On equilibrium, stability and dynamics of ITER-like plasmas

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Overview

1. Equilibrium
   - Construction of ITER-like equilibria with sheared flow
2. Stability
   - Application of a sufficient criterion for linear stability to equilibria with parallel flow
   - Resistive wall mode (RWM)
3. Dynamics
   - A multi-fluid code for burning fusion plasmas

Motivation

It has been established in several fusion devices that sheared flows play a role in the transitions to improved confinement regimes as the L-H transition and the internal transport barriers (ITBs).

Axisymmetric equilibria with sheared flows satisfy generalized Grad-Shafranov equations (GGSEs).

The stability of fluids and plasmas in the presence of equilibrium flows non parallel to the magnetic field remains a tough problem reflecting to the lack of necessary and sufficient conditions even in the framework of hydrodynamics.

It is important to study the dynamics of a burning fusion plasma in connection with the ITER-objective of producing ten times higher fusion power than the input power.

GGSE with incompressible flow

[Tasso & GNT, PoP 1998]; [GNT, Tasso & Poulipoulis, JPP 2008]

$$(1-M^2)\Delta^\star\psi - \underline{\frac{1}{2}(M^2)'|\nabla\psi|^2} + \frac{1}{2}\left(\frac{X^2}{1-M^2}\right)' + \mu_0 R^2 P_s' + \mu_0 \frac{R^4}{2}\left(\frac{\varrho(\Phi')^2}{1-M^2}\right)' = 0 \qquad (1)$$

$M(\psi)$: Mach function of the poloidal velocity with respect to the poloidal-magnetic-field Alfvén velocity

$X(\psi)$: relates to the toroidal magnetic field $[B_\phi = X/[R(1-M^2)]]$

$P_s(\psi)$: quasistatic pressure

Bernoulli relation for the pressure : $P = P_s(\psi) - \varrho\left[\frac{v^2}{2} - \frac{R^2(\Phi')^2}{1-M^2}\right]$ (2)

$\Phi(\psi)$: electrostatic potential

$\varrho(\psi)$: plasma density

Transformed GGSE

GGSE under the transformation [Morrison, BAPS 1986]; [Clemente, NF 1993]

$$u(\psi) = \int_0^{\psi} \left[1 - M^2(g)\right]^{1/2} dg \tag{3}$$

reduces to

$$\Delta^\star u + \frac{1}{2}\frac{d}{du}\left(\frac{X^2}{1-M^2}\right) + \mu_0 R^2 \frac{dP_s}{du} + \mu_0 \frac{R^4}{2}\frac{d}{du}\left[\varrho\left(\frac{d\Phi}{du}\right)^2\right] = 0. \tag{4}$$

Also, Bernoulli relation is put in the form

$$P = P_s(u) - \varrho\left[\frac{v^2}{2} - R^2\left(\frac{d\Phi}{du}\right)^2\right]. \tag{5}$$

- Eq. (4) is free of a quadratic term as $|\nabla u|^2$.
- Transformation (3) does not affect the magnetic surfaces, it just relabels them.

Construction of fusion relevant equilibria

Assigning the free function-terms in the GGSE we have constructed a variety of linear and nonlinear solutions analytically, quasianalytically and numerically.

Example: Application for a non linear axisymmetric equilibrium with reversed magnetic shear [Kuiroukidis & GNT, JPP 2015]

