Kinetic solution of a collisionless magnetic presheath

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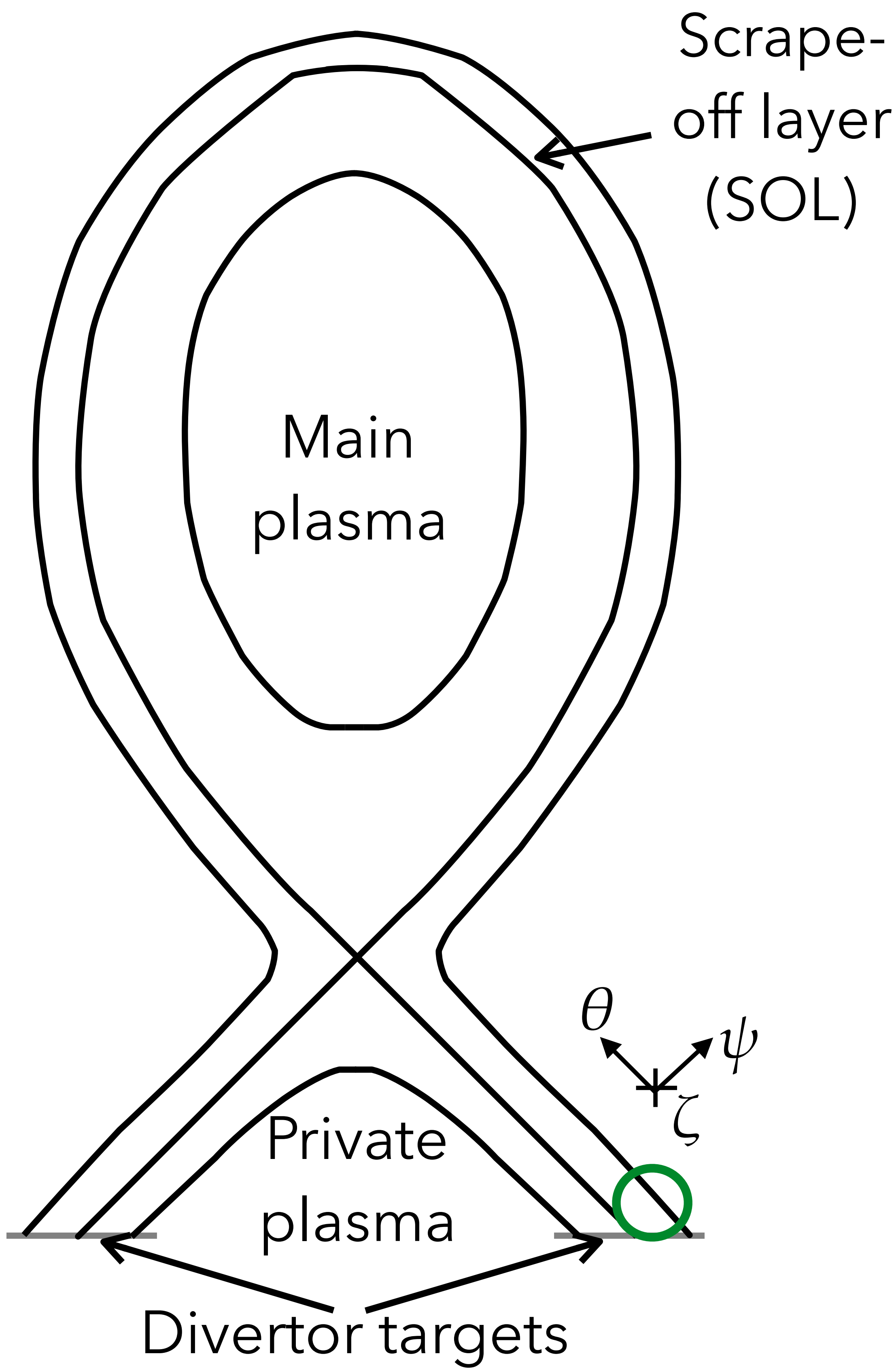
Motivation

Kinetic analysis of ions in plasma-wall boundary layer useful to:

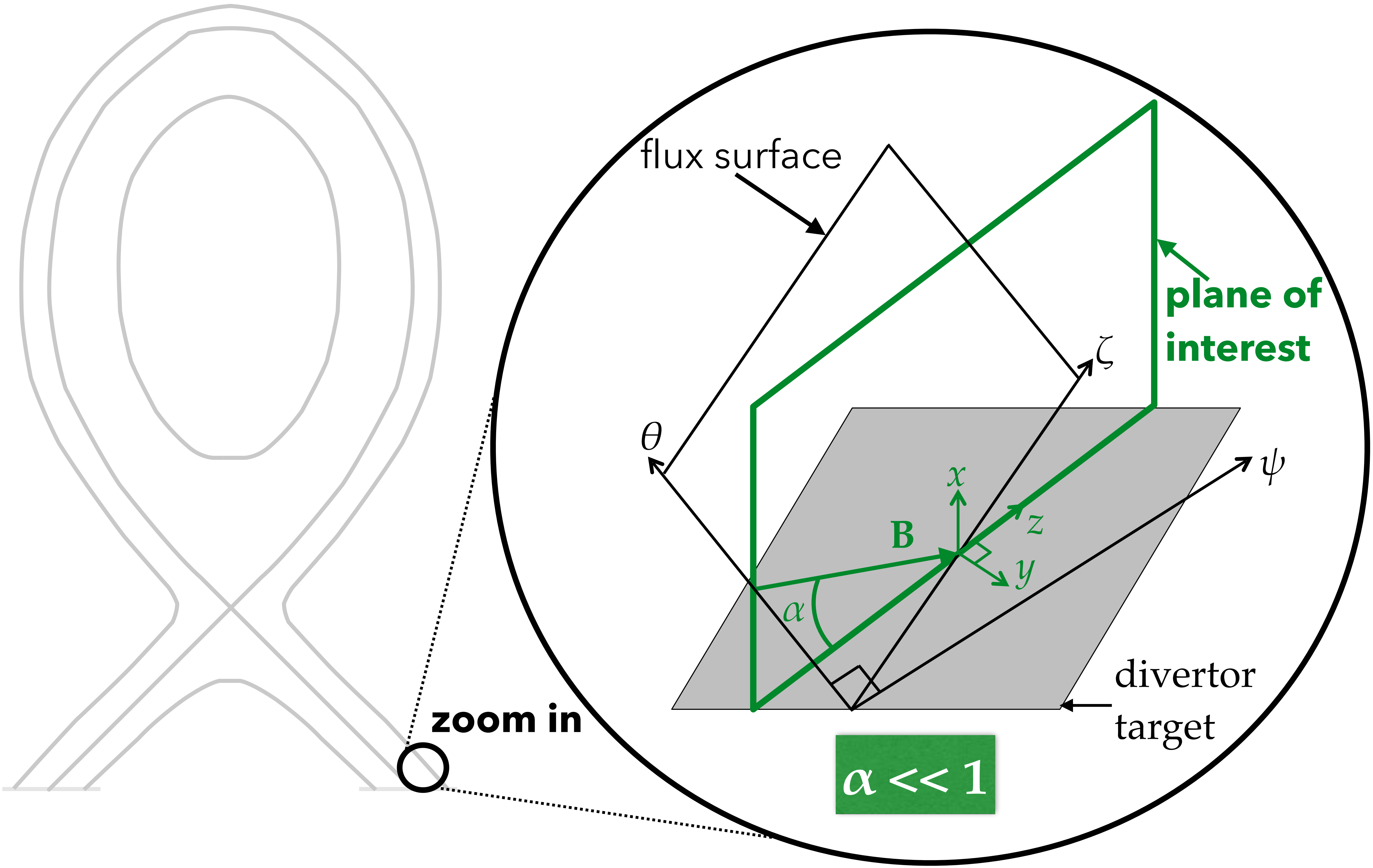
- **Determine distribution function** of ion velocities at divertor targets
- In the **long-term**: obtain **boundary conditions** (BCs) for drift-kinetic and fluid codes used to simulate the scrape-off-layer (SOL) plasma
- Theoretical interest: **generalizing gyrokinetics** to strongly **distorted orbits** in presheath geometry

Geometry



Scrape-
off layer
(SOL)
Main
plasma
θ
ψ
ζ
Private
plasma
Divertor targets

Geometry

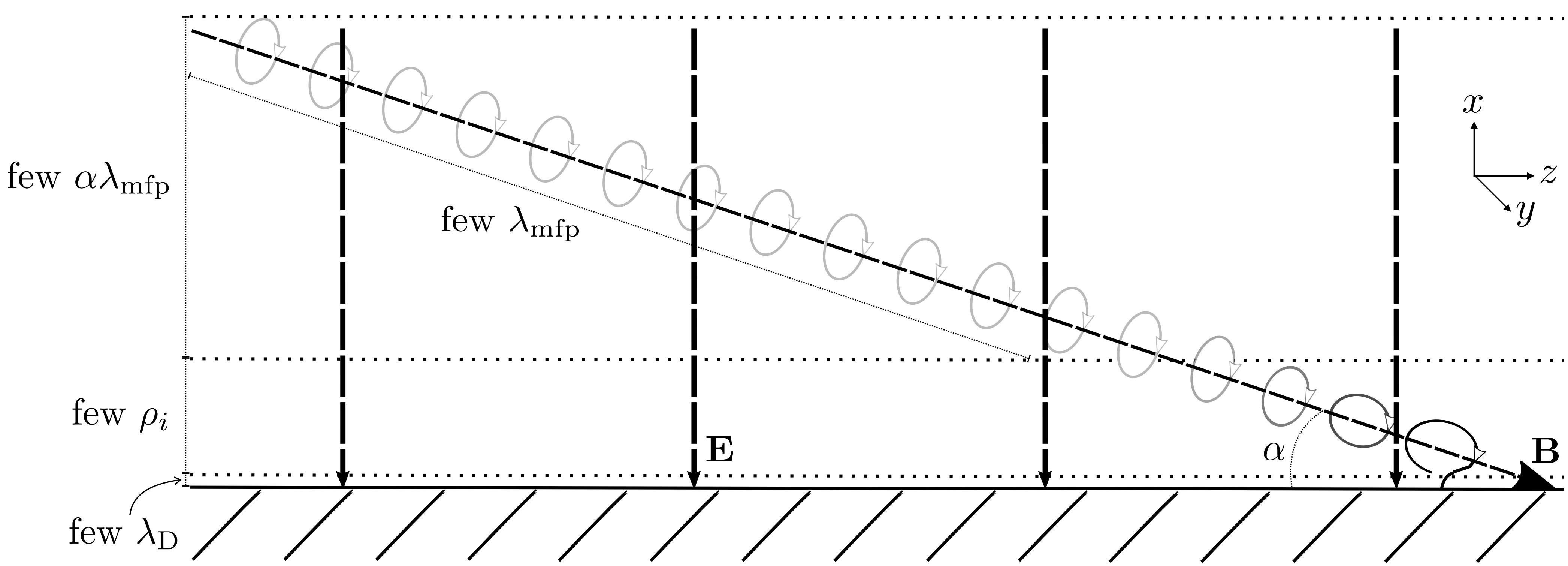

flux surface
plane of interest
ζ
θ
ψ
x
z
y
B
α
divertor target
zoom in
α << 1

Boundary layers

	Width	Estimate*
Collisional presheath	$\alpha\lambda_{mfp}$	100 mm
Magnetic presheath	ρ_i	0.7 mm
Debye sheath	λ_D	0.02 mm

*Using data from F. Militello and W. Fundamenski, *Plasma Phys. Control. Fusion* **53**, 095002 (2011)

$\lambda_D << \rho_i << \alpha\lambda_{mfp}$



References: R Chodura, *Phys. Fluids* **25** (1982); K.-U. Riemann, *Phys. Plasmas* **1**, 552 (1994); K.-U. Riemann, *J. Phys. D: Appl. Phys.* **24**, 493-518 (1991).

