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Physics of Energetic Particles and Alfvén Waves*

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Institute for Fusion Theory and Simulation, Zhejiang University

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Theory and simulation of energetic particle dynamics and ensuing collective behaviors in fusion plasmas [NonLinear Energetic particle Dynamics (NLED) ER15-ENEA-03*]

NLED Wiki Page: https://www2.euro-fusion.org/erwiki/index.php?title=ER15-ENEA-03

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Outline

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(II) **The General Fishbone Like Dispersion Relation**:
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Test Particle and self-consistent Energetic Particle Transport in the presence of multiple fluctuations

(V) **Conclusions and Discussion**

(I) Introduction

- Hannes Alfvén [1942] discovered e&m waves can propagate in a conducting fluid (plasma) in the presence of finite $\boldsymbol{B}_0$
  - ⇒ Magneto-Hydro-Dynamic Alfvén waves ⇒ prevalent in nature & laboratory plasmas; *e.g.*
    - Alfvén instabilities due to energetic particles in fusion plasmas
    - Geomagnetic oscillations in solar-wind disturbed magnetosphere

- Significance of Alfvén waves:
  finite $\delta\boldsymbol{E}$ & $\delta\boldsymbol{B}$ ⇒ energy and momentum exchange with charged particles
  - ⇒ Acceleration/heating/transport of charged particles: Solar corona heating, Transport across Earth's magnetosphere; and fast ion losses in burning fusion plasmas (ITER)
  - ⇒ Nonlinear coupling of fusion reactivity profiles with plasma stability and transport: long time-scale complex behavior mediated by energetic/alpha particles [C&Z Rev. Mod. Phys. **88**, 015008 (2016); "Springer Monograph" in preparation]

Shear Alfvén continuous spectrum (1D)

- Shear Alfvén waves (SAW) are characterized by a continuous spectrum when the wave frequency varies continuously due to non-uniformity in the x-direction ($\boldsymbol{B}_0 = \hat{\boldsymbol{z}} B_0(x)$). They correspond to local (singular) fluctuations.

$$\omega^2 = \omega^2_{\mathcal{A}\ell}(x) \equiv k^2_{\|\ell} v^2_A(x) \, . \qquad \boldsymbol{v}_g || \boldsymbol{B}_0 \, , v^2_A(x) = B^2_0 / 4\pi \varrho_m$$

- We are interested in the $|\omega_{\mathcal{A}\ell} t| \gg 1$ time asymptotic behaviors. Assuming, to be justified a posteriori, $|\partial_x| \gg |k_y|$, $\boldsymbol{\nabla} \cdot \delta \boldsymbol{j} = 0$ (vorticity Eq.) becomes

$$\frac{\partial}{\partial x} \left[\frac{\partial^2}{\partial t^2} + \omega^2_{\mathcal{A}\ell}(x) \right] \frac{\partial}{\partial x} \delta \hat{\xi}_{x\ell}(x,t) \; = \; 0 \, .$$

- This equation can be straightforwardly integrated once and it yields

$$\frac{\partial}{\partial x} \delta \hat{\xi}_{x\ell}(x,t) \; = \; \hat{C}_\ell(x) e^{\pm i \omega_{\mathcal{A}\ell}(x) t} \, ,$$

- Here, $\hat{C}_\ell(x)$ is a function depending on equilibrium non-uniformities. Now, note that, as $\omega_{\mathcal{A}\ell}t \to \infty$ (the assumption $|\partial_x| \gg |k_y|$ is justified),

$$\partial_x \cong \pm i\omega'_{\mathcal{A}\ell}(x)t \;, \qquad (k_x \to \infty)$$

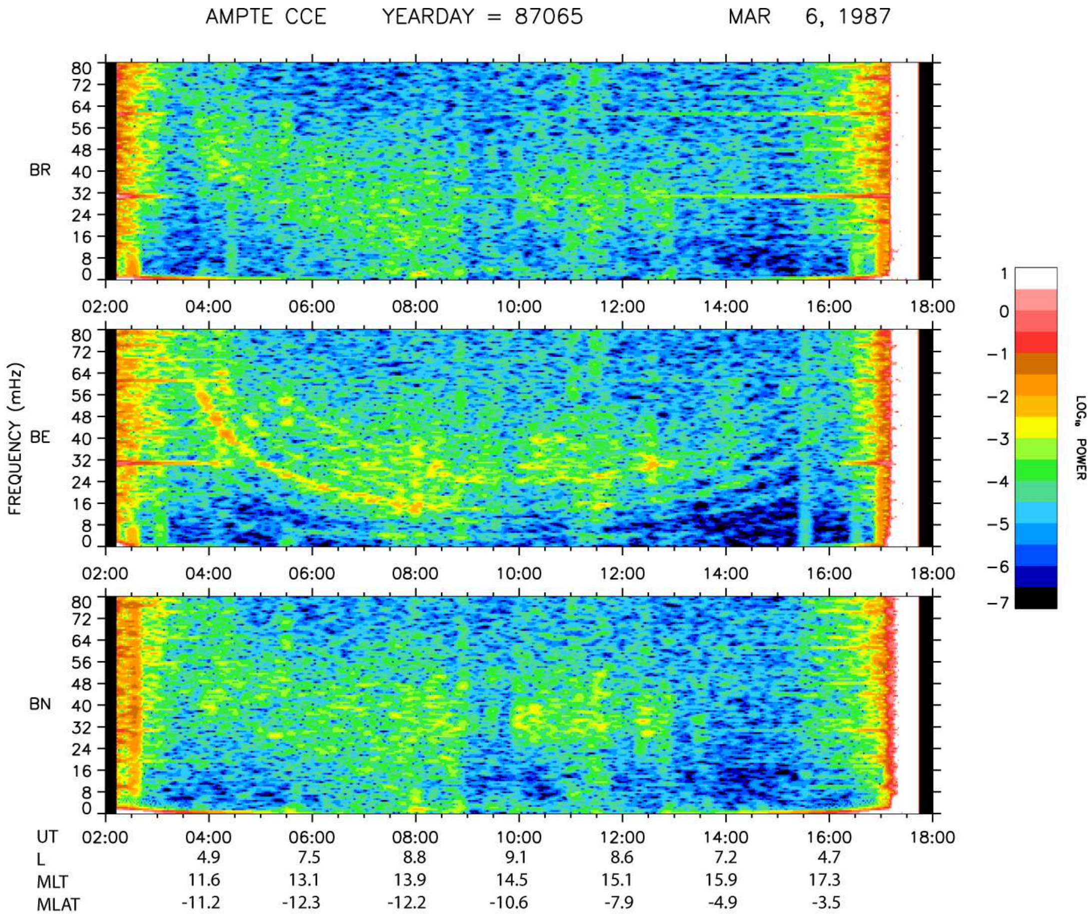
$$\delta\hat{\xi}_{x\ell}(x,t) \;=\; \mp i\frac{\hat{C}_\ell(x)}{\omega'_{\mathcal{A}\ell}(x)t}e^{\pm i\omega_{\mathcal{A}\ell}(x)t}\;.$$

- From the condition $\boldsymbol{\nabla}\cdot\delta\hat{\boldsymbol{\xi}}_{\perp\ell} = 0$, one derives $\delta\hat{\xi}_{y\ell} = (i/k_y)\partial_x\delta\hat{\xi}_{x\ell}$; i.e.,

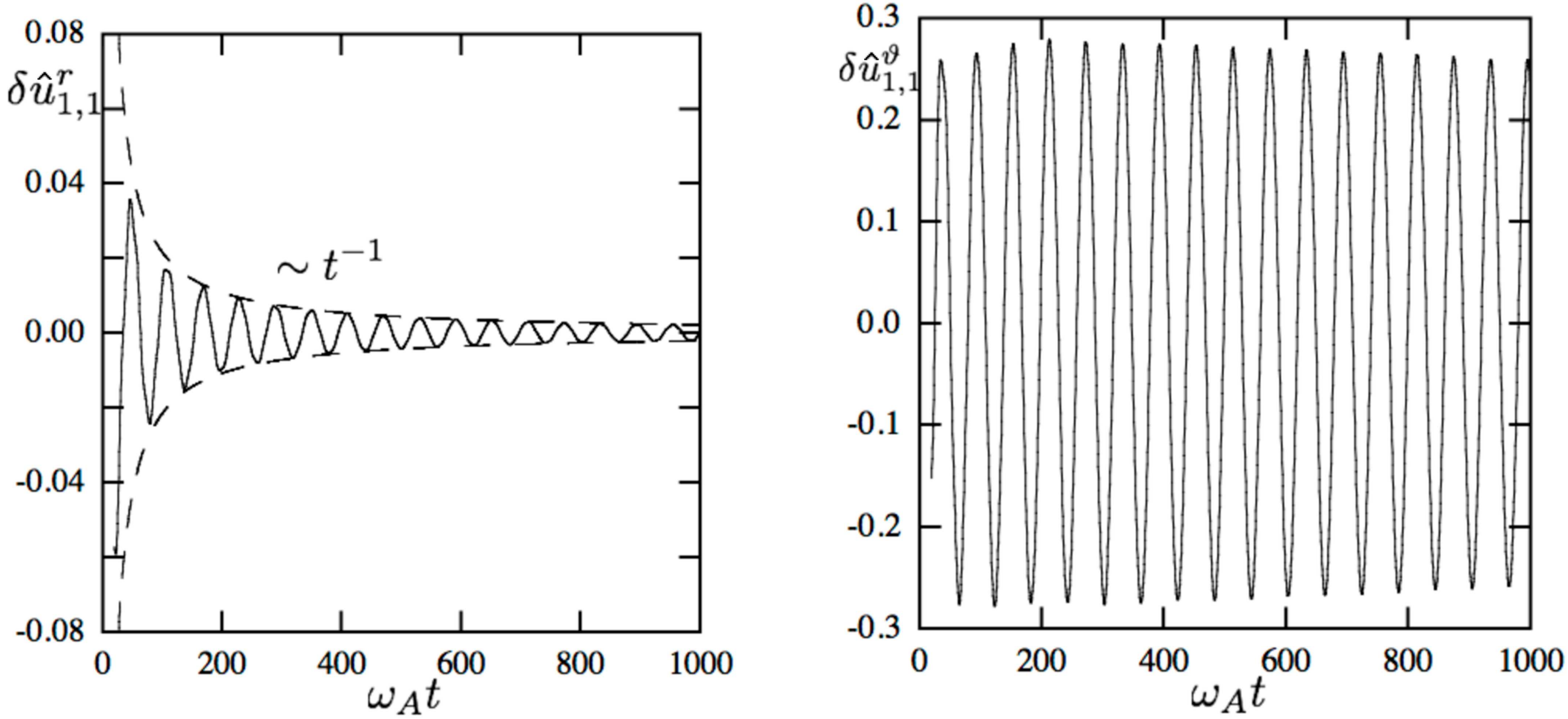
$$\delta\hat{\xi}_{y\ell}(x,t) \;=\; \frac{i}{k_y}\hat{C}_\ell(x)e^{\pm i\omega_{\mathcal{A}\ell}(x)t}\;.$$

- Insight in the dynamics associated with the resonant excitation of the SAW continuum and resonant wave absorption ($\propto \omega'_{\mathcal{A}\ell}(x)$ [Chen74, Chen95]):
  - the $\delta\hat{\xi}_{x\ell}$ component exhibits the characteristic $(1/t)$ decay via phase mixing of the continuous spectrum
  - $\delta\hat{\xi}_{y\ell}$ shows undamped oscillations at local SAW frequencies