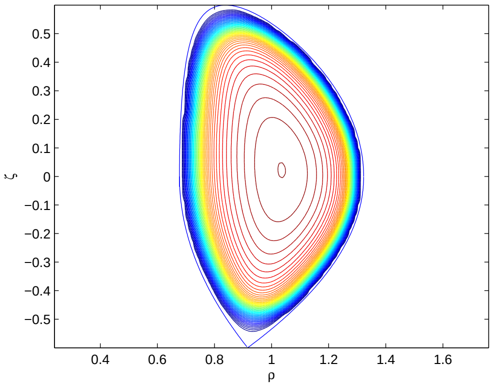
Choice of the free function profiles in the GGSE:

$$\begin{aligned} \frac{X^2}{1-M^2} &= X_0^2 + 2X_1(u_c - u) + X_2(u_c - u)^2 + \frac{2}{3}X_3(u_c - u)^3 \\ P_s(u) &= \frac{P_0}{2}\left[\tanh(\alpha(u_s - u_b)) - \tanh(\alpha(u_s - u))\right] \end{aligned} \tag{6}$$

where $u_c,\ u_b,\ u_s,\ \alpha,\ P_0,\ X_0,\ X_1,\ X_2,\ X_3$ are free parameters.

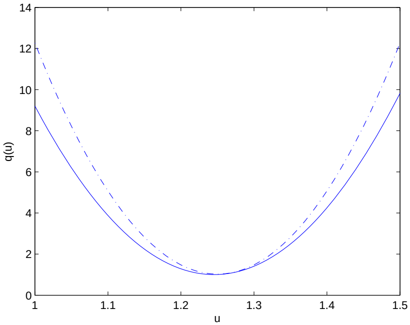
The resulting form of the GGSE equation is solved numerically for a prescribed boundary with a single lower X-point.

ITER equilibrium configuration



Equilibrium with ITER-like elongation, $\kappa = 1.86$, triangularity, $\delta = 0.5$, and parallel flow constructed numerically by imposing analytically the boundary.

Correlation of equilibrium nonlinearity and flow with reversed magnetic shear



The safety factors for an equilibrium state with $\alpha = 10$ *and* $M_0^2 = 0.005$ *(solid line) and for another equilibrium with* $\alpha = 15$ *and* $M_0^2 = 0.02$ *(dashed line).*

The nonlinearity in conjunction with a weaker contribution from the flow result in an increase of the negative magnetic shear.

Other equilibrium studies

- Remapping HELENA to incompressible plasma rotation parallel to the magnetic field
  [Poulipoulis, GNT, Konz & ITM-TF Contributors, PoP 2016]

- Hamiltonian construction of translationally symmetric extended MHD with equilibrium applications
  [Kaltsas, GNT & Morrison, POP 2017]
  EFTC-17 Poster: P1.6

- Analytic anisotropic-pressure equilibria with incompressible flow in helically symmetric geometry
  [Evangelias, GNT, accepted in PPCF 2017]
  EFTC-17 Poster: P1.15

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Sufficient condition for linear stability

[GNT & Tasso, PoP 2007]

A general steady state of a plasma of constant density and incompressible flow parallel to **B** is linearly stable to small three-dimensional perturbations if the flow is sub-Alfvénic ($M^2 < 1$) and $A \geq 0$, where

$$\begin{aligned} A &= -(1-M^2)\left[(\mathbf{j}\times\nabla\psi)^2 - (\mathbf{j}\times\nabla\psi)\cdot(\nabla\psi\cdot\nabla)\mathbf{B}\right] \\ &\quad -\frac{(M^2)'}{2}|\nabla\psi|^2\left(\nabla\psi\cdot\frac{\nabla B^2}{2} + g|\nabla\psi|^2\right), \\ g(\psi, B^2) &= \frac{P_s'}{1-M^2} - \frac{(M^2)'}{1-M^2}\frac{B^2}{2}. \end{aligned} \tag{7}$$

$$\mathbf{j}: \text{ equilibrium current density.} \tag{8}$$

- Internal modes
- No restriction on the equilibrium geometry

Physical inspection of the sufficient condition ($A \geq 0$)

$$A = A_1 + A_2 + A_3 + A_4$$

$$A_1 = -(\mathbf{j}\times\boldsymbol{\nabla} u)^2 :$$ destabilizing ($A_1 < 0$)