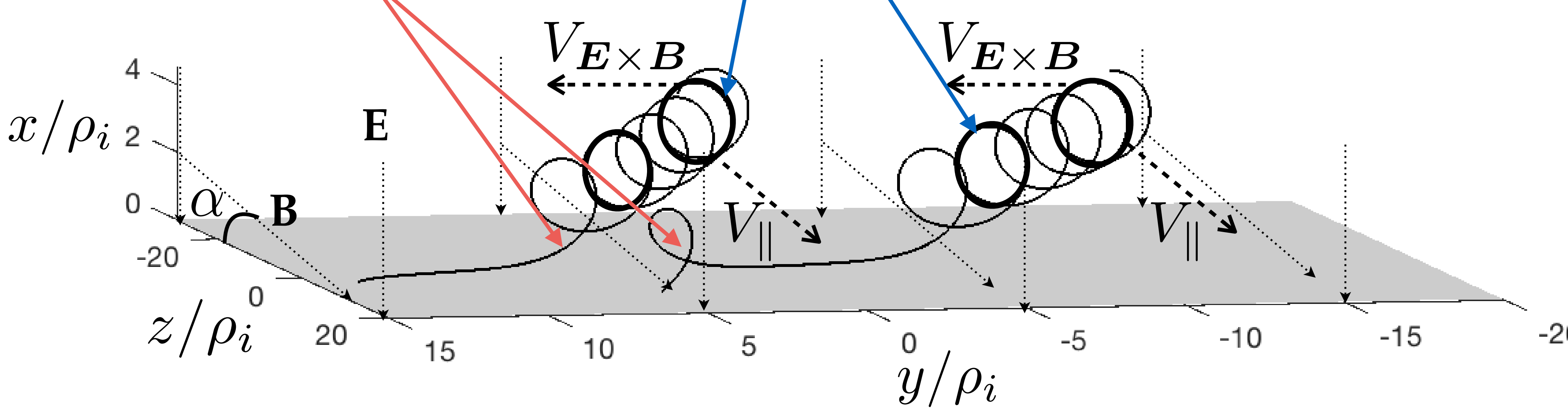
The magnetic presheath

- **Collisionless** and **quasineutral**
- **Boltzmann** electrons

$$n_{\mathrm{e}}(x) = n_{\mathrm{e}\infty} \exp\left(\frac{e\phi(x)}{T_{\mathrm{e}}}\right)$$

Ion trajectories

- Calculate **ion trajectories** by expanding in $\alpha << 1$
- Have **approximately closed orbits** (gyrokinetics)
- The final piece which cannot be approximated by a periodic orbit is an **open orbit**



α=0: closed orbits

Orbit parameters:

Orbit position	$\bar{x} = x + (1/\Omega)v_y$
Perpendicular energy	$U_\perp = ½v_x^2 + ½v_y^2 + Ze\phi/m_i$
Total energy	$U = U_\perp + ½v_z^2$

$$v_x = \pm V_x\left(\bar{x}, U_\perp, \bar{x}\right) = \pm\sqrt{2\left(U_\perp - \chi\left(x, \bar{x}\right)\right)}$$

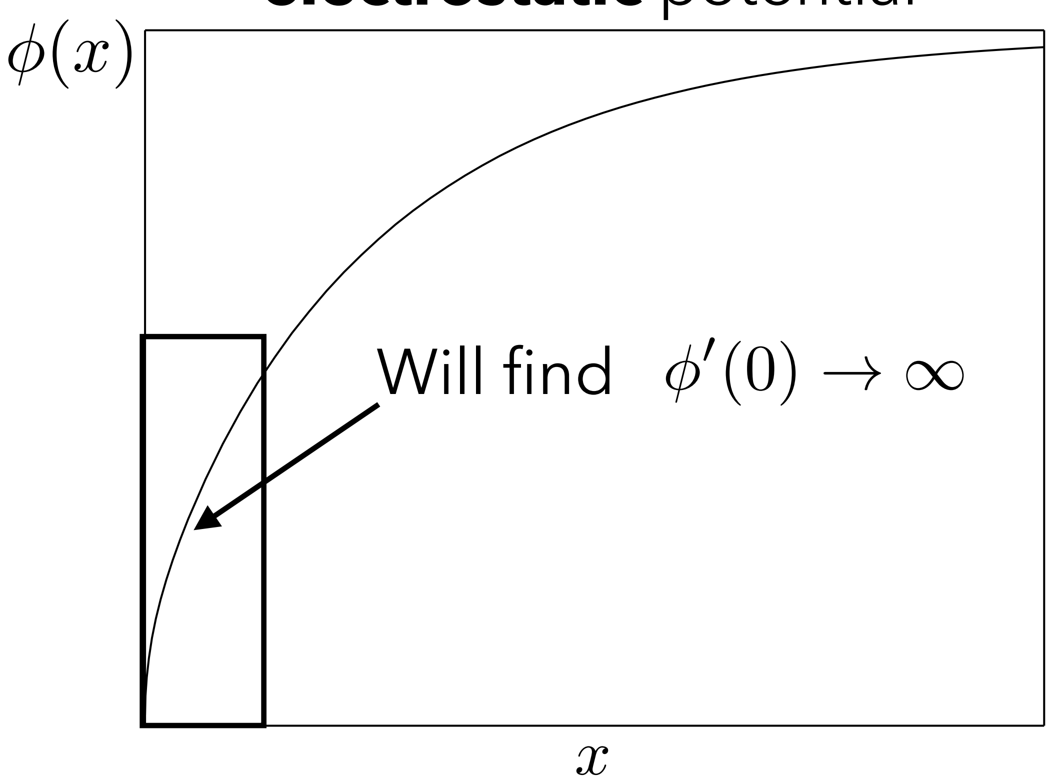
$$v_y = \Omega\left(\bar{x} - x\right)$$

$$v_z = V_{\|}\left(U_\perp, U\right) = \sqrt{2\left(U - U_\perp\right)}$$

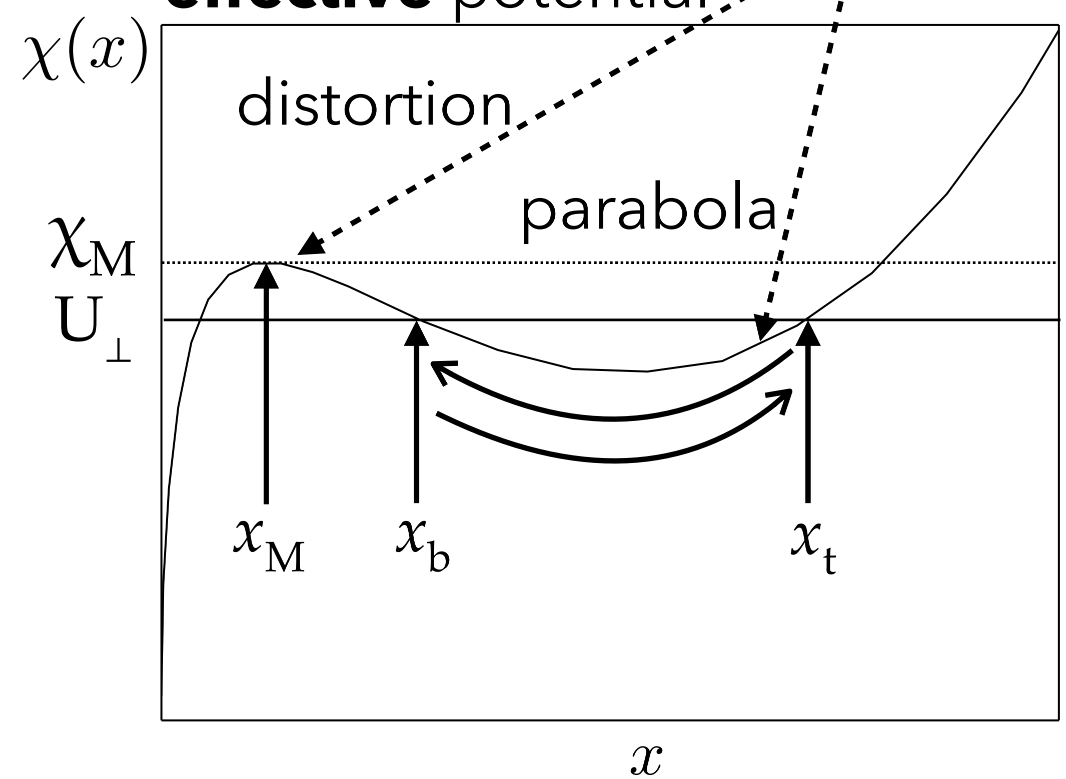
with $\chi(x, \bar{x}) = \frac{1}{2}\Omega^2(x - \bar{x})^2 + \frac{\Omega\phi(x)}{B}$