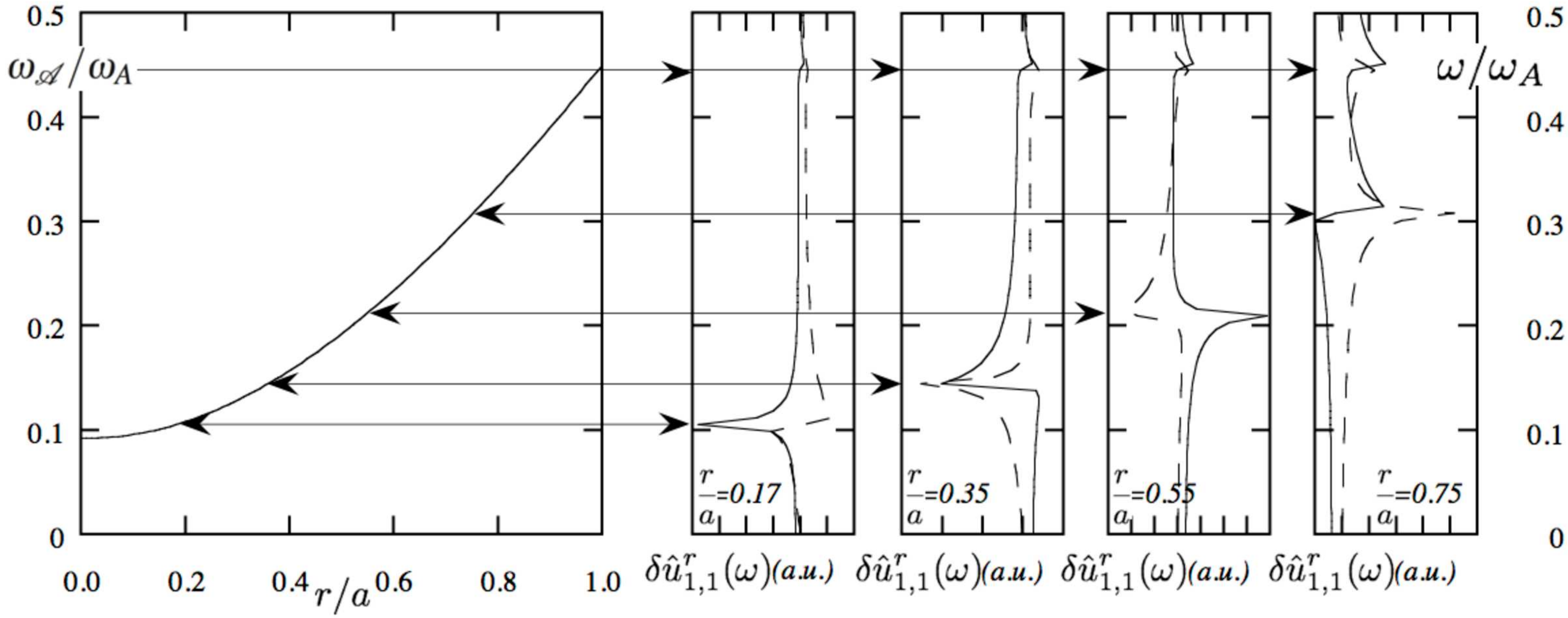
- These properties are nicely demonstrated by the satellite observations of magnetic field fluctuations in the Earth's magnetosphere [Engebretson 87]. B_R, B_E and B_N correspond to, respectively, $\delta B_x(\delta\hat{\xi}_x), \delta B_y(\delta\hat{\xi}_y), \delta B_\parallel$.



- The same behaviors have also been demonstrated by ideal MHD initial value numerical simulations of SAW dynamics in a cylindrical plasma [Vlad99].
- Introducing (r, ϑ, z) as coordinate system in a cylindrical plasma of periodic length $2\pi R_0$ and radius a, $\delta \boldsymbol{u}(r, \vartheta, z, t) \equiv \partial_t \delta \boldsymbol{\xi} = e^{\mathrm{i}(nz/R_0 - m\vartheta)} \delta \hat{\boldsymbol{u}}_{m,n}(r, t)$.



- The existence of local singular oscillations of the SAW continuous spectrum is further demonstrated by the Fourier frequency spectrum of $\delta\hat{u}^r_{1,1}(r,t)$ at several radial locations, reflecting the spatial structure of the continuum frequency $\omega_{\mathcal{A}}(r/a)/\omega_A$ [Vlad99].



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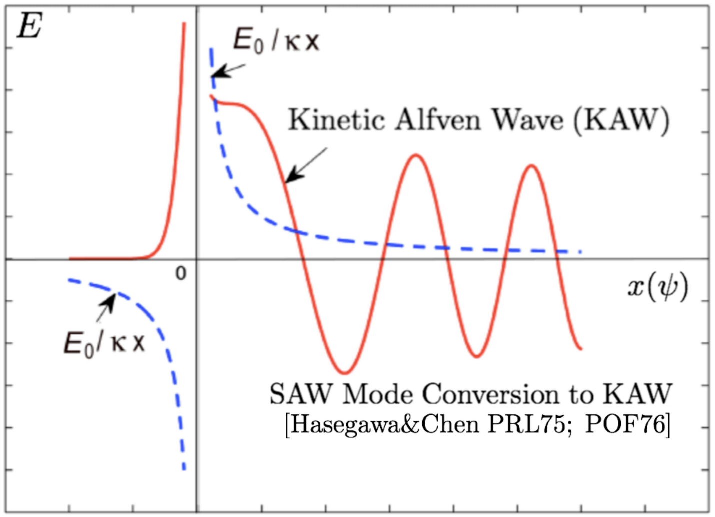
Kinetic theory I: thermal plasma response

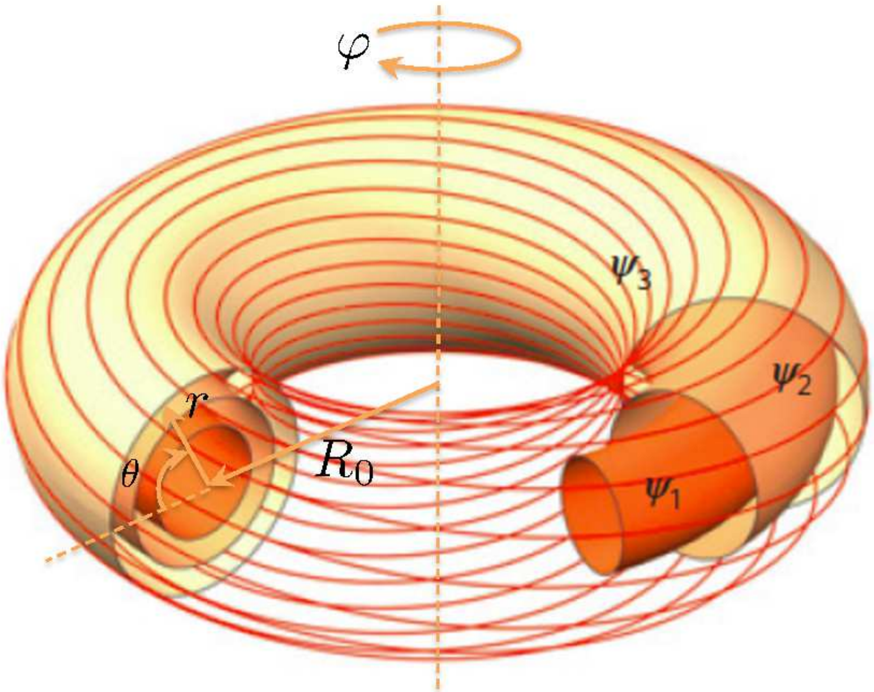
- Break down of the ideal MHD assumption ($k_\perp^2\rho_i^2 \ll 1$) is suggested by $|k_x| \simeq |\omega'_A(x)t| \to \infty$, when approaching the SAW continuum.
- Proper treatment of Larmor radius scale SAW require kinetic theory analysis and yield the Kinetic Alfvén Wave (KAW) dispersion relation

$$\omega^2 = k_\parallel^2 v_A^2(1+k_\perp^2\rho_K^2) = \omega_A^2(1+k_\perp^2\rho_K^2)\ , \qquad \rho_K^2 \propto \rho_i^2$$



- Most relevant new (w.r.t ideal MHD) dynamics on short scales are due to charge separation
  - finite $\delta E_\parallel$ due to, *e.g.*, FLR, finite electron inertia and plasma resistivity
  - finite $\delta E_\parallel$ & wave-particle interactions $\Rightarrow$ Landau damping (col.less dissip.)
  - singularities removed on short scales by finite radial energy propagation

Shear Alfvén waves in toroidal systems (2D)



Adapted from [Fasoli et al NAT16]

- SAW continuous spectrum in one dimensional non-uniform plasma

$$\omega^2 = k_\parallel^2 v_A^2 = \omega_A^2(\psi) \; .$$

- Radial (ψ) singular mode structures and resonant absorption (continuum damping) $\propto \omega_A'(\psi)$ [Chen74, Chen95].
- In higher dimensionality systems, such as nearly two-dimensional (2D) or three-dimensional (3D) toroidal devices, the main additional complication is due to the modulations of v_A along $\mathbf{B_0}$.

- This causes the loss of translational symmetry for SAW traveling along $\mathbf{B_0}$ and sampling regions of periodically varying v_A.
- Similarly to electron wave packets traveling in a one-dimensional periodic lattice of period L, SAW in toroidal systems are characterized by gaps in their continuous spectrum, corresponding to the formation of standing waves at the Bragg reflection condition; *i.e.*,

$$2L = \ell\lambda \; , \qquad \lambda \equiv \frac{2\pi}{k} \; , \qquad \ell \in \mathbb{N} \; , \qquad \begin{cases} L = 2\pi L_0 = 2\pi q R_0 \\ \text{connection length} \end{cases}$$

- Considering that $k \leftrightarrow k_\parallel$ in this analogy, the Bragg reflection condition becomes

$$k_\parallel = \frac{\ell}{2L_0} \; , \qquad \omega^2 = \frac{\ell^2 v_A^2}{4L_0^2} \; , \qquad \ell \in \mathbb{N} \; , \; \text{modulation periodicity}$$

with v_A representing now some "typical" value of the Alfvén speed on the reference magnetic surface.

- Examples:
  - $\ell = 1$: Toroidal Alfvén Eigenmode (TAE) gap [Pogutse 78, D'Ippolito 80, Kieras 82]
  - $\ell = 2$: Ellipticity induced Alfvén Eigenmode (EAE) gap [Betti 91]
  - $\ell = 3$: Non-circularity induced Alfvén Eigenmode (NAE) gap [Betti 91]

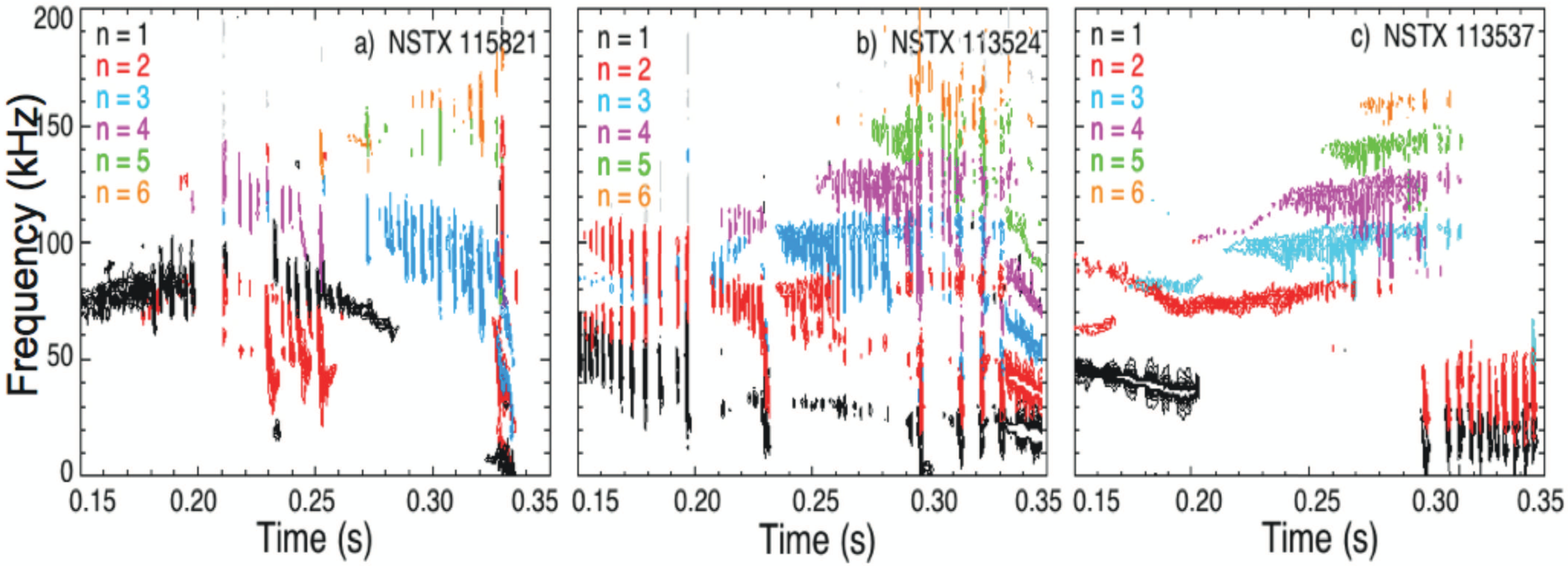
- A low-frequency ($\ell = 0$) gap also exists because of finite plasma compressibility [Chu92, Turnbull 93] at $\omega \simeq \beta_i^{1/2} v_A / R_0$. Beta-induced AE (BAE).

