Intermediate-mass ratio inspirals in globular clusters

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Abstract
Intermediate Mass Black Holes (IMBH) are believed to exist at the centers of some globular clusters. We have performed N-body simulations of young star clusters containing an IMBH at their centers for studying the interactions of the IMBH with stellar-mass black holes (BHs) and the formation and evolution of hard IMBH-BH binaries. For this we used the direct summation code Myriad, which apart from the Newtonian dynamics, contains the post-Newtonian corrections to the acceleration and its derivative for the dynamical evolution of hard BH-BH binaries. In one simulation, we find that the IMBH-BH binary that formed at $t \sim 50\,\text{Myr}$ has been evolved to an inspiral and merger in a very short time scale. We studied the gravitational radiation of the inspiral and concluded that an event like this would provide a clear and sharp signal for future space-based detectors of gravitational radiation.

Key words:

1 Introduction

Intermediate-mass black holes (IMBH; with masses $10^2 - 10^3\,M_\odot$) are theoretically predicted to fill the gap between solar-mass black holes (BHs; with masses up to $\sim 30\,M_\odot$) and supermassive black holes (SMBHs; with masses $\geq 10^6\,M_\odot$). Although there is no conclusive observational evidence for the existence of IMBHs, the list of potential candidates is increasing. The chief problem in identifying an IMBH is that we cannot resolve the sphere of influence of the hole, where the potential is dominated by the central object. This is because IMBHs are expected to form and live at the dense centers of globular clusters, where it is difficult to resolve the motions of individual stars, using the present day biggest telescopes.

One of the observational hints that IMBHs exist are the ultra-luminous X-ray sources (ULXs) detected deep in the cores of various star clusters. ULXs may be explained as the result of sub-Eddington accretion onto a BH with mass greater than a typical stellar-mass BH (Miller & Colbert). The most promising case is the ULX at the center of the massive cluster G1 of M31, where observations suggest that an IMBH of mass $\sim 2 \times 10^4\,M_\odot$ should exist. In or Centauri, a globular cluster of our own Galaxy, observations suggest that an IMBH with mass greater than $4 \times 10^4\,M_\odot$ should exist, because of the observed ULX. On the other hand, found that the presence of an IMBH is only possible with half of that mass. According to a recent study by , the mass of the potential IMBH is $4.7 \pm 1.0 \times 10^5\,M_\odot$.

Another hint of the existence of IMBHs in star clusters is the observed velocity dispersion at the core of some of the galactic globular clusters. This can be explained well if an IMBH is assumed to be present at the centers of those clusters, but for the observed core dispersion velocity there exists an alternative scenario does not require an IMBH. According to this scenario, a system of dark remnants (neutron stars, stellar-mass BHs) located at the core, would also explain the observed velocity dispersion. Up to now, the observed kinematics of the core of globular clusters provide us with only upper limits on the mass of the possible IMBH.

Although their existence is not well established, IMBHs are a topic of considerable scientific interest. One reason for that is that their existence would have important consequences in the structure, properties and dynamical evolution of the host cluster. Also, IMBHs would emit gravitational radiation detectable by future detectors such as LISA and Advanced LIGO/VIRGO as they interact with field stars and BHs.

There are at least three distinct mechanisms for the
formation of an IMBH in star clusters. An detailed review about them can be found in (Milla 2003; Miller & Colbert 2001). The first one is the collapse of a massive Pop III star. Pop III stars (Abel et al. 1998; Nakamura & Umemura 2001) have extremely low metallicity, which allows them to grow very massive, and retain most of their mass during their evolution. Also, during the supernova explosion most of the mass of the star is kept in the resulting BH. This way a seed BH with mass up to $\sim 250 M_\odot$ may be formed and then accretion and mergers with other massive stars would increase the mass to the bounds of an IMBH.

Another mechanism is the successive collisions of stellar mass BHs that are packed close to the center of a star cluster, through mass segregation. Simulations show that early in the dynamical evolution of a star cluster the most massive stars tend to sink to the center, where they interact strongly with each other. When these stars reach the core and depending on the initial virial radius of the cluster, they might already be evolved to BHs and depending on the density of the core, they could physically collide with each other. Successive collisions of BHs could lead to the formation of an IMBH.

On the other hand, if the initial half-mass radius of the cluster is smaller than $1 - 2 pc$, core collapse of the cluster would happen very fast (Gurkan et al. 2003), and most of the heavy stars would reach the core before they evolve to BHs. This would give a dense core rich in massive main sequence stars; heavy stars collide with each other and form a star with rapidly growing mass (Portegies Zwart et al. 1999; Portegies Zwart & McMillan 2003). This “runaway process” stops just after core collapse, which happens in times shorter than $\sim 5 Myr$, leaving behind a very massive star (VMS) with mass up to $\sim 250 - 1000 M_\odot$. There is not a clear consensus about the evolution of such a star, but in most of the publications found in the literature it is assumed that it collapses to a BH with limited mass-loss. Then, gas accretion could increase the mass of the resulting BH by a factor of $\sim 100$ (Vesperini et al. 2010).

In any IMBHs form at the centers of a star clusters, they are dynamically active during the first $\sim 1 Gyr$ of their life. Because of mass segregation, many of the heavy stars sink to the center early, and there they become BHs that, together with other low-mass stars, interact strongly with the IMBH. Due to 3-body interactions, it is expected, that a few Myrs after its formation, the IMBH will form a hard binary with another BH. This IMBH-BH binary will have continuous interactions with other BHs and stars that will possibly lead to a merger.

The last stages before merger of an IMBH-BH binary, and the merger itself, cannot be described by the Newtonian theory. One needs General Theory of Relativity or approximate methods, such as the Post-Newtonian formalism to describe them. According to the recent developments in Numerical Relativity (Preparca 2006), a merger between two spinning black holes with different masses, would emit gravitational radiation asymmetrically, and this would give a relatively large recoil velocity to the product of the merger. The recoil velocity depends strongly on the mass ratio of the two BHs, on the magnitude of their spins and on their directions with respect to the plane of the orbit REF. For equal-mass, maximally spinning BHs, if the spins are antialigned and on the plane of the orbit, the recoil velocity of the merger-product would be as high as 4000 km/s (cf. in the case of a merger between an IMBH with a BH, the recoil velocity would be smaller, because of the relatively large mass-ratio, but still, under certain circumstances, it could be up to hundreds of km s$^{-1}$, while the escape velocity at the center of a typical star cluster is 50 km s$^{-1}$.