- **Closed orbit** (period~$1/\Omega$ with $\Omega = ZeB/m_i$) if particle is trapped around a minimum of the **effective potential** $\chi(x)$

electrostatic potential



effective potential



α<<1: approximately closed orbits

- Orbit parameters $\bar{x}$ and $U_\perp$ slowly varying (timescale ~$1/\alpha\Omega$), hence we have an **adiabatic invariant**: $\mu\left(\bar{x}, U_\perp\right) = \frac{1}{\pi}\int_{x_b}^{x_t}\sqrt{2\left(U_\perp - \chi\left(s, \bar{x}\right)\right)}ds$
- Distribution function F constant when written in terms of μ and U
- Density of ions in approximately closed orbits is

$$n_{i,\mathrm{closed}}\left(x\right) = \int_{\bar{x}_m(x)}^{\infty}\underbrace{\Omega d\bar{x}}_{dv_y}\int_{\chi(x,\bar{x})}^{\chi_M(\bar{x})}\underbrace{\frac{2dU_\perp}{\sqrt{2\left(U_\perp - \chi\left(x, \bar{x}\right)\right)}}}_{dv_x}\int_{U_\perp}^{\infty}\underbrace{\frac{dU}{\sqrt{2\left(U - U_\perp\right)}}}_{dv_z}F\left(\mu\left(\bar{x}, U_\perp\right), U\right)$$

- **Problem** at x=0: $n_{\mathrm{i,closed}}(0)=0=n_{\mathrm{e}}(0)$ leads to

$$\frac{e\phi(0)}{T_e} = \ln\left(\frac{n_{\mathrm{i,closed}}(0)}{n_\infty}\right) = -\infty$$

solved by including open orbits

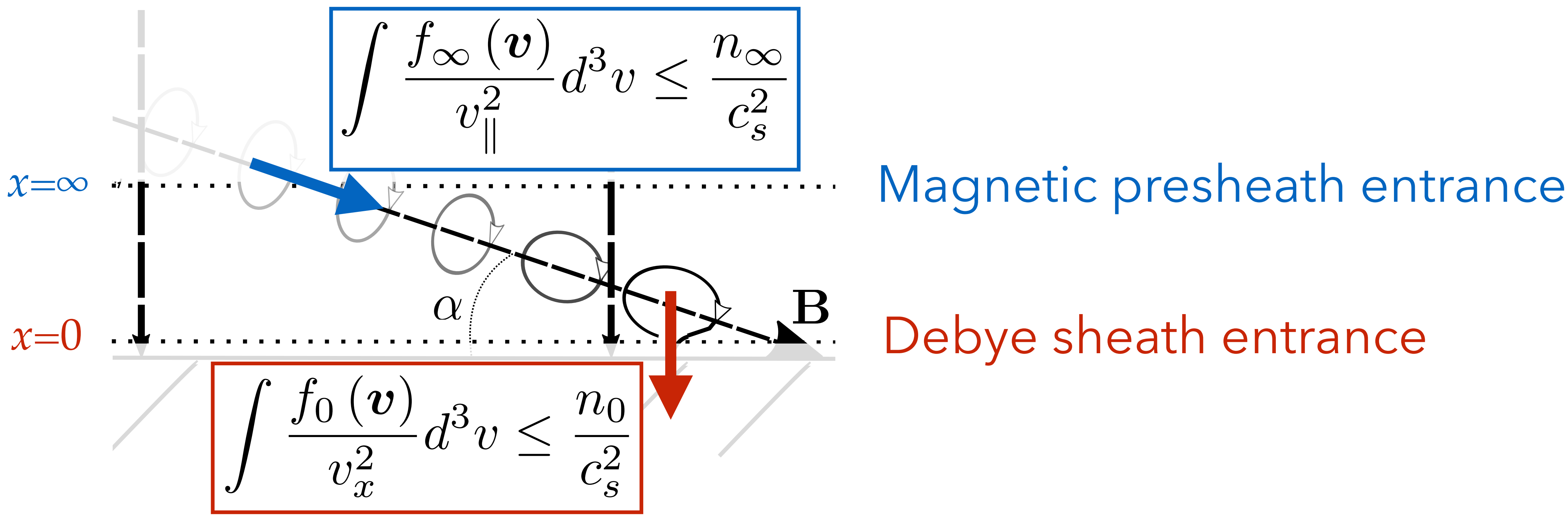
- **No problem** far away from x=0, quasineutrality is $Zn_{\mathrm{i,closed}}(x)=n_{\mathrm{e}}(x)$

References: R.H. Cohen and D.D. Ryutov, *Phys. Plasmas* **5**, 808 (1998);
A. Geraldini, F. I. Parra and F. Militello, *Plasma Phys. Control. Fusion* **59**, 025015 (2017)

Solvability condition

- Expand $Zn_{\mathrm{i,closed}}(x) = n_{\mathrm{e}}(x)$ near $x \to \infty$ using $e\phi/T_{\mathrm{e}} \ll 1$ to obtain **kinetic Chodura condition** at magnetic presheath entrance (Geraldini, *in preparation*)
- Analogous to **kinetic Bohm condition** (Harrison & Thompson, 1959) at Debye sheath entrance

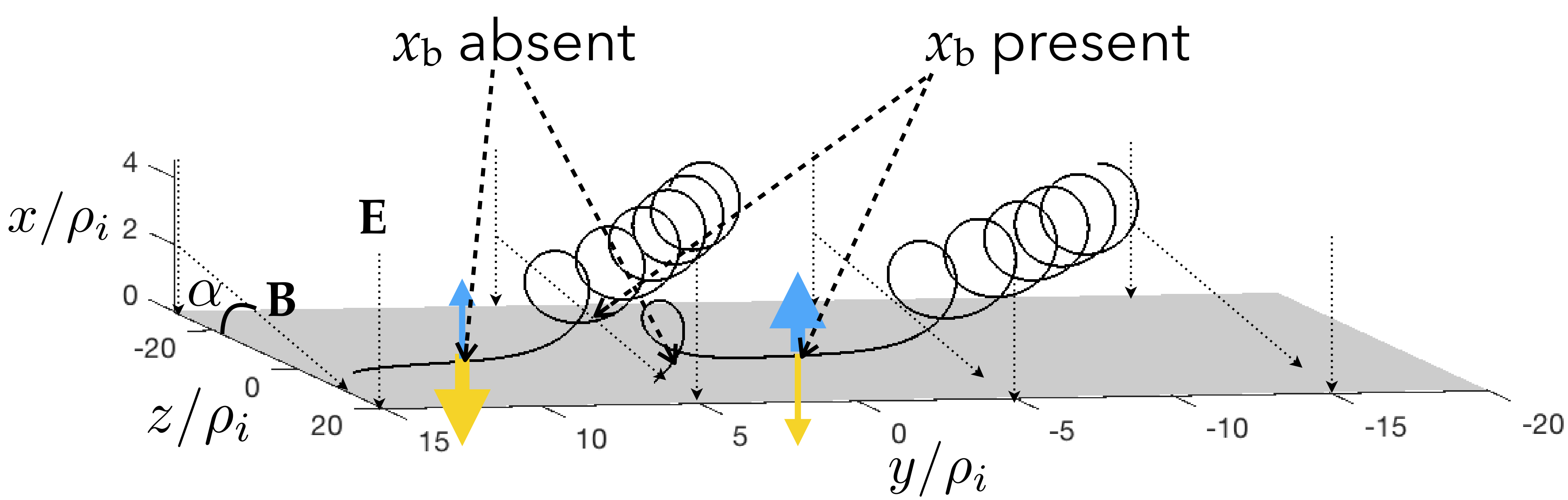
$$c_s = \sqrt{(ZT_{\mathrm{e}}/m_{\mathrm{i}})} = \textbf{Bohm speed}$$



References: A. Geraldini, F. I. Parra, F. Militello, "Solution to a collisionless magnetic presheath with kinetic ions" (*in preparation);* E. R. Harrison and W. B. Thompson, Proc. Phys. Soc. **74**, 145 (1959);

