Timing of the triple pulsar system PSR J1623-2631 using TEMP02

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M. Sc. Thesis

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Cover illustrations

Background image : A γ ray pulsar and the regions of emitted radiation. Both γ ray radiation (violet) from outer magnetosphere and radio waves (green) from magnetic poles are illustrated (NASA/Fermi/Cruz de Wilde).

Top right inset : The globular cluster M4 in the constellation of Scorpius taken by the National Optical Astronomy Observatory (NOAO/AURA/NSF).

Bottom right inset : An artistic view of the triple pulsar system PSR J1623-2631 inside the globular cluster M4. The pulsar and orbiting white dwarf are seen at lower left. The Jupiter mass planet fills the view (NASA and G. Bacon (STScI)).

Abstract

Pulsars are rapidly rotating neutron stars. Despite the fact that they have been discovered almost fifty years ago there are still a lot of questions to be answered about their interior structure and their radiation processes. Their unique properties make them ideal for studying a wide variety of phenomena and testing theories of Physics. Their almost perfect rotational stability, comparable with atomic clocks, is the property in which our work is based on.

The goal of our thesis is to investigate timing irregularities in the PSR J1623-2631 triple system, in order to identify the orbital parameters of the second Keplerian orbit. PSR J1623-2631 is a millisecond pulsar that is located in the M4 globular cluster. Very soon after its discovery (1988), modulations in the period of the pulsar indicated the existence of a white dwarf companion. Precise timing measurements revealed a Jupiter mass planetary companion. Optical observations put further limits on the mass of the white dwarf. Until today we have not arrived at accurate orbital parameter values for the outer companion, primarily due to the fact that our span of observations covers only a small fraction of the period of the outer orbit.

All our timing observations were performed with the 'Effelsberg 100-m Radio Telescope of the Max-Planck-Institute for Radioastronomy (MPIfR), Bonn, Germany' at 1410 and 1360 MHz. The PSRCHIVE software package was used for cleaning the data from Radio Frequency Interference (RFI), creating a pulsar template and calculating time of arrivals (TOAs).

Our work is based on Thorsett et al. (1999) previous research. We start our analysis assuming that we have only one Keplerian orbit, we consider the white dwarf and the planet as one body. The Blandford & Teukolsky (1976) model was used. With the double Keplerian model we tested the triple system hypothesis. Comparing the differences between the timing model and the observed arrival time (timing residuals) we derive conclusions about the validity of our hypothesis. After adding the BT1P model in the TEMPO2 timing package we apply to our data the fitting procedure that this package provides us and adjust the orbital parameters of the second orbit with our binary model. We make an initial assumption about the orbital eccentricity and change the second orbital parameters using a brute force method. Our results provide us with best root mean square values. Our results are in a good agreement with Thorsett's.

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Pulsars

1.1 Discovery

The history of pulsars starts long before their discovery. In 1934, 2 years after the discovery of neutrons, Walter Baade and Fritz Zwicky independently, proposed the existence of a new form of star, the neutron star. This star would be the outcome of a supernova explosion. Its interior would be comprised mainly of neutrons, with density up to 10^{15} g cm⁻³ and diameter approximately 20 kilometers. In this Chapter we give a brief overview of pulsars, more details can be found on Manchester & Taylor (1977), Lyne & Graham-Smith (2006) and Lorimer & Kramer (2005).

Despite the growth of radio astronomy during the Second War, thirty three years passed until the discovery of pulsars. That was partly due to the fact that radio astronomers were not expecting to find rapid period fluctuations in the signals from any celestial source. Indeed, most radio receivers were designed to reject or smooth out impulsive signals and to measure only steady signals, averaged over several seconds of integration time.

During the investigation of the interplanetary scintillations at Cambridge¹ the first pulsar was discovered. Anthony Hewish and Jocelyn Bell had constructed a large receiving antenna, containing 2,048 full-wave dipoles operating at long radio wavelengths (3.7 m). At this wavelength radio interplanetary scintillation effects are large but they only occur for radio sources with a very small angular diameter.

In July 1967, a month after the beginning of recording, Jocelyn Bell noticed large fluctuations in the signal from a specific location in the sky. Then, for several nights no signals appeared. These fluctuations were seen later at about the same location on successive days and looked like terrestrial interferences rather than scintillations. When they reappeared and it was noticed that the fluctuations returned four minutes earlier each day, there was no doubt that the signals had celestial origin. Then a recorder with an even faster response time was used and in November 1967 it observed regular pulses with period equal to 1.337 s.

¹Radio scintillations is a similar phenomenon with the twinkling of visible stars. With that project scientists at Cambridge tried to investigate the radio scintillations which occur in the terrestrial ionosphere, in the ionised interplanetary gas in the Solar system and in ionised interstellar gas of the Galaxy.

In February 1968 the discovery of the first pulsar, now known as PSR 1919+21, was published in Nature (Hewish et al., 1968). This announcement also contained an interpretation of the nature of the source of the signal. Initially the fact that the parallax of the source was not higher than 2 arc minutes led to the conclusion that the source lies outside the Solar System. Also, the regularity and the rapidity of the pulsation showed that the source must be a small, condensed star, presumably either a white dwarf or a neutron star.



Figure 1.1: (a) First observations of pulsar, CP 1919+21 ("CP" refers to Cambridge Pulsar). (b) Single pulses observations of the same pulsar (Hewish et al., 1968). This pulsar is now referred to as, PSR B1919+21, following the official IAU designation.

The announcement of the discovery sparked a series of observations from many radio observatories. So prominent was the discovery of pulsars that by the end of 1968 more than 100 papers reporting the discovery or discussing about the properties of new pulsars. In addition to the search of new pulsars, major efforts were made to measure the characteristics of individual pulses and subpulses, the pulse profiles, the polarisation, observations at higher frequencies etc. All these observations gave the opportunity to the pulsar theorists to propose explanations about the nature of pulsars. There are three mechanisms that explain the regular pulsed emission from pulsars: pulsating, binary and rotating stars. Pulsations were the first to be proposed. The white dwarfs and neutron stars could oscillate with periods between 100 and 1000 s and between $\sim 10^{-4}$ and 10^{-2} s, respectively. The discovery of the Vela and Crab pulsars, with periods less than 0.1 s, ruled out the slowly pulsating white dwarfs and rapidly pulsating neutron stars.

The second mechanism, the orbiting white dwarfs or neutron stars, was also ruled out. The white dwarfs in order to have orbital periods as small as the observed ones (0.1 s) their separation should be smaller than their radius. The neutron stars were also ruled out because as they orbit each other they lose energy in the form of gravitational radiation, their distance decreases and their orbital period as well. This comes in contradiction with the observed increase in the periods of pulsars.

Rotating white dwarfs and neutron stars were the only viable explanation. The equatorial angular velocity of a rotating star (centripetal force) should not exceed the gravitational velocity so that the star will not disintegrate. Rotating white dwarfs were ruled out because for rotation periods less than 1 s the centrifugal forces would destroy them. Only a rotating neutron star can explain the observed periods of pulsars.

In 1967 in Nature, only a few months before the discovery of pulsars, Franco Pacini had published a paper in which he showed that a rapidly rotating neutron star, with a strong dipolar magnetic field, would act as a very energetic generator that could provide a source of energy for radiation from a surrounding nebula, such as the Crab Nebula.

In 1968, T. Gold with his paper in Nature was the first who identified the pulsars with rotating neutron stars. According to his model the 'lighthouse effect' we observe is due to the fact that rotating neutron stars have a strong magnetic field and an ionised magnetosphere which co rotates with the star. It is within the magnetosphere that the beam of radiation originates. Furthermore, Gold predicted that the loss of energy in form of magnetic dipole radiation would have as a result a decrease of the period. The discovery of a slowdown in the period of the Crab pulsar (1969) equal to 36.5 ns per day verified that pulsars are rotating neutron stars.

Pulsar observations in other frequencies, besides radio, have been carried out throughout the history of pulsars. Soon after the discovery of pulsars at radio frequencies Cocke et al. (1969) identified the Crab pulsar in optical waves as well. In X and γ rays observations should be held outside the Earth's atmosphere. Since at the time no X and γ ray telescopes orbiting the Earth existed the year that followed the discovery of pulsars, balloon experiments and rocket flights detected the first high energy pulsations from pulsars. Soon after the discovery of optical pulses from the Crab pulsar, X ray pulsations were detected from the same nebula. Today the Chandra (1999 until today) and the XMM-Newton telescope (1999 until today) continue the observations.

In the 1970's γ ray balloon experiments investigated the Crab nebula. Later, the Compton Gamma Ray Observatory (1991-2000) and especially the Fermi Gamma-ray Space Telescope (2008 until today) increased the number of γ ray pulsars to more than 100 providing us with valuable information about the emission process

of pulsars.

1.2 Theoretical introduction

Pulsars, as we have mentioned, are energetic, rapidly rotating neutron stars. In this section we will give a brief introduction on their formation process, the interior structure and the magnetic field of pulsars. Questions about the exact composition of the internal structure, the exact mechanism of the radio emission and the origin of high energy emission have not been fully answered until today.

1.2.1 Formation

When a main sequence star of 6 to 15 M_{\odot} exhausts its nuclear fuel gravity takes over, the core contracts and its density increases. If the mass of the core is not greater than 1.4 M_{\odot} (Chandrasekhar limit) the final pressure of the core's degenerate electrons is high enough to balance the gravitational pressure and the star is smoothly converted to a white dwarf.

As smooth as the creation of white dwarf is, so violent the creation of a neutron star is. When the mass of the core exceeds the Chandrasekhar limit the end of the main sequence star is a Type II supernova explosion. The outer layers of the main sequence star are expelled and the mass of the remaining core of the star is equal to $1 - 2 M_{\odot}$. The star's degenerate neutron pressure resists gravity and a neutron star is created.



Figure 1.2: Left: The Crab Nebula, the remnant of the supernova explosion after which the pulsar was born. Right: Time sequence showing the fading-in and fading-out of the Crab pulsar (N.A.Sharp/NOAO/AURA/NSF).

Neutron stars are very compact stellar remnants. All the mass of a neutron star is contained in a sphere with a 10 to 15 km radius. The central density can become equal to $\sim 10^{15}$ g cm⁻³, larger than the density of an atomic nucleus, 2.3×10^{14} g cm⁻³.

Their high rotation speed is obtained by conservation of the angular momentum of their progenitor star. The progenitor star has higher radius and smaller rotation speed. After the supernova explosion, the neutron star retains a significant fraction of its initial angular momentum and since its radius is sharply reduced the rotation speed is very high.

1.2.2 Interior structure

The interior of a neutron star is a mixture of nuclei, electrons, protons, neutrons and possibly quarks or more exotic particles. We can divide the interior of the neutron star into four major regions. The exact characteristics of each region depend on the equation of state that we assume.

Outer crust. Near the surface where the density is $\geq 10^6$ g cm⁻³ the crust is a crystalline lattice probably of ${}_{26}^{56}$ Fe. As we move inward the density increases. When $\rho \sim 10^6$ g cm⁻³ the electrons become relativistic, penetrate to iron nuclei and convert protons to neutrons ($p^+ + e^- \rightarrow n + \nu_e$). New neutron rich nuclei are produced, ${}_{28}^{62}$ Ni, ${}_{28}^{64}$ Ni, ${}_{36}^{66}$ Kr, ..., ${}_{36}^{118}$ Kr. These neutrons cannot revert to protons via β -decay process due to the fact that there are no available states for the emitted electrons to occupy (Pauli exclusion principle). In the boundary of the outer and inner crust, $\rho \sim 4 \times 10^{11}$ g cm⁻³ (neutron drip point) we can find free neutrons outside the nuclei.

Inner crust. The free neutrons are combined producing bosons which do not obey to the Pauli exclusive principle and occupy the lowest energy state without losing energy. The free neutrons are a superfluid without viscosity. When $\rho \sim 10^{14}$ g cm⁻³ the nuclei dissolve, the pressure of neutrons dominates and also protons pair forming a proton fluid.

Interior. The density in the interior of a neutron star ranges from $\sim \times 10^{14}$ to $\sim \times 10^{15}$ g cm⁻³. It consists of neutron and proton superfluids and relativistic electrons. The number of neutrons dominates.

Core. The conditions in the core in a neutron star is not yet completed understood. Some equations of state suggest the existence of elementary particles like pions (π) in the core of neutron star or the existence of quarks and gluons.

1.2.3 Emission of electromagnetic radiation

Pulsars have strong magnetic fields. The magnetic field ranges from $\sim 10^{13}$ - 10^{14} G in young pulsars, $\sim 10^{10}$ - 10^{12} G in regular pulsars to $\sim 10^8$ G in millisecond pulsars.

At the surface of a neutron star this huge magnetic field can overcome gravity and push charged particles away from the polar regions of the pulsar. Depending on the direction of the magnetic field these particles can be electrons or positive ions. Their velocities cannot exceed the velocity of light. As a result, the charged particles form a magnetosphere which surrounds and co-rotates with the pulsar. The size of the co-rotating part of the magnetosphere can not exceed the size of the light cylinder (the radius of the light cylinder is $R_c = c/\omega = cP/2\pi$, P refers to the rotational period of the pulsar) (Figure 1.3).



Figure 1.3: Structure of magnetosphere and emission regions of pulsars (MAGIC Collaboration).

The upper limit in the velocity of the particles divides the magnetic field lines into open and closed. The radiation emission regions are two, *polar cap* and *outer gap* and both refer to these magnetic field lines, open and closed respectively.

The electromagnetic radiation emission in the polar cap region is caused by curvature radiation. Electrons accelerate along the open magnetic field lines and emit γ ray photons through curvature radiation. So high is the energy of γ ray photons (> 511 KeV) that an annihilation process can be initiated ($\gamma \rightarrow e^- + e^+$). This is the beginning of a cascade process. The produced electrons and positrons are accelerated, emitting γ ray photons. This process is repeated. The energy of the emitted secondary particles gradually decreases. It is probable that the radio emission is produced in this secondary plasma. Harding et al. (2002) proposed that γ rays are emitted from this region due to inverse Compton scattering of thermal photons.

The radiation from the outer cap is curvature and synchrotron radiation. As in the polar cap, electrons are accelerated along the magnetic field lines and emit γ ray photons. The energy of the electrons and positrons is very high. Some photons escape from the magnetic field and may start a pair production ($\gamma + \gamma \rightarrow e^+ + e^-$). Some of the produced electrons follow circular motion around the magnetic field lines and emit γ ray photons with lower energy through synchrotron radiation, providing new photons for pair creation. The outer gap region is associated with high energy emission from pulsars. Romani's (1996) model predicts that γ rays are produced in the outer region of the magnetosphere (outer gap model). Harding and Muslimov (2003) developed an alternative model in which γ rays are emitted in the *slot gap* region (slot gap model).

As we can see the emitted radiation from pulsars covers a very wide range of frequencies from ~ 100 MHz to 100 GHz. The exact radio and high energy emission

processes are yet to be fully understood. The Fermi Gamma-ray Space Telescope provides us with valuable data about the γ ray emission mechanism in pulsars. More than 100 γ ray pulsars have been discovered, apparently ruling out that the polar cap is the primary γ ray emission region. However, both the outer gap and slot gap models can explain the emission of pulsars. Pulsars like the Crab and Vela seem to behave according to the outer gap model and remain undetectable in γ rays outside the pulses. The Geminga and γ - Cygni pulsars behave according to the slot gap model. They are detectable in γ rays throughout their entire rotation.

1.3 General characteristics

1.3.1 The ' $P - \dot{P}$ diagram'

The rotation period of pulsars is between 0.00139 s (PSR J1748-2446ad) and 11.77894 s (PSR J1841-0456). The great majority of pulsars have periods between 0.5 s and 2.5 s (Figure 1.4).



Figure 1.4: Histogram of the distribution of the periods for 1864 pulsars. With different colors the binary, magnetars and X-ray pulsars are illustrated (Wang et al., 2011).

The electromagnetic radiation that a pulsar emits has as a result the gradual reduction of the rotational energy and a slow-down of its rotation $(\dot{P} > 0)$. From the period (P) and its first derivative (\dot{P}) we can obtain other physical parameters, such as the magnetic field strength and the characteristic age.

If we assume that a pulsar is a magnetic dipole with angular velocity $\Omega = 2\pi\nu$, and $\dot{\Omega} = -\kappa \Omega^n$ due to rotation slow down, κ is a constant and n is the breaking index (Kaspi & Helfand, 2002), then the strength of the magnetic field is equal to

$$B \propto \sqrt{P\dot{P}}$$
 (1.1)

and the characteristic age is given by

$$\tau = \frac{1}{(n-1)} \frac{P}{\dot{P}} \,. \tag{1.2}$$

Also, the rate of loss of the rotational kinetic energy ('spin-down luminosity'), if assumed to be equal to the emission energy, is

$$\dot{E} = -I \Omega \dot{\Omega} \propto \frac{P}{P},$$
(1.3)

where I is the moment of inertia.

Knowing P and \dot{P} we have a first estimate about the magnetic field, the age of the pulsar and its luminosity. All this knowledge can be summarized on the 'P - \dot{P} diagram' shown in Figure 1.5.



Figure 1.5: The P - \dot{P} diagram of known pulsars (2005) with lines of constant characteristic age, magnetic field and spin-down luminosity. Normal pulsars (dots), binary pulsars (circles) and pulsar-supernova remnant associations (stars) are shown (Lorimer & Kramer, 2005).

As we can see on the 'P - \dot{P} diagram' normal and millisecond pulsars are gathered in two clearly distinct regions. In the middle right part of the diagram we find normal pulsars with $P\sim0.7~{\rm s}$, $\dot{P}\sim10^{-15}~{\rm s}~{\rm s}^{-1}$ and $B\sim10^{12}$ G. In the lower left, not as many as normal pulsars, millisecond pulsars are located with $P\sim3ms,~\dot{P}\sim10^{-20}ss^{-1}$ and $B\sim10^8-10^9~{\rm G}.$

As we know pulsars start their lives with short periods and gradually their periods become larger and larger. But what happens to millisecond pulsars? They have very short periods but they are not as young as normal pulsars (the age for millisecond pulsars it is 10^8 yr - 10^{10} yr and normal for pulsars it is $\sim 10^7$ yr). The answer can be found in the formation process and evolution of these pulsars.

1.3.2 Evolution of binary systems - Formation of millisecond pulsars

Most of our Galactic stars belong to double or multiple systems. So, most of the times, the evolution of pulsars starts with two main sequence stars. The more massive of the two stars evolves first, passes from the main sequence state and if its mass is between 8 M_{\odot} and 15 M_{\odot} , it explodes in a supernova (Type II) to form a neutron star. This explosion may cause the disruption of the system leaving a high velocity neutron star and a runaway OB star (shown in Figure 1.6). Most of the isolated neutron stars that we observe are made through this process. Only 10% or less of binary systems can survive from a supernova explosion. Low-mass X-ray binary systems (LMXB) or high-mass X-ray binary systems (HMXB) are the highly bound survival remnants.

If the mass of the neutron star companion is low (M < 8 M_☉), then we have a low-mass X-ray binary system. The system remains bound for about $10^6 - 10^7$ yr. All these years the pulsar spins down until the companion star passes from the main sequence to the red giant phase. When the matter of the red giant overflows the Roche lobe, the strong gravitational field of the pulsar attracts it. The mass that is transfered through the accretion disk makes the system visible in X-rays and spins the pulsar up. This revived pulsar (recycled pulsar) with short period and old age occupies the lower left part of the '*P* - *P* diagram', and becomes a millisecond pulsar. The companion gradually sheds all its outer layers becoming a white dwarf. Eventually, the resulting system of low-mass X-ray binary is a millisecond pulsar - white dwarf binary. Studies of the orbital eccentricity for a millisecond pulsar - white dwarf system indicate that it ranges from 10^{-5} to 10^{-1} , approximately ($10^{-5} \leq e \leq 10^{-1}$).

In the case that the neutron star's companion is a high mass star (M > 8 M_{\odot}), we have a high-mass X-ray binary system. The first stage of evolution of this star is the same as above, except that the mass transfer state will not last as long as in low-mass X-ray binary systems. The massive companion star explodes as a supernova and a new neutron star is formed. If the system survives this second explosion, the result will be a binary neutron star system. The orbit of a binary neutron star system is more eccentric than in a millisecond pulsar - white dwarf system, $0.1 \leq e \leq 0.9$. If not, the system will be separated and the remnant will be a slow, isolated millisecond pulsar and a young isolated normal pulsar.