partly related to current driven modes

$$A_2 = (\mathbf{j}\times\boldsymbol{\nabla} u)\cdot(\boldsymbol{\nabla} u\cdot\boldsymbol{\nabla})\,\mathbf{B} :$$ indefinite sign

variation of $\mathbf{B}$ $\perp$ to the magnetic surfaces

$$A_3 = -\frac{1}{2}\frac{dM^2}{du}\left(1-M^2\right)^{-1}|\boldsymbol{\nabla} u|^2\,\boldsymbol{\nabla} u\cdot\frac{\boldsymbol{\nabla} B^2}{2} \tag{9}$$

$$A_4 = -\frac{1}{2}\frac{dM^2}{du}\left(1-M^2\right)^{-3/2}|\boldsymbol{\nabla} u|^4\, g \tag{10}$$

A_3, A_4 : flow and flow-shear terms of indefinite sign (11)

$$g = (1-M^2)^{-1/2}\left(\frac{dP_s}{du}-\frac{dM^2}{du}\frac{B^2}{2}\right) \tag{12}$$

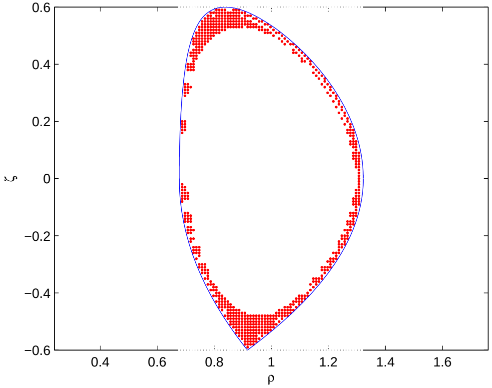
Applications of the stability condition

Applied to a variety of translational symmetric and axisymmetric equilibria the sufficient condition led to the following conclusions:

- In several cases the flow and flow shear in conjunction with the equilibrium nonlinearity play a stabilizing role.
- In equilibria with reversed magnetic shear there are synergetic effects of the equilibrium nonlinearity and reversed magnetic shear on stability.
- The stability is affected by the up-down asymmetry of the configuration.

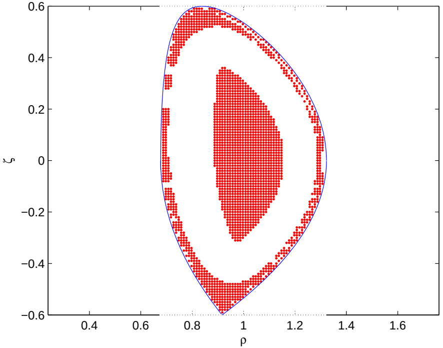
However, it should be clarified that the effects of flow on stability are far from universal; there are cases in which they play a destabilizing role.

Example: Application for the non linear axisymmetric equilibrium with reversed magnetic shear



The stability function A for a state with $\alpha = 10$ *and* $M_0^2 = 0.005$. *The red coloured region is where the sufficient condition* $A \geq 0$ *is satisfied.*

Impact of nonlinearity in conjunction with flow on stability



The stability function A for a state with $\alpha = 15$ and $M_0^2 = 0.02$. The red coloured region is where the sufficient condition $A \geq 0$ is satisfied

For stronger nonlinearity and flow the condition $A \geq 0$ is satisfied in an appreciably larger part of the plasma in the central region.

A proposed scenario for the formation of ITBs

- In a quasistatic approximation of dynamics the plasma may evolve from a low confinement state, corresponding to the L-mode in the L-H transition, to an improved confinement state, corresponding to the H-mode, through successive states with increasing reversed magnetic shear, nonlinearity and flow.

A synergism of reversed magnetic shear and sheared rotation, consisting in that on the one hand the reversed magnetic shear plays a role in triggering the ITBs development while on the other hand the rotation has an impact on the subsequent growth and allows the formation of strong ITBs, was observed in JET [De Vries et al., NF 2009] and DIII-D [Shafer et al., PRL 2009].