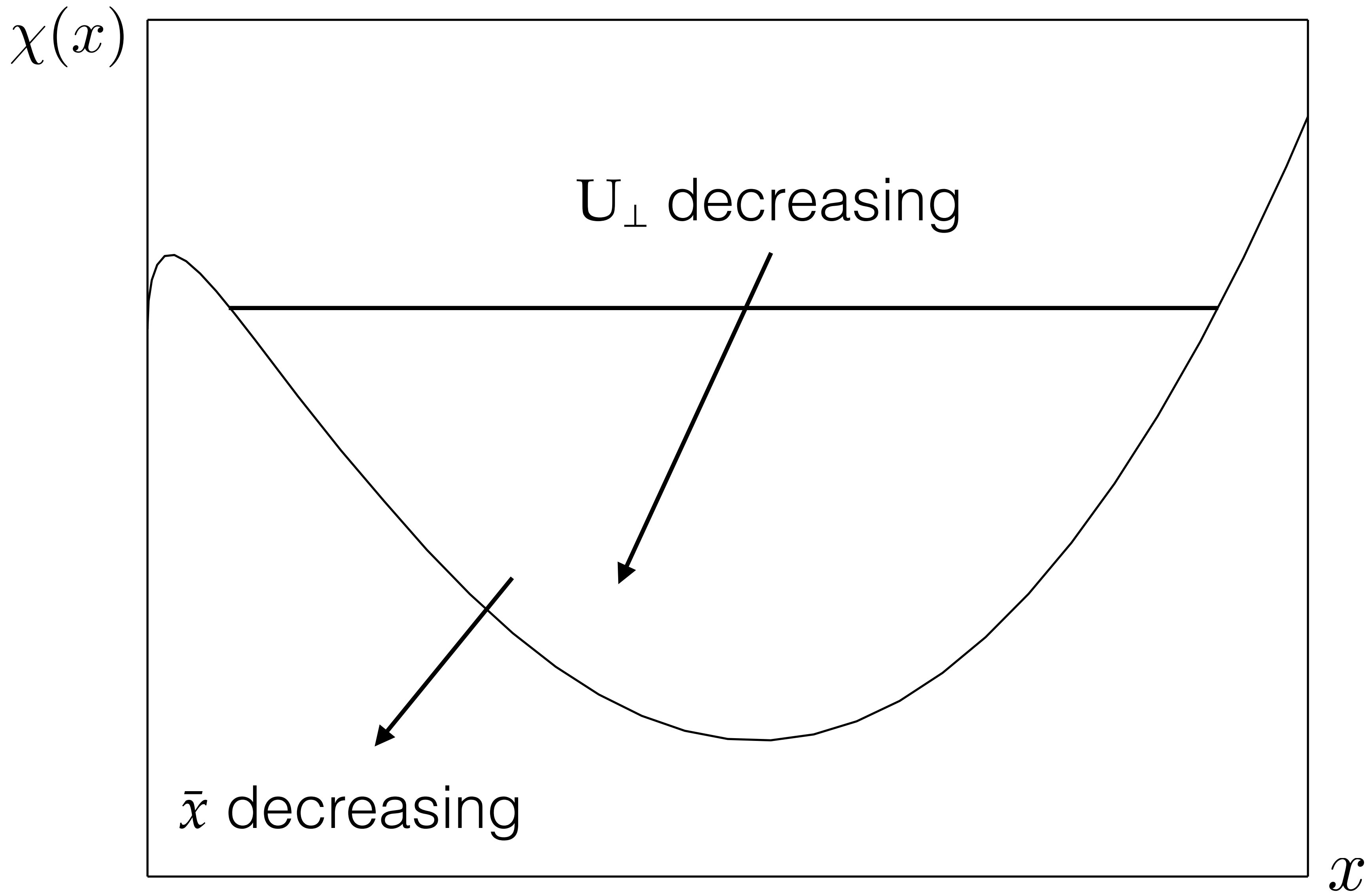
α<<1: open orbits

- After **last bounce** from x_b ion is considered in open orbit
- **Bounce point** x_b present if **magnetic force** > **electric force** when $v_x=0$
- **Magnetic force** ~ $v_y B$, **electric force** ~ $\phi'(x)$
- Time derivative $\dot{x}_b < 0$ as orbit approaches wall
- $\phi'(x)$ diverges at $x=0$, so eventually **electric force** > **magnetic force** always and x_b disappears



α<<1: open orbits

- Disappearance of x_b due to electric force beating magnetic force seen with change of **effective potential curve** and **perpendicular energy**



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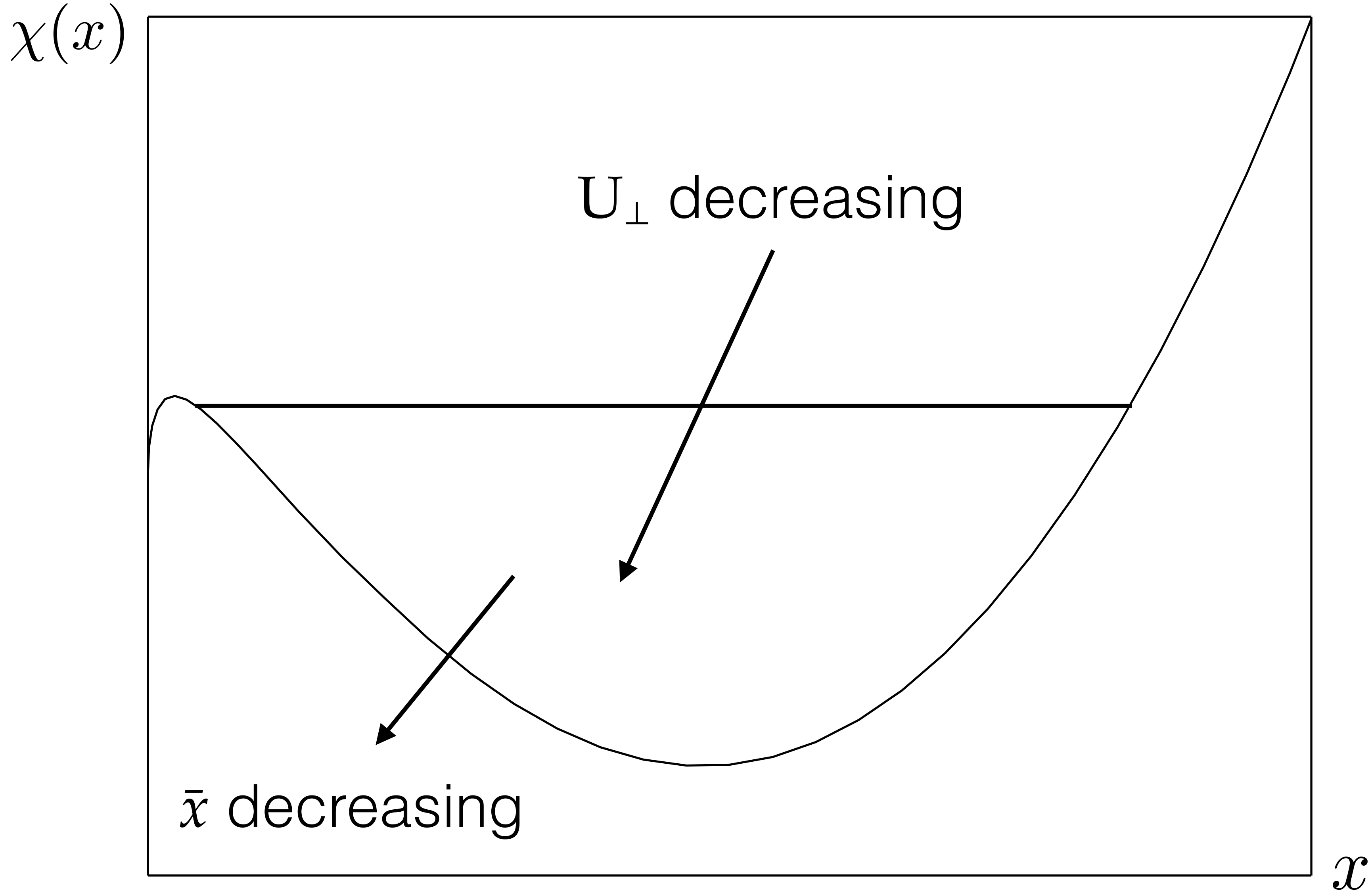

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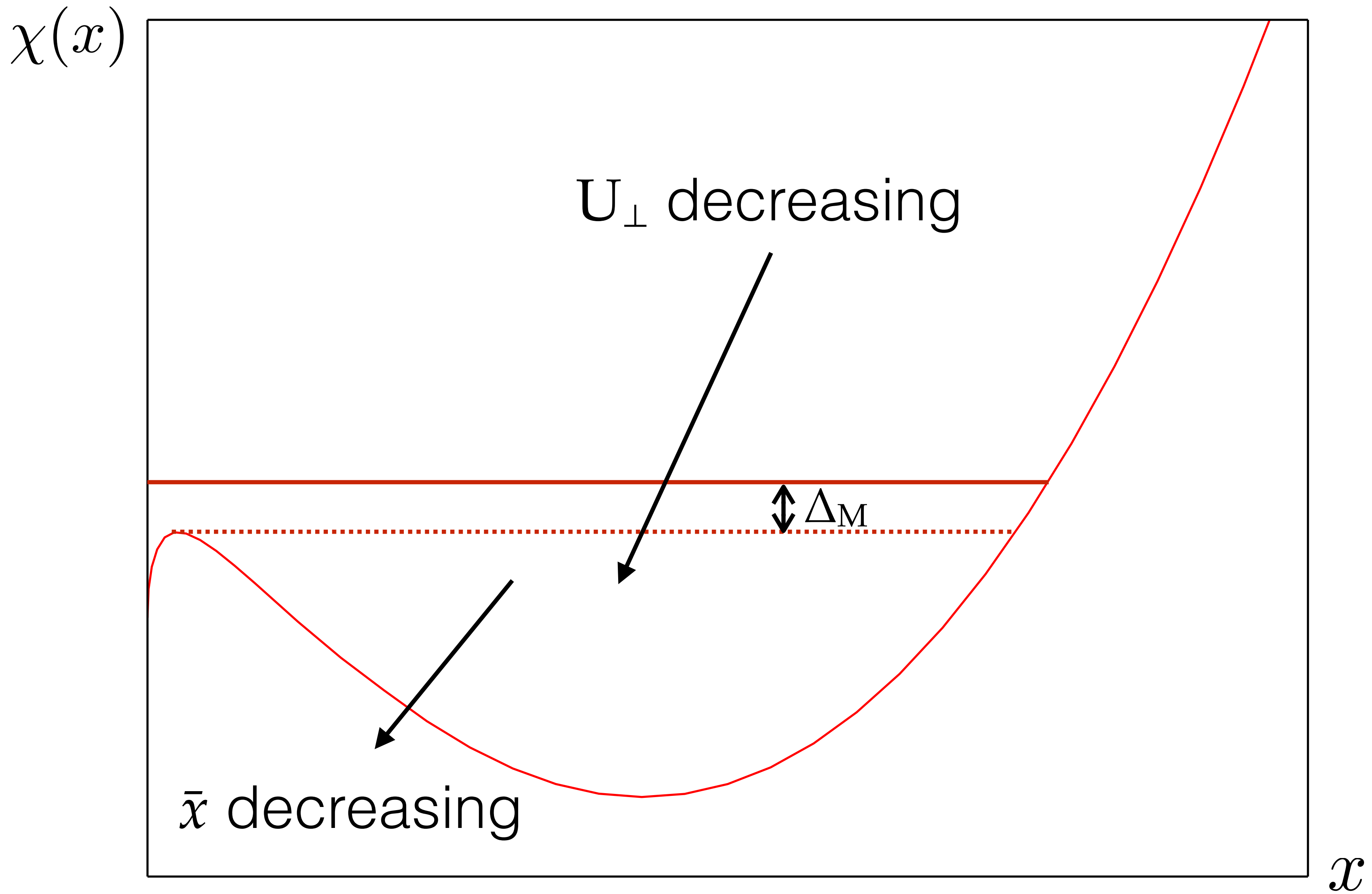