1.3.3 Galactic population of normal and millisecond pulsars

In Figure 1.7 the galactic distribution of normal and millisecond pulsars is shown. Today, more than 2200 pulsars^2 have been observed. However we know that propagation effects in the interstellar medium prevent the light from pulsars to

²To be accurate as of May 2013 2213 pulsars have been published in the ATNF pulsar catalog (http://www.atnf.csiro.au/people/pulsar/psrcat/).



Figure 1.6: Four formation scenarios for solitary normal pulsars, solitary millisecond pulsars, low-mass X-ray binary systems (millisecond pulsar - white dwarf) and high-mass X-ray binary systems (double neutron star systems) (Lorimer, 2005).

reach the Earth, so the estimated population of pulsars in our Galaxy is probably 10^6 .

The majority of normal pulsars are found near about ± 1 kpc from the galactic plane. But the progenitors of pulsars are massive O or B type stars which are located along the Galactic plane. The fact that a pulsar can be located even ± 1 kpc above the Galactic plane means that it has a large initial velocity. Pulsars receive their velocity at birth from the supernova explosion that they were formed from. Theoretical calculations and proper motion measurements indicate that the velocities of pulsars are several hundred km s⁻¹. Hobbs et al. (2005) studing the proper motion of 233 pulsar with statistical methods conclude that the mean transverse velocity of normal pulsars is 246 ± 22 km s⁻¹ and for pulsars younger than 3 Myr the 3D velocity is equal to 400 ± 40 km s⁻¹.

On the other hand the population of millisecond pulsars seems to be isotropically distributed in the Milky Way. In fact, due to scattering effects, only local (d <



Figure 1.7: The galactic distribution of normal (black), solitary millisecond (red) and binary millisecond pulsars (opened red circles) in galactic coordinates (Lorimer, 2005).

3 kpc) millisecond pulsars can be detected. The birthrates of millisecond pulsars are probably lower than those of normal pulsars (1 every 10^5 years), but their characteristic age is longer. Normal and millisecond pulsar populations are expected to be similar in size (Lyne et al., 1998). The transverse velocities of millisecond pulsars are 87 ± 13 km s⁻¹, not as high as the velocity of normal pulsars.

1.4 The structure of emitted pulses

The magnetic axis of pulsars is inclined with respect to the rotational axis (Figure 1.3). As the pulsar rotates, the radiation beam may cross the observer's line of sight. In this case the pulsar is visible and in each rotation we will detect a single main pulse, see Figure 1.8. The period of these pulses is the rotational period of the pulsar. This phenomenon is called the 'lighthouse effect'.



Figure 1.8: Pulses from PSR B0329+54. Every 0.71452 s we observe a signal which varies in shape and intensity (Figure provided by Prof. Dr. John H. Seiradakis).

We can observe these pulses in a very wide frequency range, from 20 MHz to 80 GHz. The intensity of single pulses is very low, the flux density of the strongest single pulses is equal to 1 Jy^3 , and comparable with the noise intensity. In order to solve this problem and maximize the intensity we integrate hundreds or thousands of single pulses.

The flux density of pulsars depends, also, on the observing frequency. This dependency follows a power law model. At high observing frequencies the intensity of pulsars is low and at low observing frequencies it is high. At 2 GHz and 400 MHz, for example, the maximum observed intensities are 14 mJy and 5 Jy, respectively (ATNF catalog).

Studying the characteristics and the irregularities of individual pulses and integrated profiles valuable information about the emission process and emission regions can be obtained. In the next section we will give a brief introduction to the structure of integrated and individual pulses

1.4.1 Integrated pulsar profiles

As we have mentioned earlier, in order to maximize the intensity of single pulses, we add hundreds or thousands of pulses. The result is an *integrated pulsar profile*. Unlike the single pulses, the integrated profiles are very steady morphologically. Almost at every frequency pulsars have a signature integrated profile that characterizes and makes them distinct from the other pulsars.

The length of the integrated profiles are typically 10° to 20° of the rotational longitude. However, there are pulsars with integrated profiles of only 1° and others which are constantly visible (360°). The youngest pulsars tend to have narrower profiles and the millisecond pulsars the widest. The number of components varies too. Typically the integrated profiles consist of 1 to 5 components (Figure 1.9). Millisecond pulsars tend to have more components than normal pulsars (Kramer et al., 1998).



Figure 1.9: Integrated profiles of pulsars with different number of components at 21 cm (Seiradakis et al., 1995).

 $^{^{3}}$ 1 Jy = 10⁻²⁶ W m⁻² Hz⁻¹

The differences in the number of components and the width of the profiles are mainly caused by intrinsic radiation irregularities and by the way that the line of sight cuts the radiation beam.

There are many models which try to describe the beam structure in order to explain the number of components that we observe. Backer (1976) proposed the existence of two beams of emission, the core and the cone. Rankin (1993) suggest the existence of more than one cone structures consisting of discrete regions of emission. Lyne & Manchester (1988) assert that the emission is patchy and the radiation beam is not divided into core and conal region but the whole radiation beam consists of discrete emission regions. The most recent model was proposed by Karastergiou & Johnston (2007). According to this the height of emission is inversely proportional to the age of the pulsar. The younger the pulsar, the highest the altitude of emission.

The width of the profile is affected by the observing frequency and it is usually inversely proportional to the frequency. At low frequencies, both the width of the profile and the space between the components expand. This effect is called *radius to frequency mapping* (Ruderman & Sutherland, 1975). It is probably caused by the fact that the height of emission depends on frequency. According to this theory, at high frequencies the radiation is emitted in heights closer to pulsar.

The way that the line of sight cuts the beam is probably responsible for some specific characteristics of the integrated profiles. When the magnetic axis is perpendicular to the rotation axis we observe an *interpulse* at about 180° from the main pulse. The interpulse is caused by the radiation beam from the opposite magnetic pole from which the main pulse is produced. Interpulses may also be generated by the edges of a very wide radiation beam (Manchester & Lyne, 1977).

1.4.2 Individual pulses

As we can see in Figure 1.10 the individual pulses vary in intensity and morphology. In contrast to the integrated profiles, the components of the individual pulses (sub-pulses) represent the emission from small, distinct emission regions which do not cover the whole radiation beam.

In the next section we will present some characteristic structures of individual pulses from which valuable informations about the emission process can be obtained.

Mode changing and nulling

Backer (1970) was the first to notice that the integrated profile of the PSR B1237+25 takes more than one form at the same observing frequency. This phenomenon is called *mode changing* and we can observe it in other pulsars besides the PSR B1237+25. The mechanism that is responsible for this phenomenon is not clear yet. Rankin (1986) noticed that we observe mode changing more frequently in old pulsars with multi-component profiles.

The intensity of the individual pulses is not steady. There are some extreme causes where for some number of pulses the intensity drops to very low levels (the



Figure 1.10: The integrated profile (the average of 500 pulses) and individual pulses from PSR 1133+16 at 600 MHz (Cordes, 1979).

pulsar 'switches off'). *Nulling* was first observed by Backer (1970). The duration of this phenomenon varies. The nulling state lasts for only 2 or 3 pulses for some pulsars and for others it lasts more than half of the total emission time.

Ritchings (1976) studying this phenomenon realised that it is more common in pulsars with old age. Rankin (1986) suggested that nulling is observed more frequently in pulsars with more components (cone emission). She did not find a relationship between nulling and the age of pulsar. Biggs (1992) tried to correlate nulling with the orbital period, the age and the geometry of emission of pulsars. He concluded that most of the nulling pulsars have long period derivatives, older age and small *inclination angle* (the angle between the rotational and magnetic axis).

There are studies which indicate that mode changing and nulling are uniform phenomena (Wang et al., 2007). In some pulsars mode changing and nulling have a close relationship. For example, in PSR B0826-34 the null state is actually a mode change of the profile with very low emission intensity. Kramer et al. (2006) and Lyne et al. (2009) noticed that the rotation of PSR B1931+24 (J1933+2421) and PSR J1832+0029 slows down faster when they are visible. Especially in the rotation of PSR B1931+24 there is a 50% difference in the slow down rate of the visible and nulling states. This phenomenon involves changes in the structure of the magnetosphere of pulsars. Timokhin (2010) proposed that the switching of magnetospheres between different states (different geometries or/and different distributions of currents) has as a result the observed different spin down rates and nulling. Different emission beam geometries (we observe different parts of the emission beam or no beam) mainly produce mode changes or nulling.

Gajjar, Joshi and Kramer (2012) studied the nulling behavior of PSR B2319+60

at four different frequencies (325, 610, 1420 and 5100 MHz) and concluded that the degree of nulling at these four frequencies is similar, ruling out previous studies which suggest geometric effects as the explanation of nulling effects (Herfindal & Rankin, 2009). In this pulsar, global failure of magnetospheric currents maybe the cause of nulling.

Drifting

Most of the time, the longitude at which we observe a sub-pulse is uncorrelated with the longitude that this sub-pulse appeared in the previous individual pulse. However, in some pulsars the sub-pulses drift at lower or higher longitudes. This phenomenon was noticed first by Drake and Craft (1968) in PSR 1919+21 and PSR 2016+28. The direction of drifting may vary from pulsar to pulsar. The drifting rate (D = P_2/P_3 , where P_2 is the separation between sub-pulses and P_3 the period in which a pattern appears at same longitude) in most pulsars is stable but there are also some exceptions. Drifting is observed more often in the outer components of the profile. Due to this fact, it is probable that this phenomenon is related to the outer cone of emission. Drifting is not affected by nulling. After nulling, drifting continues from its last value before nulling took place.

1.5 Interstellar medium

The interstellar medium (ISM) consists mainly of ionised, atomic or molecular gas and dust. The ISM affects the propagation of the emitted signal of pulsars and the quality of observations (signal to noise ratio). In the next section, we will briefly refer to interstellar scattering and scintillation.

1.5.1 Interstellar scattering

As we can see in Figure 1.11 at low frequencies we observe a pulse broadening. Interstellar scattering of the pulsar signal is the cause of this phenomenon.

We assume that the ISM is inhomogeneous with length scale α . The distance between the pulsar and the Earth is d. As the signal travels through the ISM, it is deviated from a straight trajectory by an angle θ_o . The scattered pulses are received by the observer on Earth at different angles (θ), forming a broadened shape with angular separation from the center equal to θ_s .

The interstellar scattering causes, as a result, a delay in the arrival time of pulses compared to those which travel unaffected. The delay is equal to

$$\Delta t(\theta) = \frac{\theta^2 d}{c} \,. \tag{1.4}$$

The scattering time scale τ_s is

$$\tau_s = \frac{\theta_s^2 d}{c} = \frac{e^4}{4\pi^2 m_e^2} \frac{\Delta n_e^2}{\alpha} \frac{d^2}{f^4},$$
(1.5)

where n_e is the electron number density, e is the charge of the electron and m_e is the mass of electron.

From Equation (1.5), the delay is affected by the size of the ISM pattern (α) , the distance between the pulsar and Earth (d) and the observation frequency (f). The pulsars which are at large distances from Earth are affected more from the interstellar scattering. Finally, the delay increases when the observations are performed at low frequencies.





Figure 1.11: The effect of interstellar scattering on pulsar integrated profiles for PSR J1740-3052 at different observation frequency (Stairs, 2003).

1.5.2 Interstellar scintillation

A phenomenon similar to the twinkling of visible stars is the interstellar scintillation of radio waves. Scheuer (1968) tried to describe this phenomenon assuming that the turbulent ISM is a thin screen midway between the pulsar and the Earth. As the signal passes through the ISM, it is scattered. If the phase of scattered waves does not differ more than 1 radian then we will have interferences. As a result, the condition that should be fulfilled in order to have scintillations is

$$2\pi\Delta f \,\tau_s \simeq 1\,. \tag{1.6}$$

The size of the region, where scintillations occur, is called field coherence scale s_0 . The size of the first Fresnel zone (s_F) and the *field coherence scale* indicate the kind of scintillation that we will observe. The quantity that is used in order to define the type of scintillation is the *scintillation strength*, u (Rickett, 1990)

$$u = \frac{s_F}{s_0} \propto f^{-1.7} \, d^{1.1} \,, \tag{1.7}$$

where d is the distance between the pulsar and the Earth and f the observation frequency. If the field coherence scale is greater that the radius of the first Fresnel

zone ($s_0 >> s_F$ or u < 1) we observe *weak scintillation*. In the opposite case, $s_0 << s_F$ or u > 1, we observe *strong scintillation*.

The frequency below and above which we observe strong and weak scintillation, respectively, corresponds to u = 1. From Equation (1.7) this frequency is equal to

$$f \propto d^{0.65} \,. \tag{1.8}$$

At high frequencies and small distances we observe weak scintillations. Strong scintillations are most frequently observed and can be divided into *diffractive* and *refractive scintillations*.

Diffractive scintillation is the result of small scale irregularities in the ISM. They are responsible for intensity and frequency variations that last from minutes to hours. On the other hand, refractive scintillation is responsible for the slow intensity variations on a timescale of weeks (Rickett, Coles and Bourgois, 1984). This phenomenon is assumed to have intrinsic origin. Refractive scintillation can be observed only in compact radio sources.

Chapter 2

Timing of pulsars

Pulsars have unique properties that make them perfect for studying a wide range of phenomena. Their excellent rotational stability comparable with atomic clocks is one of those properties which provide us information about the external phenomena that affect the propagation of pulses and the interior of pulsars. Also, it gives a valuable help for gravitational wave detection.

The technique through which we record a precise measurement of the arrival time of pulsar radiation is called *pulsar timing*. In this Chapter we give a brief overview of pulsar timing, mainly based on Lorimer & Kramer (2005), Blandford & Teukolsky (1976) and Hobbs et al. (2006).

2.1 Time of arrivals (TOAs)

The pulses that a radio telescope detect from a pulsar are amplified, de-dispersed and folded to create a *mean pulse profile*. Then, adding only the high signal to noise (S/N) pulses (of the same frequency) we create the *template profile*. We choose a specific reference point (*fiducial point*) in the mean profile. Ideally, this is the point where the rotation, magnetic axes and the line of sight are located at the same plane (shown in Figure 2.1). The cross-correlation of the template profile and the mean pulse profile with respect to the fiducial point gives us the *time of arrivals* (TOAs).

The precision of TOA measurement depends on the width of the pulsar beam and the S/N of the mean profile, as it is shown in Equation (2.1) (Lorimer & Kramer, 2005).

$$\sigma_{TOA} \simeq \frac{W}{S/N} \propto \frac{S_{sys}}{\sqrt{t_{obs}\Delta f}} \times \frac{P\delta^{3/2}}{S_{mean}}, \qquad (2.1)$$

where S_{sys} is the system equivalent flux density, t_{obs} is the integration time, Δf is the observing bandwidth, P is the pulsar period, $\delta = W/P$ is the pulse duty circle and S_{mean} is the mean flux density of the pulsar. As a result, strong, fast pulsars with narrow pulsar profiles provide the best arrival time. Also, in order to maximize the signal to noise radio, we increase the added number of pulses which create the mean pulse profile ($\sigma_{TOA} \propto \sqrt{1/N_{pulses}}$).



Figure 2.1: The geometry of the pulsar emission beam. Ω is the rotation axis, μ is the magnetic axes and $\phi = 0$ is the fiducial point (Zhang et al., 2007).

Initially, the TOAs are measured using the telescope clock (*topocentric arrival time*). In order to minimize the relativistic time effects of the massive objects in the Solar System and also to compare TOAs of different observatories, we transfer our measurements from topocenter to Solar System Barycenter (*barycentric arrival time* (SSB)).

2.1.1 TOAs for a solitary pulsar

For a solitary pulsar the TOA measured with respect to the SSB is:

$$t_{SSB} = t_{topo} + t_{corr} - \frac{\Delta D}{f^2} + \Delta_{R\odot} + \Delta_{S\odot} + \Delta_{E\odot} .$$
(2.2)

The SSB arrival time, Equation (2.2), coincides with the proper time of pulse emission as it is measured by a clock on the pulsar (T_p)

$$T_p = t_{SSB} \,. \tag{2.3}$$

The first term, t_{topo} , is the TOAs measurements to the topocenter. The second term, t_{corr} , is the clock corrections. As we have mentioned, the TOAs are firstly measured with the observatory clock. The aim of clock corrections is to transform topocentric arrival time which is measured with a non-uniform clock to Terrestrial Time (TT).

Firstly, the Coordinated Universal Time (UTC) that the observatory clock measures is transformed to the Global Positioning System (GPS) clock (UTC(GPS)), also is referred as UTC(NITS) from the National Institute of Standards and Technology (NITS) which measures it. Then, we transform the UTC(GPS) to International Atomic Time (TAI). TAI is a weighted average of the measured time of over 200 atomic clocks in over 50 national laboratories worldwide. It is a high-precision atomic time based on the notional passage of proper time on Earth's geoid. On the other hand, the UTC is based on Earth's rotation and on measurements with atomic clocks. However, due to irregularities of Earth's rotation, a leap second should be inserted occasionally to UTC. So, the difference between TAI and UTC is:

$$TAI = UTC + \Delta t \,, \tag{2.4}$$

where Δt is the sum of leap seconds. Finally, we convert TAI to Terrestrial Time (TT). TT is a non-real measured time. It is measured by an ideal atomic clock on the geoid. The difference between TT and TAI is based on historical reasons and is equal to:

$$TT = TAI + 32.184s$$
. (2.5)

With clock corrections we convert topocentric arrival time to TT. Pulse arrival time must be converted to Solar System Barycenter (SSB) and the last three terms of Equation (2.2) perform this conversion.

The fourth term, $\Delta_{R\odot}$, is the Roemer delay. The Roemer delay is the vacuum light travel time between the pulse arrival time at the observatory and the equivalent arrival time at the SSB

$$\Delta_{R_{\odot}} = -\frac{1}{c} \vec{r} \cdot \hat{s} = -\frac{1}{c} (\vec{r}_{SSB} + \vec{r}_{EO}) \cdot \hat{s} \,. \tag{2.6}$$

In order to calculate Roemer delay we should know the exact position of the SSB (\vec{r}_{SSB}) and therefore the positions of major bodies in the Solar System. We use the Jet Propulsion Laboratories (JPL) ephemeris to calculate them. Also, we should know the exact position of the observatory (\vec{r}_{EO}) and the pulsar (\hat{s}) . The position of the pulsar may be affected by its proper motion or by an external gravitational field.

The fifth term, $\Delta_{S\odot}$, is the Shapiro delay. It is a relativistic correction. It is the pulse delay caused by the passage of the pulse through strong gravitational fields (curvature of space-time). The objects in the Solar System that cause the largest delays are the Sun (< 110 μ s), Jupiter (< 180 ns), Saturn (< 58 ns), Neptune (< 12 ns) and Uranus (< 10 ns). The Shapiro delay is the sum of the delays caused by all Solar System bodies.

The sixth term, $\Delta_{E\odot}$, is the Einstein delay. Like the Shapiro delay, it is a relativistic correction. It is caused by the time dilation from the motion of Earth (special relativity) and gravitational redshift from the gravitational field of other bodies in the Solar System (general relativity).

The third term, $\Delta D/f^2$, is the delay due to frequency dispersion in arrival time from the ionised interstellar medium. ΔD is equal to $D \times DM$. DM is the dispersion measure which measures the total electron content between the pulsar and the observer

$$DM = \int_0^L n_e dl \quad (pc \, cm^{-3}) \,. \tag{2.7}$$

D is the dispersion constant:

$$D \equiv \frac{e^2}{2\pi m_e c} = (4.148808 \pm 0.000003) \times 10^3 \quad (MHz^2 p c^{-1} cm^3 s) \,. \tag{2.8}$$



Figure 2.2: Shapiro delay due to Jupiter for PSP J1022+1001 (http://www.atnf.csiro.au).



Figure 2.3: Frequency dispersion in PSR B1356-60 arrival time. The dispersion measure is 295 $pc \, cm^{-3}$ (Lorimer & Kramer, 2005).

From Equations (2.7) and (2.8) the time delay due to frequency dispersion is:

$$\Delta t \simeq \frac{4.15 \times 10^3 \times DM}{f_{1(MHz)}^2 - f_{2(MHz)}^2} \quad (s) \,. \tag{2.9}$$

As we see, the delay decreases with the frequency, the higher frequencies arrive earlier at the telescope (an example is shown in Figure 2.3).

