasilinear nonstationarity/ inhomogeneity effects cancel the nonlinear coupling of the resonant and nonresonant waves. Thus, an adiabatic invariant, the number of nonresonant quanta, is conserved, and there is no up- or down-conversion of resonant energy to nonresonant waves. In open plasma systems, the conversion becomes possible

- Because of the universal nature of the plasma-maser process, it can appear in many problems involving wave-particle interactions. Although not of the lowest order, this process can saturate, or smooth out, the predominance of the discrete or resonant nature of the natural modes and their interactions

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