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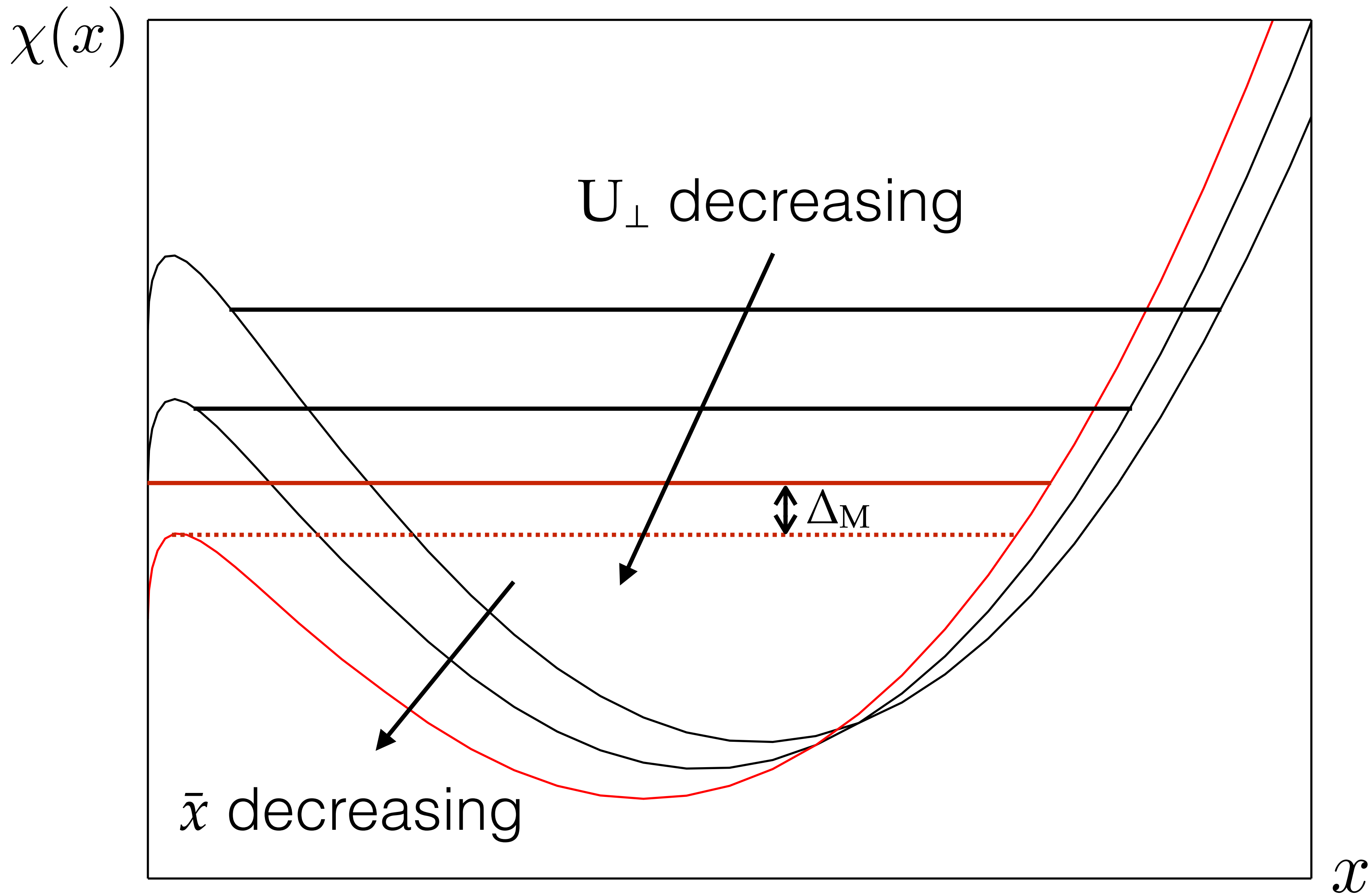


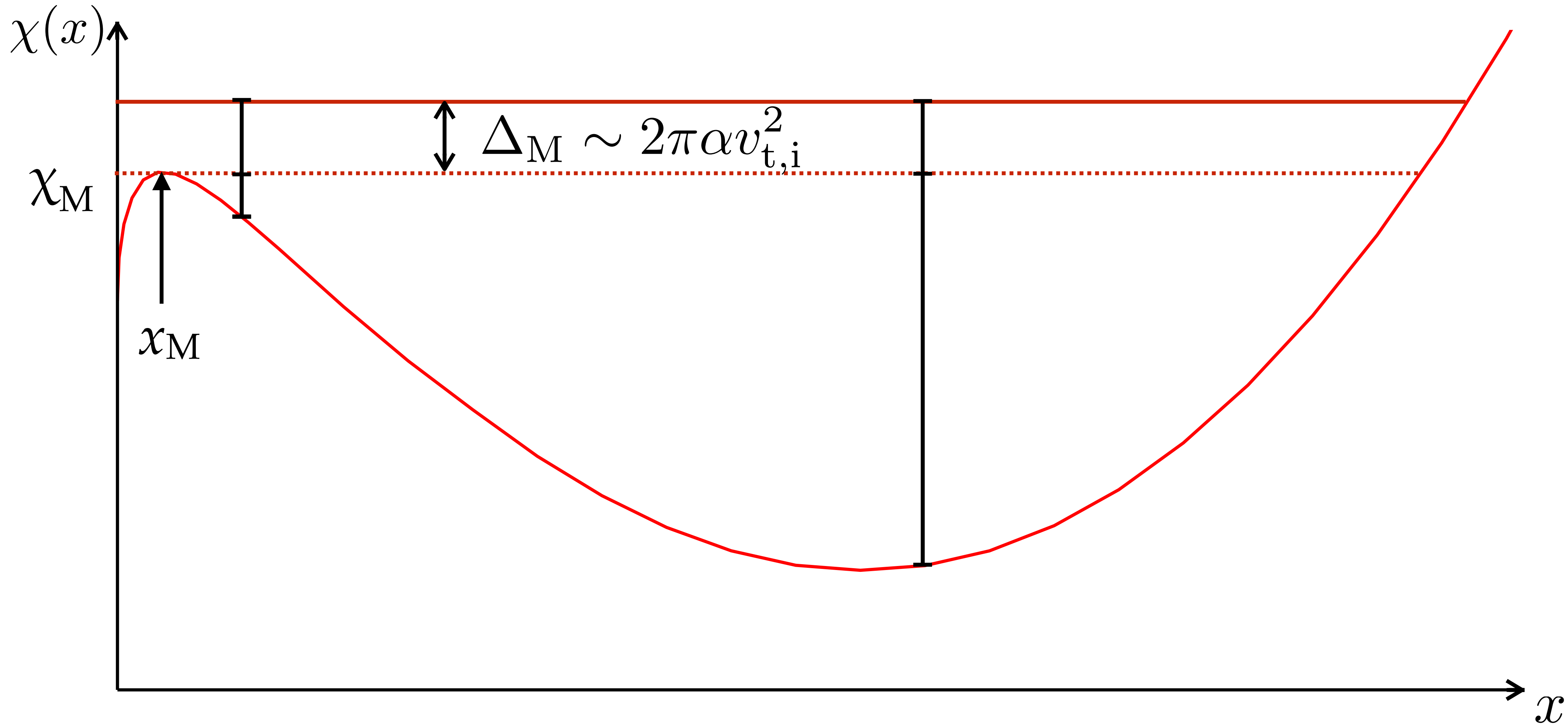
α<<1: open orbits

- $\Delta_{\rm M}$ = range of possible values of $U_\perp$ for open orbit at some $\bar{x}$ and $V_\parallel$

$$\Delta_{\rm M} = \alpha V_\parallel \int_{x_{\rm M}}^{x_{\rm t}} \frac{2\Omega^2 (s - x_{\rm M})}{\sqrt{2\left(\chi_{\rm M} - \chi(s)\right)}} ds \sim 2\pi\alpha v_{\rm t,i}^2$$

- Allows to obtain possible v_x at some $\bar{x}$, $V_\parallel$ and x

recall $\quad v_x = \pm V_x\left(\bar{x}, U_\perp, x\right) = \pm\sqrt{2\left(U_\perp - \chi\left(x, \bar{x}\right)\right)}$



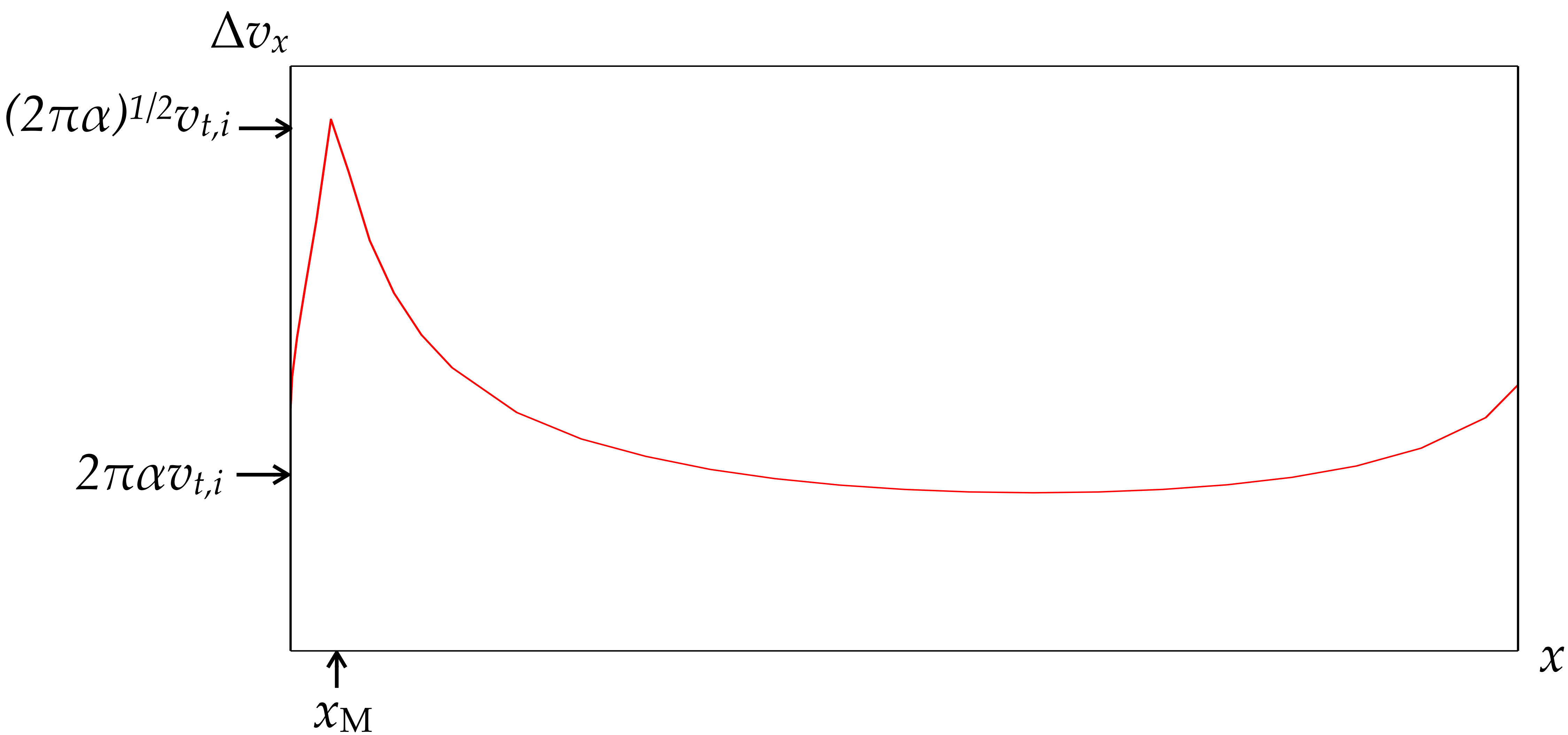
$\alpha \ll 1$: open orbits

- Distribution function of open orbits is $F(\mu(\bar{x},\chi_M(\bar{x})), U)$ and density is

$$n_{\mathrm{i,open}}(x) = \int_{\bar{x}_{\mathrm{m,o}}}^{\infty} \underbrace{\Omega d\bar{x}}_{dv_y} \int_{\chi_M(\bar{x})}^{\infty} \underbrace{\frac{dU}{\sqrt{2\left(U-\chi_{\mathrm{M}}(\bar{x})\right)}}}_{dv_z} F(\mu(\bar{x},\chi_M(\bar{x}),U)\underbrace{\Delta v_x}_{\int dv_x}$$