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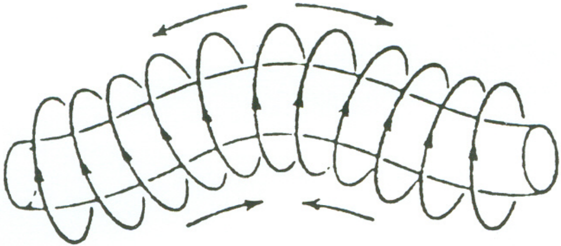
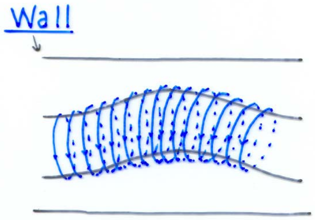
The kink mode

This is an MHD instability driven primarily by the toroidal current.



The kink instability is associated with a bending of the plasma column (z-pinch).

Resistive wall mode



- The kink mode can be stabilized if the plasma is surrounded by a perfectly conducting wall because the magnetic flux is trapped in the vacuum region in between the plasma and the wall.
- The substitution of the perfectly conducting wall by a wall of finite electrical conductivity destroys stability [Pfirsch & Tasso, NF 1971].
- The so called "resistive wall mode" is an MHD mode whose growth rate is, however, reduced by wall resistivity.
- In view of a fusion reactor the resistive wall mode is dangerous because it can lead on resistive time scale to confinement degradation.

Methods of stabilization of the resistive wall mode

- Feedback (active control)
- Dynamic stabilization (optimal control)
  Modulation of the wall resistivity might decrease the growth rate of the mode [Tasso & GNT, PoP 2002; PLA 2003] though in this case the mode remains Lyapunov unstable [Tasso & GNT, PoP 2004].
- Plasma flow
  However, the mode remains Lyapunov linearly unstable too if the flowing plasma is ideal (of infinite electrical conductivity) [Tasso & GNT, PoP 2011].
- Wall flow (liquid metal wall)

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Dynamics of a burning fusion plasma

$$D + T \longrightarrow \, ^4\text{He}(3.5 \text{ MeV}) + \text{n}(14.1 \text{ MeV})$$

In a fusion reactor the neutrons leaving the plasma is expected in part to breed tritium, by reacting with lithium, and in part produce electricity.

The alpha particles is expected to be confined by the magnetic field and through collisions to heat the plasma species.

If this alpha heating rate is equal to the plasma energy loss rate then the plasma will ignite, and the plasma burning process will be self-sustaining.

Apparently MHD is inappropriate to describe a burning plasma.

Development of multi-fluid burning plasma models on the basis of a self consistent treatment of time-dependent continuity, momentum and energy equations for each fluid species.

0th dimensional four-fluid code-1

[Lalousis, GNT & Poulipoulis, 43rd EPS-PPCF, Leuven, 2016]

Mass conservation equations:

$$\frac{dn_e}{dt} = -\frac{n_e}{\tau_p} + S_f \quad (13)$$

$$\frac{dn_D}{dt} = -\frac{n_D}{\tau_p} - S_r + S_f \quad (14)$$

$$\frac{dn_T}{dt} = -\frac{n_T}{\tau_p} - S_r + S_f \quad (15)$$

$$\frac{dn_a}{dt} = -\frac{n_a}{\tau_p} + S_r \quad (16)$$

n_e, n_d, n_T, n_a: electron-, deuterium-, tritium- and alpha-fluid densities

τ_p: particle confinement time

S_r: reaction rate

S_f: fueling rate

0th-dimensional four-fluid code-2

Energy conservation equations:

$$\frac{dE_e}{dt} = -\frac{E_e}{\tau_E} + Q_{fe} + Q_{pe} \quad (17)$$

$$\frac{dE_D}{dt} = -\frac{E_D}{\tau_E} - Q_{rD} + Q_{fD} + Q_{pD} \quad (18)$$

$$\frac{dE_T}{dt} = -\frac{E_T}{\tau_E} - Q_{rT} + Q_{fT} + Q_{pT} \quad (19)$$

$$\frac{dE_a}{dt} = -\frac{E_a}{\tau_E} + Q_{ra} + Q_{pa} \quad (20)$$

E_e, E_D, E_T, E_a: electron-, D-, T- and alpha-fluid internal energies

τ_E: energy confinement time

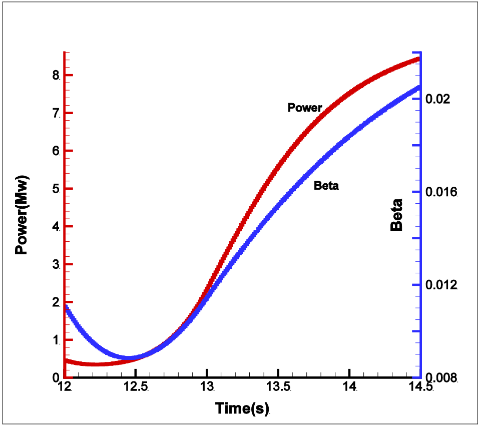
Q_{rD}, Q_{rT}: energy depletion terms due to $D - T$ reactions

$Q_{ra} = S_r \times$ 3.5 MeV: alpha fluid energy term

$Q_{pe}, Q_{pD}, Q_{pT}, Q_{pa}$: temperature equilibration terms due to Coulomb collisions

JET results (external heating included)-Power and beta

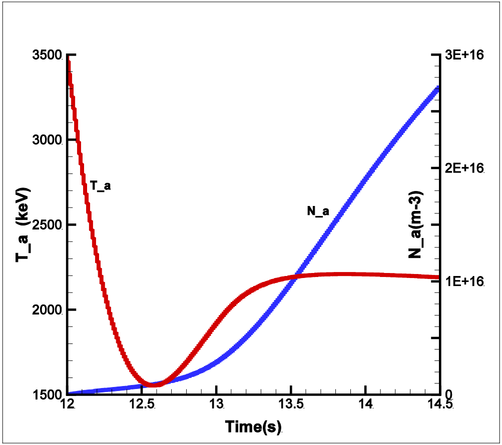
beta=Electron+ion+alpha fluid pressure/magnetic field pressure



Power generated by the neutrons, and beta as functions of time.

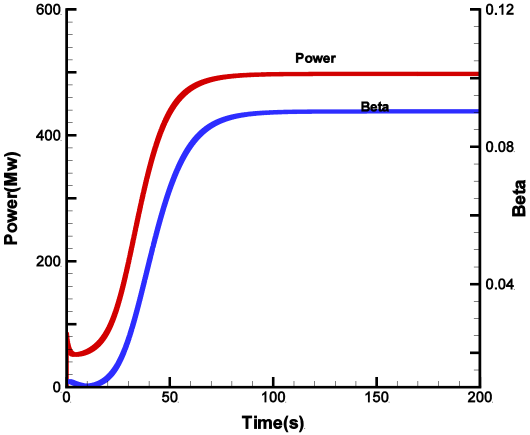
Good agreement with the 1997 JET discharges (Maximum fusion power of 6.7 MW associated with the pulse 42 847) [Thomas et al., PRL 1998]

JET results-alphas



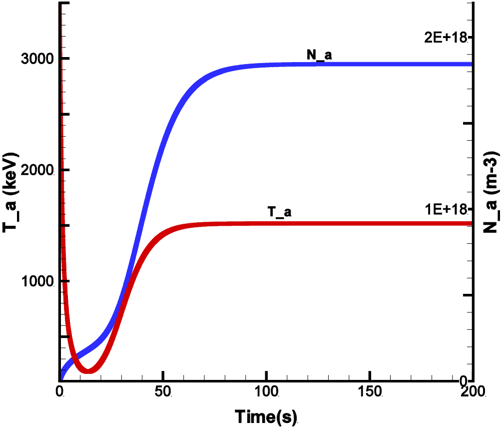
Temperature (in keV) and density of alphas as functions of time.