There is a limited number of publications that examine the retention of an IMBH due to gravitational radiation recoil. Holley-Bockelmann et al. (2007) measured the retention probability of the IMBH in a cluster, assuming that it has 25 successive collisions with other BHs. They used a large set of Monte Carlo simulations of IMBH-BH mergers to show that retaining an IMBH with mass less than $1000 M_\odot$ occurs in less than 30% of their simulations, if the masses of the BHs are selected from a Kroupa initial mass function (IMF) (Kroupa 2001), in which all stellar-mass BHs are retained in the system after formation. If the population of the stellar-mass BHs follows a Belczynski distribution (Belczynski et al. 2002), where some of them are supposed to escape the cluster due to natal kicks, then the IMBH remains in the system with a probability of 70%. Also, they found that in any case, nearly all the ejections come from collisions of the IMBH with a BH with mass $M_{bh} > 30 M_\odot$. Finally, using the results of their simulations they estimated that if all Galactic globular clusters created an IMBH at their centers, and if the surrounding stellar-mass BHs, follow a Kroupa IMF, then less than 5 of the clusters have retained their IMBH. If the IMF of the BHs is follows the more realistic Belczynski IMF, then there might be less than 25 galactic globular clusters hosting IMBHs. Also, they speculated that if the results of their simulations are correct, then there must be $\sim 100$ isolated IMBH with masses $100 - 1000 M_\odot$ moving in the Galactic halo with speeds of a few hundred km s$^{-1}$.

One way to test the results of Holley-Bockelmann et al. (2007) is to use direct-summation $N$-body techniques for simulating young star clusters containing an IMBH. $N$-body simulations have the advantage of having a small set of assumptions, which makes them appropriate to reproduce realistically the dynamical processes in a cluster. On the other hand, their disadvantage is that they need long simulation times, even if they run on fast computers. For this reason a limited number of simulations can be made, so it is not easy to cover a large area of the parameter space.

Here we present the results of an $N$-body simulation of a realistic model of a young star cluster containing an IMBH at its center. In the simulation we pay particular attention in the interactions of the IMBH with stellar-mass BHs, the formation of IMBH-BH binaries and the possible IMBH-BH mergers that could kick the IMBH from the cluster. In Section 1 we describe the numerical method, the code, the initial data and the assumptions used in the simulation. In Section 2 we present the results of the simulation. Finally, we summarize the results, estimating the number of IMBH-BH inspiral and merger-events that could be observable by future space-based gravitational wave detectors and presenting the results about the retention of the IMBH in star clusters.

The questions we address and we try to answer in this work are:

(i) What is the time-scale for the formation of the first IMBH-BH binary in a young star cluster?
(ii) Is it possible for the IMBH-BH binary that formed in a cluster to merge in a time scale less than Hubble time?

(iii) What is the gravitational wave (GW) signal of an IMBH-BH inspiral and merger? Could sources like these be detectable by existing or proposed space-or-earth-based GW detectors?

(iv) How possible is the escape of the IMBH after a merger with a stellar-mass BH due to gravitational radiation recoil?

(v) What is the effect of the IMBH escape to the structure of the cluster?

(vi) If the IMBH escapes, does it drag any stars with it?

In Section 2 we describe the numerical method paying particular attention to the initial data and the numerical code that was used. In Section 3 we present the result of our simulation, while in Section 4 we discuss the results and address the necessity of more similar simulations. In Appendix A and Appendix B, we present the post-Newtonian equation that are used by the numerical code we used for the simulations. In Appendix C, we describe the details about the recoil velocities that is assigned by te code to BHs that are created out of merging binary BHs.

2 NUMERICAL METHOD

2.1 MyriadPN

We used a modified version of the Myriad code, called MyriadPN, that includes post-Newtonian dynamics and the special treatment of collisions between black holes. Myriad has been introduced and tested in detail in Konstantinidis & Kokkotas (2010). It is a direct-summation N-body code that uses the 4th order Hermite integrator (Makino & Makino 1996) with block time steps for the time evolution of star systems. The accelerations and their derivatives are calculated using the GRAPE-6 (Makino et al. 2003) special purpose hardware. Myriad also includes special treatment of close binary and multiple systems that are integrated with high accuracy using the time-symmetric 4th order Hermite integrator (Kaplan et al. 2003) with shared, but adjustable time step, as described in Konstantinidis & Kokkotas (2010).

We extented this code for making it able to simulate close binary systems of BHs that require more than simple Newtonian dynamics. When the separation of a binary system of black holes becomes relatively small (several times their Schwarchild radius), purely newtonian gravity cannot describe their motion realistically. Effects such as the relativistic pericenter shift and the emission of gravitational radiation cannot be explained using newtonian theory. Thus, the accurate and realistic evolution of binary black holes can be done either by using the full General Relativistic Theory of Gravitation or the post-Newtonian theory. The Post-Newtonian formalism ) explains well the relativistic effects and since it has been created as an extension of normal Newtonian gravity, it is relatively easy to be included in N-body codes.

Post-Newtonian theory may be summarized by expanding the Newtonian acceleration of a particle in a series of powers of 1/c (Damour & Deruelle 1993; Saha 1996), where c is the speed of light. This way the total acceleration of particle 1 in a binary system may be written as:

\[
a_1 = \frac{\ddot{a}_{01}}{c} + \frac{1}{c^2} \ddot{a}_{21} + \frac{1}{c^4} \ddot{a}_{41} + \frac{1}{c^6} \ddot{a}_{51} + O\left(\frac{1}{c^8}\right)
\]

where:

\[
\ddot{a}_{01} = -G \frac{m_2}{r^2} \hat{n}
\]

is the Newtonian acceleration, with G the gravitational constant, m1 and m2 the masses of the two particles, r the distance between them and \(\hat{n}\) the normal vector pointing from particle 2 to particle 1. \(\ddot{a}_{21}\) is the 1PN, \(\ddot{a}_{41}\) the 2PN and \(\ddot{a}_{51}\) the 2.5PN corrections. The first two are responsible for the pericenter shift, while the last one is the dissipative term that corresponds to gravitational radiation reaction. The 2.5PN term subtracts energy from the system and because of this the semimajor axis shrinks and the two particles are guided to a merger. Note the difference between \(a_i\), which is the
total acceleration of particle $i$ and $\dddot{a}_i$, which is the $(j/2)$th post-Newtonian term of the acceleration of particle $i$. 