- Δv_x = small range of possible v_x at some $\bar{x}$, $V_\parallel$ and x

$$\Delta v_x = \sqrt{2\left(\Delta_{\mathrm{M}} + \chi_{\mathrm{M}} - \chi(x)\right)} - \sqrt{2\left(\chi_{\mathrm{M}} - \chi(x)\right)}$$



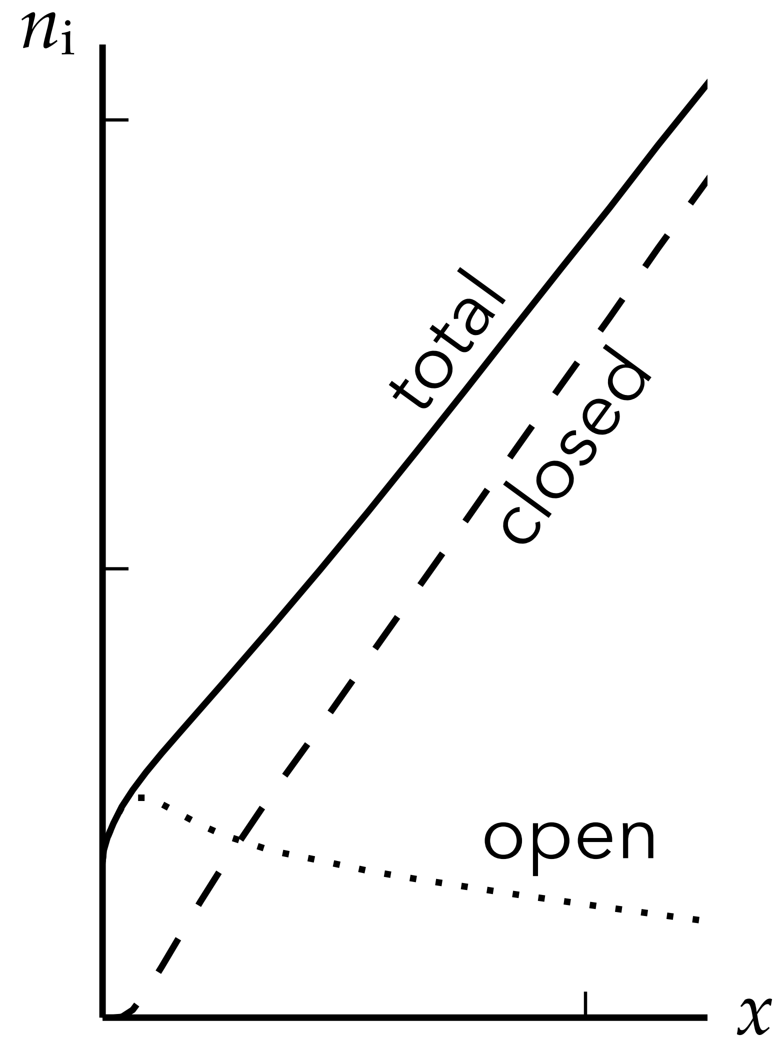
α<<1: total ion density

- Total ion density is

$$n_{\rm i}(x) = n_{\rm i,closed}(x) + n_{\rm i,open}(x)$$

- Note: $\alpha \lesssim n_{\rm i,open}(0)/n_\infty \lesssim \alpha^{1/2}$ leads to

$$\frac{e\phi(0)}{T_{\rm e}} = \ln\left(\frac{n_{\rm i,\, open}(0)}{n_\infty}\right) \sim \ln\alpha$$



Reference: A. Geraldini, F. I. Parra, F. Militello, "Solution to a collisionless magnetic presheath with kinetic ions" (*in preparation)*

Numerical Results

Boundary condition: ion distribution function

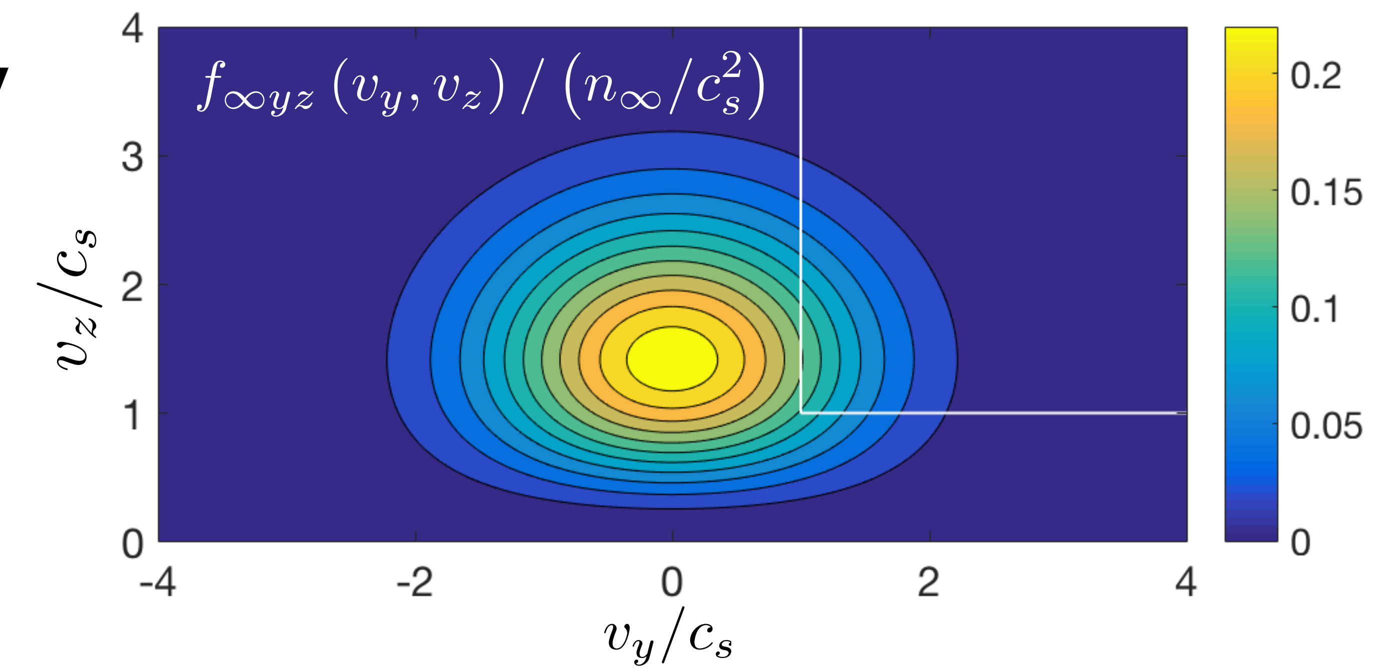
- Solved $Zn_i(x) = n_e(x)$ **numerically**
- Boundary ($x \to \infty$) distribution function **marginally** satisfies **Chodura condition**

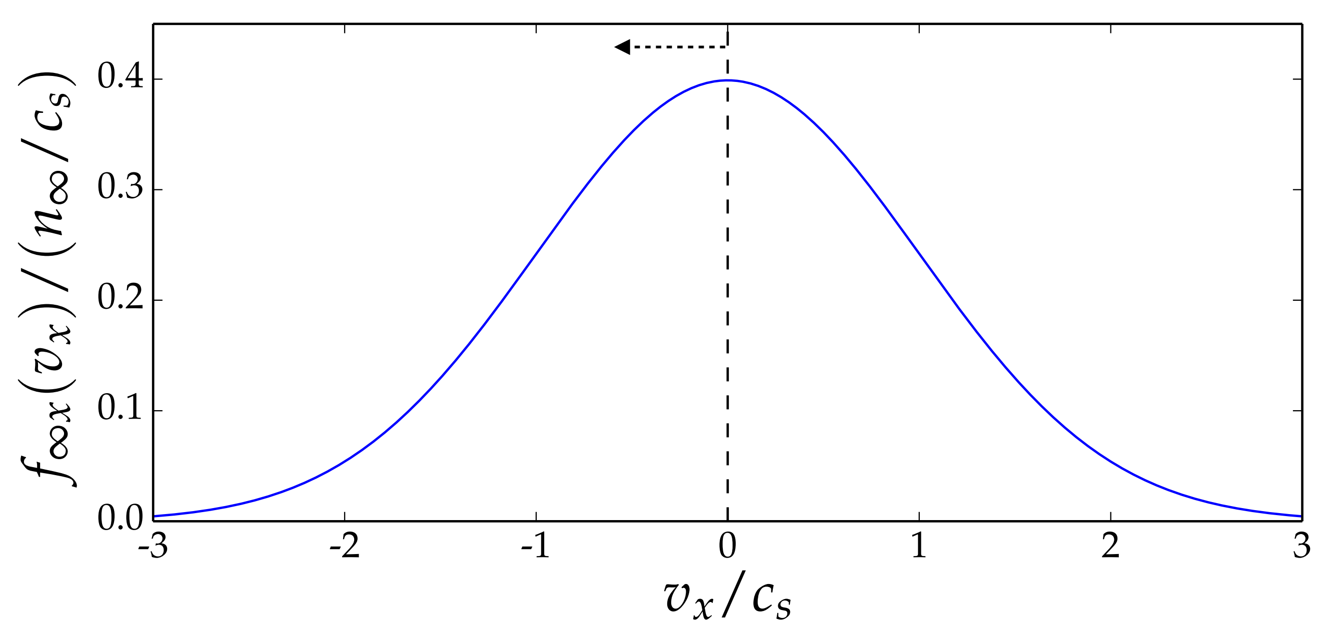
$$f_\infty(\boldsymbol{v}) \propto v_z^2 \exp\left(-\frac{m_i}{2T_i}\left(v_x^2 + v_y^2 + v_z^2\right)\right)$$