2.1.2 TOAs for a binary pulsar

When a pulsar is a part of a binary system, the emitted pulses will be affected by the gravitational field of the companion and the orbital motion. So, in the TOAs to the SSB, Equation (2.2), extra terms should be added:

$$t_{SSB} = t_{topo} + t_{corr} - \frac{\Delta D}{f^2} + \Delta_{R\odot} + \Delta_{S\odot} + \Delta_{E\odot} + \Delta_{RB} + \Delta_{SB} + \Delta_{EB} + \Delta_{AB} \,.$$
(2.10)

The binary system could be non-relativistic or relativistic.

A non-relativistic binary system can be described with Kepler's laws. We assume that the pulsar and the companion orbit around the same barycenter. In order to describe the system and measure the exact TOAs we need the *Keplerian parameters*, which are the orbital period (P_p) , the projected semi-major axis $(\alpha_p \sin i)$, the orbital eccentricity (e), the longitude of periastron (ω) and the epoch of periastron passage (T_0) .



Figure 2.4: The geometry of a Keplerian binary orbit. Longitude of periastron ω and the inclination (i) (the angle between the plane of sky and the orbital plane) are shown. The distance between the binary center of mass and the apastron is the semi-major axis (Splaver, 2004).

Kepler's equation relates the orbital parameters.

$$M = E - e\sin E, \qquad (2.11)$$

where M is the mean anomaly, E is the eccentric anomaly and e the eccentricity. The mean anomaly is equal to:

$$M(t) = 2\pi \left[\left(\frac{t - T_0}{P_b} \right) - \frac{\dot{P}_b}{2} \left(\frac{t - T_0}{P_b} \right)^2 \right].$$
 (2.12)

Knowing the mean anomaly we can solve numerically Kepler's equation, Equation (4.2) and determine E. From E we can calculate the true anomaly:

$$A(E) = 2 \tan^{-1} \left[\sqrt{\frac{1+e}{1-e}} \tan \frac{E}{2} \right].$$
 (2.13)

The longitude of periastron is equal to:

$$\omega = \omega_0 + \frac{\dot{\omega}}{\Omega_b} A(E) , \qquad (2.14)$$

where $\Omega_b = 2\pi/P_b$ is the mean angular velocity and $\dot{\omega}$ the changes of ω .

The eccentricity and projected semi-major axis are equal to:

$$e = e_0 + \dot{e}(t - T_0),$$
 (2.15)

$$x = x_0 + \dot{x}(t - T_0), \qquad (2.16)$$

where T_0 is the epoch of periastron passage and \dot{e} and \dot{x} the changes of e and x.

For a non-relativistic binary system the only additional term in BBS arrival time, Equation (2.10), is the Roemer delay. This delay is caused by the orbital motion of pulsar. For the Blandford & Teukolsky (1976) model it is:

$$\Delta_{RB} = x(\cos E - e) \sin \omega + x \sin E \sqrt{1 - e^2} \cos \omega, \qquad (2.17)$$

where $x = a_p \sin i$.

For a relativistic binary system, in the SSB arrival time we should add apart from the Roemer delay, the Shapiro delay, the Einstein delay and the aberration delay, Equation (2.10).

The Roemer delay is caused by the orbital motion of the pulsar:

$$\Delta_{RB} = x(\cos E - e_r) \sin \omega + x \sin E \sqrt{1 - e_{\theta}^2} \cos \omega , \qquad (2.18)$$

where $e_r = e(1 + \delta_r)$ and $e_{\theta} = e(1 + \delta_{\theta})$.

The Shapiro delay is caused by the gravitational field of the companion

$$\Delta_{SB} = -2r \ln\left[1 - e\cos E - s\left(\sin\omega(\cos E - e) + \sqrt{1 - e^2}\cos\omega\sin E\right)\right].$$
 (2.19)

The Einstein delay is caused by the gravitational redshift and the time dilation due to the companion's presence

$$\Delta_{EB} = \gamma \, \sin E \,. \tag{2.20}$$

The aberration delay is caused by the rotation of the pulsar

$$\Delta_{AB} = A \left[\sin[\omega + A(E)] + e \sin \omega \right] + B \left[\cos[\omega + A(E)] + e \cos \omega \right] , \qquad (2.21)$$

where the post-Keplerian parameters are:

$$\delta_r = T_{\odot}^{2/3} \left(\frac{2\pi}{P_b}\right)^{2/3} \frac{3m_p^2 + 6m_pm_c + 2m_c^2}{(m_p + m_c)^{4/3}},$$
(2.22)

$$\delta_r = T_{\odot}^{2/3} \left(\frac{2\pi}{P_b}\right)^{2/3} \frac{\frac{7}{2}m_p^2 + 6m_pm_c + 2m_c^2}{(m_p + m_c)^{4/3}},$$
(2.23)

$$s = T_{\odot}^{-1/3} \left(\frac{P_b}{2\pi}\right)^{-2/3} x \, \frac{(m_p + m_c)^{2/3}}{m_c} \,, \tag{2.24}$$

$$r = T_{\odot} m_c \,, \tag{2.25}$$

$$\gamma = T_{\odot}^{2/3} \left(\frac{P_b}{2\pi}\right)^{1/3} e \frac{m_c(m_p + 2m_c)}{(m_p + m_c)^{4/3}}.$$
(2.26)

In Section 2.2 we illustrate the Blandford and Teukolsky binary timing model (BT), the BT1P model (BT with two orbits) is based in this model, by which we will analyze the TOAs of our system.

2.2 Blandford and Teukolsky binary timing model

Blandford and Teukolsky (1976), in trying to analyze TOAs of PSR 1913+16, proposed, an one-orbit Keplerian model which also contains a redshift/ time-dilation parameter (Einstein delay, Δ_{EB}) and secular changes in the orbital parameters.

They assumed that the attraction between the two bodies is Newtonian. The relationship between proper time (T_p) , the time that the pulse is emitted measured by a hypothetical clock on the pulsar, and the time that is measured in the binary barycenter (t) is:

$$\frac{dT_p}{dt} = 1 - \frac{M_2}{r} - \frac{M_2^2}{M_1 + M_2} \frac{1}{r}.$$
(2.27)

Also from Kepler's equation:

$$E - e\sin E = 2\pi \frac{t}{P} + \sigma.$$
(2.28)

From Equations (2.27) and (2.28):

$$T_p = t - \frac{M_2(M_1 + 2M_2)}{\alpha(M_1 + M_2)} \frac{P}{2\pi},$$
(2.29)

where the subscripts 1 and 2 refers to the first and second body respectively, M is the mass, E is the eccentric anomaly and e is the eccentricity.

They calculated the arrival time of the pulses on Earth (t_{arr}) :

$$t_{arr} - t_{em} = |\mathbf{r}_e(t_{arr}) - \mathbf{r}_1(t_{em})| + \frac{D}{f^2} + 2M_2 \log\left(\frac{1 + e\cos\phi}{1 - \sin i\,\sin(\omega + \phi)}\right), \quad (2.30)$$

$$t_{arr} = t_{em} + \mathbf{r}_b + \mathbf{r}_{be}(t_{arr})\mathbf{n} + \frac{\alpha_1 \sin i (1 - e^2) \sin(\omega + \phi)}{1 + e \cos \phi} + \frac{D}{f^2},$$
 (2.31)

where \mathbf{r}_e is the position vector of the Earth in respect to the binary barycenter, \mathbf{r}_b is the position vector of the solar system barycenter with respect to the binary barycenter and \mathbf{r}_{be} is the position vector of Earth with respect to the solar system barycenter.

Then, they transformed Earth arrival time to solar system barycentric arrival time (t_{SSB}) . The frequency that we measure in the solar system barycenter (f) is not the same us the one that we calculate on Earth (f_e) . The Doppler effect, due to Earth's velocity, changes the barycentric frequency, $f = f_e(1 - \mathbf{u_en})$.

Thus, in order to transform Earth arrival time to barycentric arrival time they subtract from Equation (2.31) the position vector of Earth with respect to the barycenter $\mathbf{r}_{be}(t_{arr})\mathbf{n}$ and the dispersion measure delay as it is measured in the solar system barycenter. The barycentric arrival time is:

$$t_{SSB} = t_{arr} - \mathbf{r}_{be}(t_{arr})\mathbf{n} - \frac{D(1 - 2\mathbf{u_e}\mathbf{n})}{f_e^2}.$$
 (2.32)

From Equations (2.31) and (2.32)

$$t_{SSB} = t_{em} + \alpha_p \sin i \left[\sin \omega \left(\cos E - e \right) + \sqrt{1 - e^2} \cos \omega \sin E \right] .$$
 (2.33)

They inserted a new eccentric anomaly that relates explicitly the t_{em} and T_p :

$$\tilde{E} - e\sin\tilde{E} = 2\pi \frac{T_p}{P} + \sigma.$$
(2.34)

From Equations (2.29) and (2.34):

$$t_{em} = T_p + \frac{M_2^2(M_1 + 2M_2)}{\alpha_1(M_1 + M_2)^2} \frac{P_b}{2\pi} \sin \tilde{E}$$
(2.35)

and then Equation (2.33) takes the form:

$$t_{SSB} = T_p + \alpha_p (\cos \tilde{E} - e) + (\beta + \gamma) \sin \tilde{E}, \qquad (2.36)$$

where

$$x = \alpha_p \sin i, \qquad \alpha_p = x \sin \omega, \qquad \beta = \sqrt{1 - e^2} x \cos \omega, \qquad \gamma = \frac{M_2^2 (M_1 + 2M_2)}{\alpha_1 (M_1 + M_2)^2} \,.$$

They inserted a third eccentric anomaly E' for which:

$$E' - e\sin E' = 2\pi \frac{t_{SSB}}{P_b} + \sigma$$
 (2.37)

Thus, Equation (2.36) takes the form:

$$T_{p} = t_{SSB} - \alpha_{p}(\cos E' - e) - (\beta + \gamma)\sin E'$$

$$- \frac{(\alpha_{p}\sin E' - \beta\cos E')[\alpha_{p}(\cos E' - e) + (\beta + \gamma)\sin E']}{P_{b}(1 - e\cos E')},$$
(2.38)

which is the time that the pulse is emitted, measured by a clock on the pulsar (proper time).

2.3 Timing residuals

We know that the pulsar signal is periodic and that the emitted electromagnetic radiation has as a result the reduction of its rotation energy and a slow-down in rotation (\dot{P}) . Thus, the theoretical evolution of the pulsar's phase with time $(\nu = d\phi/dt)$ is given by a Taylor series:

$$\phi(t) = \phi_0 + \sum_{n \ge 1} \frac{\nu^{(n-1)}}{n!} \left(T_p - t_{epoch} \right)^n,$$
(2.39)

where $\nu = 1/P$ is the pulsar rotation frequency and its derivatives, T_p is the pulse emission time and t_{epoch} is a reference epoch in which the phase of pulsar is ϕ_0 and $\nu = d\phi/dt$.

2.3.1 Pre-fit residuals

As we have seen, given the orbital parameters of a solitary pulsar or of a binary system we can calculate theoretically the emission time of the pulses (proper time) using an appropriate timing model. Thus, for every observed arrival time we have the equivalent theoretical emission time. From the theoretical emission time we can measure the time evolution of pulse phase, Equation (2.39). The difference

$$R_i = \frac{\phi_i - N_i}{\nu} \tag{2.40}$$

is the timing residual, where 'i' refers to the *i*' th observation, ϕ_i is the pulse phase of the *i*'th observation, N_i is the nearest integer to ϕ_i and ν is the pulsar rotation frequency in the reference epoch, see also Figure (2.5).



Figure 2.5: The method with which the timing residuals are measured.

2.3.2 Post-fit residuals

In order to improve our timing residuals, to minimize the difference between observed and theoretical TOAs, we should recalculate the parameters of the pulsar applying a least squares fitting of the timing model to pulsar timing data. In this section we will cover the linear least squares fitting method that is used by the TEMPO2 software package¹ so that the post-fit residuals will be calculated.

Before we describe the linear least squares fitting method we should decide which parameters should be fitted in order to improve our residuals. The best prefit residuals have a Gaussian distribution around zero with a root mean square comparable with the uncertainties of TOAs (Lorimer & Kramer, 2005). An example is shown in Figure 2.6 (a). In Figure 2.6 we illustrate specific examples of prefit residuals with systematic errors. By fitting the parameters which cause these errors we improve the residuals. Pre-fit residuals with a parabolic form are produced by incorrect measurement of the second period derivative (Figure 2.6 (b)). The decreasing branch of the parabola indicates that the theoretical period of a pulsar is smaller than the observed ($P_{model} < P_{obs}$), the increasing branch indicates the opposite. Incorrect measurement (declination and right ascension) of the position of the pulsar has, as a result, sinusoidal residuals (Figure 2.6 (c)). Incorrect measurement of proper motion produces sinusoidal residuals with increasing magnitude (Figure 2.6 (d)).

In what follows, we will describe the least squares fitting method used by TEMP02. We assume that $y_i = R_i$ and $x_i = t_{bbat} - t_{pepoch}$, where *i* refers to the *i*'th observation, R_i is the timing residual and t_{bbat} is the arrival time in the barycenter of the binary system. We apply to our data the linear least squares fitting method,

¹ темро and темро2 are pulsar timing softwares. The темро package was developed by the Australia Telescope National Facilities and Pricenton University and is written in fortran. The темро2 package is re-written in C++ based on the темро code (Hobbs et al., 2006).


Figure 2.6: The timing residuals of PSR B1133+16. (a) The residuals which are obtained when a perfect timing model is applied. (b) The residuals after the removal of the second period derivative from a perfect timing model. (c) The residuals when we apply to the timing model a 1 arcmin offset in the position of the pulsar. (d) The residuals after neglecting the proper motion (Lorimer, 2005).

therefore, the combination of x_i and y_i is

$$y(x) = \sum_{k=1}^{M} \alpha_k X_k(x)$$
, (2.41)

where α_k are the parameters that we want to be fitted, k is the number of parameters and X_k are the *basis functions*.

The basis functions are the partial derivatives of the fitting parameters. In the case that we analyze the TOAs with the Blandford and Teukolsky binary model (BT) the basis functions of the Keplerian parameters have the form:

$$X(a_p \sin i) = \sin \omega (\cos E - e) + \sqrt{1 - e^2} \cos \omega \sin E, \qquad (2.42)$$

$$X(\omega) = a_p \sin i \left[\cos \omega (\cos E - e) - \sqrt{1 - e^2} \sin \omega \sin E \right] , \qquad (2.43)$$

$$X(e) = -W\sin E + a_p \sin i + \sqrt{1 - e^2} a_p \sin i e \cos \omega \sin E, \qquad (2.44)$$

$$X(P_p) = \frac{W}{P_p}(E - e\sin E),$$
 (2.45)

where $W = \alpha_p \sin i \left[\sin \omega \sin E - \sqrt{1 - e^2} \cos \omega \cos E \right] / (1 - e \cos E).$

We pick as the best parameters those that minimize χ^2 :

$$\chi^{2} = \sum_{i=1}^{N} \left[\frac{y_{i} - \sum_{k=1}^{M} \alpha_{k} X_{k}(x_{i})}{\sigma_{i}} \right]^{2}, \qquad (2.46)$$

where σ_i is the error of the *i*'th observation.

We define the *design matrix*

$$A_{ij} = \frac{X_j(x_i)}{\sigma_i} \tag{2.47}$$

and a vector

$$b_i = \frac{y_i}{\sigma_i}, \qquad (2.48)$$

where *i* refers to the number of observations (i = 1, ...N) and j refers to the number of parameters (j = 1, ...M). The minimization of Equation (2.46) can be written as

$$\chi^2 = |\mathbf{A} \cdot \mathbf{a} - \mathbf{b}|^2 \,. \tag{2.49}$$

The technique that TEMPO2 uses in order to minimize (2.49) and find the best values of the parameters is the *Singular Value Decomposition (SVD)*. With this technique the design matrix (A) is a factorization of the form

$$A_{ij} = U_{ii} W_{ij} V_{jj}^T, \quad i = 1, ...N \text{ and } j = 1, ...M,$$
 (2.50)

where U is a $n \times n$ unitary matrix, W is a $n \times m$ diagonal matrix and V^T is a $m \times m$ unitary matrix.

Therefore the parameters (α) that minimize the Equation (2.49) are given by

$$\alpha(j) = V_{jj} \left[diag \frac{1}{w_j} \right] \left(U_{ii}^T \cdot b_i \right), \qquad i = 1, \dots N \text{ and } j = 1, \dots M.$$
(2.51)

Using these parameters we recalculate the theoretical time of arrivals and improve the timing residuals (post-fit residuals).

Chapter 3

PSR J1623-2631 (B1620-26)¹

3.1 Discovery

PSR J1623-2631 is a millisecond pulsar which is located inside the M4 globular cluster. It is the second millisecond pulsar that is found in a globular cluster (Lyne et al., 1988).

Hamilton, Helfand and Becker (1985) using the Very Large Array (VLA) looked for millisecond pulsars inside globular clusters. Their research not only resulted in the discovery of a millisecond pulsar inside M28 but also encouraged future searches for millisecond pulsars inside globular clusters like the one that was held by Lyne et al (1988) and led to the discovery of PSR J1623-2631. Using the Lovell telescope they estimated that the period of the pulsar is 11 ms and its distance is 2.2 ± 0.8 kpc (based on an observed value of DM of 63 cm⁻³ pc and the Lyne, Manchester and Taylor (1985) electron distribution model).

They also discovered a fluctuation in the pulsar's period, indicating the existence of a companion. Timing measurements by Backer et al. (1993) confirmed the existence of a companion with a mass equal to $0.3 M_{\odot}$ (probably a white dwarf) and a period of 191 days. In that survey, also, a large second derivative in the pulsar's rotation rate was measured. The best explanation that Backer et al. suggested was the existence of another weakly bound companion. The orbit of this second companion is ~ 100 years and its mass is approximately 10 M_J, where M_J refers to Jupiter mass. The PSR J1623-2631 is the first pulsar that has been found in a triple system.

Further timing observations improved the properties of the second orbit (Rasio 1994, Sigurdsson 1995, Arzoumanian et al 1996, Joshi & Rasio 1997). The second companion has a mass of $\sim 0.01~M_{\odot}$, (typical for a brown dwarf or a planet) and is in a $\sim 40~AU$ orbit.

¹'B' and 'J' refers to different coordinates. 'B' name is based on Besselian coordinates while 'J' name is based on J2000 coordinates.

3.2 From the binary millisecond pulsar to three body system

The most extensive study of PSR J1623-2631, including a two-orbit analysis, was presented in Thorsett et al. (1999). They applied a polynomial and a double Keplerian model to calculate the parameters of the second companion's orbit.

As Backer et al. (1993) noticed, when we use a ν , $\dot{\nu}$ model the timing residuals have a cubic form. In order to minimize this effect we add to the model more frequency derivatives. The Thorsett et al. (1999) calculations indicated a large second frequency derivate. The most reliable explanation is a changing gravitational acceleration due to a massive body external to the binary (Thorsett et al., 1993). The low central mass density and velocity dispersion in the M4 globular cluster rule out the acceleration by the mean cluster field.

Thorsett et al. (1999), using a $\nu^{(5)}$ polynomial model of one Keplerian orbit and linear variations in orbital elements (e.g. $\omega = \omega_0 + \dot{\omega}(t - T_0)$), calculated the timing parameters of PSR J1623-2631. The results were in agreement with previous calculations. A very important result was the very large derivative of the projected semimajor axis ($x_a/\dot{x_a} \approx 3$ Myr), another evidence for the existence of a second companion.

To deal with the triple system they assumed that the pulsar orbits around the common center of mass with the first companion in a Keplerian orbit. Then, this inner binary orbits about the center mass of the triplet. The parameters of the first and the second orbit are x_a , P_a , e_a , ω_a , T_a and x_b , P_b , e_b , ω_b , T_b respectively. In order to calculate the mass and orbital parameters of the second companion they applied Joshi & Rasio's (1997) method. With this method we use the measured frequency derivatives in order to calculate the five orbital parameters: x (projected semimajor axis), P (orbital period), e (eccentricity), ω (argument of periastron), T (epoch of periastron).