The analytic equations for the post-Newtonian terms are found in Blanchet (2009); Kup et al. (2009) and described in detail in Appendix A. 

In a similar way one can write the post-Newtonian expansion for the derivative of the acceleration:

$$\dddot{a}_i = \dddot{a}_{i0} + \frac{1}{c^2}\dddot{a}_{i1} + \frac{1}{c^4}\dddot{a}_{i2} + \frac{1}{c^6}\dddot{a}_{i3} + O(\frac{1}{c^8})$$ (3)

where:

$$\dddot{a}_{i0} = \frac{Gm_i}{r^2}[\dddot{v}_{12} - 3(\dddot{v}_{12} \cdot \dddot{n}_{12})\dddot{n}_{12}]$$ (4)

is the Newtonian part, while the equations for the lengthy post-Newtonian terms $\dddot{a}_{i1}$, $\dddot{a}_{i2}$ and $\dddot{a}_{i3}$ are given in Appendix B. 

In order to verify the equations for the post-Newtonian terms of the derivative of the acceleration we included in the code the numerical computation of those terms for comparison with the analytical formulas. The numerical and analytical results for the derivative of the acceleration of a BH which is a member of a binary BH-BH system with equal mass BHs, are presented in Figure ???. As seen in this figure there is an excellent agreement between the numerical results and the results from the calculated analytical formulas. 

We have also performed a set of simulations up to merger of equal-mass binary BHs using the post-Newtonian terms. As a time of merger we considered the time when the semimajor axis of the system became 5 times the sum of their Schwarzschild radii. We repeated the simulations for different values of the initial eccentricity. For each value of the eccentricity we performed two simulations, using in the first only the 2.5$PN$ term and in the second all the $PN$ terms up to 2.5$PN$. Our result was the time of merger, which can be tested against the theoretical result of Peters (1964). In Peters (1964) they calculated that the set of equations that describe the semimajor axis $a$ and the eccentricity $e$ of a binary BH is:

$$\frac{da}{dt} = -\frac{64}{5} \frac{G^3m_1^2m_2^2(m_1 + m_2)^{1/2}}{c^6a^3(1 - e^2)^{7/2}} \left(1 + \frac{73}{24} e^2 + \frac{37}{96} e^4\right),$$ (5)

$$\frac{de}{dt} = -\frac{304}{15} \frac{G^3m_1^2m_2^2(m_1 + m_2)}{c^6a^3(1 - e^2)^{5/2}} \left(1 + \frac{121}{304} e^2\right),$$ (6)

and

$$\frac{da}{de} = \frac{12}{19} \frac{\alpha}{c^4} \left(1 + \frac{24}{25} e^2 + \frac{37}{20} e^4\right).$$ (7)

Where $G$ is the gravitational constant, $c$ the speed of light and $m_1$, $m_2$ the masses of the two BHs. We solved the above set of equations for each binary of our simulations and found the theoretical value of the time of merger for the binary. 

The results are presented in Figure 3. It is obvious that the theoretical result is in excellent agreement with the result of the simulations where only the 2.5$PN$ was used. The simulations that have all the $PN$ terms, up to 2.5$PN$ have smaller merger times than those where only the 2.5$PN$ term was used. This is expected and explained in REF. According to their explanation, the 1$PN$ and 2$PN$ terms are responsible for the periastron shift and in general this shift leads to more pericenter passages in a certain amount of time. This means that in the simulations with all the $PN$ terms the binary passes through its pericenter more times in a given time than in the simulations where only the 2.5$PN$ term is used. Most of the energy and angular momentum is lost from the system, due to the emission of gravitational radiation, during the periastron passages, so more periastron passages lead to faster shrinking of the semimajor axis of the system, and smaller merger times. 

In Figure 3 and Figure 4 we present the evolution of the semimajor axis and eccentricity of one of the simulations mentioned above. Again, the agreement between the simulation that uses only the 2.5$PN$ term with the theoretical result is excellent, while the simulation using all the $PN$ terms lead to a faster merger. 

2.2 Initial Data

The parameters that can be varied in the initial data are:
(i) The mass of the IMBH.
(ii) The initial mass function (IMF) of the cluster.
(iii) The masses of stellar-mass BHs existing in the cluster.
(iv) The number of stellar-mass BHs.
(v) The distribution of stars and stellar-mass BHs in the initial configuration.

Let’s investigate those parameters one by one, using previous works, some assumptions and ideas.

### 2.2.1 The mass of the IMBH

Since we have no direct proof for the existence of an IMBH at the centers of star clusters, we can only use the upper limits coming from observations and theoretical limits coming from calculations for the mass of the IMBH. Theoretically an IMBH has mass between $10^2$ and $10^4 M_\odot$ so this is the mass-range that we should use. The exact mass for the IMBH for our initial data might be calculated using different methods. In Holley-Bockelmann et al. (2007), they investigate the retention of the IMBH in star clusters, using 3-body interactions, they vary the mass of the IMBH according to its assumed formation scenario. Their mass-range for the IMBH is $10 - 3000 M_\odot$. They use:

- (i) $M_{\text{IMBH}} \sim 1000 M_\odot$, if the IMBH is assumed to come from runaway mergers of main sequence stars.
- (ii) $M_{\text{IMBH}} \sim 100 M_\odot$, if the IMBH is assumed to come from collisions of stellar-mass BHs.
- (iii) $M_{\text{IMBH}} \sim 200 - 400 M_\odot$, if the IMBH is assumed to be the remnant of a Pop III star.