- Major breakthrough: nearly undamped fluctuations exist in frequency gaps near SAW acc. points (vanishing continuum damping $\omega'_A(\psi) = 0$)
  - Theoretical prediction [Cheng, Chen, Chance 85]
  - Radial (local) potential well due to geometry and equilibrium non-uniformity [C&Z POP14,RMP16]
    EP effects may be non-perturbative [Chen 84,94] (X.Wang,Z.Lu)
  - Experimental observation [Heidbrink et al 91; Wong et al 91]

Alfvén Eigenmodes and Energetic Particles Modes

- Toroidal Alfvén Eigenmodes (TAEs) [Cheng, Chen Chance 1985] and Energetic Particle Modes (EPMs) [Chen 1994] observed in toroidal devices

NSTX $\delta\boldsymbol{B}$ [Fredrickson et al. 2006]

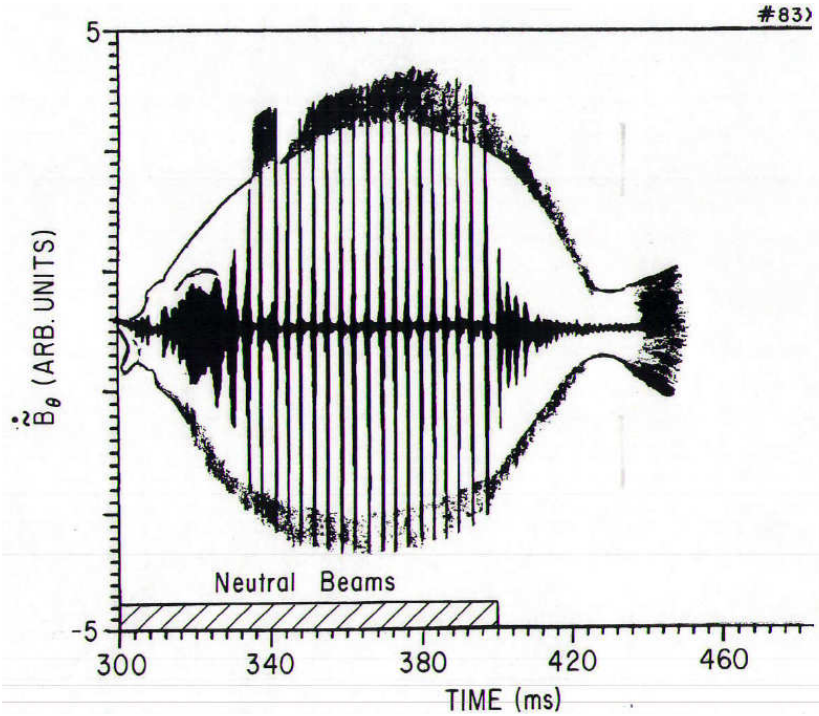


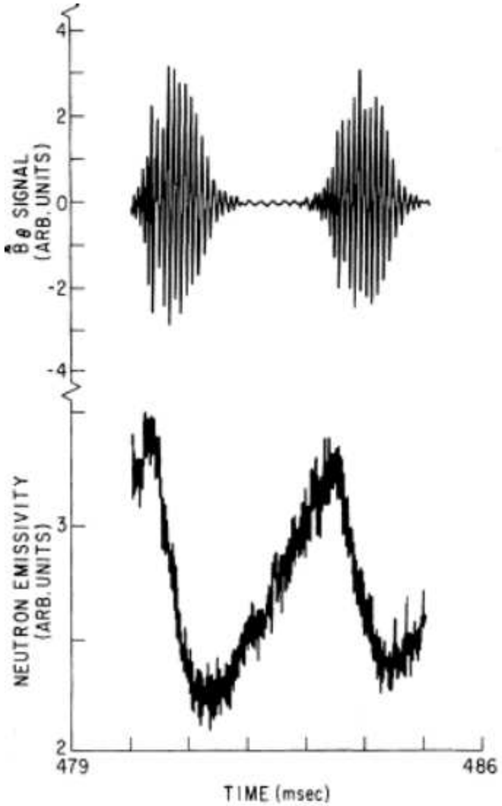
- On left, bursting, chirping EPM-like modes (non-perturbative).
- Evolutions to nearly coherent, TAE-like modes on right.

Kinetic theory II: excitation by En. Particles

"Fishbone" instability in PDX [McGuire; 1983]

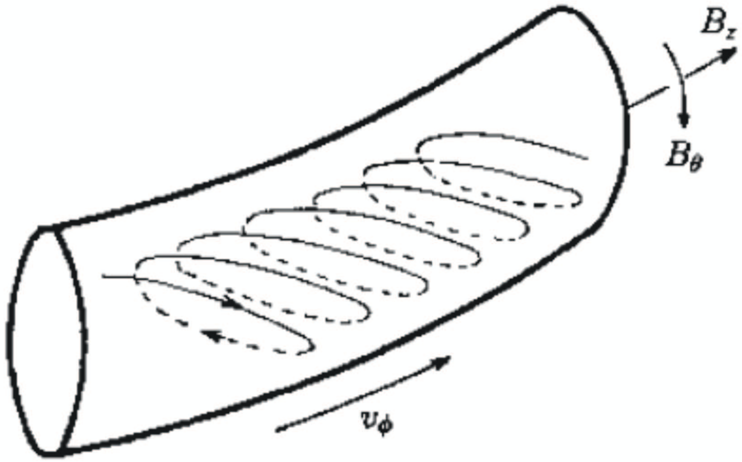
- ⇒ Suprathermal SAW fluctuations excited during perpendicular (to $\boldsymbol{B}$) beam-injection experiments.
- ⇒ Symmetry-breaking perturbations ⇒ significant ($\sim$ 30%) fraction loss of beam ions!!





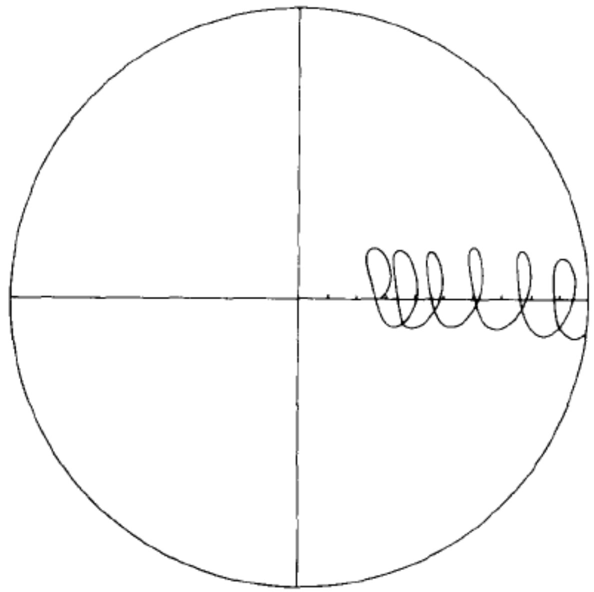
Insights from PDX Fishbone observations: [White et al PF83]

- Excitations via wave-particle interactions tapping EP's finite $|\nabla P_{EP}|$ expansion free energy.



⇒ Magnetically trapped charged particles precess in ϕ

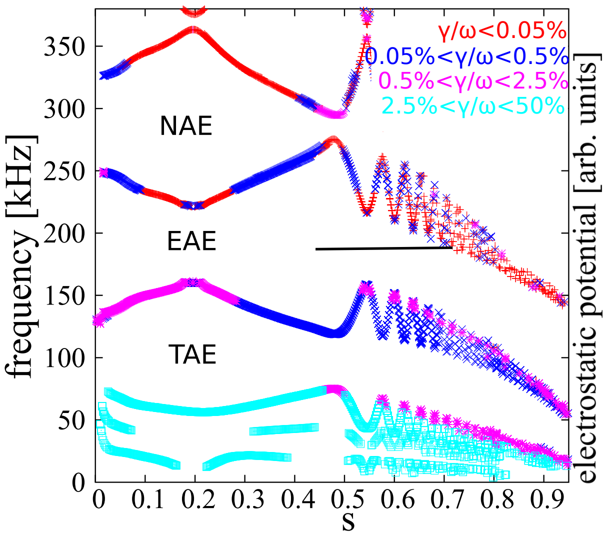
⇒ Precessional frequency $\bar{\omega}_d \propto \mathcal{E} = v^2/2$

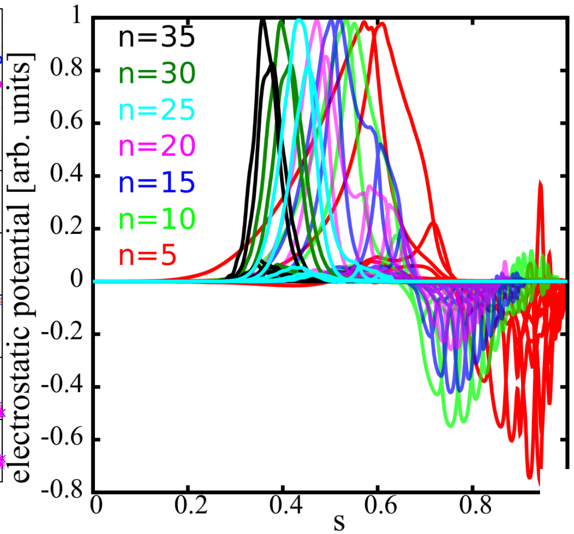


⇒ $\omega = \bar{\omega}_d$ resonant particles secularly move in the radial (R) direction

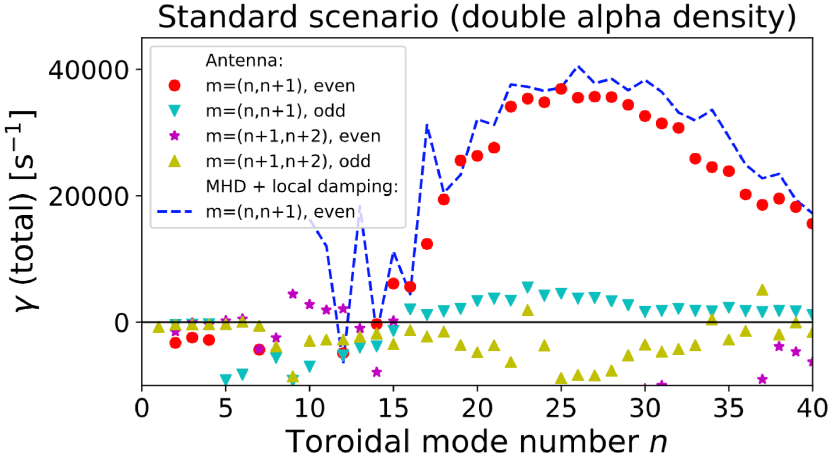
Gyrokinetic simulation of SAW (1) – (NLED)

- AE spectrum in ITER [Gorelenkov 14, Lauber 15] (LIGKA [Lauber 05]).