The frequency derivatives are equal to:

$$\dot{\nu} = -\nu \frac{\alpha \cdot \hat{n}}{c}, ..., \nu^{(n)} = -\nu \frac{\alpha^{(n-1)} \cdot \hat{n}}{c},$$
(3.1)

where *c* is the speed of light, α is the acceleration of the pulsar and \hat{n} is a unit vector in the direction of the line of sight. The acceleration α and the unit vector \hat{n} can be related to the five unknown parameters of the orbit. We already know the frequency derivatives from observations and solving the nonlinear Equations (3.1) we can calculate the parameters of the orbit. With this method if we know all the five frequency derivatives we can directly measure all the five parameters of the orbit.

Thorsett et al. (1999) used for their calculations four frequency derivatives. In this case they assumed values for one parameter and calculated the remaining four. The final nonlinear system that they had to solve was:

$$\ddot{\nu} = \frac{B\,\dot{\lambda}_1\,\dot{\nu}}{A^2\,\sin(\lambda_1 + \bar{\omega}_1)}\,,\tag{3.2}$$

$$\nu^{(3)} = \frac{C\,\dot{\lambda}_1^2\,\dot{\nu}}{A^2\,\sin(\lambda_1 + \bar{\omega}_1)}\,,\tag{3.3}$$

$$\nu^{(4)} = \frac{D\,\hat{\lambda}_1^3\,\dot{\nu}}{A^2\,\sin(\lambda_1 + \bar{\omega}_1)}\,,\tag{3.4}$$

where '1' refers to the pulsar and the first companion (binary system) and '2' refers to the second companion, λ is the longitude at the reference epoch (measured from the pericenter), $\bar{\omega}$ is the longitude of the pericenter and :

$$A = 1 + e_2 \cos \lambda_1,$$

$$B = 2AA' \sin(\lambda_1 + \omega_1) + A^2 \cos(\lambda_1 + \omega_1),$$

$$C = B' + \frac{2BA'}{A}, \quad D = C' + \frac{4CA'}{A},$$

also, $\lambda_1 = \lambda_2$ and $\omega_1 = \omega_2 + 180^{\circ}$.

Changing the values of the eccentricity from 0 to 1 Thorsett et al. (1999) calculated the orbital parameters and $\nu^{(5)}$ and compared it with the measured values. They also repeated the calculations twice. The first time they neglected any contribution from the intrinsic pulsar spin-down ($\dot{\nu} = \dot{\nu}_{acc}$). The second time they assumed that 90 % of the spin-down rate was intrinsic ($0.1 \dot{\nu} = \dot{\nu}_{acc}$). The results are shown in Figure 3.1.

As we can see, in the case that we neglect the intrinsic pulsar spin-down ($\dot{\nu} = \dot{\nu}_{acc}$) the mass of the second companion is ~0.01 M_☉. On the other hand, when $0.1 \dot{\nu} = \dot{\nu}_{acc}$ the mass of the second companion can be as small as ~ 10^{-3} M_☉. As a result, the second companion cannot be a star (hydrogen-burning limit ~0.08 M_☉) but it can be a brown-dwarf (deuterium-burning limit ~0.015 M_☉) or a planet (Thorsett et al., 1999).

Thorsett et al. (1999) also applied a double Keplerian model to their timing observations. In this model, the pulsar orbits in a Kepleriant orbit around the common center of mass with the first companion. The pulsar and first companion orbit as a binary system orbit in a second Keplerian orbit around the common center of mass with the second companion.

First, they assumed that the second orbit is circular. The parameters of a circular second orbit are in excellent agreement with these of a polynomial model. The resulting high magnetic field of 2.6×10^9 G is below the upper limit for 11 ms pulsars.

Furthermore, using the assumption that the pulsar's magnetic field is small (~ 3×10^8 G) and because $\dot{\nu}$ is equal to 0, they performed calculations for dif-



Figure 3.1: Parameter solutions based on Joshi & Rasio's (1997) method. For the bold line solutions no contribution from intrinsic pulsar spin-down was assumed. For the lighter line, 90 % of the spin-down rate was intrinsic (Thorsett et al., 1999).

ferent eccentricities. Their solutions are in better than 10 % agreement with the polynomial ones.

3.3 Optical observations

Both optical observations with the Hubble Space Telescope (HST) and photometric calculations with the Canada-France-Hawaii Telescope (CFHT) were done in order to calculate the position and proper motion of the white dwarf, the first companion of PSR J1623-2631.

Richer et al. (2003), using ground-based observations in combination with the USNO-B1.0 catalog, concluded that the position of the white dwarf at right accession and inclination is $16^{h} 23^{m} 38^{s}.217$ and $-26^{o} 31' 53".662$ respectively, $0".126 \pm 0".13$ from the pulsar, which is another fact that supports the association between the two. They, also calculated the proper motion of the white dwarf in respect to the cluster to be 0.9 ± 1.1 mas yr⁻¹. This calculation does not exceed the cluster's (M4) proper motion dispersion (0.6 mas yr⁻¹) more than 2 σ , but still the exact value of the white dwarf's proper motion is still unknown.

Apart from Richer et al. (2003), Bassa et al. (2004) used optical observations to identify X-ray sources in the M4 globular cluster. Their position calculations of PSR J1623-2631 and the white dwarf companion are in a excellent agreement with these of Richer et al. (2003).

Sigurdsson et al. (2003) identified the position of the pulsar using multiepoch WFPC2 HST images (Figure 3.2). They also calculated the color and magnitude of the white dwarf. As we can see from the color-magnitude diagram (Figure 3.3), the



Figure 3.2: The position of the pulsar inside the M4 globular cluster. The radius of the circle is 0.7". Each image represents a different bandpass, the first is the U (F336W), the second is the V (F555W) and the third is the I (F814W) (Sigurdsson et al., 2003).

mass of white dwarf is about 0.45 M_\odot . In combination with the absolute magnitude of ${\sim}0.1$ magnitudes, the mass is constrained to 0.34 \pm 0.04 M_\odot and its age is 4.8 \times 10⁸ \pm 1.4 \times 10⁸ years.

From the new mass of the white dwarf, combined with the observed mass function and assuming that the pulsar mass is 1.35 M_{\odot} , they concluded that the inclination of the pulsar-white dwarf binary to the line of sight is 55^{14}_{-8} , the semimajor axis of the second orbit is 23 AU and the mass of the second companion is $\sim 2.5 \pm 1 \ M_J$.



Figure 3.3: Color-magnitude diagram of M4 cluster's stars. In blue the white dwarf companion is shown. The green dashed line is the cooling curve for a 0.5 M_{\odot} carbon-oxygen white-dwarf. The red curves are the cooling curves for 0.3 M_{\odot} and 0.4 M_{\odot} helium-core white dwarfs with hydrogen envelopes (Sigurdsson et al., 2003).

The optical observations provide us more information about the white dwarf (mass, age and orbital inclination) and about the second companion (mass and semimajor axis of the second orbit). The formation scenarios that probably explain the measured parameters of the triple system are demonstrated in the next section.

3.4 Formation scenarios

As we know, the white dwarf (the first companion of our the triple system) has a low mass and a large orbital eccentricity. The inclination between the inner binary plane and the orbital plane of the planet is high, but the eccentricity of the planet orbit is low (probably ~ 0.2). Sigurdsson and Phinney (1995) illustrated a formation scenario which explains the parameters of the system (Sigurdsson et al., 2003, Sigurdsson & Thorsett, 2005), *the canonical formation scenario*.

According to this scenario, in the beginning, the companion of the pulsar was a heavy white dwarf. Mass transformation from the white dwarf spins up the pulsar converting it to a millisecond pulsar. The white dwarf probably has a $\sim 0.7~M_{\odot}$ mass and an orbit with semimajor axis of 0.3 AU. The system spent most of its life in the cluster core. 1 to 2 Gyears ago an encounter with a main-sequence star resulted in an exchange between the two. The main-sequence star took the place of the white dwarf in the system. The mass of that new star was higher than the original white dwarf. The current white dwarf is the descendant of that main-sequence star.



Figure 3.4: The canonical formation scenario as it has been illustrated by Sigurdsson & Thorsett (2005) (http://hubblesite.org).

After that point, the main sequence star started to evolve. It passed from the

main sequence state to a red giant. The transformation of its mass resulted in the recycling of the pulsar for the second time. The orbit circularized and expanded. The RGB phase has not been completed, so the mass of the resulting helium white dwarf was lower than its progenitor.

The planet started to orbit the main sequence star and after the exchange it remained in the system. A triple system was formed (Figure 3.4).

The eccentricity of the planet orbit that this scenario predicts is \simeq 0.3-0.7, and the inclination is high.

Other scenarios have been proposed so as to explain the origin of the triple system. In a double exchange scenario (Rasio et al 1995) two exchanges took place, a white dwarf-main-sequence star (as above) and a planet exchange from another system. But, the eccentricity of the planet put a limit to the order of events. The low eccentricity indicates that the orbit circularized after the exchange. We also know that the white dwarf formed recently, thus the planet must have become a part of the system before the evolution of the white dwarf in order for the planetary orbit to have the observed eccentricity.

In other scenarios, the planet formed in site. The accretion of the disk around the inner system may have led to the creation of the planet. We already know that no helium burning took place, thus the essential materials for that formation were not abundant.

Chapter 4

Data analysis

4.1 Observations

All of our timing observations were obtained with the 'Effelsberg 100-m Radio Telescope of the Max-Planck-Institute for Radioastronomy (MPIfR), Bonn, Germany' at 21 cm (1410 MHz and 1360 MHz). At this frequency this kind of timing observations were made once a month. A cooled High-Electron-Mobility Transistor (HEMT) receiver was used with system temperatures from 30 to 40 K, depending on weather conditions and telescope elevation (Janssen et al., 2008).

The total used bandwidth was 100 MHz. For left-hand circular (LHC) and righthand circular (RHC) polarisation the band was split into four sub-bands each of which was subdivided into eight digitally sampled channels. The output signals of these 32 bands were fed into dedisperser boards for coherent on-line dedispersion and were synchronously folded with the topocentric period (Lazaridis et al., 2009). The Effelsberg - Berkeley Pulsar Processor (EBPP) that executed the coherent online dedispersion process is shown in Figure 4.1.

Initially, the arrival time of pulses is measured with an H-maser clock at the observatory and then it is converted to Coordinated Universal Time (UTC) using recorded maser offset information from Global Positioning System (GPS) satellites. In general timing observations consists of three single observation scans each of which lasts from 5 to 15 min. Thus, every session provides us with three TOAs for the pulsar. The procedure that we will follow in order to obtain TOAs is presented in detail in the next sections but in few words, TOAs are produced by the cross correlation of the average profiles of each scan with a high signal-to-noise (S/N) ratio template, made with a Gaussian fitting method.

The observations of PSR J1623-2631 lasted from 1999 to 2011 in irregular intervals. On a total of 60 days of observations 358 scans have been gathered. As a result in 60 days of PSR J1623-2631 observations 358 TOAs have been obtained.



EBPP Coherent De-dispersing Backend

Figure 4.1: Pulsar coherence dedispersion with the Effelsberg - Berkeley Pulsar Processor (EBPP). (http://www.astron.nl)

4.2 Data cleaning techniques

PSR J1623-2631 is very faint. The mean signal to noise ratio of our 358 observations is 4.45 ± 1.17 . We can increase the intensity and minimize the error of TOA observations by reducing the Radio Frequency Interference (RFI) and summing observations.

A major problem in radio astronomical observations is RFI. The source of RFI is of human (radio stations, TV stations, radars, cell phones, electric power transmission lines), terrestrial (lightnings) and cosmic (Sun) origin. In some cases the telescope itself causes RFI.

In order to minimize the effect of RFI we check each of the 32 frequency channels using $PSRCHIVE^1$ (van Straten, 2012). If the intensity of interference in one or more channels is strong enough to affect the S/N ratio and consequently the TOA error (Figure 4.2) we smooth the flux of the affected channels by assigning them a zero weight.

As we have seen above, TOA uncertainties depend on the added number of

¹PSRCHIVE is an open-source, data analysis software library written in C++ for analysis of pulsar data.

pulses, Equation (2.1).

$$\sigma_{TOA} \propto \sqrt{1/N_{pulses}}$$

By adding more that one pulse, we can increase the arrival time precision. In our analysis we sum the observations that were being held at the same day provided that the integration time is not longer than 50 min. Our TOA observations decreased from 358 to 61.



Figure 4.2: Channel intensity amplitude against frequency and phase. The problematic channels that have to be removed are 24 to 30. After having assigned zero weight to these channels, the S/N ratio has been improved (S/N = 13.132).

Finally, we scrunch the data in frequency, time and polarization (FTp) and dedisperse them (D). We have to combine the data in that fashion in order for the pulse arrival times to be compared properly.

4.3 Template

In order for the TOAs to be created a pulsar template is required. We choose scrunched and free of RFI data with the highest S/N ratio, observed at the same frequency. Specifically, data with S/N ratio higher than 15 have been used. Thereafter, we align the chosen data in phase. The summed profile is shown in Figure 4.3 with S/N ratio equal to 70.055.

The PSRCHIVE package is used for the PSR J1623-2631 template to be created. PSRCHIVE fits the summed data with multiple Gaussian functions, trying to create a template that matches our observations.



Figure 4.3: The profile that emerges after summing the observations with S/N higher than 15. The template that matches better to our observations was created with PSRCHIVE.



Figure 4.4: The integrated profile of PSR J1623-2631 at 1460 MHz (EPN (http://www.jb.man.ac.uk)).

The output noise-free template that we create together with our observational integrated profile are shown in Figure 4.3. In Figure 4.4 we present an Effelsberg integrated profile of PSR J1623-2631 at 1460 MHz. The profile of this pulsar consists of 1 prominent and two minor components. In our data we are able to observe only the prominent component.

In Appendix A we present the PSRCHIVE commands that we used in order to clean RFI, scrunch the data and create a pulsar template.

4.4 Time of arrivals (TOAs)

We have described above the process that we follow in order to create a template. Each observed profile (p(t)) is related to the noise-free template (t(t)) by the equation:

$$p(t) = a + bt(t - \tau) + n(t)$$
(4.1)

where a and b are constants, n(t) is noise and τ is the time shift between the observed profile and the template. The time shift should be calculated with the greatest accuracy so that TOAs will be defined properly.

The high S/N template is cross correlated with each observation profile, with respect to a fiducial point, so that the TOAs will be obtained. For determining the time shift (τ), PSRCHIVE uses a technique which is called *Fourier Phase Gradient (PGS)* (Taylor, 1992). PGS is a frequency domain method of discrete Fourier transforms of observed profile and template. Using the 'shift theorem', template and observed profiles differ by a linear phase gradient. Minimizing the goodness-of-fit (χ^2) between the two profiles we can obtain the time shift τ and the constant *b*.

Our purpose is to obtain TOAs with the minimum possible error. We check the error before and after removing RFI. We choose the .tim file which provides, us with the minimum error of TOAs.

With PSRCHIVE we can select the format that the output file (.tim file) containing the TOAs will have. In our case we choose a TEMPO and TEMPO2² format. Next, we present a part of our resulted TOAs in TEMPO2 format (Table 4.1). As we can see the .tim file contains the names of observation files, the frequency at which the observations were done, the TOAs at the telescope (SAT), the error of SAT and finally the telescope ID.

Our .tim file contains also time corrections. The 'TIME' command refers to telescope time corrections. Their values vary from 1 to 60 s (e.g. TIME +1, +60, ...). We can identify the value of time corrections from timing residuals. The distribution of TOA errors has a Gaussian form. Residual values outside Gaussian distribution should be checked for time corrections. We execute trials with different time corrections until we find the one that gives the best rms and χ^2 values. In the next section we will visualize these trials.

²TEMPO and TEMPO2 are pulsar timing softwares. The TEMPO package was developed by the Australia Telescope National Facilities and Pricenton University and is written in fortran. The TEMPO2 package is re-written in C++ based on the TEMPO code (Hobbs et al., 2006).

Filename	f (MHz)	Site arrival time (SAT) (MJD)	satErr (us)	tel ID
FORMAT 1				
51988.FTDp	1410.000	51988.091921525404428	7.892	g
52046.FTDp	1410.000	52046.946018643352633	11.902	g
52101.FTDp	1410.000	52101.831713062243743	4.903	g
52134.FTDp	1410.000	52134.755301152997495	6.705	g
52306.FTDp	1410.000	52306.223726973795250	5.179	g
52394.FTDp	1410.000	52394.009433044214005	3.368	g
52436.FTDp	1410.000	52436.888611296422547	3.984	g
52481.FTDp	1410.000	52481.804108963798036	4.742	g
52538.FTDp	1410.000	52538.598055703903714	8.277	g
52540.FTDp	1410.000	52540.634780226928239	10.792	g
TIME +1				
52598.FTDp	1410.000	52598.476990924483392	6.643	g
TIME -1				
52728.FTDp	1410.000	52728.131574190149247	11.551	g

Table 4.1: Part of file that contains our TOAs in TEMPO2 format.

Generally, a .tim file can also contain jump corrections and other time corrections.

4.5 Timing residuals

As we have mentioned in a previous chapter, timing residuals are equal to

$$R_i = \frac{\phi_i - N_i}{\nu},$$

where 'i' refers to the *i*'th observation, ϕ_i is the pulse phase of *i*'th observation, N_i is the nearest integer to ϕ_i and ν is the pulsar frequency in the reference epoch.

The pulse phase of the i'th observation is

$$\phi_i = \left[(bbat - t_{pepoch}) + (T_p - t_{SSB}) \right] \cdot \nu - \left[(int)bbat - (int)t_{pepoch} \right] \cdot (int)\nu + \sum_{n \ge 2} \frac{\nu^{(n-1)}}{n!} \left[(bbat - t_{pepoch}) + (T_p - t_{SSB}) \right]^n,$$

where bbat is the binary barycentric arrival time, t_{pepoch} is the reference epoch in which $\phi(0) = 0$, t_{SSB} is the arrival time at SSB, T_p is the pulse emission time calculated theoretically, ν_0 is the pulsar rotation frequency and $\nu^{(n-1)}$ is the pulsar rotation frequency derivatives.

From residuals we test the validity of the theoretical model with which we try to describe the characteristics of our pulsar system.

We obtained the timing residuals using the TEMPO2 package. A short description of the procedure that TEMPO2 used to calculate the residuals is shown in Appendix B.

We fit our system with two models, a single Keplerian model (BT) and double Keplerian model (BT1P).

4.5.1 Single Keplerian model (BT)

The first model that we try to describe our system with is a single Keplerian model (Blandford & Teukolsky, 1976) with only the first derivative of the rotation frequency. In this model we assume that we only have one orbit (the orbit of the pulsar) while the white dwarf and the planet are being considered as a single body. In the parameters of the pulsar's orbit, secular changes and redshift/time-dilation will be included.

Next we will describe the exact procedure that we follow in order to calculate the theoretical emission time of the pulsar's signal. First of all, we solve Kepler's equation

$$E(t) - esinE(t) = M(t), \qquad (4.2)$$

where E is the eccentric anomaly, M is the mean anomaly and e the eccentricity. The eccentric anomaly is the angular distance from periastron of a fictitious pulsar that would have been located in a point of a circular orbit, with radius α_p (semi-major axis) that has vertical distance from the real position of the pulsar on the ellipse. The mean anomaly M is the angular distance from periastron which a fictitious pulsar would have if it moved on the circle of radius α_p (semi-major axis) with a constant angular velocity and with the same orbital period P_p as the real pulsar moving on the ellipse. In other words, mean anomaly is the time that has passed since the last pass of the pulsar from periastron:

$$M(t_{bat}) = 2\pi \left[\frac{t_{bat} - T_0}{P_p} - \frac{\dot{P}_p}{2} \left(\frac{t_{bat} - T_0}{P_p} \right)^2 \right].$$
 (4.3)

In the case that we have a relativistic system we add a term for orbital changes due to gravitational wave emission (\dot{P}_p^{GR}):

$$M(t_{bat}) = 2\pi \left[\frac{t_{bat} - T_0}{P_p} - \frac{\dot{P}_p - \dot{P}_p^{GR}}{2} \left(\frac{t_{bat} - T_0}{P_p} \right)^2 \right],$$
(4.4)

where t_{bat} is the arrival time in the barycenter of the orbit, T_0 is the time of periastron passage of the pulsar (it is produced by us) and P_p is the orbital period of the pulsar.