In ?, where they discuss the properties of globular clusters that harbor IMBH, they use a mass-range of $125 - 1000 M_\odot$ for the mass of the IMBH in their simulations. The limits come from their assumption that the mass of the IMBH is in the range $0.1\% - 1\% M_\odot$, where $M_e$ is the total mass of the cluster. This assumption comes mainly from extrapolating the $M_{\text{BH}} - M_{\text{bulge}}$ relation that applies to galaxies, down to the mass of globular clusters. Replacing the mass $M_{\text{bulge}}$ with the total mass of the cluster $M_e$, gives a result of $0.1\% M_e$ for the mass of the IMBH. The question if this relation applies to globular clusters is still under debate.

Using higher mass than predicted from the $M_{\text{BH}} - M_{\text{bulge}}$ relation aims to making more realistic the interactions between the IMBH and the stellar-mass BHs. This is described in a very nice way in ?, where they study the evolution of star clusters containing primordial binaries and an IMBH at their centers. In their paragraph 2.3 they explain their choice for the IMBH mass. They use a mass range of $0.8\% - 3\% M_e$ for the mass of the IMBH which is slightly higher than the mass predicted from extrapolating the $M_{\text{BH}} - M_{\text{bulge}}$ relation. As they note, the expected result from the $M_{\text{BH}} - M_{\text{bulge}}$ relation would have been a mass-range of $0.3\% - 1\% M_e$, assuming a velocity dispersion of $10 km/s$ for a typical globular cluster. The mass of the IMBH would then be around $10^4 M_\odot$.

The problem is that none can simulate a full globular cluster, containing $10^5 - 10^6$ individual stars. In the simulations that they do in they use $4K - 16K$ stars (1K=1024) due to the limits of the software and mainly hardware. In our simulations, a total number of $32K - 64K$ could be used, but no more, because of the limiting power of the available hardware. So, there is a scaling in the total number of stars in the simulations, and there has to be a scaling for the mass of the IMBH used. There are two choices:

(i) Keep the mass of the IMBH close to the limits provided by the $M_{\text{BH}} - M_e$ relation, using as $M_e$ the mass of the simulated cluster.
(ii) Keep constant the ratio of the mass of the IMBH to the stellar mass.

The first option would give a smaller mass for the IMBH, but it would be scaled according to the mass of the cluster, which could produce results that could be compared with observations more easily. The effects of the IMBH to the cluster structure might be studied and compared with observational results. The disadvantage of this choice is that the local interactions of the IMBH with main sequence stars and BHs would be unrealistic, because the mass of the IMBH would be unrealistic, while the masses of all other particles of the cluster would come from a realistic distribution of masses.

The second option would give a realistic mass for the IMBH, independently of the size of the cluster, and thus, the interactions of the IMBH with its local environment would be realistic. On the other hand, comparisons with observations would be not easy, since the scaling with the size of the cluster, would be unrealistic.

In our study we are mostly interested in the local interactions of the IMBH with its environment, and not so much on the effects of the existence, or escape, of the IMBH on the structure of the cluster. When a IMBH-BH forms, we need to have the realistic masses for its components, and when there is a merger of the IMBH with a stellar mass BH, we need to include the realistic mass-ratio in the equations for the recoil velocity of the merger product. On the other hand, we need to compare this recoil velocity to the escape velocity of a real globular cluster (which might be as high as $50 km/s$) and not to the escape velocity of the simulated cluster which might be a few times lower due to the small $N$ used in the simulation.

### 2.2.2 The IMF of the cluster

For the IMF of the cluster there are some different approaches that can be used. This is closely related to the population and masses of the stellar-mass BHs that will be included in the initial configuration. In ?, where they study the formation of BH-BH binaries in star clusters without an IMBH at their centers, they use a low-mass Kroupa IMF. The stellar mass is in the limits of $0.5 \leq m \leq 1.0M_\odot$. They use a total number of $10^6$ particles distributed according to a Plummer density profile. For the masses of the stellar-mass BHs they use a fixed value of $10 M_\odot$. They note that according to studies the mass of the stellar-mass BHs should be expected to lie between $8M_\odot$ and $12M_\odot$, but the upper limit might be even larger. This will be discussed in the next section.

A different approach can be found in Portegies Zwart & McMillan (2001); where they study the formation of a very massive star (VMS) from runaway mergers of main-sequence stars in star clusters. In their initial data they use
a Scalo or Kroupa IMF with a lower mass of $0.1 - 0.3 M_\odot$ and a higher mass of $100 M_\odot$. The VMS that is produced is of a mass of $\sim 0.1 M_\odot$ and it is created in less than $10 M_{\odot}yr$. In ?, where they discuss the properties of globular clusters that harbor IMBH, they use a Kroupa IMF with limits $0.1 \leq m \leq 30 M_\odot$ and they do not include any stellar-mass BHs. Finally, in ?, where they study the black-hole binaries in globular clusters, without IMBH at their centers, they use a Kroupa IMF with a mass range of $0.1 \leq m \leq 150 M_\odot$. They evolve the stars according to Belczynski paper and produce the stellar-mass BHs with the appropriate masses.

We adopted a similar method for creating initial data for our simulations. We chose an initial mass function with index $\alpha = 1.3$ for $m \leq 0.5 M_\odot$ and $\alpha = 2.4$ for $m > 0.5 M_\odot$. The lower mass limit is set to $m_{\text{low}} = 0.2 M_\odot$, while the upper mass limit to $m_{\text{upper}} = 150 M_\odot$. We then evolved the system using the stellar evolution method of ? for $5 M_{\odot}yr$ assuming that it takes that much time for the IMBH to form at the center of the cluster via runaway mergers. The final result is all stars with masses above $\sim 10 M_\odot$ have evolved away from the main sequence. We neglect the possible neutron stars of white dwarfs produced by the stellar evolution code, by assuming that they are still stars of the cluster. Also, BHs with masses less than $\sim 10 M_\odot$ are not taken into account. In this way, there have formed 62 stellar-mass BHs in the system with masses that range $\sim 10 - 26.5 M_\odot$. The upper limit in the mass of the BHs is coming from the stellar dynamics code and the assumption it is used there for the stellar winds during supernov explosions. According to this approach, the choice of the IMF and the evolution of the IMF in time gives also the next two parameters of the initial data. Finally, for the distribution of stars and BHs in the cluster we assumed a King King profile with concentration parameter $W_0 = 7$. Also, no primordial mass segregation is used, so the BHs formed in all the distances from the center of the cluster.