$$\text{with } T_i = T_e$$

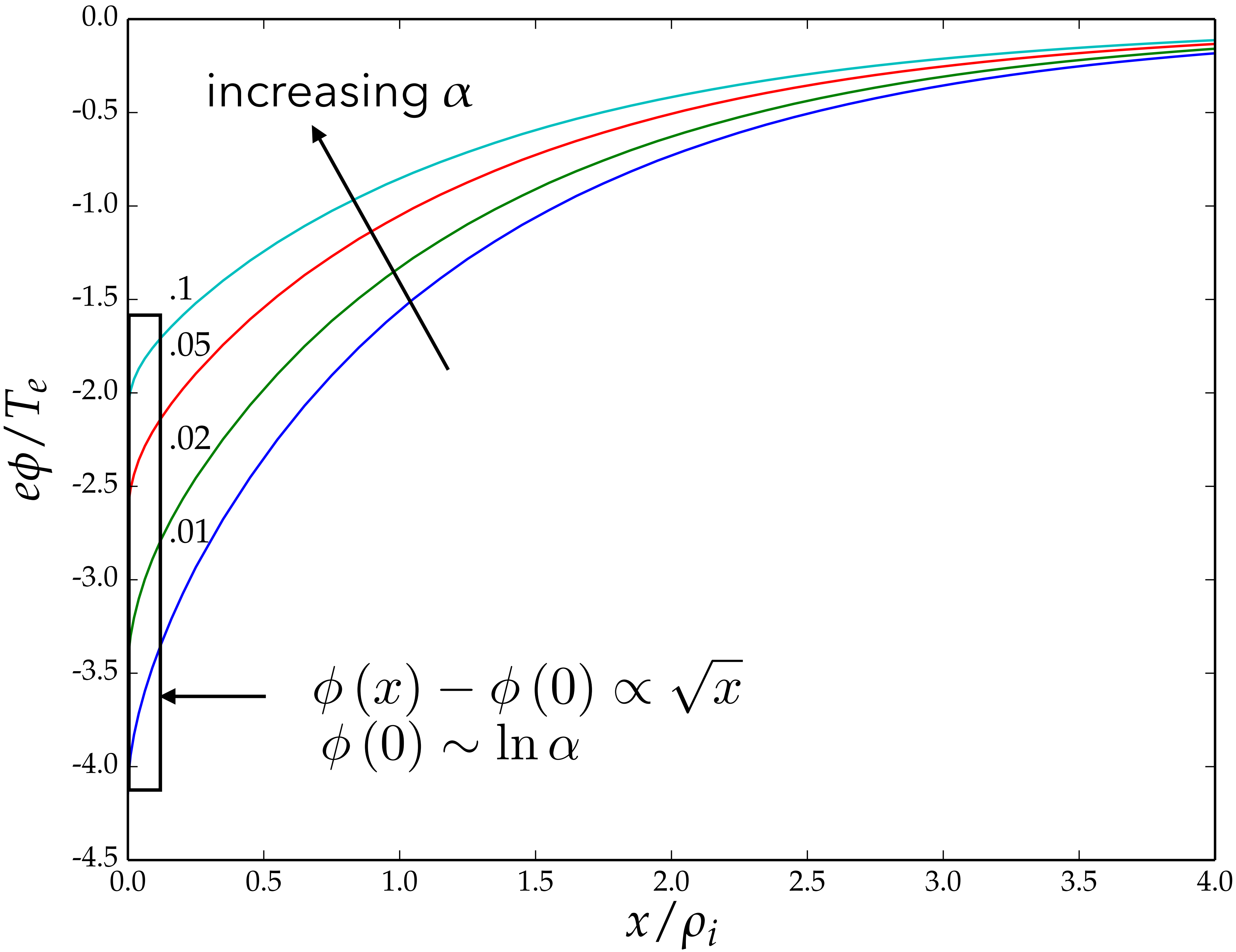
$$F(\mu, U) \propto (U - \Omega\mu) \exp\left(-\frac{m_i U}{T_i}\right)$$

$$u_{\parallel\infty} = 2\sqrt{\frac{2}{\pi}} c_s \simeq 1.60 c_s$$

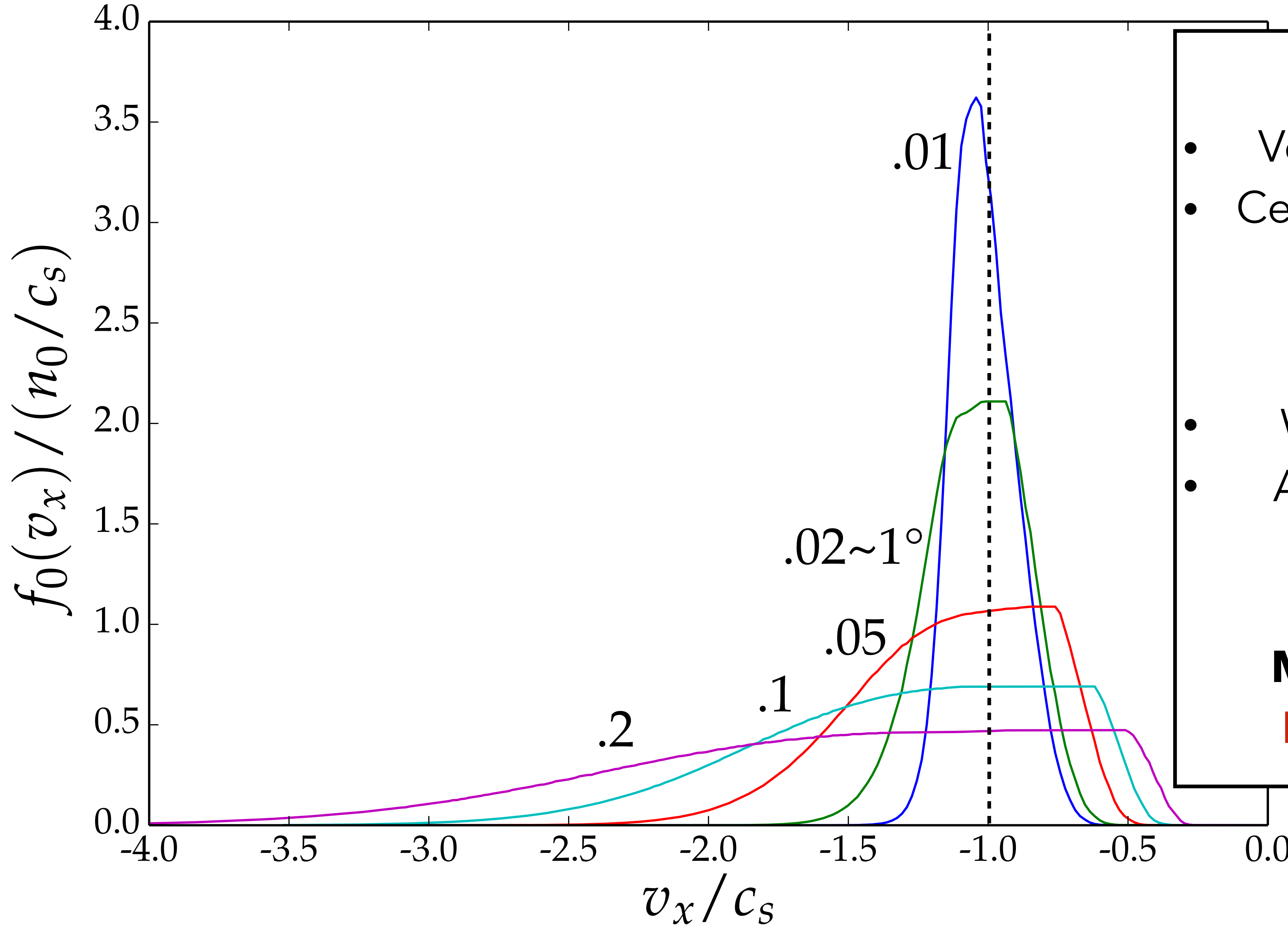




Electrostatic potential



Distribution function at x=0 (velocity normal to wall)



α = 0.01-0.02:

- Very thin & symmetric
- Centred at Bohm speed

α = 0.05-0.2:

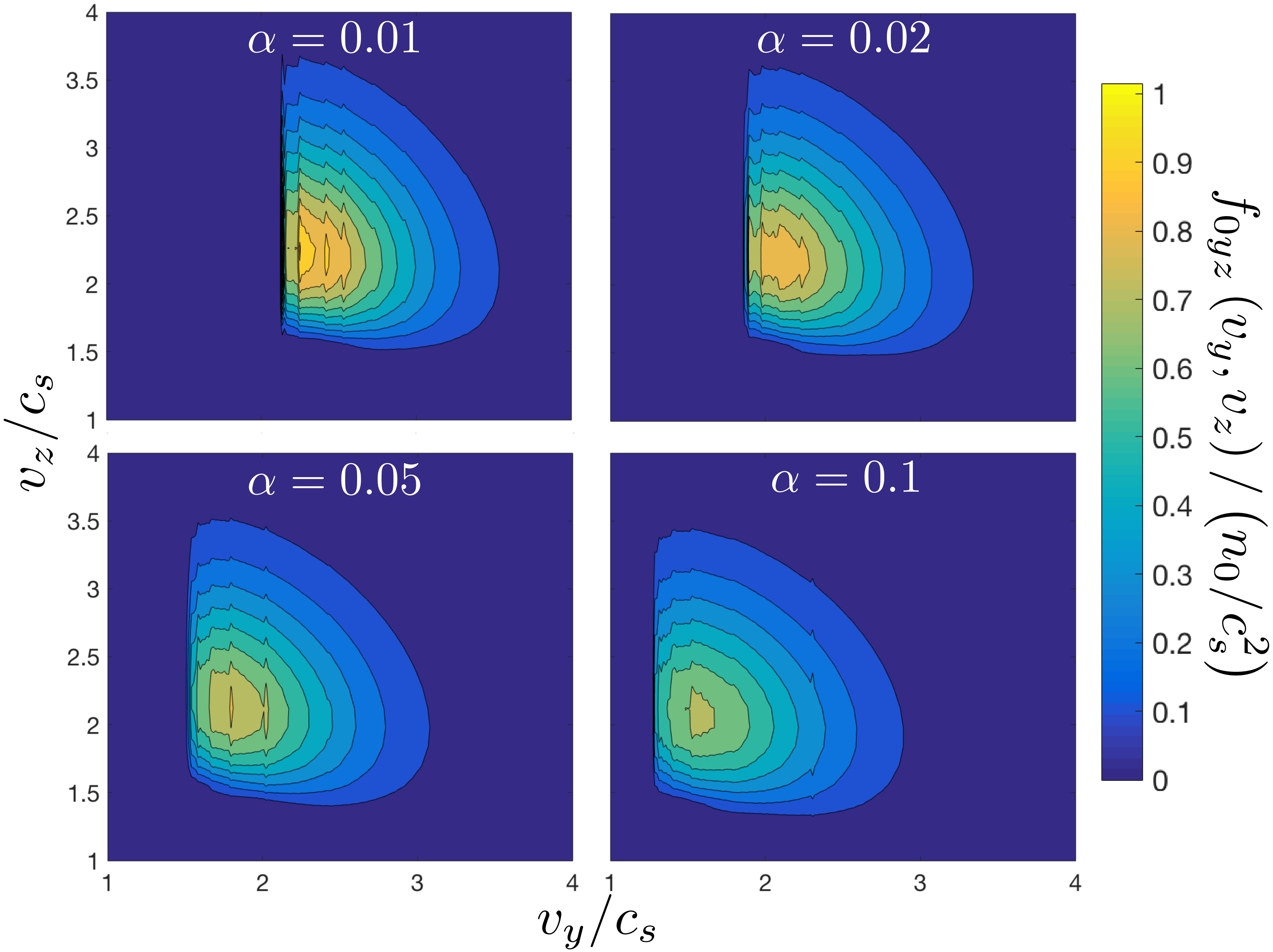
- Wider & asymmetric
- Average flow > sonic

All:
Marginally satisfy
Bohm condition

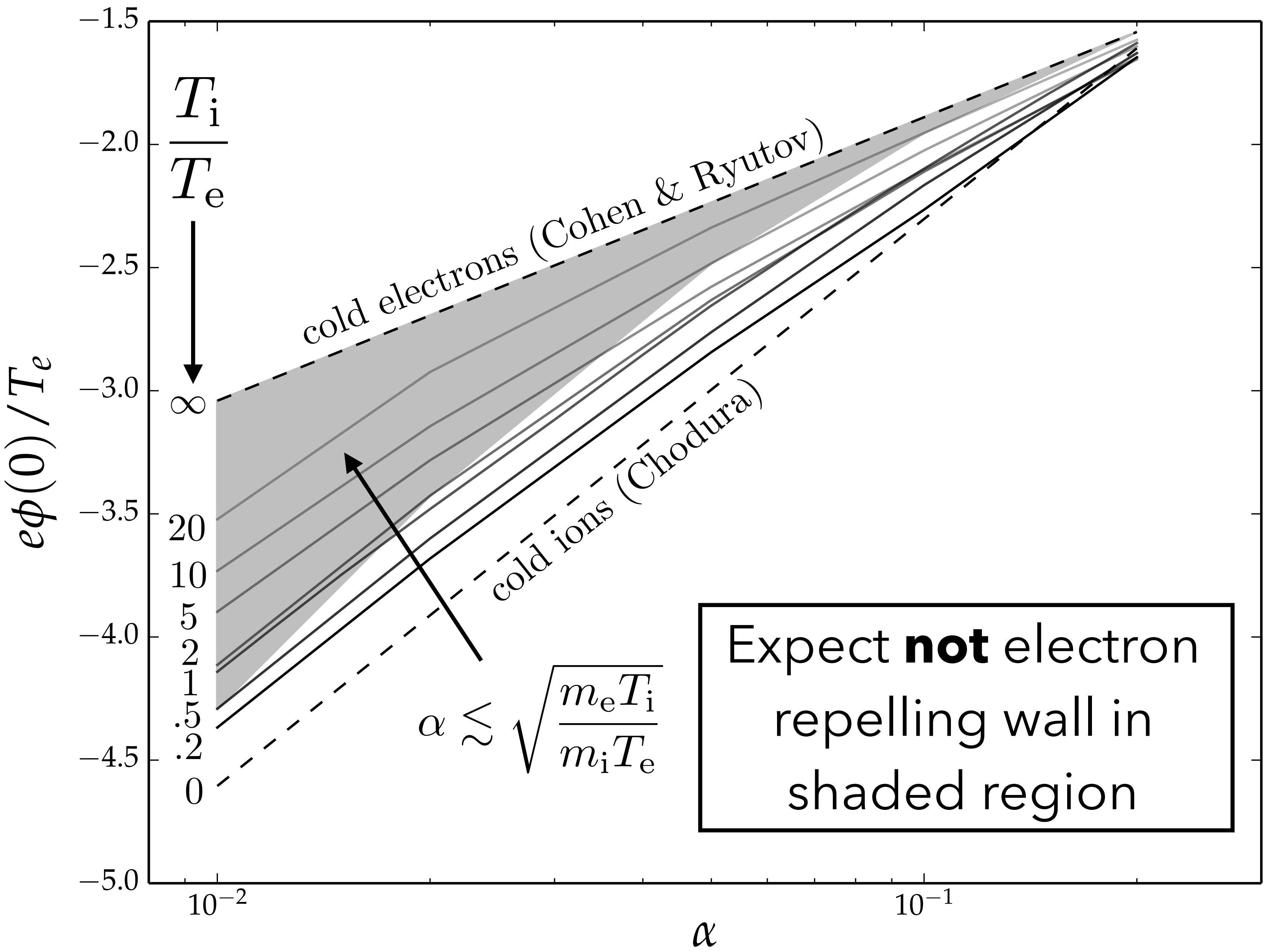
Potentially **less sputtering** at **smaller angles**, due to smaller number of ions with large velocity component normal to wall

Distribution function at x=0 (velocity parallel to wall)

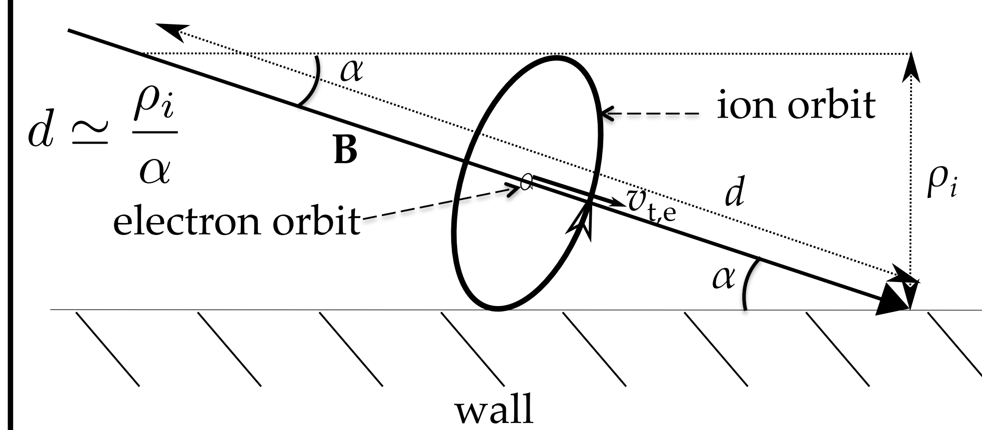
Combination of large $\mathbf{E}\times\mathbf{B}$ drift (v_y) and parallel velocity (v_z)



Current work: temperature dependence



- Consider ion and electron near wall



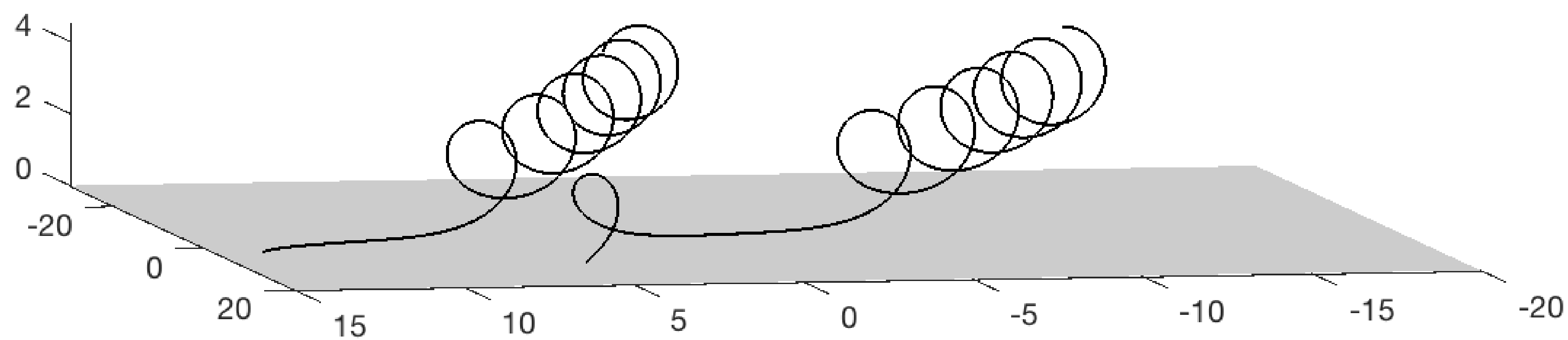
- Wall electron repelling if electrons reach it faster

$$\frac{\rho_i}{\alpha v_{t,e}} \ll \frac{\rho_i}{v_{t,i}}$$

$$\Rightarrow \quad \alpha \gg \sqrt{\frac{m_e T_i}{m_i T_e}}$$

Conclusion

- Exploited **small α** expansion of ion trajectories to solve for ion distribution function
- For a given potential profile, found expressions for lowest order ion density across magnetic presheath including crucial contribution of **open orbits** near $x=0$
- Derived **kinetic** generalization of **Chodura** condition
- Developed **code** that computes ion density and iterates over potential until quasineutrality is solved (with Boltzmann electrons)
- Numerical results consistent with kinetic Bohm condition at Debye sheath entrance
- For $\alpha \lesssim 0.05$ found substantially fewer ions travelling with large normal component of velocity towards wall —> **less damage to divertor targets**



Current work:

- Solve magnetic presheath numerically for different T_i/T_e ratios

Future work:

- Numerically study propagation of turbulence in the magnetic presheath