To all the other orbital parameters we add linear variations caused by the other bodies of the system. Thus,

$$e = e + \dot{e} (t_{bat} - T_0),$$
 (4.5)

$$x = x + \dot{x} \ (t_{bat} - T_0), \tag{4.6}$$

$$\omega = \omega + \dot{\omega} \ (t_{bat} - T_0), \tag{4.7}$$

where e is the eccentricity, x is the projected semimajor axis and ω is the argument of periastron.

We apply a Newton-Raphson numerical method with accuracy 10^{-12} in Kepler's equation and we find the roots of the eccentric anomaly

$$E_1 = E_0 - \frac{f(E_0)}{f'(E_0)}.$$
(4.8)

As we have seen in the theoretical introduction the emission time of the pulsar (T_p) as is calculated by applying the Blandford and Teukolsky timing model that it is equal to

$$T_p = t_{SSB} - \alpha_p (\cos E - e) - (\beta + \gamma) \sin E \qquad (4.9)$$
$$- \frac{(\alpha_p \sin E - \beta \cos E) [\alpha_p (\cos E - e) + (\beta + \gamma) \sin E']}{P_b (1 - e \cos E)},$$

where $\beta = \sqrt{1 - e^2} x \cos \omega$ and γ is the Lorentz factor for the time dilation and the gravitational redshift.

After fitting our parameters to this single Keplerian model we obtain the parameter file that is shown in Table 4.2. The parameter file and the TOAs file are the two files that TEMPO and TEMPO2 need to calculate the timing residuals.

Table 4.2: Timing parameters (.par file) of PSR J1623-2631 after fitting with the BT model (TEMPO format).

Fit and dataset				
Pulsar name	J1623-2631			
MJD range	51396.8 - 55822.7			
Number of TOAs	56			
Measured Quantities				
Pulse frequency, ν (s ⁻¹)	90.28733048(3)			
First derivative of pulse frequency, $\dot{\nu}$ (s ⁻²)	$2.21(7) \times 10^{-15}$			
Set Quantities				
Right ascension, α	16:23:38.21774700			
Declination, δ	-26:31:54.0821600			
Epoch of frequency determination (MJD)	48725			
Dispersion measure, DM (cm ^{-3} pc)	62.8633			
Proper motion in right ascension, μ_{lpha} (mas yr ⁻¹)	-7.9885			
Proper motion in declination, μ_{δ} (mas yr ⁻¹)	5.035			
Orbital period, P_b (d)	191.443			
Epoch of periastron, T_0 (MJD)	48728.3			
Projected semi-major axis of orbit, x (lt-s)	64.8095			
Longitude of periastron, ω_0 (deg)	117.128			
Orbital eccentricity, <i>e</i>	0.0253154			
First derivative of orbital period, \dot{P}_b	-9.8674×10^{-10}			
First derivative of x , \dot{x} (10 ⁻¹²)	-7.00588×10^{-13}			
Assumptions				
Clock correction procedure	UTC(NIST)			
Solar system ephemeris model	DE405			
Binary model	BT			
Model version number	5.00			

Note: Figures in parentheses are the nominal 1σ TEMPO2 uncertainties in the least-significant digits quoted.

The least squares process that is followed from TEMPO and TEMPO2 has been presented in the theoretical introduction. In this part we use TEMPO for our analysis due to the fact that after fitting our data they loose phase connection and only



Figure 4.5: The timing residuals after fitting our data with the single Keplerian model (BT) with only the first derivative of rotation frequency. We observe the characteristic cubic form that both Backer et al. (1993) and Thorsett et al. (1999) observed.

TEMPO has the right plugin which minimizes this effect. The output residuals are presented in Figure 4.5. The cubic form of the residuals is clear. This behavior has been noticed from Backer et al. (1993) and Thorsett et al. (1999). One possible explanation is that these fluctuations are random and similar behavior can be observed in young pulsars (Cordes, 1993). It is called red timing noise and is caused by stochastic interactions between the crust of the young pulsar and the superfluid vortices in the interior (Thorsett et al., 1999). The first problem of this suggestion is that PSR J1623-2631 is a millisecond pulsar and not a young pulsar. The second problem is that the phase of these pulsars varies several milliseconds in a period of a few weeks. The most viable explanation is that the system has one or more other bodies apart from the pulsar and one companion.

Multi frequency single Keplerian model

The BT model (single Keplerian model) with only the first frequency derivative purely fits our data. The easier way to force the model to converge to our data is to add more frequency derivatives in our analysis. Frequency derivatives add polynomial terms in residuals

$$\phi(t) = \phi_0 + \sum_{n \ge 1} \frac{\nu^{(n-1)}}{n!} \, (T_p - t_{epoch})^n.$$

This method has been previously used by Thorsett et al. (1993) and Joshi & Rasio (1997) in order to analyze timing residuals of PSR J1623-2631. The resulting residuals are shown in Figure 4.6. The quality of TOAs is indicated by their error

distribution and root mean square value. The TOA error should follow a Gaussian distribution around zero. Our results are presented in Figure 4.7. As we can see only a few TOAs are not compatible with this theory. In the rest of our analysis we are going to exclude these problematic data. Finally, after removal of these data the root mean square value is comparable with the error of our timing residuals and the TOA error distribution is Gaussian (Figures 4.8 and 4.9).



Figure 4.6: Multi frequency timing residuals of PSR J1623-2631 through time after fitting with the single Keplerian model and the TEMPO2 package.

After following the previously described procedure we obtained the parameter file that is presented in Table 4.3 (TEMPO2 format).

4.5.2 Double Keplerian model (BT1P model)

Next we will examine the triple system hypothesis. For our analysis we are going to use a double Keplerian model. We assume that the pulsar and the white dwarf orbit about their common center of mass (first orbit). This inner binary orbits the planet about their common center of mass in a second Keplerian orbit. Inside TEMPO this model is called BT1P. We were able to implement this model in TEMPO2 as well. In Appendix C the C++ code is cited. Next, we will present the procedure followed with this model in order to calculate the pulsar emission time and the residuals.

The first Keplerian orbit follows the Blandford & Teukolsky (1976) model as we have described it in the previous section with linear variations in orbital parameters, Equations (4.5), (4.6) and (4.7). At first, we calculate the pulsar emission time for the first orbit, Equation (4.10). In contrast to first orbit, we do not apply



Figure 4.7: TOA errors against residuals. The Gaussian distribution and problematic TOAs are obvious.



Figure 4.8: Multi frequency timing residuals of PSR J1623-2631 through time after fitting with the single Keplerian model and removing problematic data.

1620–26 (Wrms = 50.844 μ s) post-fit



Figure 4.9: TOA errors against residuals without problematic data. The data follow a Gaussian distribution.

linear variations in the second keplerian orbit. Thus, the mean anomaly is equal to:

$$M(t_{bat}) = 2\pi \left(\frac{t_{bat} - T_0}{P_p}\right).$$
(4.10)

In the timing parameter file we produce the exact range of values that the eccentricity (e), the projected semimajor axis (x) and the argument of periastron (ω) of the second orbit will take. The resulting pulsar emission T_p time is added to the one that is calculated for the first orbit:

$$T_p = T_{p \ first \ orbit} + T_{p \ second \ orbit}$$
.

In addition, we identify the appropriate second orbit parameters that minimize the timing residuals while their mean square value is comparable with the data error. In order to calculate the orbital characteristics of the second Keplerian orbit we applied a *brute force search method* in four and two orbital parameters of the second orbit, respectively. In both cases we set an initial assumption for the eccentricity. We examined six different possible orbits, from circular to elliptical with eccentricity equal to 0.5 with a step of 0.1.

First, we apply a brute force method in four second-orbit parameters, orbital period, projected semimajor axis, argument of periastron and epoch of periastron of the second orbit. In the single orbit, we model the parameters only with a first order rotational frequency derivative while we add the above parameters and the eccentricity of the second orbit. During the procedure we faced two problems:

Fit and dataset					
Pulsar name	J1623-2631				
MJD range	51396.8 - 55822.7				
Number of TOAs	56				
Rms timing residual (μs)	11.9				
Weighted fit	Y				
Reduced χ^2 value	6.1				
Measured Quantities					
Right ascension, α	16:23:38.2169(15)				
Declination, δ	-26:31:54.09(11)				
Pulse frequency, ν (s ⁻¹)	90.287330367(5)				
First derivative of pulse frequency, $\dot{\nu}$ (s ⁻²)	$-2.552(10) \times 10^{-15}$				
Second derivative of pulse frequency, $\ddot{\nu}$ (s ⁻³)	$88(15) \times 10^{-24}$				
F3 (s ⁻⁴)	1.677(15)×10 ⁻³¹				
F4 (s ⁻⁵)	$88(9) \times 10^{-40}$				
F5 (s ⁻⁶)	$1.3(3) \times 10^{-48}$				
Proper motion in right ascension, μ_{α} (mas yr ⁻¹)	-7.1(17)				
Proper motion in declination, μ_{δ} (mas yr ⁻¹)	7(10)				
Orbital period, P_b (d)	191.442830(3)				
Epoch of periastron, T_0 (MJD)	48728.26191(13)				
Projected semi-major axis of orbit, x (lt-s)	64.80500(3)				
Longitude of periastron, ω_0 (deg)	117.1800(3)				
Orbital eccentricity, <i>e</i>	0.02531543(13)				
First derivative of orbital period, \dot{P}_b	$33(52) \times 10^{-10}$				
First derivative of x , \dot{x} (10 ⁻¹²)	$99(6) \times 10^{-13}$				
Set Quantities					
Epoch of frequency determination (MJD)	48725				
Dispersion measure, DM (cm ^{-3} pc)	62.8633				
Derived Quantities					
\log_{10} (Characteristic age, yr)	8.75				
$\log_{10}(\text{Surface magnetic field strength, G})$	9.27				
Assumptions					
Clock correction procedure	TT(TAI)				
Solar system ephemeris model	DE405				
Binary model	BT				
Model version number	5.00				

Table 4.3: Timing parameters (.par file) of PSR J1623-2631 after fitting with the BT model (TEMP02 format).

Note: Figures in parentheses are the nominal 1σ TEMPO2 uncertainties in the least-significant digits quoted.

computational time and small number of observations. Four nested loops of second orbit parameters are executed in order to accomplish all possible combinations of values. We fit only for orbital frequency and its first derivative ($\dot{\nu}$). Necessarily, due to computational time, we apply big value steps which affect our results negatively. None of our results offer satisfactory root mean square values.

Our system sensitivity to small value changes forced us to minimize the nested loops from four to two and change manually only two of four second-orbit parameters. In order to increase the accuracy and accomplish acceptable results we apply much smaller steps than the ones previously used. We try different combinations of projected semimajor axis and argument of periastron. We let the TEMPO2 fitting procedure to find the appropriate values for the other orbital parameters, orbital period, epoch of periastron and orbital frequency with its first derivative ($\dot{\nu}$). Our test value is the root mean square. We repeat the fitting procedure with smaller steps in the region that provides us with the best rms values. The aforementioned procedure is executed both in the TEMPO and TEMPO2 timing packages.

In the next chapter we present the resulted second orbit parameters as they have been calculated after fitting with TEMPO and TEMPO2.

Chapter 5

Results

Our primary goal is to measure the orbital parameters of the second Keplerian orbit. All our analysis and results are based on the residuals that are obtained after least square fitting of the observations with the theoretical model. The packages that were used are TEMPO and TEMPO2.

Previously, only Thorsett et al. (1999) have presented timing solutions about the second Keplerian orbit. Assuming that the second orbit is either circular or elliptical with eccentricity equal to 0.2 and 0.5 and small magnetic pulsar field they arrived at the results presented in Tables 5.1 and 5.2.

Table 5.1: Thorsett et al. (1999) timing solutions assuming that the second orbit is circular.

Timing parameter	Value (error)
Spin period P (ms)	11.075750687 (5)
Spin frequency f (Hz)	90.28733386 (4)
$\dot{f}(s^{-2})$	$-4.836(10) \times 10^{-15}$
Epoch of $f(JD)$	2,448,725.5
Projected semimajor axis x (s)	6.4 (2)
Orbital period P_b (yr)	61.8 (7)
Time of ascending node T_0 (JD)	2,449,104 (5)
Mass function (M_{\odot})	5.6 (4) \times 10 ⁻¹⁰

TIMING SOLUTION FOR CIRCULAR OUTER ORBIT^a

^a Intrinsic spin frequency derivatives beyond the first are assumed to vanish.

We extend our analysis to elliptical orbits with eccentricity of 0.1, 0.3 and 0.4. Our results are presented in Tables 5.3 and 5.4, and analytically in Tables 5.5 and 5.6. It is obvious that they are in a good agreement with Thorsett's.

The differences between TEMPO and TEMPO2 are expected due to improved features that TEMPO2 has over TEMPO. Differences between these packages are presented in Hobbs et al. (2006) and Edwards et al. (2006). In particular the fitting procedure of TEMPO2 includes secular motion of the pulsar and parallax terms if Table 5.2: Thorsett et al. (1999) timing solutions assuming that the second orbit is elliptical.

Binary parameter	Value (error)
Eccentricity $e = 0.20$	
Projected semimajor axis x (s) Orbital period P_{x} (vr)	30.4 (1.1) 129 (2)
Argument of periastron (deg)	283.9 (9)
Epoch of periastron T_0 (JD)	2,445,156 (12)
Mass function (M_{\odot})	$1.36(10) \times 10^{-8}$ $34(1) \times 10^{-3}$
Relative semimajor axis ^b a_b (AU)	30(2)
Eccentricity $e = 0.50$	
Projected semimajor axis x (s)	126(4)
Orbital period P_b (yr)	389 (5)
Argument of periastron (deg)	313.4 (5)
Epoch of periastron T_0 (JD)	2,446,624 (10)
Mass function (M_{\odot})	$1.06(8) \times 10^{-7}$
Projected mass ^a $m_3 \sin i_b (M_{\odot}) \dots$	$6.7(2) \times 10^{-3}$
Relative semimajor axis ^b a_b (AU)	64 (3)

REPRESENTATIVE SOLUTIONS FOR ELLIPTICAL OUTER ORBIT

^a Assuming inner binary mass $m_1 + m_2 = 1.7 M_{\odot}$. ^b The semimajor axis of the relative orbit, $a_b = a_{1b} + a_{2b}$, is nearly independent of sin i_b .

Table 5.3: TEMPO results

Eccentricity	0.0	0.1	0.2
Spin frequency (s^{-1})	90.287332943(4)	90.287330832(3)	90.287329259(5)
First frequency derivate ($ imes 10^{-15} s^{-2}$)	1.411(1)	4.281(6.9)	9.214(6.6)
Projected semimajor axis (s)	8.6	4.55(5)	20.8
Orbital period (days)	25516(2.08)	19095.2(9)	33814(1.7)
Argument of periastron (deg)	0.0	209.3 209.8 a	209.1
Epoch of periastron (MJD)	53097(7.1)	47342(4)	46504(4.5)
root mean square (rms)	29.63	20.27	20.80
Eccentricity	0.3	0.4	0.5
Spin frequency (s^{-1})	90.28732435(5)	90.28731939(7)	90.2873193(1)
First frequency derivate ($ imes 10^{-15} s^{-2}$)	14.6(2)	17.55(5)	16.25(5)
Projected semimajor axis (s)	40.1(3)	83.8	134.5(4)
Orbital period (days)	45310(10)	54200(200)	93800(200)
Argument of periastron (deg)	167(1)	162.9(5)	186.3(4)
Epoch of periastron (MJD)	37210(400)	44740(10)	45220(20)
root mean square (rms)	20.314	20.526	20.775

^aTwo discrete values

we have a binary system. Also, TEMPO2 includes atmospheric propagation delays and Shapiro delays not only from the Sun but also from major planets.

As we can see in TEMP02 the final rms values are between ~ 16 and ~ 25 calculated with TEMP02. In comparison, the rms values of the multi frequency single Keplerian orbit model is 11.9. Most likely, this is due to the fact that the second Keplerian orbit has an eccentricity between 0.1 and 0.2.

The projected semi major axis and the orbital period increase gradually as eccentricity increases. Furthermore, the pulsar spin frequency is measured to an accuracy of the fourth significant digit. The reason is that we have only 56 observations that makes the fitting procedure vulnerable to small changes.

The fact that the span of our observations covers only a small fraction of the period of the outer orbit sets constrains to our results.

Eccentricity	0.0	0.1	0.2
Spin frequency (s^{-1})	90.287331540(3.6)	90.287329495(3)	90.287322784(4.7)
First frequency derivate ($\times 10^{-15} s^{-2}$)	1.417	4.546(5.7)	9.237(2)
Projected semimajor axis (s)	9.0	5.6	20.9
Orbital period (days)	25517(1.7)	20820.7(87)	33847(8)
Argument of periastron (deg)	0.0	214.8	208.95(5)
Epoch of periastron (MJD)	53101(6)	47213(3.7)	46490(2)
root mean square (rms)	25.451	16.827	16.813
Eccentricity	0.3	0.4	0.5
Spin frequency (s^{-1})	90.28732412(2)	90.287320(1)	90.287319933(6.6)
First frequency derivate ($\times 10^{-15} s^{-2}$)	13.57(2)	15.2(1)	15.354(5)
Projected semimajor axis (s)	44.75(5)	83.5 83.6 83.8 ^a	132.0
Orbital period (days)	44100(100)	64500(500)	100847(4)
Argument of periastron (deg)	186.4(3)	188(1.5)	198.0
Epoch of periastron (MJD)	45565(5)	45360(30)	45596(3.7)
root mean square (rms)	16.878	17.027	17.186

Table 5.4: TEMPO2 results

^aThree discrete values

The procedure developed during this work works well and taking into account the limited number of data is sufficiently accurate. If further data become available they can be easily incorporated in the analysis and the orbital parameters will be more accurately established.