2.3 Assumptions

In all simulations presented here, all stars are assumed to be point particles. The real dimensions of a star or a stellar remnant such as a black hole, are taking into account only in the case of collisions. Also, we assumed that there is no gas in the cluster and no stellar evolution after the simulation starts. The latter is not as bad a choice, since the age of the cluster at the beginning of a simulation is $5 M_{\odot}yr$, and most of the heavy stars have already transformed into BHs. In the following Myrs, just a few stars will evolve to black holes or neutron stars, with masses less than $10 M_\odot$, which is too low to eject the IMBH after a collision. Also, we assume no binary-stellar evolution for BH-Star binaries that form during the simulations. We assumed that all BHs formed before the beginning of the simulation remain in the system, and do not escape due to natal kicks. Finally, all BHs are assumed to be maximally spinning with random directions of their spins. The spins have no impact on the BH-BH interactions, as they are considered only when two BHs collide.

3 RESULTS

Initially, the IMBH interacts strongly with a sub-group of stars and BHs that contains approximately 20 members. As the system evolves the members of this sub-group change. Soon, most of the stellar-mass BHs of the system, sink towards the center and star to interact with the IMBH and its environment. During this process, some of them already receive big kicks due to 3-body interactions and are thrown away of the center of the cluster or the cluster itself. After $t \sim 3 M_{\odot}yr$ the first stable IMBH-BH binary forms. The companion of the IMBH is a stellar-mass BH with mass $M_{\text{bh}} = 23.9 M_\odot$ and the initial semi-major axis of this binary is $a \sim 88 AU$. At $t \sim 9.2 M_{\odot}yr$ this binary has a strong interaction with another BH of the system. The interaction leads to a change of companion for the IMBH, which now forms a binary with a BH that has mass $M_{\text{bh}} = 20.1 M_\odot$. The initial semi-major axis of the new binary is $a \sim 17.6 AU$. This binary is the most stable binary formed in the simu-
The IMBH escapes in a short time scale from the system. In Fig. (f) we show the Lagrange radii of the cluster during the simulation. We stopped the simulation \( \sim 10\text{Myr} \) after the ejection of the IMBH. As is shown the Lagrange radii increase instantaneously when the IMBH leaves the system, but they start decreasing slowly just afterwards. This decrease, would lead to a new core collapse of the system, unless some hard binaries will be formed soon. The IMBH-BH binaries provided a “heating” source to the system, transferring kinetic energy to the stars or BHs that passed close from the center of the cluster. The lack of such a binary, makes the system core and internal radii decrease in size, until another heating source will be formed and stop further shrinkage of the system. During the \( \sim 10\text{Myr} \) after the ejection of the IMBH, no such a source has been created.

In Fig. (f) the evolution of the distances of the 10 more massive BHs from the center of the system are presented. It is obvious that most of the BHs inspiral the center very rapidly, as long as the IMBH exists in the cluster. Some of them escape the system, after passing very close to the central binary. After the IMBH is removed from the system, the trajectories of the remaining BHs are not so steep as they orbit the center without sinking rapidly to it. In Fig. (g) the distances of the 9 more massive stellar-mass BHs from the IMBH are presented as functions of time. At it is shown clearly, all the ejected BHs, before their ejection, approach the IMBH in small distances and interact strongly with the IMBH-BH binary.

In Figs. (5) and (11) we present the evolution of the semimajor axis and of the eccentricity of the IMBH-BH binary that exists at all times at the center. The same diagrams are presented in Figs. (4) and (10) in which we have zoomed to the last 10000 years of the last binary formed. In the last two figures, the influence of the post-newtonian dynamics used is obvious. Similar diagrams but for the period of the binary are presented in (14) and (13). During the last stages before merger the binary had a period of the order of seconds.

In Fig. (16) we show the time evolution of the frequency of the gravitational radiation emitted from the IMBH-BH binary from the beginning of the simulation. In Fig. (17) we...
Figure 9. Zoom in at the final $\sim 10000$ years of the diagram of Fig. 1.

Figure 10. The evolution of the eccentricity of the IMBH-BH binary. Different colors indicate different companions for the IMBH.

Figure 11. Zoom in at the final $\sim 10000$ years of the diagram of Fig. 12.

Figure 12. The evolution of the period of the IMBH-BH binary. Different colors indicate different companions for the IMBH.

Figure 13. Zoom in at the final $\sim 10000$ years of the diagram of Fig. 12.

Figure 14. The evolution of the frequency of the gravitational radiation emitted from the IMBH-BH binary from the beginning of the simulation. Different colors indicate different companion BHs for the IMBH. The black line indicates the lower limiting frequency of LISA.
focus on the last 10000 yr and we show the limiting frequency of gravitational radiation that could be observable by LISA. Also, in Fig. (15) we show the way the frequency of the gravitational radiation changes with the changing eccentricity of the system. The spiral nature of the trajectory of the two BHs is shown in Fig. (17). The caption that follows this figure describes the different features that are shown.

From the above it is natural to address the question of whether or not such sources could be detectable by figure space-based gravitational radiation detectors. For this we did a study of the waveforms of sources with similar mass ratio and initial eccentricity and the results are shown in Figures (15) and (16) in which we show the sensitivity curves of LISA and ALIA, which is an improved version of LISA that has already been studied. As shown from the figures, such a binary would be a strong source for those detectors, even if it exists in distances as high as 1Gpc. We didn’t want to do any estimates about the number of such sources that could be detectable during a year, because we believe that such a calculation would be too risky, given the lack of proof for the existence of IMBH and of information about the fraction of star cluster that could contain an IMBH at their centers. If the existence of IMBH is confirmed and if the fraction of clusters that harbor an IMBH is not that low, then in a volume of 1Gpc there should be many sources like the one we discovered in our N-body simulation, given the large number of galaxies that is contained and the fact that every galaxy contains tens up to hundreds or thousands of young star clusters.