ITER results-Power and beta



Neutron generated power and beta as functions of time.

ITER results-alphas



Temperature (in keV) and density of alphas as functions of time.

Summary

- Since sheared flows play a role in the establishment of improved confinement regimes in tokamaks, we have constructed and studied several ITER-pertinent equilibria with incompressible sheared flow by solving a generalized GS equation.
- Application of a sufficient condition for linear stability for the equlibria constructed implies that in certain cases the equilibrium nonlinearity, reversed magnetic shear and sheared flow have synergetic stabilizing effects.
- For ideal plasmas with weak (linear) flows the resistive wall mode remains Lyapunov unstable. Therefore in order that a tokamak relevant flow has potential stabilizing effects, certain plasma dissipation as viscosity is necessary.
- We have been developing a multi-fluid dynamical code for a fusion burning plasma.

Current projects

1. Helically symmetric equilibria with anisotropic pressure and flow (PhD thesis of A. Evangelias)
2. Study of equilibrium and stability of a magnetized plasma by variational principles including Hamiltonian methods and Casimirs with applications to magnetically confined plasmas (PhD thesis of D. A. Kaltsas)
3. Extension of the HELENA code for anisotropic pressure or/and non parallel incompressible flow

Collaborators

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References-1

- H. Tasso and G. N. Throumoulopoulos, Phys. Plasmas **4**, 2378 (1998)
- G. N. Throumoulopoulos, H. Tasso, G. Poulipoulis, J. Plasma Physics **74**, 327 (2008)
- P. J. Morrison, Bull. Am. Phys. Soc. **31**, 1609 (1986)
- R. A. Clemente, Nucl. Fusion **33**, 963 (1993)
- G. Poulipoulis, G. N. Throumoulopoulos, C. Konz and ITM-TF Contributors, Phys. Plasmas **23**, 072507 (2016)
- D. A.Kaltsas, G. N.Throumoulopoulos and P. J. Morrison, Phys. Plasmas **24**, 092504 (2017)
- A. Evangelias G. N.Throumoulopoulos, accepted in Plasma Phys. Control. Fusion (2017)

References-2

- G. N. Throumoulopoulos and H. Tasso, Phys. Plasmas **14**, 122104 (2007)
- G. N. Throumoulopoulos, G. Poulipoulis, H. Tasso, J. Plasma Physics **74**, 327 (2008)
- Ap Kuiroukidis and G. N. Throumoulopoulos, J. Plasma Physics **81**, 905810404 (2015)
- P. C. de Vries, E. Joffrin, M. Brix, C. D. Challis et al., Nucl. Fusion **49**, 075007 (2009)
- M. W. Shafer, G. R. McKee, M. E. Austin, K. H. Burrell et al., Phys. Rev. Lett. **103**, 075004 (2009)
- D. Pfirsch and H. Tasso, Nucl. Fusion **11**, 259 (1971)
- H. Tasso and G. N. Throumoulopoulos, Phys. Plasmas **9**, 2662 (2002); Phys. Lett. A **307**, 304 (2003)
- H. Tasso and G. N. Throumoulopoulos, Phys. Plasmas **11**, 334 (2004)

References-3

H. Tasso and G. N. Throumoulopoulos, Phys. Plasmas **18**, 070702 (2011)

P. Lalousis, G. N. Throumoulopoulos, G. Poulipoulis, *43rd European Physical Society Conference on Plasma Physics, 4-8 July 2016, Leuven*, ECA, P5.069 (2016)

P. R. Thomas, P. Andrew, B. Balet et al., Phys. Rev. Lett. **80**, 5548 (1998)