\mathbf{a}_n sin i (s)	v (s ⁻¹)	$\dot{v} \times 10^{-15} s^{-2}$	\mathbf{P}_n (days)	ω (deg)	T ₀ (MJD)	
Fccentricity 0.0						
86	00.287222042(4)		25516(2)	0.0	52007(7)	
0.0	90.287332943(4)		25510(2)	0.0	55097(7)	
	1	Eccentricity 0	.1	1		
4.5	90.287330832(3)	4.281(6.9)	19095.2(9)	209.3	47342(4)	
4.6	90.287330832(3)	4.281(6.9)	19259.4(9)	209.8	47326(4)	
		Eccentricity 0	.2			
20.8	90.287329259(6)	9.214(6.6)	33814(1.7)	209.1	46504(4)	
	•	Eccentricity 0	.3	-		
39.8	90.287324315(8)	14.834(6)	36901(1.9)	165.9	45307(4)	
	90.287324328(8)	14.482(6)	36987(1.9)	166.1	45305(4)	
	90.287324334(8)	14.481(6)	37015(1.9)	166.2	45308(4)	
39.9	90.287324341(8)	14.480(6)	37043(1.9)	166.3	45310(4)	
	90.287324348(8)	14.479(6)	37070(1.9)	166.4	45313(4)	
	90.287324342(8)	14.480(6)	37102(1.9)	166.4	45306(4)	
	90.287324349(8)	14.479(6)	37129(1.9)	166.5	45309(4)	
40.0	90.287324355(8)	14.478(6)	37157(1.9)	166.6	45311(4)	
	90.287324362(8)	14.477(6)	37185(1.9)	166.7	45314(4)	
	90.287324356(8)	14.789(6)	37216(1.9)	166.7	45307(4)	
	90.287324363(8)	14.780(6)	37244(1.9)	166.8	45310(4)	
40.1	90.287324370(8)	14.771(6)	37272(1.9)	166.9	45312(4)	
	90.287324376(8)	14.761(6)	37299(1.9)	167.0	45315(4)	
	90.287324383(8)	14.752(6)	37327(1.9)	167.1	45318(4)	
	90.287324378(8)	14.762(6)	37358(1.9)	167.1	45311(4)	
40.2	90.287324384(8)	14.753(6)	37386(1.9)	167.2	45314(4)	
40.2	90.287324391(8)	14.743(6)	37414(1.9)	167.3	45316(4)	
	90.287324398(8)	14.734(6)	37442(1.9)	167.4	45319(4)	
	90.287324392(8)	14.744(6)	37473(1.9)	167.4	45312(4)	
40.3	90.287324399(8)	14.735(6)	37501(1.9)	167.5	45315(4)	
	90.287324406(8)	14.726(6)	37528(1.9)	167.6	45318(4)	
	90.287324412(8)	14.716(6)	37556(1.9)	167.7	45320(4)	
40.4	90.287324412(8)	14.708(6)	37643(1.9)	167.9	45319(4)	
	90.287324427(8)	14.469(6)	37671(1.9)	168.0	45321(4)	
		Eccentricity 0	.4			
	90.287319328(9)	17.620(6)	53975(2.8)	162.3	44730(4)	
	90.287319353(9)	17.599(6)	54067(2.8)	162.5	44734(4)	
	90.287319366(9)	17.589(6)	54113(2.8)	162.6	44736(4)	
	90.287319378(9)	17.578(6)	54158(2.8)	162.7	44738(4)	
	90.287319391(9)	17.568(6)	54204(2.8)	162.8	44740(4)	
83.8	90.287319404(9)	17.557(6)	54250(2.8)	162.9	44742(4)	
	90.287319416(9)	17.547(6)	54296(2.8)	163.0	44744(4)	
	90.287319429(9)	17.536(6)	54341(2.8)	163.1	44746(4)	
	90.287319442(9)	17.526(6)	54387(2.8)	163.2	44749(4)	
	90.287319455(9)	17.515(6)	54433(2.8)	163.3	44751(4)	
	90.287319467(9)	17.505(6)	54478(2.8)	163.4	44753(4)	
	1	Eccentricity 0	.5	1	1	
134.1	90.287319427(8)	16.216(6)	94184(4.9)	186.7	45243(4.5)	
134.2	90.287319407(8)	16.226(6)	94142(4.9)	186.6	45238(4.5)	
134.3	90.287319387(8)	16.237(6)	94101(4.9)	186.5	45234(4.5)	
134.4	90.287319366(8)	16.247(6)	94059(4.9)	186.4	45229(4.5)	
134.5	90.287319346(8)	16.258(6)	94017(4.9)	186.3	45225(4.5)	

Table 5.5: TEMPO results

134.6	90.287319309(8)	16.276(6)	93910(4.9)	186.1	45217(4.5)
134.7	90.287319289(8)	16.286(6)	93869(4.9)	186.0	45213(4.5)
134.8	90.287319252(8)	16.304(6)	93761(4.9)	185.8	45206(4.5)
134.9	90.287319215(8)	16.322(6)	93654(4.9)	185.6	45198(4.5)

Table 5.6: TEMPO2 results

\mathbf{a}_p sin i (s)	v (s ⁻¹)	$\dot{v} \times 10^{-15} s^{-2}$	\mathbf{P}_p (days)	ω (deg)	\mathbf{T}_0 (MJD)
Eccentricity 0.0					
0.0	90.287331540(3.6)	1.417(9)	25517(1.7)	0.0	53101(6)
		Eccentricity 0	.1		
5.6	90.287329495(3)	4.546(5.7)	20820.7(87)	214.8	47213(3.7)
		Eccentricity 0	.2	•	
20.0	90.287322784(4.7)	9.239(5)	33840(1)	208.9	46488(3.6)
20.9	90.287322785(4.7)	9.235(5)	33855(1)	209.0	46492(3.7)
		Eccentricity 0	.3		
	90.287324108(6)	13.593(5)	44032(1.9)	186.0	45560(3.7)
44.7	90.287324115(6)	13.585(5)	44060(1.9)	186.1	45563(3.6)
	90.287324123(6)	13.578(5)	44087(1.9)	186.2	45566(3.7)
44.0	90.287324149(6)	13.555(5)	44230(1.9)	186.6	45572(3.7)
44.8	90.287324149(6)	13.555(5)	44230(1.9)	186.7	45572(3.7)
		Eccentricity 0	.4		
	90.287321052(6)	15.356(5)	64959(2.8)	187.2	45337(3.7)
	90.287321065(6)	15.349(5)	65002(2.8)	187.3	45340(3.7)
	90.287321077(6)	15.341(5)	65045(2.8)	187.4	45343(3.7)
	90.287321089(6)	15.333(5)	65088(2.8)	187.5	45345(3.7)
	90.287321102(7)	15.325(5)	65131(2.8)	187.6	45348(3.7)
	90.287321114(7)	15.317(5)	65173(2.8)	187.7	45351(3.7)
	90.287321127(7)	15.309(5)	65216(2.8)	187.8	45354(3.7)
	90.287321139(7)	15.301(5)	65259(2.8)	187.9	45356(3.7)
	90.287321151(7)	15.293(5)	65302(2.8)	188.0	45359(3.7)
	90.287321164(7)	15.285(5)	65345(2.8)	188.1	45362(3.7)
	90.287321176(7)	15.277(5)	65388(2.8)	188.2	45364(3.7)
83.8	90.287321188(7)	15.270(5)	65430(2.8)	188.3	45367(3.7)
00.0	90.287321201(7)	15.262(5)	65473(2.8)	188.4	45370(3,7)
	90.287321213(7)	15.254(5)	65516(2.8)	188.5	45373(3.7)
	90.287321225(7)	15.246(5)	65559(2.8)	188.6	45375(3.7)
	90.287321238(7)	15.238(5)	65601(2.8)	188.7	45378(3.7)
	90.287321250(7)	15.230(5)	65644(2.8)	188.8	45381(3.7)
	90.287321262(7)	15.223(5)	65687(2.8)	188.9	45384(3.7)
	90.287321275(7)	15.215(5)	65730(2.8)	189.0	45386(3.7)
	90.287321287(7)	15.207(5)	65772(2.8)	189.1	45389(3.7)
	90.287321299(7)	15.199(5)	65815(2.8)	189.2	45392(3.7)
	90.287321312(7)	15.191(5)	65858(2.8)	189.3	45395(3.7)
	90.287321324(7)	15.183(5)	65900(2.8)	189.4	45397(3.7)
	90.287321337(7)	15.176(5)	65943(2.8)	189.5	45400(3.7)
	90.287320974(7)	15.409(5)	64727(2.8)	186.6	45318(3.7)
	90.287320987(7)	15.401(5)	64770(2.8)	186.7	45320(3.7)
	90.287320999(7)	15.393(5)	64813(2.8)	186.8	45323(3.7)
	90.287321011(7)	15.385(5)	64856(2.8)	186.9	45326(3.7)
	90.287321024(7)	15.377(5)	64899(2.8)	187.0	45328(3.7)