4 DISCUSSION

According to our N-body simulation the interactions of an IMBH with stellar-mass BHs in a young star cluster, might lead to the formation of a very hard binary. This binary would merge in a timescale of 10000 yr, while emitting strong gravitational radiation. An event like this would be observable by the proposed future space-based gravitational radiation detectors. Also, it would have a strong impact on the cluster that hosts the IMBH. More precisely, according to simulations that we performed, in which we measure the recoil velocity of the merger product of an IMBH and a stellar-mass BH, with mass ratio of 20, we found that there is only 30% probability that the final product would remain bound in the cluster, while in most of the cases the recoil velocity is high enough to eject it from the system. This is probably another reason for which some globular clusters do...
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Figure 18. Waveforms of the various harmonics of the gravitational radiation as they reach the earth. The binary IMBH-BH has been placed in distances of 1Mpc, 100pc, 1Gpc and 10Gpc from the earth. The sensitivity curve of LISA is also shown in the diagrams for comparison.

not seem to contain an IMBH at their centers: if an IMBH formed while they were young clusters, it might escape after a merger with a stellar-mass BH.

As it is obvious, a single simulation cannot be a proof of the existence of IMBH-BH binaries that merge in clusters in timescales less than Hubble time. We need to run more simulations with different initial data and try to cover some part of the parameter space, which is large and impossible to be fully covered using N-body simulations. For this reason, we run 3 more simulations with different parameters. The most important parameter that we changed is the number of stellar-mass BHs in the system. Here the number is high enough to assume that 100% of the BHs remain in the system after their formation, but this is not normally the case. BHs receive large recoil velocities as they form due to the asymmetries of the supernova explosion that give birth to them. These natal kicks can be high enough that the resulting BH escapes the system, so not all of the BHs should be retained in the cluster.
APPENDIX A: IMPLEMENTING POST-NEWTONIAN EQUATIONS IN MYRIAD

The equations for the post-Newtonian terms of the acceleration are found in (Blancet, Pau-Amaro), while the ones for the terms of the derivative of the acceleration have been calculated analytically. In all the equations we have used:

\[
\begin{align*}
\vec{r}_{12} &= \vec{r}_1 - \vec{r}_2, \tau = |\vec{r}_{12}|, \vec{n}_{12} = \vec{r}_{12}/\tau, \\
\vec{v}_{12} &= \vec{v}_1 - \vec{v}_2, \vec{v}_{12} = |\vec{v}_{12}|, \\
\vec{a}_{12} &= \vec{a}_1 - \vec{a}_2, \vec{a}_{12} = |\vec{a}_{12}|
\end{align*}
\]

(A1)

The vector product of two vectors \(\vec{x}_1\) and \(\vec{x}_2\) is denoted as \(x_1x_2\) (i.e. \(n_{12}v_1 = \vec{n}_{12} \cdot \vec{v}_1\)). Note the difference between \(a_i\), which is the total acceleration of particle \(i\) and \(\vec{a}_i\), which is the \(i\)-th post-Newtonian correction of particle \(1\). The equations of the post-Newtonian corrections for particle 2 can be found if we make the switch \(1 \leftrightarrow 2\) noting that:

\[
\begin{align*}
\vec{n}_{21} &= -\vec{n}_{12} \\
\vec{v}_{21} &= -\vec{v}_{12}
\end{align*}
\]

(A2)

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The 1PN corrections is:

\[
\tilde{a}_{21} = \frac{Gm_2}{r^2}\left\{ \bar{n}_{12}\left[ -v_1^2 - 2v_2^2 + 4v_1 v_2 + \frac{3}{2}(n_{12}v_2)^2 + \frac{5}{2}\frac{Gm_1}{r} + 4\frac{Gm_2}{r}\right] + \dot{v}_{12}(4n_{12}v_1 - 3n_{12}v_2) \right\}.
\]

(A3)

The 2PN correction is:

\[
\tilde{a}_{41} = \frac{Gm_3}{r^3}\left\{ \bar{n}_{12}\left[ - 2v_1^2 + 4v_2^2(v_1v_2) - 2(v_1v_2)^2 + \frac{3}{2}v_1^2(n_{12}v_2)^2 + \frac{9}{2}v_1^2(n_{12}v_2)^2 - 6(v_1v_2)^2(n_{12}v_2)^2 - \frac{15}{8}(n_{12}v_2)^4 \right. \\
- \frac{3}{2}(v_1v_2)^2 + \frac{5}{3}v_1^2 - \frac{5}{2}v_1 v_2 + \frac{39}{2}(n_{12}v_1)^2 \\
- 39(n_{12}v_1)(n_{12}v_2) + \frac{17}{2}(n_{12}v_2)^2 \right] \\
+ \frac{Gm_2}{r^2}\left( 4v_2^2 - 8v_1 v_2 + 2(n_{12}v_1)^2 - 4(n_{12}v_1)(n_{12}v_2) - 6(n_{12}v_2)^2 \right) \\
+ \frac{Gm_3}{r^3}\left( v_1^2(n_{12}v_2) + 4v_2^2(n_{12}v_1) - 5v_1^2(n_{12}v_1) - 4(v_1v_2)(n_{12}v_1) + 4(v_1v_2)(n_{12}v_2) - 6(n_{12}v_1)(n_{12}v_2)^2 + \frac{9}{2}(n_{12}v_2)^3 + \frac{Gm_1}{r}\left( - \frac{63}{4}n_{12}v_1 + \frac{55}{4}n_{12}v_2 + \frac{Gm_2}{r}\left( -2n_{12}v_1 - 2n_{12}v_2 \right) \right) \right\} \\
\frac{G^4m_2}{r^4}n\left[ - \frac{57}{2}m_1^2 - 9m_2^2 + \frac{69}{2}m_1m_2 \right].
\]