- ITER 15 MA scenario (LIGKA) [Lauber 15]
- SAW continuum with local damping as color code
- Typical TAE mode structures

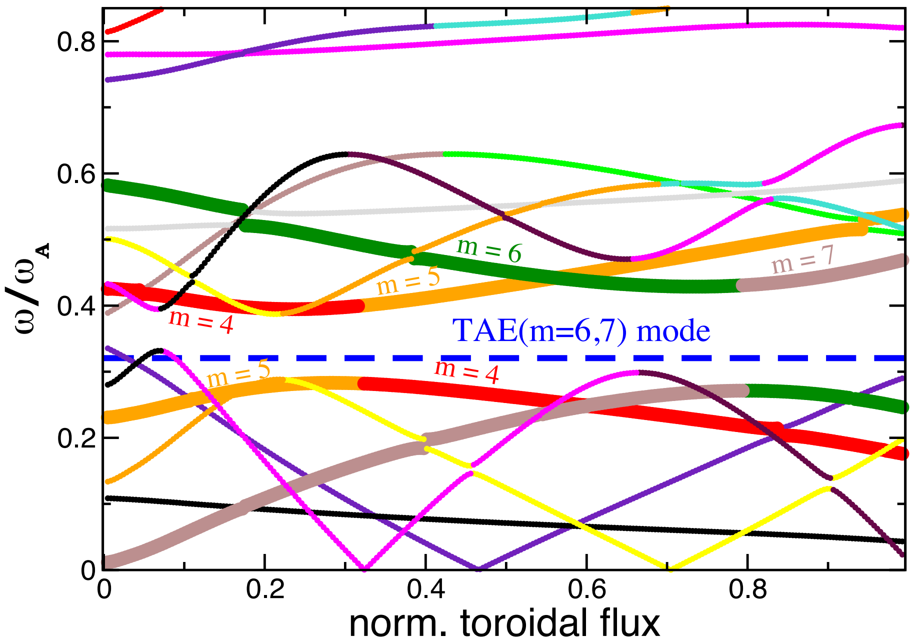


- Automatically obtained growth rates of various TAE branches using HAGIS-LIGKA code [T. Hayward-Schneider&Ph. Lauber 2017]

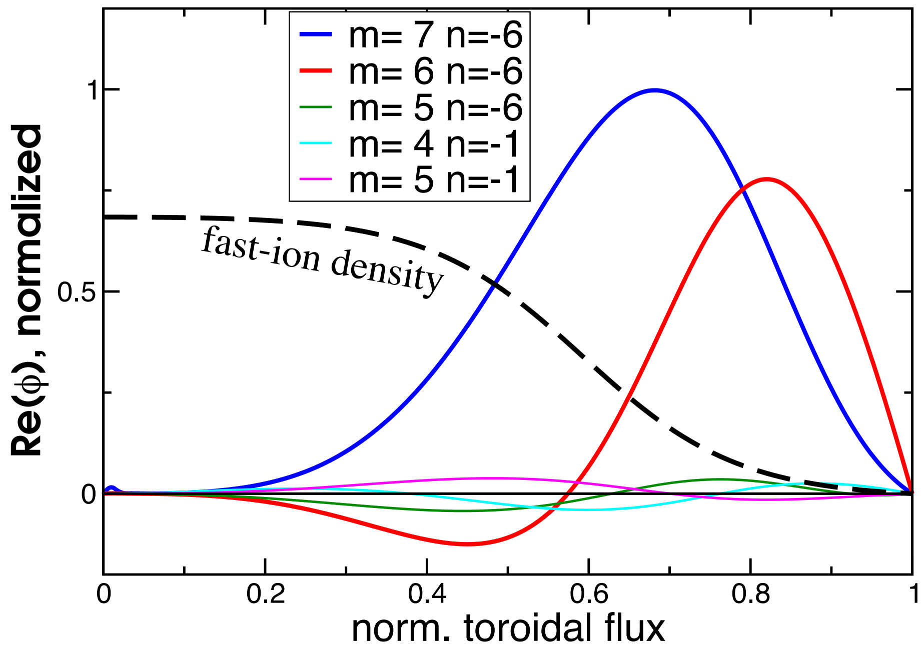
Gyrokinetic simulation of SAW (2) – (NLED)

- AE spectrum in W7-X [A. Könies et al 2015, 2017] (CKA-EUTERPE [A. Könies et al 2015]). (A. Mishchenko)

SAW Continuous Spectrum



AE mode structures



- In collisionless fusion plasmas, hybrid-kinetic and/or gyrokinetic descriptions are necessary and are becoming routine [C&Z RMP16].

(II) General Fishbone Like Dispersion Relation

- Kinetic Energy Principle ⇒ General Fishbone Like Dispersion Relation (GFLDR) provides a unified theoretical framework [Z&C POP14]

$$\boxed{i|s|\Lambda(\omega) = \delta\hat{W}_f + \delta\hat{W}_k(\omega)} \qquad s = \text{magnetic shear}$$

- $\Lambda(\omega)$: "inertia" (kinetic energy) due to background plasma
  ⇒ Structures of continuum, gaps, and resonant absorption
- $\delta\hat{W}_f$: "δW" (potential energy) due to background plasmas
  ⇒ existence of discrete AEs (different types; depending on equilibrium)
- $\delta\hat{W}_k$: "δW" (active potential energy) due to EPs
  ⇒ instability mechanisms & new unstable modes: EPMs [Chen 1994]
- Simple limit [Chen, White, Rosenbluth 1984]; [Coppi & Porcelli 1986]
  ⇒ $\Lambda(\omega) = \omega/\omega_A$ $(\omega_A = v_A/qR_0)$, $\delta\hat{W}_f \approx 0$ ⇒ fishbone

$$\Rightarrow \delta\hat{W}_k \propto \left\langle \frac{\mathcal{E}\bar{\omega}_d}{\bar{\omega}_d - \omega} \frac{\partial F_{EP}}{\partial r} \right\rangle_{\boldsymbol{v}}$$

- $F_{EP}(r, \mathcal{E})$: EP distribution function

- The fishbone-like dispersion relation demonstrates the existence of two types of modes (note: $\Lambda^2 = k_\parallel^2 q^2 R_0^2$ is SAW continuum):
  - a discrete gap mode, or Alfvén Eigenmode (AE), for $\mathbb{R}\mathrm{e}\Lambda^2 < 0$;
  - an Energetic Particle continuum Mode (EPM) for $\mathbb{R}\mathrm{e}\Lambda^2 > 0$.

- For AE $\Rightarrow$ Core plasma and non-resonant fast ion response provide a real frequency shift away from continuum accumulation point $(\Lambda = 0)$;
  $\Rightarrow$ resonant wave-particle interaction gives the mode drive.
  - $\delta \hat{W}_f + \mathbb{R}\mathrm{e}\delta \hat{W}_k > 0$: AE frequency is above the accumulation point
  - $\delta \hat{W}_f + \mathbb{R}\mathrm{e}\delta \hat{W}_k < 0$: AE frequency is below the accumulation point

- For EPM $\Rightarrow$ iΛ represents continuum damping [Chen et al 84, Chen 94]

$$\mathbb{R}\mathrm{e}\delta \hat{W}_k(\omega_r) + \delta \hat{W}_f = 0 \quad \Rightarrow \text{ determines } \omega_r \; , \qquad (\text{non} - \text{perturbative})$$

$$\gamma/\omega_r = (-\omega_r \partial_{\omega_r} \mathbb{R}\mathrm{e}\delta \hat{W}_k)^{-1} (\mathbb{I}\mathrm{m}\delta \hat{W}_k - |s|\Lambda) \quad \Rightarrow \text{ determines } \gamma/\omega_r$$

Non-perturbative energetic particle response

- Comparing the GFLDR and structure of SAW continuum [Z&C POP14, C&Z RMP16, POP17]. (X.Wang,Z.Lu)

GFLDR for AE/EPM

$$i|s|\Lambda(\omega) = \delta\hat{W}_f + \delta\hat{W}_k(\omega)$$

SAW continuous spectrum

$$\Lambda^2(\omega) = k_\parallel^2(\psi)q^2R_0^2 = (nq-m)^2$$

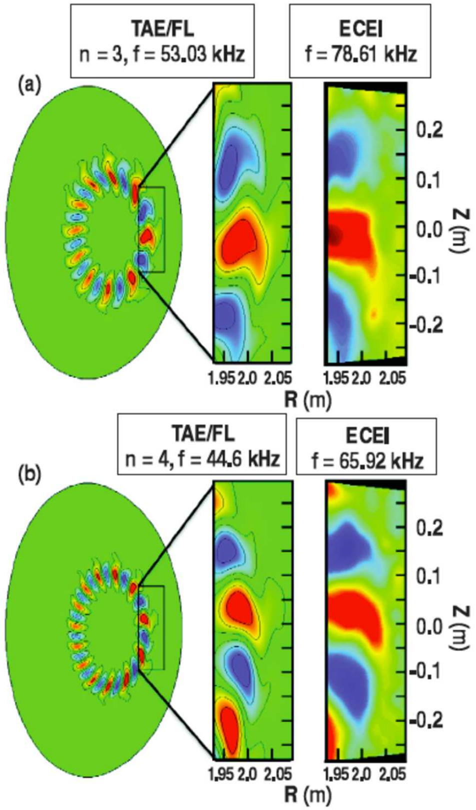
- Equilibrium geometry and non-uniformity of both core plasma ($\delta\hat{W}_f$) and energetic particles ($\delta\hat{W}_k(\omega)$) determine effective $k_\parallel$ of radially bound state $\Rightarrow$ potential well that determines mode frequency and mode structure.

- Recent interest for BAE and Alfvén Acoustic (BAAE [Gorelenkov et al 2007]) fluctuations: $\mathbb{R}\mathrm{e}(\delta\hat{W}_f + \delta\hat{W}_k) < 0$
  $\Rightarrow$ Non-perturbative EP response in $\delta\hat{W}_f > 0$ core plasma [C&Z POP17]

- Confirmed in GK simulations [Liu et al NF17], [Bierwage and Lauber NF17] that also emphasize crucial role of kinetic core plasma response, consistent with theory [Chavdarovski 09,14].

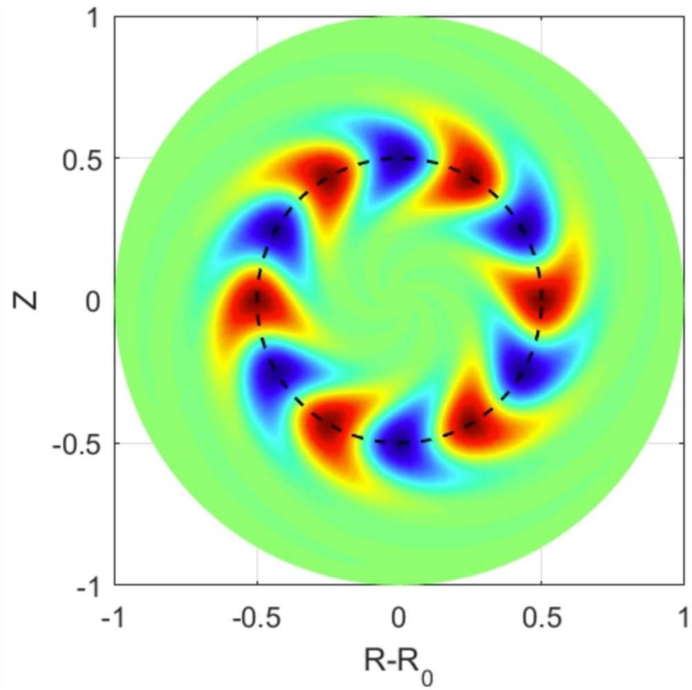
- Anti-hermitian EP response crucial for breaking the radial/parallel symmetry of mode structure [R. Ma et al POP15; Z. Lu et al 17].
  Also important for momentum transport [Z. Lu et al 17].