90.287321036(7) 15.369(5) 64942(2.8) 187.1 45331(3.7) 90.287321048(7) 15.362(5) 64985(2.8) 187.2 45334(3.7) 90.287321073(7) 15.346(5) 65071(2.8) 187.4 45339(3.7) 90.287321085(7) 15.338(5) 65114(2.8) 187.5 45342(3.7) 90.287321098(7) 15.338(5) 65157(2.8) 187.6 45345(3.7) 90.28732113(7) 15.314(5) 65200(2.8) 187.7 45347(3.7) 90.287321147(7) 15.298(5) 65328(2.8) 188.0 45356(3.7) 90.287321147(7) 15.298(5) 65324(2.8) 188.1 45358(3.7) 90.287321147(7) 15.298(5) 65414(2.8) 188.2 45364(3.7) 90.287321197(7) 15.267(5) 65542(2.8) 188.5 45369(3.7) 90.287321217(7) 15.267(5) 65542(2.8) 188.5 45364(3.7) 90.287321247(7) 15.267(5) 65542(2.8) 188.5 45369(3.7) 90.287321247(7) 15.267(5) 65542(2.8) 188.5 45369(3.7)						
90.287321048(7) 15.362(5) 64985(2.8) 187.2 45334(3.7) 90.287321073(7) 15.346(5) 6507(12.8) 187.4 45339(3.7) 90.287321085(7) 15.330(5) 65114(2.8) 187.5 45345(3.7) 90.2873211007) 15.330(5) 65157(2.8) 187.6 45345(3.7) 90.287321130(7) 15.30(5) 65280(2.8) 187.7 45353(3.7) 90.287321135(7) 15.306(5) 65280(2.8) 187.9 45353(3.7) 90.287321147(7) 15.290(5) 65371(2.8) 188.0 45356(3.7) 90.287321147(7) 15.290(5) 65547(2.8) 188.1 45364(3.7) 90.287321147(7) 15.259(5) 65547(2.8) 188.4 45367(3.7) 90.287321209(7) 15.259(5) 65542(2.8) 188.4 45367(3.7) 90.287321221(7) 15.245(5) 65671(2.8) 188.4 45367(3.7) 90.287321247(7) 15.245(5) 65671(2.8) 188.7 45380(3.7) 90.287321247(7) 15.224(5) 6579(2.8) 188.1 45380(3.7)		90.287321036(7)	15.369(5)	64942(2.8)	187.1	45331(3.7)
90.287321061(7) 15.354(5) 65071(2.8) 187.3 45337(3.7) 90.287321073(7) 15.336(5) 65071(2.8) 187.4 45339(3.7) 90.287321085(7) 15.330(5) 65114(2.8) 187.5 45342(3.7) 90.287321120(7) 15.322(5) 65200(2.8) 187.7 45347(3.7) 90.287321123(7) 15.306(5) 65286(2.8) 187.9 45335(3.7) 90.287321147(7) 15.298(5) 65371(2.8) 188.1 45356(3.7) 90.287321160(7) 15.290(5) 655414(2.8) 188.1 45364(3.7) 90.287321197(7) 15.258(5) 65414(2.8) 188.1 45367(3.7) 90.28732120(7) 15.251(5) 65550(2.8) 188.4 45367(3.7) 90.28732120(7) 15.251(5) 65557(2.8) 188.5 45372(3.7) 90.28732120(7) 15.251(5) 65550(2.8) 188.5 45373(3.7) 90.287321234(7) 15.220(5) 6571(2.8) 188.9 4538(3.7) 90.287321236(7) 15.220(5) 65799(2.8) 189.1 45386(3.7)		90.287321048(7)	15.362(5)	64985(2.8)	187.2	45334(3.7)
90.287321073(7) 15.346(5) 65071(2.8) 187.4 45339(3.7) 90.287321085(7) 15.338(5) 65114(2.8) 187.5 45342(3.7) 90.287321110(7) 15.322(5) 65200(2.8) 187.7 45347(3.7) 90.287321133(7) 15.314(5) 65286(2.8) 187.9 45353(3.7) 90.287321135(7) 15.298(5) 65328(2.8) 188.0 45356(3.7) 90.287321136(7) 15.298(5) 65327(2.8) 188.1 45358(3.7) 90.287321172(7) 15.298(5) 65547(2.8) 188.4 45367(3.7) 90.287321197(7) 15.267(5) 65547(2.8) 188.4 45367(3.7) 90.287321209(7) 15.267(5) 65542(2.8) 188.4 4537(3.7) 90.287321234(7) 15.252(5) 655712(2.8) 188.7 45336(3.7) 90.287321234(7) 15.228(5) 65713(2.8) 188.9 4537(3.7) 90.28732123(7) 15.228(5) 65713(2.8) 188.9 4538(3.7) 90.28732123(7) 15.204(5) 6584(2.8) 189.1 4538(3.7) </td <td></td> <td>90.287321061(7)</td> <td>15.354(5)</td> <td>65028(2.8)</td> <td>187.3</td> <td>45337(3.7)</td>		90.287321061(7)	15.354(5)	65028(2.8)	187.3	45337(3.7)
90.287321085(7) 15.338(5) 65114(2.8) 187.5 45342(3.7) 90.287321109(7) 15.330(5) 65157(2.8) 187.6 45345(3.7) 90.287321123(7) 15.314(5) 65204(2.8) 187.9 45335(3.7) 90.287321135(7) 15.306(5) 65284(2.8) 187.9 45353(3.7) 90.287321147(7) 15.290(5) 65371(2.8) 188.1 45356(3.7) 90.287321147(7) 15.290(5) 6547(2.8) 188.0 45366(3.7) 90.287321197(7) 15.250(5) 65547(2.8) 188.4 45367(3.7) 90.28732120(7) 15.259(5) 65542(2.8) 188.4 4537(3.7) 90.287321234(7) 15.259(5) 65542(2.8) 188.4 4537(3.7) 90.287321246(7) 15.226(5) 6571(2.8) 188.4 4537(3.7) 90.287321247(7) 15.228(5) 6571(2.8) 188.9 4538(3.7) 90.287321230(7) 15.228(5) 65712(2.8) 189.1 45386(3.7) 90.287321230(7) 15.228(5) 65712(2.8) 188.9 45338(3.7) <td></td> <td>90.287321073(7)</td> <td>15.346(5)</td> <td>65071(2.8)</td> <td>187.4</td> <td>45339(3.7)</td>		90.287321073(7)	15.346(5)	65071(2.8)	187.4	45339(3.7)
90.287321098(7) 15.330(5) 65157(2.8) 187.6 45345(3.7) 90.287321123(7) 15.322(5) 65200(2.8) 187.7 45537(3.7) 90.287321135(7) 15.306(5) 65246(2.8) 187.9 45535(3.7) 90.287321160(7) 15.298(5) 65328(2.8) 188.0 45356(3.7) 90.287321160(7) 15.290(5) 65371(2.8) 188.1 45368(3.7) 90.28732118(7) 15.275(5) 65457(2.8) 188.2 45364(3.7) 90.287321209(7) 15.267(5) 65542(2.8) 188.4 45367(3.7) 90.287321209(7) 15.243(5) 65542(2.8) 188.6 4537(3.7) 90.28732123(7) 15.243(5) 65576(2.8) 188.6 4537(3.7) 90.28732124(7) 15.228(5) 65713(2.8) 188.9 45386(3.7) 90.28732128(7) 15.215(5) 65542(2.8) 189.0 45386(3.7) 90.28732128(7) 15.224(5) 65713(2.8) 188.9 45391(3.7) 90.28732128(7) 15.243(5) 65542(2.8) 189.0 45386(3.7) <td></td> <td>90.287321085(7)</td> <td>15.338(5)</td> <td>65114(2.8)</td> <td>187.5</td> <td>45342(3.7)</td>		90.287321085(7)	15.338(5)	65114(2.8)	187.5	45342(3.7)
90.28732110(7) 15.322(5) 65200(2.8) 187.7 45347(3.7) 90.287321133(7) 15.314(5) 65243(2.8) 187.8 45350(3.7) 90.287321135(7) 15.306(5) 65286(2.8) 187.9 45353(3.7) 90.287321147(7) 15.298(5) 65321(2.8) 188.0 45356(3.7) 90.28732117(7) 15.283(5) 65414(2.8) 188.1 45358(3.7) 90.287321197(7) 15.267(5) 65457(2.8) 188.3 45364(3.7) 90.287321209(7) 15.259(5) 65542(2.8) 188.4 45367(3.7) 90.287321221(7) 15.251(5) 65585(2.8) 188.4 45372(3.7) 90.287321224(7) 15.235(5) 656713(2.8) 188.7 45375(3.7) 90.28732124(7) 15.220(5) 657612.8) 188.9 45380(3.7) 90.287321259(7) 15.220(5) 6579(2.8) 189.0 45380(3.7) 90.28732126(7) 15.245(5) 6584(2.8) 189.1 45380(3.7) 90.28732103(7) 15.39(5) 65742(2.8) 189.3 45391(3.7) <td></td> <td>90.287321098(7)</td> <td>15.330(5)</td> <td>65157(2.8)</td> <td>187.6</td> <td>45345(3.7)</td>		90.287321098(7)	15.330(5)	65157(2.8)	187.6	45345(3.7)
90.287321123(7) 15.314(5) 65243(2.8) 187.8 45350(3.7) 90.287321135(7) 15.306(5) 65286(2.8) 188.0 45356(3.7) 90.287321147(7) 15.298(5) 65328(2.8) 188.1 45358(3.7) 90.287321172(7) 15.290(5) 65371(2.8) 188.1 45358(3.7) 90.287321172(7) 15.267(5) 65457(2.8) 188.3 45364(3.7) 90.287321197(7) 15.259(5) 65542(2.8) 188.4 45367(3.7) 90.287321221(7) 15.251(5) 65585(2.8) 188.4 45372(3.7) 90.287321234(7) 15.235(5) 65671(2.8) 188.9 45380(3.7) 90.287321246(7) 15.226(5) 65713(2.8) 188.9 45380(3.7) 90.287321259(7) 15.226(5) 65714(2.8) 188.9 45380(3.7) 90.287321283(7) 15.212(5) 6579(2.8) 189.0 45383(3.7) 90.28732103(7) 15.24(5) 65842(2.8) 189.1 45383(3.7) 90.28732103(7) 15.396(5) 65842(2.8) 189.1 45328(3.7)		90.287321110(7)	15.322(5)	65200(2.8)	187.7	45347(3.7)
90.287321135(7) 15.306(5) 65286(2.8) 187.9 45353(3.7) 90.287321147(7) 15.298(5) 65328(2.8) 188.0 45356(3.7) 90.287321160(7) 15.290(5) 65371(2.8) 188.1 45358(3.7) 90.287321172(7) 15.263(5) 65457(2.8) 188.3 45364(3.7) 90.287321209(7) 15.259(5) 65542(2.8) 188.4 45367(3.7) 90.287321221(7) 15.259(5) 655542(2.8) 188.4 45377(3.7) 90.28732124(7) 15.243(5) 65671(2.8) 188.7 45380(3.7) 90.287321246(7) 15.228(5) 6571(2.8) 188.9 45380(3.7) 90.287321259(7) 15.228(5) 6579(2.8) 189.0 45380(3.7) 90.287321271(7) 15.204(5) 6579(2.8) 189.0 45380(3.7) 90.28732128(7) 15.212(5) 6579(2.8) 189.0 45380(3.7) 90.28732103(7) 15.30(5) 64882(2.8) 189.3 45391(3.7) 90.28732103(7) 15.383(5) 64925(2.8) 187.0 45322(3.7) <td></td> <td>90.287321123(7)</td> <td>15.314(5)</td> <td>65243(2.8)</td> <td>187.8</td> <td>45350(3.7)</td>		90.287321123(7)	15.314(5)	65243(2.8)	187.8	45350(3.7)
90.287321147(7) 15.298(5) 65328(2.8) 188.0 45356(3.7) 90.287321160(7) 15.290(5) 65371(2.8) 188.1 45358(3.7) 90.287321172(7) 15.283(5) 65414(2.8) 188.2 45361(3.7) 90.287321197(7) 15.267(5) 65557(2.8) 188.4 45367(3.7) 90.287321209(7) 15.251(5) 65558(2.8) 188.6 45372(3.7) 90.287321221(7) 15.251(5) 65628(2.8) 188.6 45372(3.7) 90.287321246(7) 15.228(5) 6571(2.8) 188.8 45378(3.7) 90.287321259(7) 15.228(5) 65756(2.8) 188.9 45380(3.7) 90.287321283(7) 15.204(5) 65842(2.8) 189.0 45389(3.7) 90.287321283(7) 15.204(5) 65842(2.8) 189.0 45380(3.7) 90.28732103(7) 15.204(5) 65842(2.8) 189.3 45391(3.7) 90.28732103(7) 15.39(5) 6542(2.8) 187.0 45328(3.7) 90.28732103(7) 15.357(5) 6504(2.8) 187.4 45330(3.7) <td></td> <td>90.287321135(7)</td> <td>15.306(5)</td> <td>65286(2.8)</td> <td>187.9</td> <td>45353(3.7)</td>		90.287321135(7)	15.306(5)	65286(2.8)	187.9	45353(3.7)
90.287321160(7) 15.290(5) 65371(2.8) 188.1 45358(3.7) 90.287321172(7) 15.283(5) 65414(2.8) 188.2 45361(3.7) 90.287321197(7) 15.267(5) 65550(2.8) 188.4 45367(3.7) 90.287321209(7) 15.259(5) 65542(2.8) 188.5 45369(3.7) 90.287321209(7) 15.259(5) 65542(2.8) 188.6 45372(3.7) 90.287321234(7) 15.243(5) 65628(2.8) 188.7 45375(3.7) 90.287321259(7) 15.228(5) 6571(2.8) 188.9 45380(3.7) 90.287321259(7) 15.228(5) 6579(2.8) 189.0 45380(3.7) 90.287321259(7) 15.22(5) 6579(2.8) 189.0 45380(3.7) 90.287321296(7) 15.21(5) 6579(2.8) 189.1 45386(3.7) 90.287321007(7) 15.390(5) 64842(2.8) 189.2 45389(3.7) 90.28732103(7) 15.383(5) 6492(5.8) 187.1 45325(3.7) 90.28732103(7) 15.367(5) 65011(2.8) 187.4 45323(3.7)		90.287321147(7)	15.298(5)	65328(2.8)	188.0	45356(3.7)
90.287321172(7) 15.283(5) 65414(2.8) 188.2 45361(3.7) 90.287321184(7) 15.275(5) 65457(2.8) 188.3 45364(3.7) 90.287321209(7) 15.259(5) 65550(2.8) 188.4 45367(3.7) 90.287321221(7) 15.251(5) 65585(2.8) 188.6 45372(3.7) 90.287321234(7) 15.243(5) 65628(2.8) 188.7 45375(3.7) 90.287321259(7) 15.228(5) 6571(2.8) 188.9 45380(3.7) 90.287321234(7) 15.220(5) 65756(2.8) 188.9 45380(3.7) 90.287321296(7) 15.212(5) 65791(2.8) 189.0 45383(3.7) 90.287321296(7) 15.204(5) 65842(2.8) 189.1 45389(3.7) 90.287321030(7) 15.390(5) 64884(2.8) 189.1 45389(3.7) 90.287321020(7) 15.390(5) 64884(2.8) 189.1 45325(3.7) 90.28732103(7) 15.390(5) 64925(2.8) 187.1 45328(3.7) 90.28732103(7) 15.351(5) 6507(2.8) 187.4 45336(3.7)		90.287321160(7)	15.290(5)	65371(2.8)	188.1	45358(3.7)
90.287321184(7) 15.275(5) 65457(2.8) 188.3 45364(3.7) 90.287321197(7) 15.267(5) 65500(2.8) 188.4 45367(3.7) 90.287321209(7) 15.251(5) 65585(2.8) 188.6 45372(3.7) 90.287321234(7) 15.251(5) 656628(2.8) 188.6 45372(3.7) 90.287321246(7) 15.235(5) 65671(2.8) 188.8 45377(3.7) 90.287321246(7) 15.228(5) 65713(2.8) 188.9 45380(3.7) 90.287321259(7) 15.20(5) 65756(2.8) 189.0 45383(3.7) 90.287321283(7) 15.212(5) 65794(2.8) 189.1 45386(3.7) 90.287321296(7) 15.204(5) 65842(2.8) 189.1 45386(3.7) 90.287321007(7) 15.309(5) 65884(2.8) 186.9 4532(3.7) 90.287321020(7) 15.367(5) 64982(2.8) 187.1 45336(3.7) 90.287321020(7) 15.367(5) 65074(2.8) 187.4 45336(3.7) 90.28732103(7) 15.351(5) 65054(2.8) 187.4 45336(3.7)		90.287321172(7)	15.283(5)	65414(2.8)	188.2	45361(3.7)
90.287321197(7) 15.267(5) 65500(2.8) 188.4 45367(3.7) 90.287321209(7) 15.259(5) 65542(2.8) 188.5 45369(3.7) 90.287321221(7) 15.251(5) 65585(2.8) 188.6 45372(3.7) 90.287321234(7) 15.243(5) 65671(2.8) 188.7 45375(3.7) 90.287321259(7) 15.228(5) 65771(2.8) 188.9 45380(3.7) 90.287321271(7) 15.220(5) 65776(2.8) 189.0 45383(3.7) 90.287321283(7) 15.212(5) 65799(2.8) 189.0 45389(3.7) 90.287321296(7) 15.204(5) 6584(2.8) 189.2 45389(3.7) 90.287321030(7) 15.196(5) 6584(2.8) 189.3 45391(3.7) 90.287321007(7) 15.390(5) 64882(2.8) 187.0 45322(3.7) 90.28732103(7) 15.353(5) 64968(2.8) 187.1 45333(3.7) 90.28732103(7) 15.354(5) 65054(2.8) 187.3 45333(3.7) 90.287321057(7) 15.351(5) 65054(2.8) 187.4 45336(3.7)		90.287321184(7)	15.275(5)	65457(2.8)	188.3	45364(3.7)
90.287321209(7) 15.259(5) 65542(2.8) 188.5 45369(3.7) 90.287321221(7) 15.251(5) 65585(2.8) 188.6 45372(3.7) 90.287321234(7) 15.243(5) 65628(2.8) 188.7 45375(3.7) 90.287321246(7) 15.235(5) 65671(2.8) 188.9 45380(3.7) 90.287321259(7) 15.228(5) 65771(2.8) 188.9 45380(3.7) 90.287321283(7) 15.212(5) 65769(2.8) 189.0 45383(3.7) 90.287321283(7) 15.204(5) 6584(2.8) 189.1 45386(3.7) 90.287321030(7) 15.196(5) 6584(2.8) 189.3 45391(3.7) 90.287321030(7) 15.390(5) 64882(2.8) 189.3 4532(3.7) 90.287321030(7) 15.390(5) 64968(2.8) 187.1 45328(3.7) 90.28732103(7) 15.357(5) 64968(2.8) 187.3 45333(3.7) 90.28732104(7) 15.351(5) 65054(2.8) 187.4 45336(3.7) 90.287321057(7) 15.351(5) 65054(2.8) 187.4 45333(3.7) <		90.287321197(7)	15.267(5)	65500(2.8)	188.4	45367(3.7)
90.287321221(7) 15.251(5) 65585(2.8) 188.6 45372(3.7) 90.287321234(7) 15.243(5) 65628(2.8) 188.7 45375(3.7) 90.287321259(7) 15.228(5) 65713(2.8) 188.8 45377(3.7) 90.287321259(7) 15.228(5) 65756(2.8) 188.9 45380(3.7) 90.287321283(7) 15.212(5) 65756(2.8) 189.0 45383(3.7) 90.287321283(7) 15.212(5) 6579(2.8) 189.1 45386(3.7) 90.287321308(7) 15.204(5) 65842(2.8) 189.3 45391(3.7) 90.287321007(7) 15.390(5) 64882(2.8) 186.9 45322(3.7) 90.287321020(7) 15.390(5) 64968(2.8) 187.0 45328(3.7) 90.287321020(7) 15.350(5) 65011(2.8) 187.1 45330(3.7) 90.28732103(7) 15.351(5) 6504(2.8) 187.4 45330(3.7) 90.28732105(7) 15.351(5) 6504(2.8) 187.4 45339(3.7) 90.28732106(7) 15.351(5) 65140(2.8) 187.4 45344(3.7) </td <td></td> <td>90.287321209(7)</td> <td>15.259(5)</td> <td>65542(2.8)</td> <td>188.5</td> <td>45369(3.7)</td>		90.287321209(7)	15.259(5)	65542(2.8)	188.5	45369(3.7)
90.287321234(7) 15.243(5) 65628(2.8) 188.7 45375(3.7) 90.287321246(7) 15.235(5) 65671(2.8) 188.8 45377(3.7) 90.287321259(7) 15.228(5) 65713(2.8) 188.9 45380(3.7) 90.287321283(7) 15.212(5) 65756(2.8) 189.0 45383(3.7) 90.287321296(7) 15.204(5) 65842(2.8) 189.2 45389(3.7) 90.287321036(7) 15.196(5) 65884(2.8) 189.2 45389(3.7) 90.287321020(7) 15.390(5) 64882(2.8) 189.3 45391(3.7) 90.287321020(7) 15.390(5) 64925(2.8) 187.0 45322(3.7) 90.287321020(7) 15.375(5) 64968(2.8) 187.1 45330(3.7) 90.287321020(7) 15.359(5) 65011(2.8) 187.2 45330(3.7) 90.287321020(7) 15.359(5) 6504(2.8) 187.4 45336(3.7) 90.28732104(7) 15.351(5) 65097(2.8) 187.4 45336(3.7) 90.28732108(7) 15.327(5) 65269(2.8) 187.7 45344(3.7)		90.287321221(7)	15.251(5)	65585(2.8)	188.6	45372(3.7)
90.287321246(7) 15.235(5) 65671(2.8) 188.8 45377(3.7) 90.287321259(7) 15.228(5) 65713(2.8) 188.9 45380(3.7) 90.287321271(7) 15.220(5) 65756(2.8) 189.0 45383(3.7) 90.287321283(7) 15.212(5) 65799(2.8) 189.1 45386(3.7) 90.287321296(7) 15.204(5) 65842(2.8) 189.2 45389(3.7) 90.287321308(7) 15.196(5) 65884(2.8) 189.3 45321(3.7) 90.287321007(7) 15.390(5) 64882(2.8) 187.0 45322(3.7) 90.287321032(7) 15.375(5) 64968(2.8) 187.1 45328(3.7) 90.287321032(7) 15.367(5) 65011(2.8) 187.2 45330(3.7) 90.287321044(7) 15.351(5) 6504(2.8) 187.4 45336(3.7) 90.287321057(7) 15.351(5) 6504(2.8) 187.4 45336(3.7) 90.28732104(7) 15.351(5) 6504(2.8) 187.4 45336(3.7) 90.28732106(7) 15.351(5) 65140(2.8) 187.4 4534(3.7) </td <td></td> <td>90.287321234(7)</td> <td>15.243(5)</td> <td>65628(2.8)</td> <td>188.7</td> <td>45375(3.7)</td>		90.287321234(7)	15.243(5)	65628(2.8)	188.7	45375(3.7)
90.287321259(7) 15.228(5) 65713(2.8) 188.9 45380(3.7) 90.287321271(7) 15.220(5) 65756(2.8) 189.0 45383(3.7) 90.287321283(7) 15.212(5) 65799(2.8) 189.1 45386(3.7) 90.287321296(7) 15.204(5) 65842(2.8) 189.2 45389(3.7) 90.287321308(7) 15.196(5) 65884(2.8) 189.3 45391(3.7) 90.287321007(7) 15.390(5) 64882(2.8) 186.9 45322(3.7) 90.28732102(7) 15.383(5) 64925(2.8) 187.0 45328(3.7) 90.287321032(7) 15.375(5) 65011(2.8) 187.1 45330(3.7) 90.28732103(7) 15.367(5) 65054(2.8) 187.3 4533(3.7) 90.28732105(7) 15.359(5) 65054(2.8) 187.4 45336(3.7) 90.28732106(7) 15.351(5) 65097(2.8) 187.4 45339(3.7) 90.287321094(7) 15.319(5) 65183(2.8) 187.4 45341(3.7) 90.287321106(7) 15.319(5) 65269(2.8) 187.4 45349(3.7) <		90.287321246(7)	15.235(5)	65671(2.8)	188.8	45377(3.7)
90.287321271(7) 15.220(5) 65756(2.8) 189.0 45383(3.7) 90.287321283(7) 15.212(5) 65799(2.8) 189.1 45386(3.7) 90.287321296(7) 15.204(5) 65842(2.8) 189.2 4539(3.7) 90.287321308(7) 15.196(5) 65884(2.8) 189.3 45391(3.7) 90.287321007(7) 15.390(5) 64882(2.8) 186.9 45322(3.7) 90.287321020(7) 15.383(5) 64925(2.8) 187.0 45328(3.7) 90.287321032(7) 15.375(5) 64968(2.8) 187.1 45328(3.7) 90.287321057(7) 15.357(5) 65054(2.8) 187.2 45330(3.7) 90.287321057(7) 15.351(5) 65097(2.8) 187.4 45336(3.7) 90.287321069(7) 15.335(5) 65140(2.8) 187.5 45339(3.7) 90.287321094(7) 15.335(5) 65183(2.8) 187.4 45336(3.7) 90.287321094(7) 15.319(5) 65269(2.8) 187.5 45341(3.7) 90.28732113(7) 15.31(5) 65312(2.8) 187.4 45344(3.7)		90.287321259(7)	15.228(5)	65713(2.8)	188.9	45380(3.7)
90.287321283(7) 15.212(5) 65799(2.8) 189.1 45386(3.7) 90.287321296(7) 15.204(5) 65842(2.8) 189.2 45389(3.7) 90.287321308(7) 15.196(5) 65884(2.8) 189.3 45391(3.7) 90.287321007(7) 15.390(5) 64882(2.8) 186.9 45322(3.7) 90.287321020(7) 15.383(5) 64925(2.8) 187.0 45325(3.7) 90.287321032(7) 15.375(5) 64968(2.8) 187.1 45328(3.7) 90.287321044(7) 15.367(5) 65011(2.8) 187.2 45330(3.7) 90.287321057(7) 15.351(5) 65097(2.8) 187.3 45333(3.7) 90.28732106(7) 15.351(5) 65140(2.8) 187.4 45336(3.7) 90.28732109(7) 15.351(5) 65183(2.8) 187.4 45339(3.7) 90.28732109(7) 15.351(5) 65183(2.8) 187.7 45344(3.7) 90.28732109(7) 15.327(5) 65269(2.8) 187.7 45349(3.7) 90.28732119(7) 15.311(5) 65352(2.8) 188.0 45352(3.7) <		90.287321271(7)	15.220(5)	65756(2.8)	189.0	45383(3.7)
90.287321296(7) 15.204(5) 65842(2.8) 189.2 45389(3.7) 90.287321308(7) 15.196(5) 65884(2.8) 189.3 45391(3.7) 90.287321007(7) 15.390(5) 64882(2.8) 186.9 45322(3.7) 90.287321020(7) 15.383(5) 64925(2.8) 187.0 45325(3.7) 90.287321032(7) 15.375(5) 65011(2.8) 187.1 45328(3.7) 90.28732105(7) 15.357(5) 65054(2.8) 187.3 45330(3.7) 90.28732105(7) 15.359(5) 65054(2.8) 187.4 45336(3.7) 90.287321069(7) 15.351(5) 65097(2.8) 187.4 45336(3.7) 90.287321081(7) 15.335(5) 65183(2.8) 187.7 45344(3.7) 90.287321094(7) 15.327(5) 65226(2.8) 187.7 45344(3.7) 90.287321106(7) 15.319(5) 6526(2.8) 187.7 45344(3.7) 90.287321131(7) 15.311(5) 65312(2.8) 187.9 45349(3.7) 90.287321136(7) 15.295(5) 65398(2.8) 188.1 45355(3.7)		90.287321283(7)	15.212(5)	65799(2.8)	189.1	45386(3.7)
90.287321308(7)15.196(5)65884(2.8)189.345391(3.7)90.287321007(7)15.390(5)64882(2.8)186.945322(3.7)90.287321020(7)15.383(5)64925(2.8)187.045325(3.7)90.287321032(7)15.375(5)64968(2.8)187.145328(3.7)90.287321044(7)15.367(5)65011(2.8)187.245330(3.7)90.287321057(7)15.359(5)65054(2.8)187.345333(3.7)90.287321069(7)15.351(5)65097(2.8)187.445336(3.7)90.287321081(7)15.343(5)65140(2.8)187.545339(3.7)90.287321094(7)15.335(5)65183(2.8)187.645341(3.7)90.28732109(7)15.319(5)65269(2.8)187.745344(3.7)90.287321106(7)15.311(5)65312(2.8)187.845347(3.7)90.28732113(7)15.303(5)65355(2.8)188.045352(3.7)90.287321166(7)15.295(5)65398(2.8)188.145355(3.7)90.287321166(7)15.288(5)65440(2.8)188.145355(3.7)90.287321166(7)15.288(5)65440(2.8)188.245358(3.7)90.287321168(7)15.288(5)65440(2.8)188.145355(3.7)90.287321168(7)15.288(5)100847(4)198.045596(3.7)		90.287321296(7)	15.204(5)	65842(2.8)	189.2	45389(3.7)
90.287321007(7) 15.390(5) 64882(2.8) 186.9 45322(3.7) 90.287321020(7) 15.383(5) 64925(2.8) 187.0 45325(3.7) 90.287321032(7) 15.375(5) 64968(2.8) 187.1 45328(3.7) 90.287321044(7) 15.367(5) 65011(2.8) 187.2 45330(3.7) 90.287321057(7) 15.359(5) 65054(2.8) 187.3 45333(3.7) 90.287321069(7) 15.351(5) 65097(2.8) 187.4 45336(3.7) 90.287321081(7) 15.335(5) 65140(2.8) 187.5 45339(3.7) 90.287321094(7) 15.335(5) 65183(2.8) 187.6 45341(3.7) 90.287321106(7) 15.327(5) 65226(2.8) 187.7 45344(3.7) 90.287321106(7) 15.319(5) 65312(2.8) 187.8 45347(3.7) 90.287321131(7) 15.303(5) 65355(2.8) 188.0 45352(3.7) 90.287321143(7) 15.295(5) 65398(2.8) 188.1 45355(3.7) 90.287321168(7) 15.288(5) 65440(2.8) 188.2 45358(3.7)		90.287321308(7)	15.196(5)	65884(2.8)	189.3	45391(3.7)
90.287321020(7) 15.383(5) 64925(2.8) 187.0 45325(3.7) 90.287321032(7) 15.375(5) 64968(2.8) 187.1 45328(3.7) 90.287321044(7) 15.367(5) 65011(2.8) 187.2 45330(3.7) 90.287321057(7) 15.359(5) 65054(2.8) 187.3 45333(3.7) 90.287321069(7) 15.351(5) 65097(2.8) 187.4 45336(3.7) 90.287321081(7) 15.335(5) 65140(2.8) 187.5 45339(3.7) 90.287321094(7) 15.335(5) 65183(2.8) 187.6 45341(3.7) 90.287321106(7) 15.327(5) 65226(2.8) 187.7 45344(3.7) 90.28732119(7) 15.31(5) 65312(2.8) 187.8 45347(3.7) 90.28732113(7) 15.31(5) 65312(2.8) 187.9 45349(3.7) 90.28732113(7) 15.303(5) 65398(2.8) 188.0 45352(3.7) 90.287321156(7) 15.295(5) 65398(2.8) 188.1 45355(3.7) 90.287321168(7) 15.288(5) 65440(2.8) 188.2 45358(3.7) <		90.287321007(7)	15.390(5)	64882(2.8)	186.9	45322(3.7)
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90.287321044(7) 15.367(5) 65011(2.8) 187.2 45330(3.7) 90.287321057(7) 15.359(5) 65054(2.8) 187.3 45333(3.7) 90.287321069(7) 15.351(5) 65097(2.8) 187.4 45336(3.7) 90.287321081(7) 15.343(5) 65140(2.8) 187.4 45339(3.7) 90.287321094(7) 15.335(5) 65183(2.8) 187.5 45341(3.7) 90.287321094(7) 15.327(5) 65226(2.8) 187.7 45344(3.7) 90.287321106(7) 15.319(5) 65312(2.8) 187.8 45347(3.7) 90.287321131(7) 15.311(5) 65312(2.8) 187.9 45349(3.7) 90.287321131(7) 15.303(5) 65355(2.8) 188.0 45352(3.7) 90.287321143(7) 15.295(5) 65398(2.8) 188.1 45355(3.7) 90.287321168(7) 15.288(5) 65440(2.8) 188.2 45358(3.7) 90.287321168(7) 15.288(5) 65440(2.8) 188.2 45358(3.7) 90.287321168(7) 15.288(5) 100847(4) 198.0 45358(3.7)		90.287321032(7)	15.375(5)	64968(2.8)	187.1	45328(3.7)
90.287321057(7) 15.359(5) 65054(2.8) 187.3 45333(3.7) 90.287321069(7) 15.351(5) 65097(2.8) 187.4 45336(3.7) 90.287321081(7) 15.343(5) 65140(2.8) 187.5 45339(3.7) 90.287321094(7) 15.335(5) 65183(2.8) 187.6 45341(3.7) 90.287321094(7) 15.327(5) 65226(2.8) 187.7 45344(3.7) 90.287321106(7) 15.319(5) 65269(2.8) 187.8 45347(3.7) 90.287321131(7) 15.311(5) 65312(2.8) 187.9 45349(3.7) 90.287321131(7) 15.303(5) 65355(2.8) 188.0 45352(3.7) 90.287321156(7) 15.295(5) 65398(2.8) 188.1 45355(3.7) 90.287321168(7) 15.288(5) 65440(2.8) 188.2 45358(3.7) 90.287321168(7) 15.288(5) 65440(2.8) 188.2 45358(3.7) 90.287321168(7) 15.288(5) 100847(4) 198.0 45358(3.7) 132.0 90.287319933(6.6) 15.354(5) 100847(4) 198.0		90.287321044(7)	15.367(5)	65011(2.8)	187.2	45330(3.7)
90.287321069(7) 15.351(5) 65097(2.8) 187.4 45336(3.7) 90.287321081(7) 15.343(5) 65140(2.8) 187.5 45339(3.7) 90.287321094(7) 15.335(5) 65183(2.8) 187.6 45341(3.7) 90.287321106(7) 15.327(5) 65226(2.8) 187.7 45344(3.7) 90.28732119(7) 15.319(5) 65226(2.8) 187.8 45347(3.7) 90.287321131(7) 15.311(5) 65312(2.8) 187.9 45349(3.7) 90.287321131(7) 15.303(5) 65355(2.8) 188.0 45352(3.7) 90.287321156(7) 15.295(5) 65398(2.8) 188.1 45355(3.7) 90.287321168(7) 15.288(5) 65440(2.8) 188.2 45358(3.7) 90.287321168(7) 15.288(5) 65440(2.8) 188.2 45358(3.7) 90.287321168(7) 15.288(5) 65440(2.8) 188.2 45358(3.7) 132.0 90.287319933(6.6) 15.354(5) 100847(4) 198.0 45596(3.7)		90.287321057(7)	15.359(5)	65054(2.8)	187.3	45333(3.7)
83.6 90.287321081(7) 15.343(5) 65140(2.8) 187.5 45339(3.7) 90.287321094(7) 15.335(5) 65183(2.8) 187.6 45341(3.7) 90.287321106(7) 15.327(5) 65226(2.8) 187.7 45344(3.7) 90.28732119(7) 15.319(5) 65269(2.8) 187.8 45347(3.7) 90.287321131(7) 15.311(5) 65312(2.8) 187.9 45349(3.7) 90.287321131(7) 15.303(5) 65355(2.8) 188.0 45352(3.7) 90.287321156(7) 15.295(5) 65398(2.8) 188.1 45355(3.7) 90.287321168(7) 15.288(5) 65440(2.8) 188.2 45358(3.7) 90.287321168(7) 15.288(5) 65440(2.8) 188.2 45358(3.7) 90.287321168(7) 15.288(5) 65440(2.8) 188.2 45358(3.7) 132.0 90.287319933(6.6) 15.354(5) 100847(4) 198.0 45596(3.7)		90.287321069(7)	15.351(5)	65097(2.8)	187.4	45336(3.7)
90.287321094(7) 15.335(5) 65183(2.8) 187.6 45341(3.7) 90.287321106(7) 15.327(5) 65226(2.8) 187.7 45344(3.7) 90.287321119(7) 15.319(5) 65269(2.8) 187.8 45347(3.7) 90.287321131(7) 15.311(5) 65312(2.8) 187.9 45349(3.7) 90.287321143(7) 15.303(5) 65355(2.8) 188.0 45352(3.7) 90.287321156(7) 15.295(5) 65398(2.8) 188.1 45355(3.7) 90.287321168(7) 15.288(5) 65440(2.8) 188.2 45358(3.7) 90.287321168(7) 15.288(5) 65440(2.8) 188.2 45358(3.7) 132.0 90.287319933(6.6) 15.354(5) 100847(4) 198.0 45596(3.7)	83.6	90.287321081(7)	15.343(5)	65140(2.8)	187.5	45339(3.7)
90.287321106(7) 15.327(5) 65226(2.8) 187.7 45344(3.7) 90.287321119(7) 15.319(5) 65269(2.8) 187.8 45347(3.7) 90.287321131(7) 15.311(5) 65312(2.8) 187.9 45349(3.7) 90.287321143(7) 15.303(5) 65355(2.8) 188.0 45352(3.7) 90.287321156(7) 15.295(5) 65398(2.8) 188.1 45355(3.7) 90.287321168(7) 15.288(5) 65440(2.8) 188.2 45358(3.7) Eccentricity 0.5 132.0 90.287319933(6.6) 15.354(5) 100847(4) 198.0 4536(3.7)	00.0	90.287321094(7)	15.335(5)	65183(2.8)	187.6	45341(3.7)
90.287321119(7) 15.319(5) 65269(2.8) 187.8 45347(3.7) 90.287321131(7) 15.311(5) 65312(2.8) 187.9 45349(3.7) 90.287321143(7) 15.303(5) 65355(2.8) 188.0 45352(3.7) 90.287321156(7) 15.295(5) 65398(2.8) 188.1 45355(3.7) 90.287321168(7) 15.288(5) 65440(2.8) 188.2 45358(3.7) 90.287321168(7) 15.288(5) 65440(2.8) 188.2 45358(3.7) Eccentricity 0.5 132.0 90.287319933(6.6) 15.354(5) 100847(4) 198.0 45596(3.7)		90.287321106(7)	15.327(5)	65226(2.8)	187.7	45344(3.7)
90.287321131(7) 15.311(5) 65312(2.8) 187.9 45349(3.7) 90.287321143(7) 15.303(5) 65355(2.8) 188.0 45352(3.7) 90.287321156(7) 15.295(5) 65398(2.8) 188.1 45355(3.7) 90.287321168(7) 15.288(5) 65440(2.8) 188.2 45358(3.7) Eccentricity 0.5 132.0 90.287319933(6.6) 15.354(5) 100847(4) 198.0 45596(3.7)		90.287321119(7)	15.319(5)	65269(2.8)	187.8	45347(3.7)
90.287321143(7) 15.303(5) 65355(2.8) 188.0 45352(3.7) 90.287321156(7) 15.295(5) 65398(2.8) 188.1 45355(3.7) 90.287321168(7) 15.288(5) 65440(2.8) 188.2 45358(3.7) 90.287321168(7) 15.288(5) 65440(2.8) 188.2 45358(3.7) Eccentricity 0.5 132.0 90.287319933(6.6) 15.354(5) 100847(4) 198.0 45596(3.7)		90.287321131(7)	15.311(5)	65312(2.8)	187.9	45349(3.7)
90.287321156(7) 15.295(5) 65398(2.8) 188.1 45355(3.7) 90.287321168(7) 15.288(5) 65440(2.8) 188.2 45358(3.7) Eccentricity 0.5 132.0 90.287319933(6.6) 15.354(5) 100847(4) 198.0 45596(3.7)		90.287321143(7)	15.303(5)	65355(2.8)	188.0	45352(3.7)
90.287321168(7) 15.288(5) 65440(2.8) 188.2 45358(3.7) Eccentricity 0.5 132.0 90.287319933(6.6) 15.354(5) 100847(4) 198.0 45596(3.7)		90.287321156(7)	15.295(5)	65398(2.8)	188.1	45355(3.7)
Eccentricity 0.5 132.0 90.287319933(6.6) 15.354(5) 100847(4) 198.0 45596(3.7)		90.287321168(7)	15.288(5)	65440(2.8)	188.2	45358(3.7)
132.0 90.287319933(6.6) 15.354(5) 100847(4) 198.0 45596(3.7)		1	Eccentricity 0	.5		
	132.0	90.287319933(6.6)	15.354(5)	100847(4)	198.0	45596(3.7)