(A4)

and finally the dicative 2.5PN term is:

\[
\tilde{a}_{61} = \frac{4}{5}\frac{G^4m_1m_2}{r^3}\left\{ \bar{n}_{12}\left[ - (v_1 - v_2)^2 + 2\frac{Gm_1}{r} - \frac{5}{2}\frac{Gm_2}{r} \right] + \bar{n}_{12}(n_{12}v_1 - n_{12}v_2)\left[ 3(n_1 - v_2)^2 + \frac{5}{2}\frac{Gm_1}{r} \right] \right\}.
\]

(A5)

APPENDIX B: POST-NEWTONIAN
EQUATIONS UP TO 2.5PN FOR THE
DERIVATIVE OF THE ACCELERATION

Here we present the lengthy equations for the post-Newtonian terms of the derivative of the acceleration (jerk) in a binary system of black holes. Those terms are used in equation 3. As mentioned previously, in all the equations we have used:

\[
\bar{r}_{12} = \bar{r}_1 - \bar{r}_2, \bar{v}_{12} = \bar{v}_1 - \bar{v}_2, \bar{a}_{12} = \bar{a}_1 - \bar{a}_2, \bar{a}_{12} = \bar{a}_{11} - \bar{a}_{12}, \bar{a}_{11} = \bar{a}_{12} / r,
\]

(B1)

The vector product of two vectors \( \bar{x}_1 \) and \( \bar{x}_2 \) is denoted as \( x_1x_2 \) (i.e. \( n_{12}v_1 = \bar{n}_{12} \cdot \bar{v}_1 \)). Note the difference between \( a_i \) which is the total acceleration of particle 1 and \( \bar{a}_i \), which is the \( i \)-th post-Newtonian correction of particle 1. The equations of the post-Newtonian corrections for particle 2 can be found if we make the switch \( 1 \leftrightarrow 2 \) noting that:

\[
\bar{a}_{21} = -\bar{a}_{12}, \bar{v}_{21} = -\bar{v}_{12}, a_{21} = -a_{12}
\]

(B2)

The 1PN correction is:

\[
\tilde{a}_{21} = -2\frac{n_{12}v_1 - n_{12}v_2}{r} \bar{a}_{21} + \frac{Gm_2}{r^2}\left\{ \bar{v}_{12} - \bar{n}_{12}(n_{12}v_1 - n_{12}v_2) \right\} \\
\times \left[ - v_1^2 - 2v_2^2 + 4v_1 v_2 + \frac{3}{2}(n_{12}v_1)^2 + \frac{5Gm_1}{r} + \frac{4Gm_2}{r} \right] \\
+ \bar{n}_{12}\left[ -2v_1 a_{12} - 4v_2 a_{12} + 4v_1 a_{22} + 4v_2 a_{22} \right] \\
+ 5(n_{12}v_2)\left( \frac{1}{2}\bar{v}_2 \left( \bar{v}_{12} - \bar{n}_{12}(n_{12}v_1 - n_{12}v_2) \right) + n_{12}a_{22} \right) \\
+ \bar{a}_{12}\left( 4n_{12}v_1 - 3n_{12}v_2 \right) \\
+ \bar{v}_{12}\left[ \frac{1}{2}\left( \bar{v}_{12} - \bar{n}_{12}(n_{12}v_1 - n_{12}v_2) \right) \\
(4v_1 - 3v_2) + \bar{n}_{12}(4a_{21} - 3a_{22}) \right] \right\}.
\]

(B3)

The 2PN correction is:

\[
\tilde{a}_{41} = \frac{Gm_3}{r^3}\left\{ \bar{n}_{12}2l_1 + \bar{v}_{12}l_2 \right\} + \frac{G^3m_2}{r^4}n_{12}l_3
\]

(B4)

where:

\[
I_1 = -2v_1^2 + 4v_2^2(v_1v_2) - 2(v_1v_2)^2 + \frac{3}{2}(n_{12}v_2)^2 + \frac{9}{2}(n_{12}v_2)^3 - 6(v_1v_2)(n_{12}v_2)^2 - \frac{15}{8}(n_{12}v_2)^4 + \frac{Gm_1}{r}\left( - \frac{15}{4}v_1^2 + \frac{5}{4}v_2^2 + \frac{15}{4}v_1 v_2 + \frac{39}{2}(n_{12}v_1)^2 - 39(n_{12}v_1)(n_{12}v_2) + \frac{17}{2}(n_{12}v_2)^2 \right) \cdot \frac{Gm_2}{r}\left( 4v_2^2 - 8v_1 v_2 + 2(n_{12}v_1)^2 - 4(n_{12}v_1)(n_{12}v_2) - 6(n_{12}v_2)^2 \right),
\]

(B5)

\[
I_2 = v_1^2(n_{12}v_2) + 4v_2^2(n_{12}v_1) - 5v_1^2(n_{12}v_2) - 4v_1v_2(n_{12}v_2) \\
+ 4v_1v_2(n_{12}v_2) - 6(n_{12}v_1)(n_{12}v_2)^2 + \frac{9}{2}(n_{12}v_2)^3 + \frac{Gm_1}{r}\left( - \frac{63}{4}n_{12}v_1 + \frac{55}{4}n_{12}v_2 \right) + \frac{Gm_2}{r}\left( -2n_{12}v_1 - 2n_{12}v_2 \right),
\]

(B6)

and

\[
I_3 = \frac{57}{4}m_1^2 - 9m_2^2 + \frac{69}{2}m_1m_2.
\]

(B7)