RSAE ECE imaging in DIII-D
[Tobias et al PRL11]



Importance of $k_r = \mathbb{R}\text{e}k_r + i\mathbb{I}\text{m}k_r$ [Lu et al 17]
BAE mode structure ($c_r = 60, c_i = 50, r_0 = 0.5$)

$$\sim \exp\left[i(c_r + ic_i)(r - r_0)^2 + in\phi - im\theta\right]$$



(Z. Lu)

(III) Nonlinear Physics

⇒ Addresses SAW turbulence spectrum and spectral transfers

⇒ Needed for proper assessments of heating/acceleration and transports

⇒ Broad scope of ongoing research activities

□ Formal solutions of nonlinear GFLDR: $D = i|s|\Lambda^L - (\delta\hat{W}_f + \delta\hat{W}_k)^L$ [Zonca NF05, C&Z RMP16]

$$D = -i|s|\Lambda^{NL} + (\delta\hat{W}_f + \delta\hat{W}_k)^{NL}$$

(III.A) Nonlinear Wave-Wave Interactions and spectral transfers: Λ^{NL}.
⇒ Understand and describe nonlinear processes in terms of breaking of the Alfvénic state ⇔ cancellation of Reynolds and Maxwell stresses

(III.B) Nonlinear Wave-EP Interactions and transports: $\delta\hat{W}_k^{NL}$.
⇒ Nonlinear dynamics of structures in the EP phase space ⇒ phase-space zonal structures ⇒ secular nonadiabatic resonant particle transport ($\tau^{TRANSP} \sim \tau^{NL}$) on meso- and macro-scales

(III.A) Nonlinear Wave-Wave Interactions

- ⊙ The pure Alfvénic state ⇒ Nonlinear self-consistent SAW solution
  - ∘ Infinite, uniform, ideal magnetohydrodynamic (MHD) fluid
    - $\boxed{\varrho_m(\partial_t + \boldsymbol{u}\cdot\boldsymbol{\nabla})\boldsymbol{u} = -\boldsymbol{\nabla}\cdot\underline{\underline{\boldsymbol{P}}} + \boldsymbol{J}\times\boldsymbol{B}/c}$
    - $\boldsymbol{u}_0 = 0 = \boldsymbol{J}_0,\ \boldsymbol{B}_0 = B_0\hat{\boldsymbol{b}}$
- ∘ Nonlinear SAW Vorticity equation
- • Quasi-neutrality: $\boldsymbol{\nabla}\cdot\delta\boldsymbol{J} \simeq 0$ (Valid for low-frequency SAW)
- ∘ Ideal MHD constraint: $\delta E_\parallel \simeq 0 \bigoplus$ incompressible response

$$\Rightarrow \boxed{c^2\left[(\boldsymbol{b}_0\cdot\boldsymbol{\nabla})^2 - V_A^{-2}\partial_t^2\right]\nabla_\perp^2\delta\phi + 4\pi\partial_t(\boldsymbol{\nabla}\cdot\delta\boldsymbol{J}_\perp^{(2)}) = 0}$$

$$\Rightarrow \boxed{\boldsymbol{\nabla}\cdot\delta\boldsymbol{J}_\perp^{(2)} = -(c/B_0)\boldsymbol{b}_0\cdot\boldsymbol{\nabla}\times[\mathbf{Re}+\mathbf{Mx}]} \text{ (Reynolds + Maxwell)}$$

- Pure Alfvénic state: Walén relation
  - $\boxed{\delta \boldsymbol{u}_{\perp W}/V_A = \pm \delta \boldsymbol{B}_{\perp W}/B_0} \Rightarrow \boxed{[\partial_t \pm V_A \boldsymbol{b}_0 \cdot \boldsymbol{\nabla}]\delta\phi_W = 0}$

$\Rightarrow \boxed{\mathbf{Re} + \mathbf{Mx} = 0 \quad \Rightarrow \quad \boldsymbol{\nabla} \cdot \delta \boldsymbol{J}_{\perp}^{(2)} = 0}$ (wave-wave coupling suppressed)

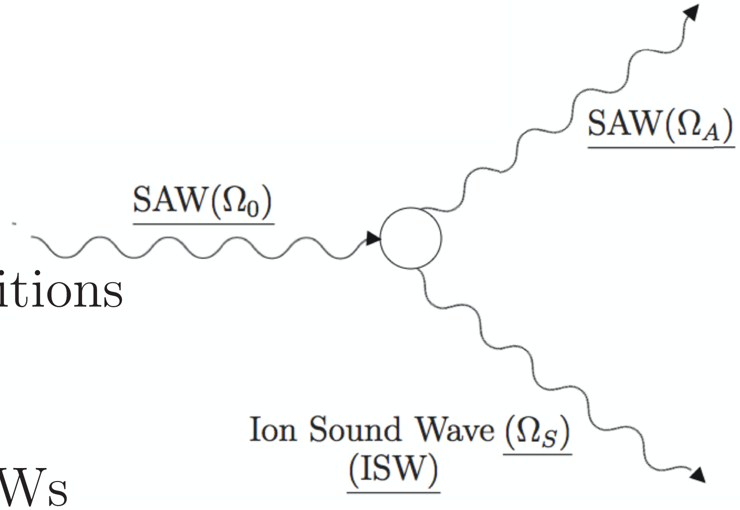
$$\Rightarrow \boxed{\left[(\boldsymbol{b}_0 \cdot \boldsymbol{\nabla})^2 - V_A^{-2}\partial_t^2\right] \delta\phi_W = 0}$$

  - $\delta\phi_W$: solution to nonlinear SAW equations

- Nonlinear wave-wave interactions ⇒ Breaking Alfvénic states: [C&Z POP13]
  - Finite ion compressibility: ion sound perturbations along $\boldsymbol{B}$
  - Microscopic-scales (ρ_i) Kinetic Alfvén Waves
    ⇒ Enhanced electron-ion decoupling ⇒ Enhanced $\delta E_{\parallel}$
  - Geometries: continuous and discrete SAW spectra
    [*e.g.*, Toroidal Alfvén Eigenmodes (TAE)].

Finite Ion Compressibility: Parametric Decays in Uniform Plasma

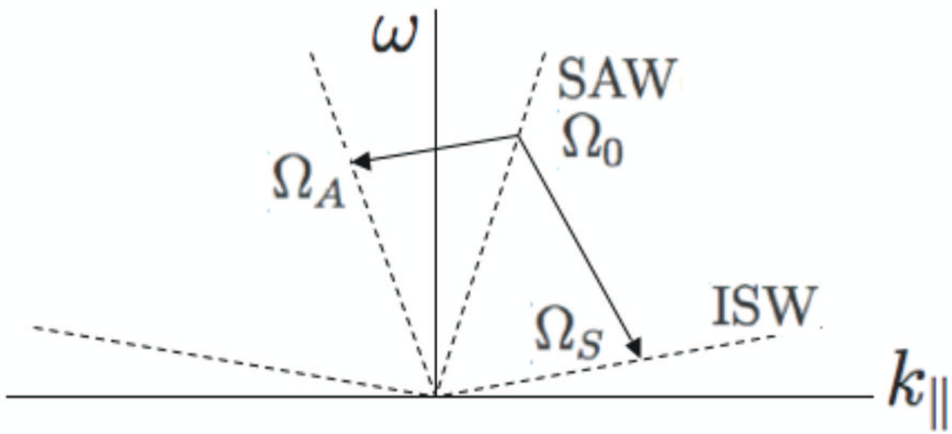
(A) Ideal MHD macro-scale theories
[Sagdeev & Galeev 1969]



- Resonant decay $\Rightarrow \omega - \boldsymbol{k}$ matching conditions

  $\Rightarrow \ \Omega_0 = (\omega_0, \boldsymbol{k}_0) = \Omega_S + \Omega_A$

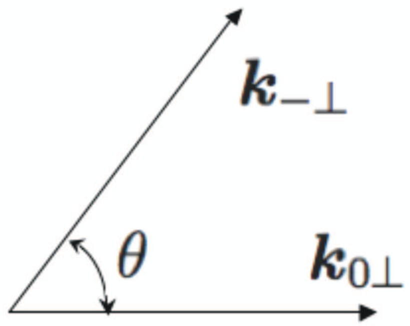
- Backscattering: Counter-propagating SAWs



⊙ Parametric Dispersion Relation

$$\epsilon_S \epsilon_{A-} = C_I |e\delta\phi_0/T_e|^2$$

- ϵ_S : ISW
- ϵ_{A-} : SAW
- $C_I \sim \mathcal{O}(k_\perp^2 \rho_i^2) \cos^2\theta$,



$\Rightarrow$ Coupling maximizes around $\theta = 0, \pi$; $\boldsymbol{k}_{0\perp} \parallel \boldsymbol{k}_{-\perp}$

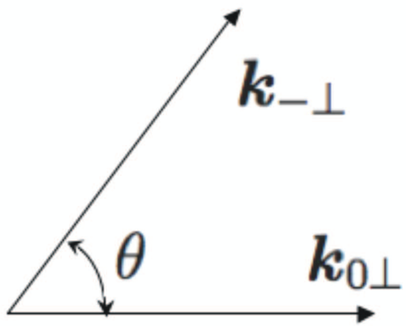
(B) Gyrokinetic micro/meso-scale theory [C&Z, EPL 2011]

$$\Rightarrow k_\perp \rho_i \sim \mathcal{O}(1)$$

⊙ Parametric Dispersion Relation

$$\epsilon_{SK}\epsilon_{A-K} = C_K |e\delta\phi_0/T_e|^2$$

- ϵ_{SK} : Kinetic ISW
- ϵ_{A-K} : KAW
- $$C_K \sim \mathcal{O}\left[\left(\frac{\Omega_i}{\omega_0}\right)^2 (k_\perp \rho_i)^6\right] \sin^2\theta,$$



$\Rightarrow$ Maximizes around $\theta \simeq \pm\pi/2$ and $|k_\perp \rho_i| \sim \mathcal{O}(1)$

- Simulation by Y. Lin et al. (PRL 2012)

(C) Quantitative & Qualitative differences [C&Z POP13]

- $|C_I| \propto \cos^2\theta \Rightarrow \boldsymbol{k}_{\perp -} \parallel \boldsymbol{k}_{\perp 0}$
  $|C_K| \propto \sin^2\theta \Rightarrow \boldsymbol{k}_{\perp -} \perp \boldsymbol{k}_{\perp 0}$

  $\Rightarrow$ Qualitative implications to turbulence and transports

  - Mode converted KAW $\Rightarrow \boldsymbol{k}_{0\perp} \simeq k_{0r}\hat{\boldsymbol{r}}$

  $\Rightarrow$ MHD regime: $\Rightarrow \boldsymbol{k}_{-\perp} \simeq k_{-r}\hat{\boldsymbol{r}} \Rightarrow$ no P_θ breaking $\Rightarrow$ little transport!

  $\Rightarrow$ Kinetic regime: $\Rightarrow \boldsymbol{k}_{-\perp} \simeq k_{-\theta}\hat{\boldsymbol{\theta}} \Rightarrow$ large P_θ breaking
  $\Rightarrow$ significant transport!
  $\Rightarrow$ cross-scale couplings with excitation by EPs!

- $\boxed{|C_K/C_I| \sim (\Omega_i/\omega_0)^2(k_\perp\rho_i)^4}$

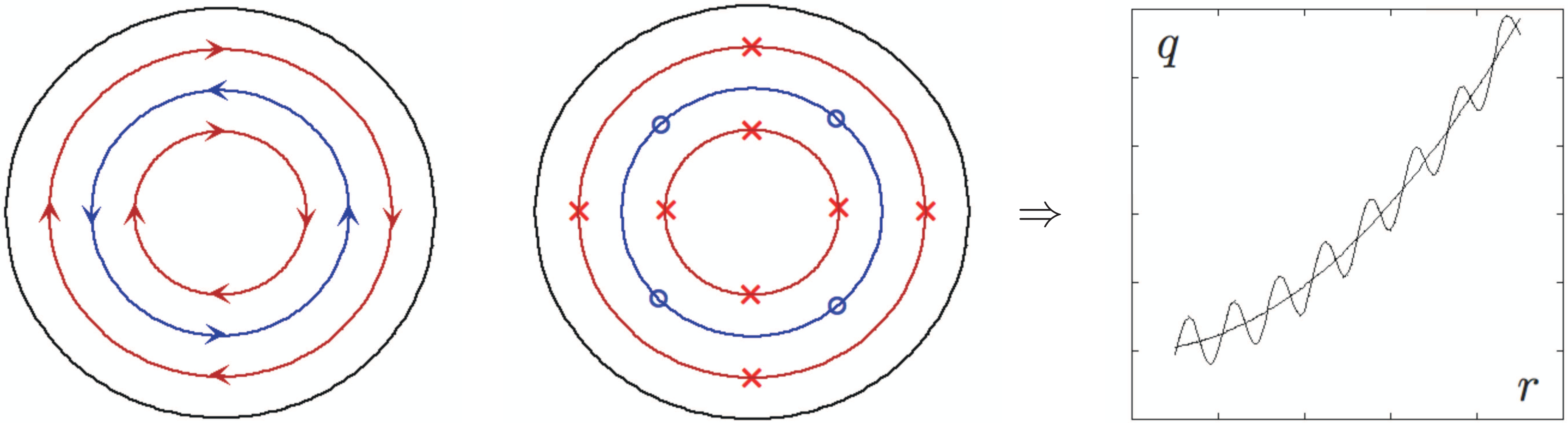
$\Longrightarrow$ For $|k_\perp\rho_i|^2 > |\omega_0|/|\Omega_i| \sim \mathcal{O}(10^{-2})$, kinetic effects are qualitatively and quantitatively crucial for SAW turbulence dynamics and associated transports.