Appendix A

A.1 Forming TOAs with **PSRCHIVE**

PSRCHIVE is an open-source library, written in object-oriented C++. Its classes are used in order to manipulate pulsar observations. In our analysis we used PSRCHIVE to RFI mitigation, display, add, dedisperse and scrunch data and generate a template profile. Below, we present the exact commands that are used.

A.1.1 Cleaning data

- 1. We plot single, scrunch in frequency, time and polarization profiles. **pav -FTDp *.dat**
- We visualize the amplitude against frequency and phase of every of 32 frequency channels. Check for interferences.
 pav -Gd *.dat
- 3. We remove the channels that are corrupted with interferences in order to improve the S/N ratio of our observations. With the extension .datpz we refer to the data from which we have removed a frequency channel. **paz -e .datpz -Z "channel(s) number" *.dat**
- 4. We add the data which was obtained in the same day provided that the integration time is not longer than 50 min. The output files have the extension .t which refers to the added files.
 - psradd -f *.t *.dat
- We scrunch data in frequency, time and polarization (FTp) and dedisperse them (D). The output files have the extension .FTDp.
 pam -e .FTDp -FTDp *.t

A.1.2 Creating a template

1. We view the parameters of our .FTDp data. The frequency and S/N ratio are the two parameters that we need.

psrstat -c name,freq,snr c*.FTDp

- 2. We choose the observations with the highest S/N ratio. We create a specific directory in which a copy of these observations is placed.
- 3. We phase align (-P) .FTDp files and then add them (-f) in one file with the extension .hour.

psradd -P -f template.hour c*.FTDp

4. We create an analytic template giving the center, concentration and height of the profile components.

paas -w 1620.1.paas -Dc "center concentration height" template.hour -s paas.1.std

The output files are 1620.1.paas, paas.1.std and paas.txt.

5. We improve the previous template (paas.1.std) recreating a new Gaussian model which fits better to our data.
paas -Dr 1620.1.paas -fw 1620.2.paas template.hour -s paas.2.std
The output files are 1620.2.paas, paas.2.std and paas.txt.

The high S/N template that we create using our clearer data is paas.2.std.

A.1.3 Creating TOAs

We cross correlate the high S/N template with each observation profile (.FTDp) with respect to a fiducial point in order to obtain the TOAs.

pat -s paas.2.std -f "tempo2" *.FTDp > name_of_tim_file.tim

Appendix B

B.2 Forming residuals with TEMPO2

TEMPO2 is a pulsar timing package which is developed at ATNF. In the next section we will cite the basic plugins that TEMPO2 uses in order to produce the timing residuals.

1. readParfile.C

With this function TEMPO2 reads the .par file, the file that contains the astronomic (right accession, declination, proper motion), pulsar (rotation frequency, dispersion measure) and Keplerian parameters (projected semimajor axis, epoch of periastron passage, period of orbit and longitude of periastron passage) with TEMPO2 or HEAD format.

The parameters that a .par file should have necessarily are: name, right ascension, declination, dispersion measure, pulsar rotation frequency and Epoch of period/frequency parameters and position.

2. readTimfile.C

With this function TEMPO2 reads the .tim file, the file that contains the TOAs (SAT), time and the jump corrections.

3. formBats.C

After having read .par and .tim files TEMPO2 calculates the barycentric arrival time (BAT) or binary barycentric arrival time (BBAT) for every SAT that the .tim file contains.

 $bat = sat + TT \ corrections + TT \ TB \ correction - Troposheric \ Delay$

+ Roemer Delay – Shapiro Delay

- Intestellar Dispersion measure delay
- Dispersion measure delay due to solar system

 $bbat = bat - Shklovskii\ corrections$

4. **model.C**

TEMPO2 reads the chosen model from the .par file and calculate the theoretical values for every TOAs that the .tim file contains.

5. formResiduals.C

The first time that we call this function we create the pre-fit residuals, using the selected timing model.



Using the timing model, TEMPO2 calculates the theoretical values of TOAs. Comparing the theoretical and observed TOAs, TEMPO2 determines the phase of the pulsar and from the phase and observation frequency ($\nu = d\phi/dt$) residuals are calculated. Apart from theoretical and observed TOAs the phase of the pulsar is affected by glitches, jumps and gravitational wave signals.

6. **doFit.C**

Only if in the .par file we have chosen some parameters to be fitted, TEMPO2 calls this function. The purpose is to recalculate the chosen parameters so that the residuals will be minimized. TEMPO2 applies the *least squares fitting method* and uses the *singular value decomposition method* to minimize the TOAs and recalculate the fitted parameters.

TEMP02 recalls the **model.C** and **formResiduals.C** functions so that post-fit TOAs will be recalculated using the fitted parameters.

Appendix C

We present the BT1P model plugin that is created and added to the main TEMPO2 source code. It is based on the Blandford & Teukolsky (1976) binary model.

C.3 Double Keplerian model (BT1P model)

```
1 double BT1Pmodel(pulsar *psr, int p, int ipos, int param, int arr)
 |2|
 3
     double torb;
     double tt0;
 4
 5
     double orbits;
     double pb;
 6
                    /* Orbital period (sec) */
 7
     double pbdot;
     double xpbdot;
 8
 9
                    /* Orbital eccentricity */
     double ecc;
10
     double edot;
11
     double asini;
12
     double xdot;
13
     double omdot;
     double omega;
14
15
     double gamma;
16
     int
            norbits;
17
18
     int i;
19
20
     double phase;
21
     double ep,dep,bige,tt,som,com;
22
     double alpha, beta, sbe, cbe, q, r, s;
23
     const char *CVS_verNum = "$Revision: 1.6 $";
\mathbf{24}
25
     if (displayCVSversion == 1) CVSdisplayVersion("BTmodel.C", "BTmodel()", CVS_verNum);
26
27
28
     torb = 0.0;
29
30
31
     if (psr[p].param[param_pbdot].paramSet[0] == 1) pbdot = psr[p].param[param_pbdot].val[0];
32
     else pbdot=0.0;
33
     if (psr[p].param[param_aldot].paramSet[0] == 1) xdot = psr[p].param[param_aldot].val[0];
34
35
     else xdot = 0.0;
36
37
     if (psr[p].param[param_omdot].paramSet[0] == 1) omdot = psr[p].param[param_omdot].val[0];
38
     else omdot = 0.0;
39
40
     if (psr[p].param[param_gamma].paramSet[0]==1) gamma = psr[p].param[param_gamma].val[0];
41
     else gamma = 0.0;
42
43
     xpbdot = 0.0;
44
```

```
45
46
47
      for (int i=0; i<=1; i++)</pre>
 48
 49
    {
50
51
        tt0 = (psr[p].obsn[ipos].bbat - psr[p].param[param_t0].val[i])*SECDAY;
52
53
 54
        if (i==0)
55
        {
56
 57
          asini = psr[p].param[param_a1].val[i] + xdot*tt0;
58
 59
          omega = (psr[p].param[param_om].val[i] + omdot*tt0/(SECDAY * 365.25))/(180.0/M_PI);
60
                 = psr[p].param[param_pb].val[i] * SECDAY;
61
          \mathbf{p}\mathbf{b}
 62
63
64
           edot = 0.0;
65
                 = psr[p].param[param_ecc].val[i] + edot*tt0;
          ecc
66
 67
          orbits = tt0/pb - 0.5*(pbdot+xpbdot)*pow(tt0/pb,2);
68
 69
70
        }
71
72
 73
        if (i!=0)
74
        {
 75
 76
77
          asini = psr[p].param[param_a1].val[i] ;
78
 79
          omega = (psr[p].param[param_om].val[i])/(180.0/M_PI);
80
81
                 = psr[p].param[param_pb].val[i] * SECDAY;
          pb
82
83
84
                 = psr[p].param[param_ecc].val[i] ;
          ecc
85
86
87
          orbits = tt0/pb;
88
 89
        }
90
91
92
        if (ecc < 0.0 || ecc > 1.0)
93
94
        {
95
          printf("BTmodel: problem with eccentricity = %Lg\n",psr[p].param[param_ecc].val[i]);
96
           exit(1);
97
        }
98
99
100
      /* Should ct be the barycentric arrival time? -- check bnrybt.f */
101
102
      norbits = (int)orbits;
103
      if (orbits < 0.0) norbits---;
104
105
      phase = 2.0*M_PI * (orbits-norbits);
106
107
      /* Using Pat Wallace's method of solving Kepler's equation --- code based on bnrybt.f */
108
      ep = phase + ecc * sin(phase) * (1.0 + ecc * cos(phase));
109
110
      /* This line is wrong in the original tempo: should be inside the do loop \ast/
111
      /* denom = 1.0 - ecc*cos(ep);*/
112
113
      dep = 0.0;
114
115
      do {
```
```
116
        dep = (phase - (ep-ecc*sin(ep)))/(1.0 - ecc*cos(ep));
117
        ep += dep;
118
      } while (fabs(dep) > 1.0e-12);
119
      bige = ep;
120
121
      tt = 1.0 - ecc * ecc;
122
      som = sin(omega);
      com = cos(omega);
123
124
125
      alpha = asini*som;
      beta = asini*com*sqrt(tt);
126
127
      sbe = sin(bige);
128
      cbe = cos(bige);
      q = alpha * (cbe-ecc) + (beta+gamma)*sbe;
129
130
      r = -alpha*sbe + beta*cbe;
131
      s = 1.0/(1.0 - ecc * cbe);
132
133
      torb = -q+(2*M_PI/pb)*q*r*s + torb;
134
135
136
137
138
      if (i==0)
139
      {
140
141
        if (arr==0)
142
        {
143
             if (param==param_pb)
144
                 ł
                 return -2.0*M_PI*r*s/pb*SECDAY*tt0/(SECDAY*pb) * SECDAY;
145
146
                 }
147
             else if (param==param_al)
148
                 {
149
                 return (som*(cbe-ecc) + com*sbe*sqrt(tt));
150
151
                 }
             else if (param==param_ecc)
152
153
             {
154
                 return -(alpha*(1.0+sbe*sbe-ecc*cbe)*tt - beta*(cbe-ecc)*sbe)*s/tt;
155
             }
156
             else if (param==param_om)
157
             {
                 return asini*(com*(cbe-ecc) - som*sqrt(tt)*sbe);
158
159
             }
160
             else if (param==param_t0)
161
             {
162
                 return -2.0*M_PI/pb*r*s*SECDAY;
163
             }
             else if (param==param_pbdot)
164
165
             {
166
                 return 0.5*(-2.0*M_PI*r*s/pb*SECDAY*tt0/(SECDAY*pb))*tt0;
167
             }
            else if (param==param_aldot)
168
169
             {
170
                 return (som*(cbe-ecc) + com*sbe*sqrt(tt))*tt0;
171
             }
             else if (param==param_omdot)
172
173
             {
174
                 return asini*(com*(cbe-ecc) - som*sqrt(tt)*sbe)*tt0;
175
             }
176
             else if (param==param_edot)
177
             {
178
                 return (-(alpha*(1.0+sbe*sbe-ecc*cbe)*tt - beta*(cbe-ecc)*sbe)*s/tt)*tt0;
179
            }
             else if (param==param_gamma)
180
181
             {
182
                return sbe:
            }
183
184
185
        }
186
```

```
187
      }
188
189
190
     if (i!=0)
191
     {
192
193
         if (arr!=0)
194
        {
195
             if (param==param_pb)
196
                -{
197
                     return -2.0*M_PI*r*s/pb*SECDAY*tt0/(SECDAY*pb) * SECDAY; /* fctn(12+j) */
198
                }
199
             else if (param==param_al)
200
                {
201
                     return (som*(cbe-ecc) + com*sbe*sqrt(tt));
                                                                                 /* fctn(9+j) */
202
203
                }
204
             else if (param==param_ecc)
205
                {
206
                     return -(alpha*(1.0+sbe*sbe-ecc*cbe)*tt - beta*(cbe-ecc)*sbe)*s/tt; /* fctn(10+j)
                         */
207
                }
208
209
             else if (param==param_om)
210
                {
211
                     return asini*(com*(cbe-ecc) - som*sqrt(tt)*sbe);
                                                                                /* fctn(13+j) */
212
                 }
213
214
             else if (param==param_t0)
215
                {
216
                    return -2.0*M_PI/pb*r*s*SECDAY;
                                                                                 /* fctn(11+j) */
217
                }
218
219
        }
220
     } //end if(i!=0)
221
222
223
224
225 }//end for
226
227
      if (param==-1) return torb;
228
229
230
      return 0.0;
231
232
    }
233
234
    void updateBT1P(pulsar *psr,double val,double err,int pos,int arr)
235 {
236
      if (pos==param_pb)
237
        {
238
          psr->param[param_pb].val[arr] += val;
239
          psr->param[param_pb].err[arr] = err;
240
241
242
        }
243
      else if (pos==param_al || pos==param_ecc || pos==param_t0 || pos==param_gamma || pos==param_edot)
244
        {
245
          psr->param[pos].val[arr] += val;
246
          psr->param[pos].err[arr] = err;
247
248
        }
249
      else if (pos==param_om)
250
        {
251
          psr->param[pos].val[arr] += val*180.0/M_PI;
          psr->param[pos].err[arr] = err*180.0/M_PI;
252
253
        }
254
      else if (pos==param_pbdot)
255
        {
256
          psr->param[pos].val[0] += val;
```

```
psr->param[pos].err[0] = err;
257
258
          }
259
       else if (pos==param_omdot)
260
         {
            psr->param[pos].val[0] += val*(SECDAY*365.25)*180.0/M_PI;
psr->param[pos].err[0] = err*(SECDAY*365.25)*180.0/M_PI;
261
262
263
         }
264
       else if (pos==param_aldot)
265
          {
            psr->param[pos].val[0] += val;
266
            psr->param[pos].err[0] = err;
267
268
269
          }
270 }
```

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