The 2.5PN correction to the jerk is:
\[ \dot{a}_{51} = \frac{3n_{12}v_1 - n_{12}v_2}{r} \dot{a}_1 \]

\[ + \frac{4G^2m_1m_2}{r^3} \left\{ \dot{a}_{512}I_4 \right\} \]

\[ \dot{v}_1 \left[ 2v_{12}a_{5AB} + \frac{1}{r^2}(n_{12}v_1 - n_{12}v_2)(-2Gm_1 + 8Gm_2) \right] \]

\[ \left\{ \frac{1}{r} \left[ v_{12} - n_{12}(n_{12}v_1 - n_{12}v_2) \right] \right\} \left[ n_{12}v_1 - n_{12}v_2 \right] \]

\[ \dot{a}_{12} \left[ \frac{1}{r} \dot{v}_{12} (\dot{v}_{12} - n_{12}(n_{12}v_1 - n_{12}v_2) + n_{12}a_{AB}) \right] \] 

\[ + n_{12}v_{12}v_1 - n_{12}v_2) \left[ (6Gm_1 - \frac{52}{3}Gm_2) \right] \]

\[ (6Gm_1 - \frac{52}{3}Gm_2) \] 

\[ \text{where} \]

\[ I_4 = -v_{12}^2 + 2 \frac{Gm_1}{r} - 8 \frac{Gm_2}{r} \] 

\[ I_5 = 3v_{12}^2 - 6 \frac{Gm_1}{r} + \frac{52}{3} \frac{Gm_2}{r} \] 

\[ \text{APPENDIX C: RECOIL VELOCITIES} \]

There are two different mechanisms that drive a binary system of two BHs to a merger. The first is the extraction of energy from the system through 3-body encounters with field stars and BHs, while the second is the gravitational radiation that is emitted from the binary system. The first mechanism is responsible for shrinking of the semimajor axis from large distances to distances of a few AU or smaller than \( \sim 1 \) AU, while the second acts on very close binary systems, leading them to a merger.

Collisions of BHs are accompanied by strong emission of gravitational radiation. According to Numerical Relativity (Pretorius 2005), when two unequal-mass, spinning black holes collide, gravitational radiation can be emitted asymmetrically. This would lead to a recoil velocity in the resulting black hole, which might be as high as \( 4000 \text{ km s}^{-1} \) (Gonzalez et al. 2007, Campanelli et al. 2007/2006, Heinly et al. 2006, Herrmann et al. 2010), depending on the mass ratio of the initial black holes and the directions of their spins, but this velocity might be suppressed a lot due to the relativistic alignment of the spins (Kesden et al. 2001). Of course, it is not easy to include Numerical Relativity in an N-body code, but there are semi-empirical formulas, coming from fitting between Numerical Relativity results and post-Newtonian theory (Lousto & Zlochower 2003, Baker et al. 2003, Lousto et al. 2011, Gonzalez et al. 2007), that give the direction and magnitude of the recoil velocity. We have included the semi-empirical formula of Gonzalez et al. (2007) in the binary module of MyriadPN for assigning recoil velocities in the merger product of a colliding binary BH system.

According to Lousto et al. (2011), the recoil velocity of the merger product of two BHs is given by:

\[ \vec{v} = (v_m + v_\perp \cos \xi) \vec{e}_1 + v_\perp \sin \xi \vec{e}_2 + v_\parallel \vec{e}_3 \]  

The index \( \perp \) and // refer to perpendicular and parallel to the orbital angular momentum \( \vec{L} \). The unit vector \( \vec{e}_1 \) is on the plane of the orbit connecting the two BHs and pointing from the heavier 1 to the lighter 2 BH. The \( \vec{e}_2 \) is the vector on the plane of the orbit and perpendicular to \( \vec{e}_1 \) so that the three unit vectors \( (\vec{e}_1, \vec{e}_2, \vec{e}_3) \) form an orthonormal system. The third vector \( \vec{e}_3 \) is the unit vector parallel to the vector \( \vec{L} \) of the angular momentum of the orbit. \( \xi \) measures the angle between the unequal mass and spin contribution to the recoil velocity in the orbital plane. According to Gonzalez et al. (2007) and references therein \( \xi \approx 45' \) for quasi-circular orbits while \( \xi \approx 90' \) for head-on collisions. In Myriad we use \( \xi \approx 145' \). \( v_m \) is the recoil velocity of the final product as if the progenitors had no spin. This is given by:

\[ v_m = \eta \frac{v_{\perp}}{1 + q} \]  

\( v_\perp \) is the recoil velocity parallel to \( \vec{L} \), i.e. on the plane of the orbit and it is given by:

\[ v_\perp = H\eta \frac{1}{1 + q} (\mu \mu_0) \]  

\( v_\parallel \) is the recoil velocity parallel to \( \vec{L} \), i.e. perpendicular to the plane of the orbit and it is given by:

\[ v_\parallel = K \eta^2 \frac{1}{1 + q} |\mu \mu_0| \cos(\theta - \Theta) \]

where \( q \) is the mass ratio of the smaller to the larger mass \( \frac{m_2}{m_1} \), \( H = \frac{\sqrt{a}}{1 + \sqrt{a}} \) is the symmetric mass ratio and \( \epsilon \) is the eccentricity of the orbit.

In Gonzalez et al. (2007) and references therein the fitting parameters are defined as: \( A = 1.2 \times 10^4 \text{ km s}^{-1} \), \( B = -0.93 \), \( H = 6.9 \times 10^3 \text{ km s}^{-1} \), \( K = 6.072 \times 10^4 \text{ km s}^{-1} \).

Finally:

\[ \vec{a} = \frac{\vec{S}}{m_1} \]  

is the dimensionless spin of the \( i - \text{th} \) BH, where \( \vec{S} \) is the spin of the \( i - \text{th} \) particle and \( m_1 \) its mass, with \( i \) taking values from 1 to 2. The angle \( \Theta \) is the angle between vector \( \vec{L} \) and \( \vec{e}_1 \), where vector \( \vec{L} \) is defined as:

\[ \vec{L} = (m_1 + m_2)(\frac{S_1}{m_2} - \frac{S_2}{m_1}) \]  

In MyriadPN, when two BHs merge, the resulting BH gets as mass the sum of the masses of the two BHs. Then the \( (e_1, e_2, e_3) \) system is found from the angular momentum \( \vec{L} \) and the direction of the merger \( e_1 \). Equation (8) calculates the recoil velocity of the merger product assuming random distribution of the spins. In addition the maximum and minimum recoil velocities for spinning BHs are computed as well as the recoil assuming no spins. The final recoil velocity is transformed back to the center of mass reference frame and assigned to the resulting BH.

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