Zonal Structures by Toroidal Alfvén Eigenmodes

[C&Z, PRL, 2012]

- Zonal structures ⇒ coherent micro/meso-scale radial corrugations of equilibrium in toroidal device plasmas.

Examples:



Zonal Flow

Zonal Current

[More generally: phase-space zonal structures] [Z&C NJP15; C&Z RMP16]

⇒ Zonal structures spontaneously excited by micro/meso-scale turbulence due to plasma instabilities.

⇒ Zonal structures scatter instability turbulence to shorter-radial wavelength stable domain ⇒ nonlinearly damp the instability.

⇒ Self-regulation of plasma instabilities!

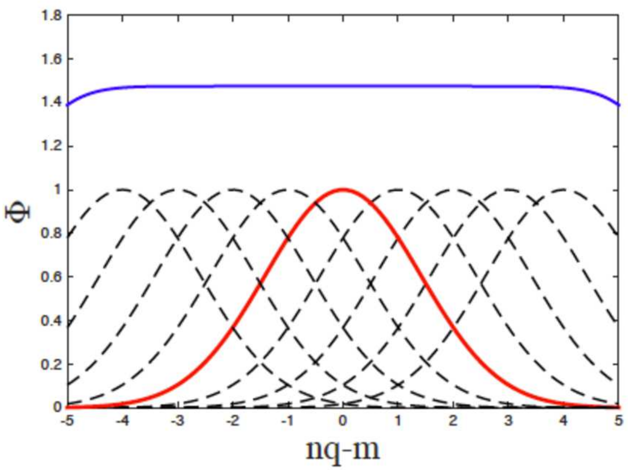
- In toroidal plasmas ⇒ continuous and discrete spectra
  - Continuous spectrum ⇒ $\omega^2 = k_\parallel^2(r)V_A^2(r) \Rightarrow \mathbf{Re} + \mathbf{Mx} \simeq 0$ ⇒ negligible nonlinear contributions (Alfvénic State)
  - Discrete spectrum ⇒ AEs ⇒ finite nonlinear contribution ⇒ Spontaneous excitation of zonal structures via modulational instability of the radial envelope of a finite-amplitude TAE pump wave.

- Modulational stability condition

  ⇒ Spontaneous excitation threshold:
  $|\delta B_r/B_0|_0 \gtrsim \mathcal{O}(10^{-4})$ ⇒ Competitive nonlinear saturation process!

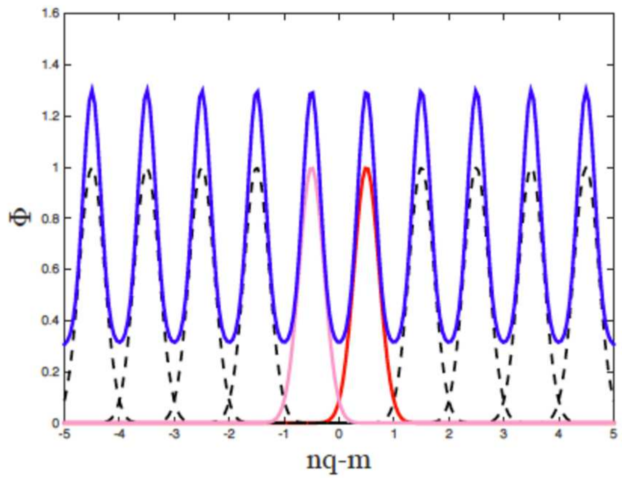
- Other discrete AEs (*e.g.*, BAE [Z.Qiu et al NF16]) can also break the Alfvénic State and stimulate interesting nonlinear wave-wave interactions.

□ Nonlinear cross section of ZS generation by AEs is enhanced by fine radial structures w.r.t. DW turbulence case [Z.Qiu et al NF17].



strongly ballooning DWs

v.s.

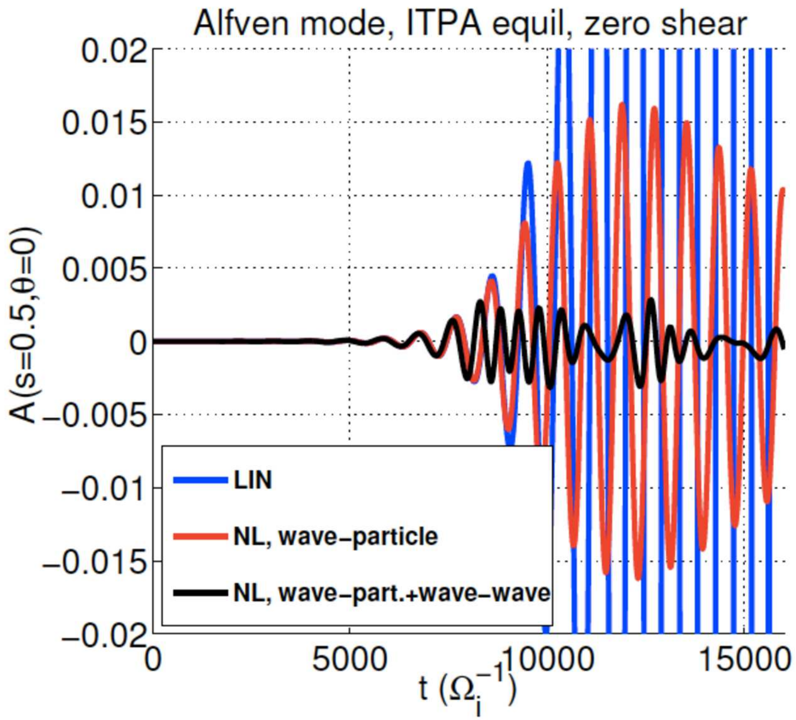


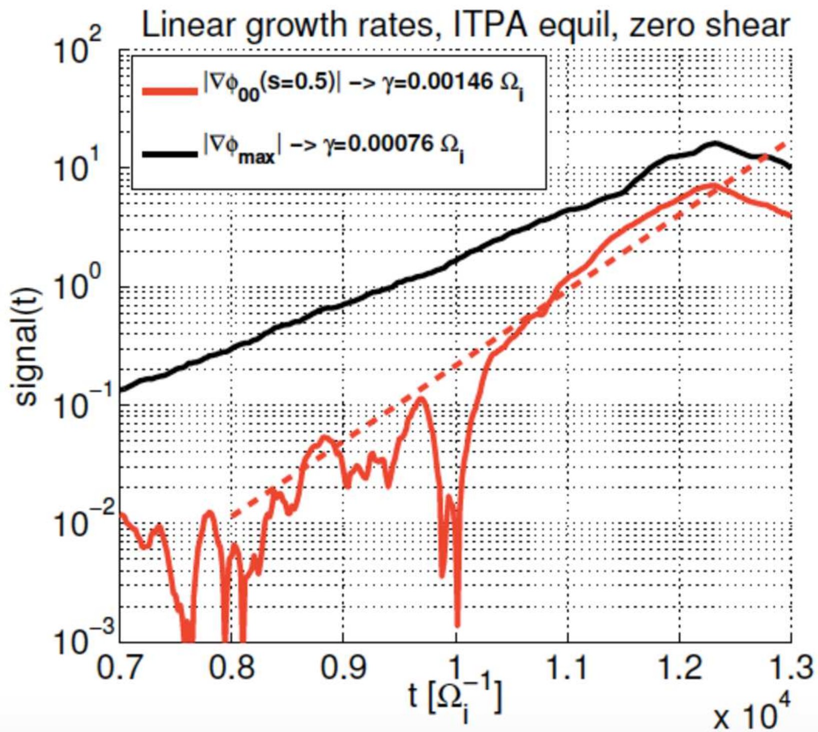
weakly ballooning AEs

- EPs, via pressure-curvature coupling ($\delta \hat{W}_k^{NL}$), can compete with core plasma (Reynolds/Maxwell stress) in ZS formation [Z.Qiu et al POP16]

⇒ Forced driven excitation of ZS! [Z.Qiu et al POP16]: consistent with earlier sim. results [Y. Todo et al NF10; Z. Wang et al 17; A. Biancalani et al 17]

- NL GK simulations [Biancalani et al IAEA16] (ORB5 code) provide evidence of forced driven excitation of GAM (geodesic acoustic modes) by EP driven AEs, consistent with theory [Z.Qiu et al NF16].





- EP driven GAM (EGAM) (anisotropic EPs) can play very important roles in cross-scale couplings with DW turbulence [D. Zarzoso et al 13–17], [R. Dumont et al 13], [J.-B. Girardo et al 14], [A. Biancalani et al 16].

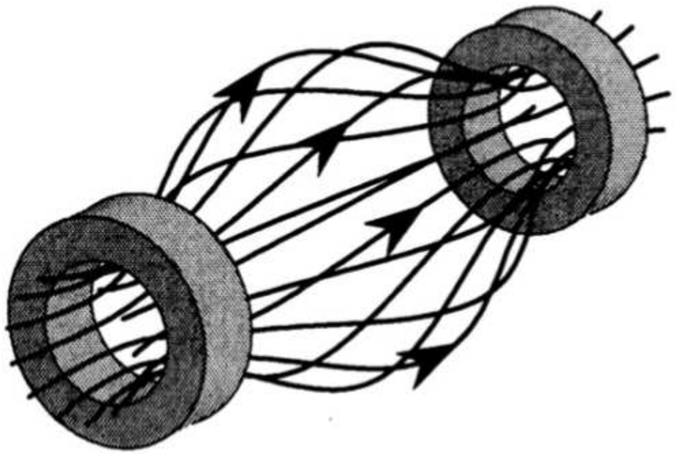
(III.B) Nonlinear Wave-EP Interactions

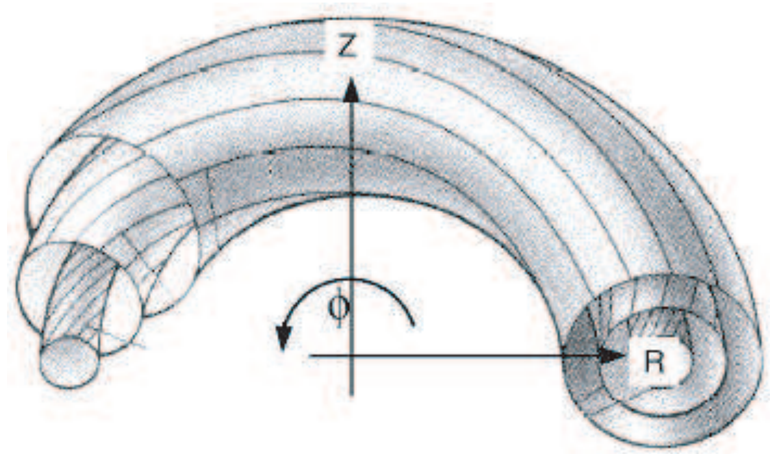
- When considering transport processes due to nearly periodic fluctuations with $\gamma/\omega \ll 1$, wave-particle resonances play a crucial role both in wave excitations as well as transport processes [Chen JGR99].

- Historically: nonlinear dynamics of 1D uniform Vlasov plasmas $\Rightarrow$ Phase-space holes (lack of density w.r.t. surrounding phase-space) and clumps (excess of density w.r.t. surrounding phase-space): extensively investigated after pioneering work by Bernstein, Greene and Kruskal (BGK) [Phys.Rev57].

- Nonlinear dynamics of phase-space holes and clumps in the presence of sources and collisions: widely adopted by Berk, Breizman and coworkers (review by [Breizman PPCF11; Sharapov et al NF13]) $\Rightarrow$ 1D uniform beam-plasma system as paradigm for nonlinear behaviors of Alfvén Eigenmodes near marginal stability [Berk PFB90].

The beam-plasma system vs. EP-SAW interactions in tokamaks



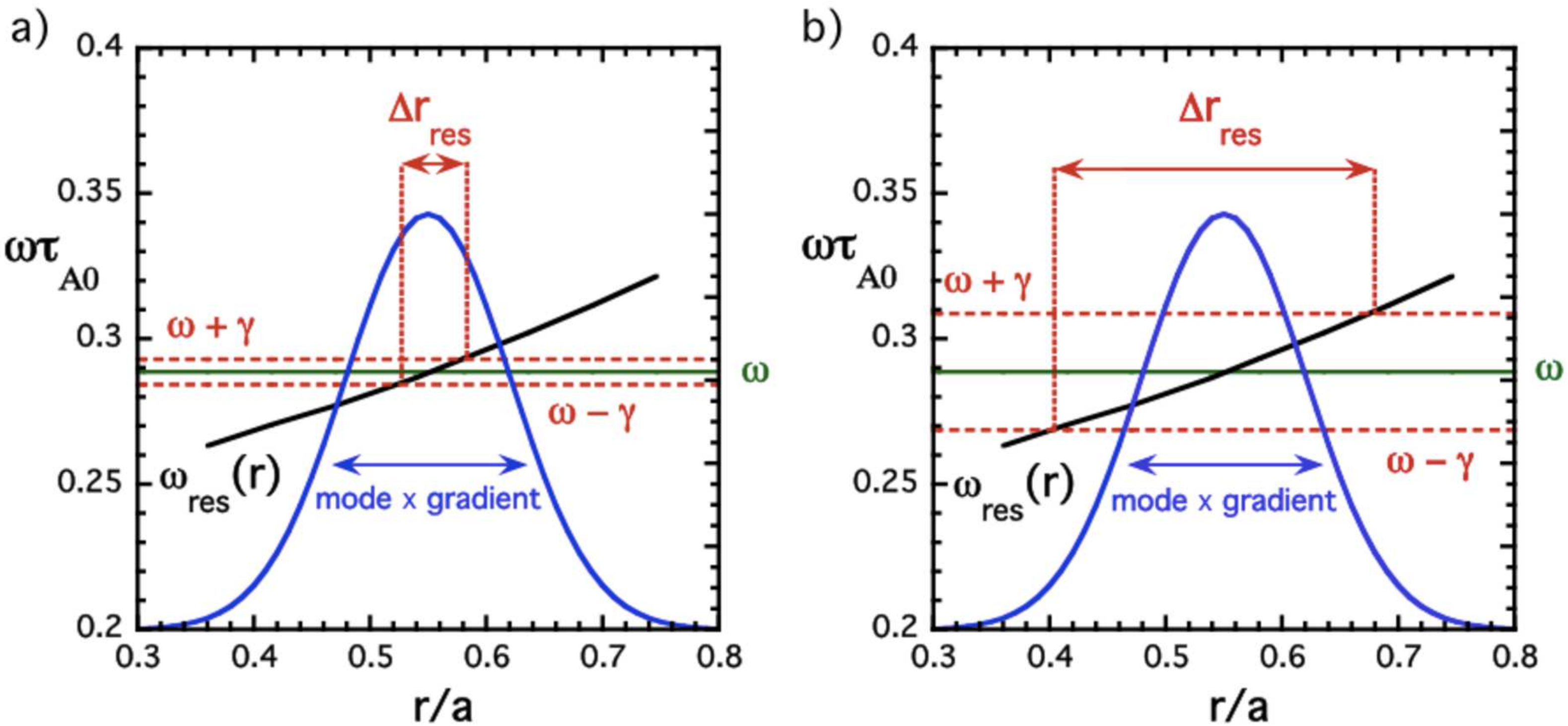


- Similarities can be drawn but strong differences and peculiarities emerge depending on drive strength [Zonca et al NF05; NJP15; C&Z RMP16]:
  - Advantages of using a simple 1-D system for complex dynamics studies [Berk PFB90]; [Review by Breizman PPCF11]
  - Roles of mode structures, non-uniformity and geometry in determining nonlinear behaviors [Zonca NF05, PPCF06]; [C&Z NF07, RMP16] for $1 \gg \gamma/|\omega| \gtrsim 10^{-3} \div 10^{-2}$ (depending on resonances)

Resonance detuning and radial decoupling

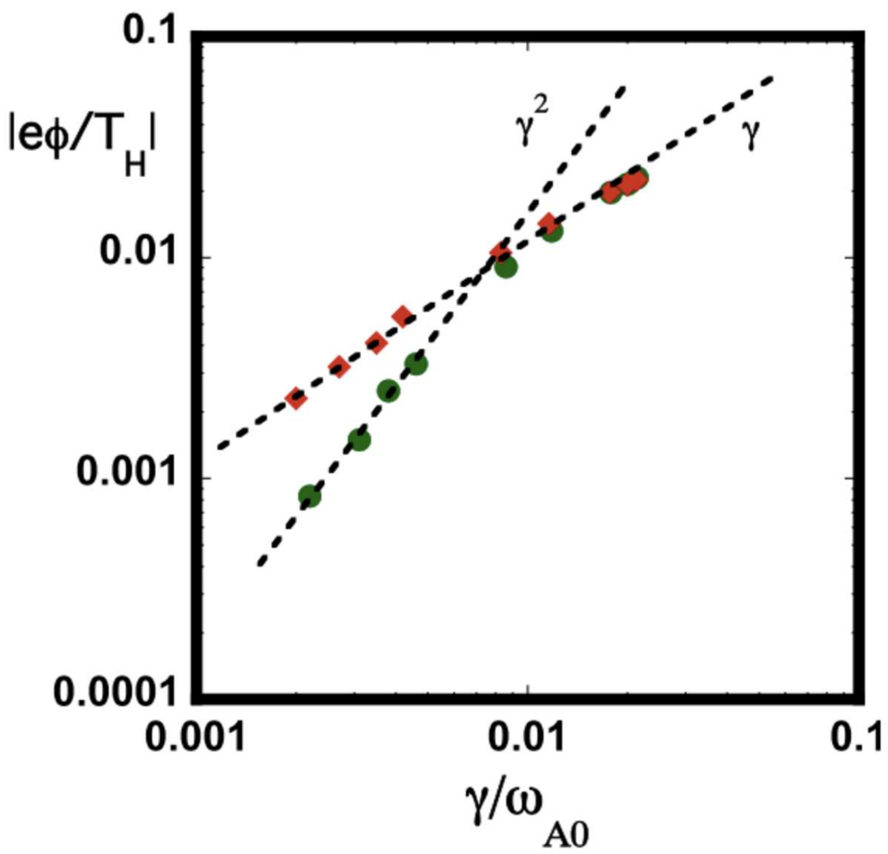
- Particle phase space diagnostics (Hamiltonian Mapping) [Briguglio et al POP14] (HMGC code [Briguglio et al POP95,98]) (X Wang)

weak drive | strong drive



resonance detuning | radial decoupling

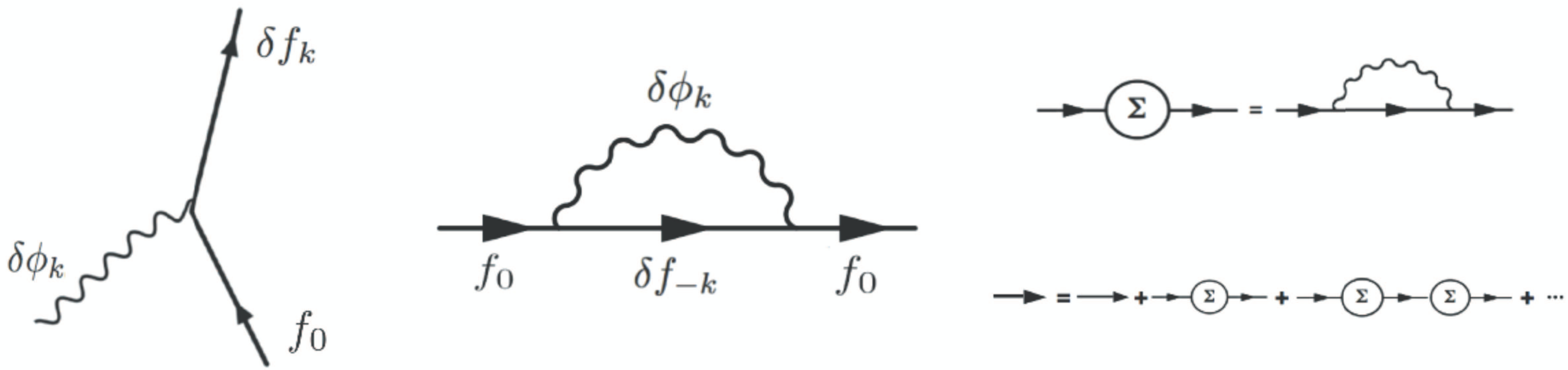
- Importance of geometry ⇒ Effect of resonance detuning/radial decoupling on growth-rate scaling of saturation amplitude [Wang et al POP16] (HMGC code [Briguglio et al POP95,98]) (X Wang)



- Different behavior of co- and counter-passing particles due to resonance frequency with broader/narrower radial profile ⇒ Geometry!
- Transition from quadratic γ-scaling (resonance detuning/wave-particle trapping) to linear γ-scaling (radial decoupling) consistent with simple theoretical model [Wang et al PRE12; POP16]
- Results confirmed by other NLED codes: EUTERPE [A. Könies et al 15]; [M. Cole et al 16].

Transport due to phase-space zonal structures

- Importance of wave-EP interactions ⇒ Role of phase space zonal structures (PSZS) that are undamped by collisionless processes [Zonca et al NJP15].
- Counterpart of zonal structures (ZS) (reflection of equilibrium corrugations/deviation from local equilibrium) ⇒ PSZS regulate transport processes on transport time scale and longer [M. Falessi ArXiV 2017].
- Evolution equation for renormalized particle distribution function (f_0) [Zonca et al NJP15]; [C&Z RMP16] ⇒ Dyson Equation



- Nearly coherent NL interaction ⇒ Importance of phase locking and phase bunching ⇒ Break down of QL description (non-perturbative/adiabatic)

- Application: "Fishbone" Nonlinear Theory [Zonca et al 2007; C&Z RMP16] based on the GFLDR: $i|s|\Lambda(\omega) = \delta\hat{W}_f + \delta\hat{W}_k(\omega|F_{0EP})$

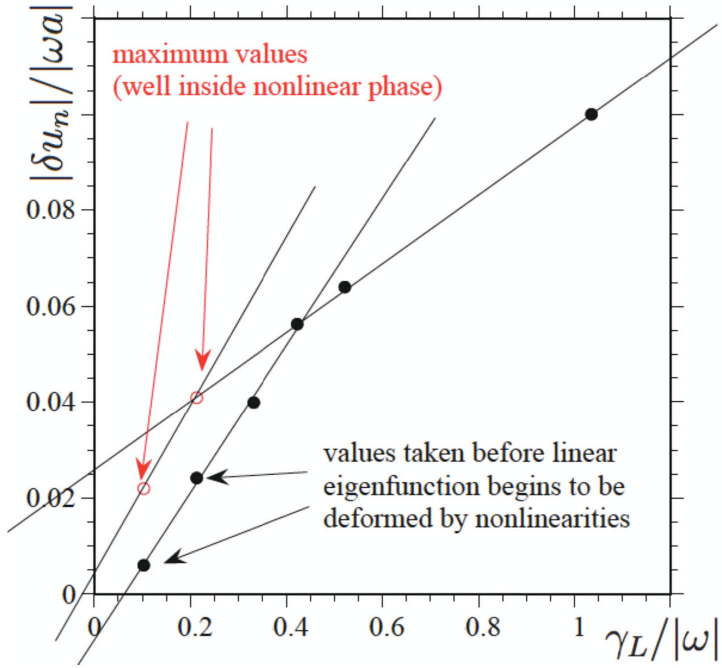
- Resonant EPs convect outward with radial speed $|\delta u_n|$ ⇒ Nonlinear saturation occurs when $|\delta u_n|/\gamma_L \sim r_s$

  [r_s ∼ mode structure width → Wave-EP interaction domain]

- Consistent with numerical simulation results by [GY Fu et al POP 2006].

- Near marginal stability regime explored by [M. Idouakass et al POP16; 2017 tbs] analytically and numerically

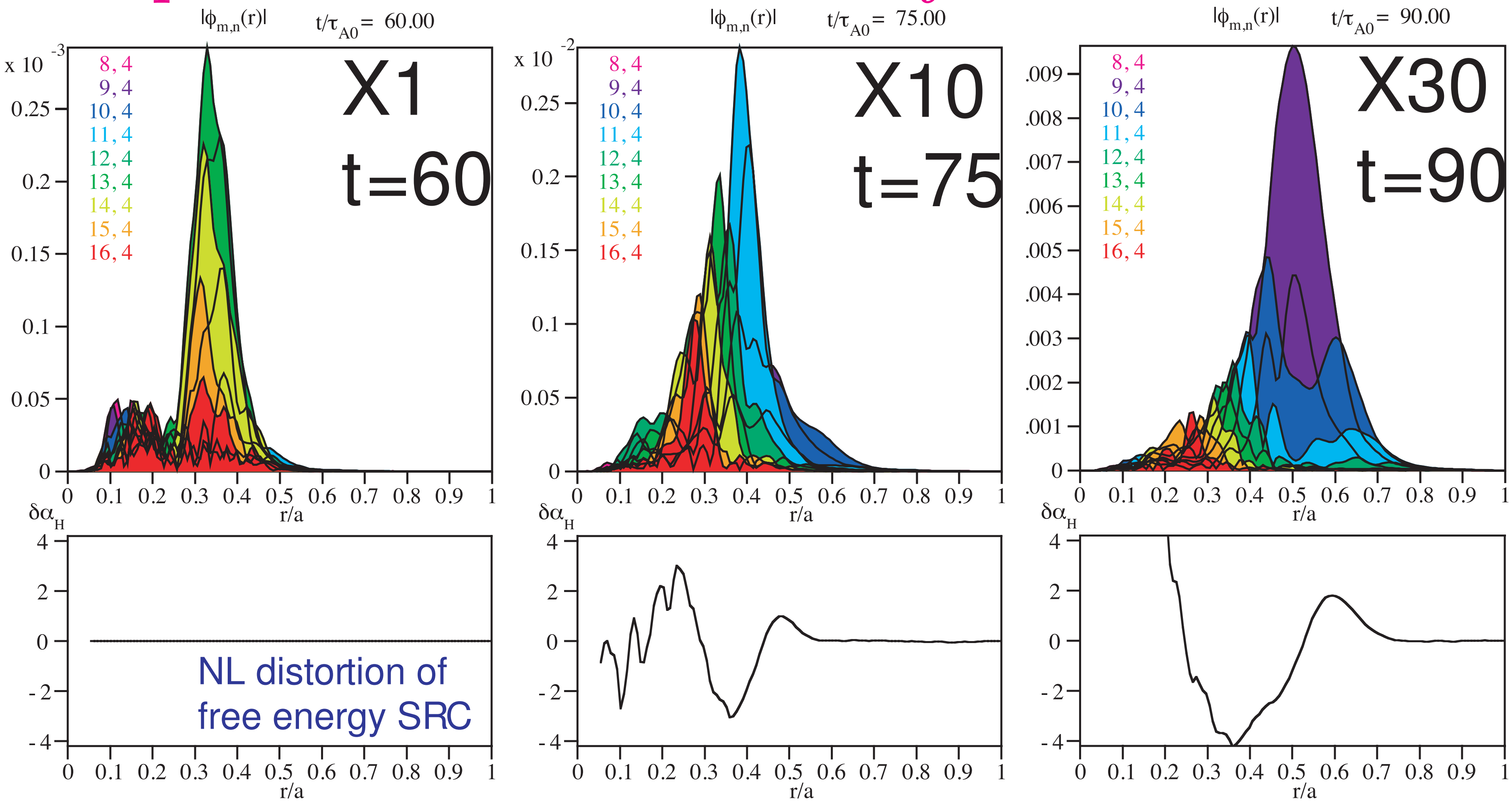
- [Vlad et al., 2012] simulation results



- Electron-fishbone simulation results with HMGC code [Vlad et al NF13]; [Vlad et al NJP16].

Zonca et al. IAEA, (2002)

Non-perturbative NL EPM dynamics



- EPM convective amplification and EP secular transport ⇒ Avalanches

(IV) Energetic Particle Transport

- Present understanding of nonlinear wave-wave and wave-particle interactions is reasonably sound and complete [C&Z RMP16].
- One crucial open issue remains the realistic prediction of global transport of EPs and fusion products and their impact on the system material walls.
- Collective oscillations excited by EPs in burning plasmas are characterized by a dense spectrum of modes with characteristic frequencies and spatial locations [C&Z NF07, RMP16].
- Standard approach usually relies on:
  - Near marginal stability Ansatz
  - Quasilinear description
  - Test-particle studies
  - Advanced NL GK analyses
  - Reduced models (advanced)

} Widely & successfully adopted
Fundamental issues remain

□ Advanced NL GK analyses:

- The natural framework for predicting EP transport by Alfvénic and MHD fluctuations
  - NL GK models [many groups/including NLED Team]: easy coupling with DWT; but technical issues remain
  - Hybrid MHD-Gyrokinetic models [Briguglio, Vlad et al]; [Todo et al]; [Gorelenkov, Fu et al]: kinetic physics for low frequency may be an issue/coupling with DWT?
  - Gyrofluid models [Spong et al.; Staebler et al.]: nonlinear closure remains an issue
- Reduced descriptions are needed (beyond QL/Test-particle)
- Outstanding theoretical issue: transport on long time scales (longer than characteristic transport time) [M. Falessi et al.]
  - transport of phase space structures (PSZS) and deviation from thermodynamic equilibrium (meso-scales)
  - interplay of collisional and fluctuation induced transport

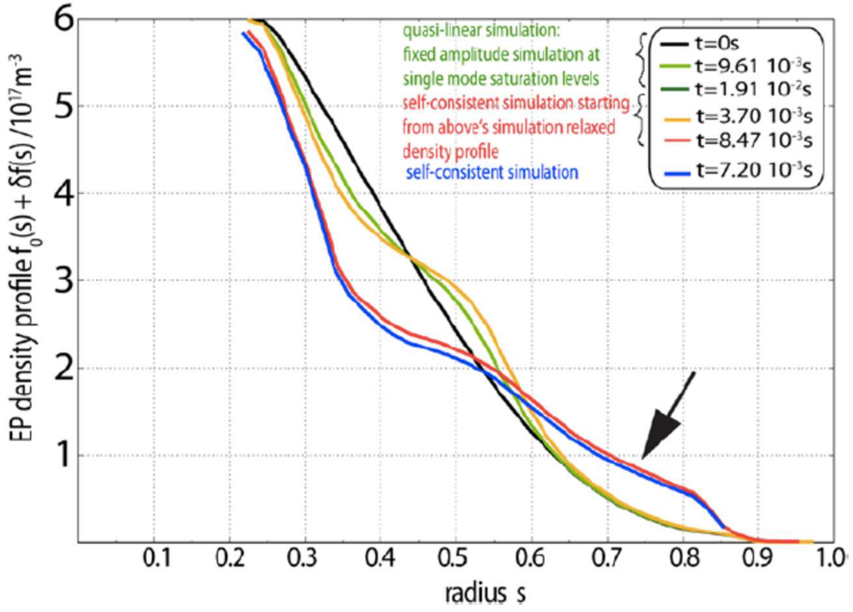
- Nonlinear energetic particle transport in the presence of multiple Alfvénic waves in ITER: reduced GK analysis with HAGIS-LIGKA [M. Schneller et al 2013-15]
  - Linear GK mode structures (LIGKA code [Lauber 05]) with non-perturbative EP ⊕ Amplitude evolution consistent with EP transport
  - Evidence of break-down of QL description

- Importance of modes of the linear stable and unstable spectrum. Confirmed in recent assessment [T. Hayward-Schneider, 2017].

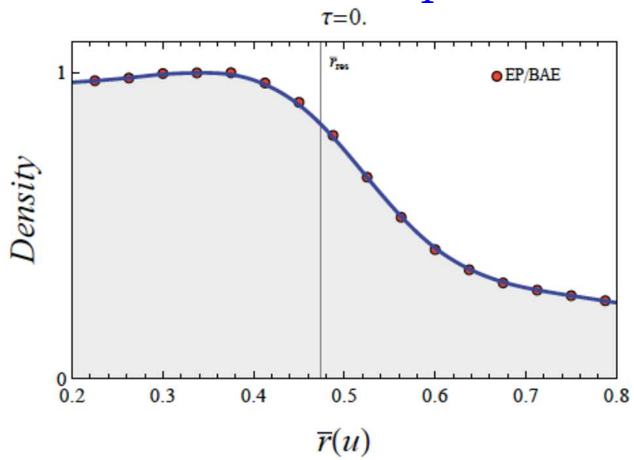
- Saturation level can be different (higher) than single-mode saturation ⇒ cross-scale coupling mediated by EP [C&Z RMP16].

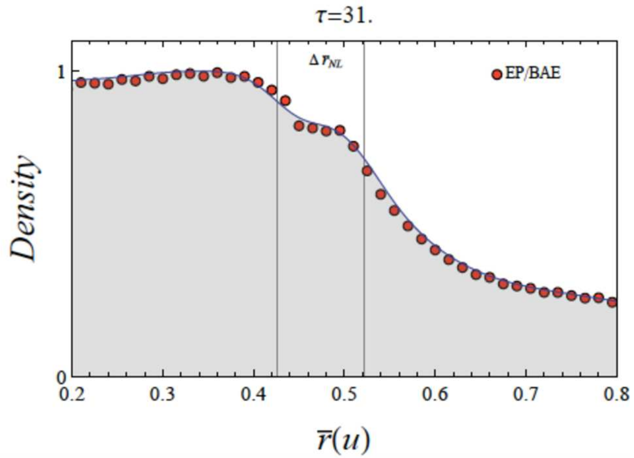
- Confirmed in Hybrid MHD-GK simulations (HMGC) [Vlad et al 2017].

[M. Schneller et al PPCF15]



- Correspondence of SAW and BoT problems: implications to EP transport (reduced model) [N. Carlevaro et al JPP15; ENT16]
  - Correspondence/mapping $r \leftrightarrow v$ must be established preserving non-linear displacement (not growth rate) [N. Carlevaro et al 2017]





- Importance of linear unstable as well as stable spectra.
- Crucial role of cross scale couplings mediated by EP: modes may saturate at higher value than single mode saturation.
- Meso-scale (spatiotemporal) behavior is due to self-consistent evolution of fluctuation intensity on the same time scale of particle transport.
- Importance of phase bunching and phase locking $\Rightarrow$ beyond the QL paradigm.

(V) Conclusions and Discussion

(V.1) SAW in plasmas confined by realistic $\boldsymbol{B}$ $\Rightarrow$ rich, interesting physics

(V.2) Nonuniformities and geometries $\Rightarrow$ Continuous spectrum, "singular" resonant absorption, mode conversion to KAW, frequency gaps, discrete Alfvén eigenmodes.

(V.3) Nonlinear physics:

- Nonlinear wave-wave interactions:
  - Compressibility, geometries, microscopic kinetic effects $\Rightarrow$ Breaking the Alfvénic states
  - Qualitative and quantitative effects on parametric decay instabilities, generation of nonlinear equilibria, excitation of convective cells, and zonal structures

- ◦ Nonlinear wave-EP interactions:
  - Frequency chirping and phase locking
    $\Rightarrow$ mode particle pumping $\Rightarrow$ EP radial redistribution
    $\Rightarrow$ wave-particle decoupling due to finite mode widths.
  - Self-consistent interplay of NL mode dynamics and EP transport
    $\Rightarrow$ EPM-EP Avalanches; Fishbones; Secular transport

(V.4) Realistic plasma nonuniformities, $\boldsymbol{B}$ geometries, mode structures

$\Rightarrow$ Play crucial roles in the linear and nonlinear SAW and non-perturbative EP dynamics!!

$\Rightarrow$ Need to go beyond Quasi-Linear and/or Test-particle EP transport

(V.5) Careful in-depth analyses and cross-checks among experiments/observation, theory, and realistic first-principle simulations

$\Rightarrow$ advances in Alfvén wave and EP physics

$\Rightarrow$ Intellectually exciting! Practically important!