Workshop on Gravitational Waves

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1. Introduction

According to general relativity any change in the gravitational field of a compact gravitating system like neutron star or black hole, will produce a ripple on the spacetime which will propagate with the speed of light. This is a *gravitational wave*. Although gravitational waves have been predicted from the very early days of general relativity they have not yet been detected directly. Nevertheless, their indirect influence on the evolution of compact binary systems has been observed almost 20 years ago, and the observational results agree to remarkable accuracy with the theoretical predictions.

Almost thirty five years ago attempts were made to detect these tiny ripples in the spacetime by constructing gravitational wave detectors in the form of resonant bars. Today these detectors are either narrowband resonant bars or spheres or broadband laser interferometers. The resonant detectors are already in operation and they have achieved sensitivities close to their limits, while the laser interferometers (LIGO, VIRGO, GEO600) are in the construction phase and expected to become operational before the end of this century. The next generation of gravitational wave detectors includes space experiments like LISA accepted by ESA as a possible cornerstone mission.

The direct detection of the gravitational waves will not only be a great triumph for Einstein's theory of relativity but is expected to signal a new era in astronomy by opening a non electromagnetic window of observation. By a network of at least three laser interferometers one will be able to determine the distance of a coalescing binary system, its

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position in the sky, the masses of the members of the system and orbital frequency at final merger of the system. This information in turn will enable the determination of the Hubble constant [1], distribution of masses of neutron stars and black holes and constraints on the neutron star equation of state.

The era of *gravitational wave astronomy* is hopefully not too much in the far future and there is an international collaboration between experts from various different backgrounds to solve the relevant problems and march to this new epoch. We see combined work from various groups around the world in the fields of numerical relativity, techniques in data analysis, approximation methods and importantly an exchange of ideas and methods between technical groups around the various detectors both bar and interferometer.

In the workshop for gravitational waves there were talks related to the theory of the gravitational waves, the methods to be used to analyse the data and extract information as well as presentations on the status of experimental work on bar and laser interferometric detectors. The most promising source of gravitational waves is the binary neutron star or black hole coalescence. Though most of the talks were focused on these systems, nevertheless, stellar collapse to neutron stars or black holes as also gravitational waves from pulsars were also discussed.

The workshop was divided into two sessions: A discussion session to summarise present status and future directions in some areas we considered important; the other a session of contributed papers to present interesting work in progress. The list of discussion topics is not exhaustive but indicative. The endeavour to detect gravitational waves probably pushes to the extreme every aspect that it involves. Technology of all kinds, vacuum, optics, isolation systems, noise control. It also has need to push theory to orders of accuracy not available today. For once in general relativity experiment drives the theory! We begin with theoretical aspects and look at the inspiralling binary system where the frontier lies at third- post-Newtonian (3PN) order. One needs to address the issue of generation on one hand and equation of motion on the other. The best calculation in the restricted two body limit is at 4PN. What next? To go beyond the approximation and perturbation techniques to the fully general relativistic regime as of date there is only the numerical route. The “grand challenge” is the problem of black hole collisions in three dimensions but the testing grounds for conceptual and numerical issues are one dimensional and two dimensional problems. All this work is driven by the need to have high phasing accuracy in the gravitational wave templates. Data analysis aspects need to be carefully assessed to separate what we really know from what we think we know, the folklore from the facts. Though the logo of our community could well be the inspiralling and coalescing binary it is important to explore weaker sources like pulsars. And finally even at this late hour reflect whether we "interfere" or "resonate".

2. Inspiralling Compact Binaries : Problems in the 3PN Program

One of the most urgent challenges on the theoretical front in the PN programme is the 3PN generation and this was discussed by Luc Blanchet. A challenging problem was to predict the time evolution of the orbital phase of an inspiralling compact binary, as due to the reaction to the emission of gravitational waves. Such prediction was needed for the future observations of the LIGO and VIRGO detectors. The demanded precision should be extremely high, namely it should take into account many high-order relativistic or PN corrections [2, 3, 4].

The computation generally relied on an energy balance argument which equated the decrease of binding energy of the binary and (minus) the energy flux generated by the binary in the form of gravitational waves. The validity of this argument had been proved up to now only to 1.5 post-Newtonian (1.5PN) order, or order c^{-3} when the speed of light $c \to \infty$ [5, 6, 7, 8, 9]. A general and important problem was to prove that this argument stayed correct to much higher order than 1.5PN.

The binding energy of the binary resulted from the equations of motion of the binary which were known presently up to 2.5PN order [10, 11]. On the other hand the energy flux at infinity was also known at 2.5PN order [12, 13, 14, 15]. *Assuming* that the energy balance argument was valid at this order, the orbital phase was therefore known presently up to 2.5PN.

To 3PN order the following problems must be (or have already been) solved.

- The multipole moments of the binary system are needed at 3PN. Some general expressions for the moments as integrals over an arbitrary source (in principle), and formally valid up to any post-Newtonian orders, are known. However the proof of the validity of these expressions still needed some clarification. The reduction of these expressions to binary systems was a hard computational task (which should be done partly on the computer). Many intricate integrals are to be evaluated. It was not clear that all of them could be obtained analytically.

– The nonlinear effects in the wave zone gave an important observational contribution. At 3PN order in the energy flux nonlinear terms were known to be composed solely of "tails of tails", which are the tails of waves generated by curvature scattering of the (quadratically nonlinear) tails themselves. This effect was cubically nonlinear. It was presently under control.

– The equations of motion of the binary should be generalized from 2.5PN to 3PN order, at least for circular orbits. The equations of motion were needed both for computing the binding energy and for reducing the time derivatives when computing the energy flux. This problem was a difficult one in part because one needed to justify at this order the use of delta functions to represent the compact objects.

These problems when solved should give a result in perfect agreement, in the test mass limit for one body, with the result of black-hole perturbation theory [16, 17].

This was followed by a remark by Piotr Jaranowski on the technical hurdles in the computation of the 3PN equation of motion which was in progress in collaboration with Gerhard Schäfer. They were currently calculating 3PN Hamiltonian for the two body point-particle system in the framework of ADM formalism.

They had encountered some serious ambiguities due to highly singular structure of the integrals involved in the calculation. There were terms for which Hadamard's "partie finie" procedure gave the result different from that obtained by using the Riesz's kernel representation of the Dirac delta distribution.

They had also recognized that regularizing integrals by means of multiplying an integrand by $r_1^\alpha r_2^\beta$ was of restricted validity, because it did not give zero for some integrals being full divergences. They should be zero due to the obvious identity $(\nabla_1 + \nabla_2)\, f(\mathbf{r}_{12}) = 0$.

3. Black Hole Perturbation Approach

Misao Sasaki reviewed the recent developments in the black-hole perturbation approach to gravitational waves from coalescing binaries. This approach, though valid only in the limit when one of the two bodies is much heavier than the other in the strict sense, has been shown to be very useful in analyzing higher post-Newtonian features of the gravitational radiation and the orbital evolution of a binary in a complementary manner as compared to the standard post-Newtonian approach.

The highest PN order analytically achieved so far is 4PN for a non-spinning particle [18] and 2.5PN for a spinning particle [19], both in the case of circular orbits around a Kerr black hole. Although the calculations would become increasingly complicated, there seemed no fundamental difficulty to go to still higher PN orders for circular orbits or those allowing perturbative expansion from a circular orbit. A matter of discussion would be whether it would be meaningful or useful to do so. In this respect, one interesting issue was the convergence property of PN expansion. This can be examined to good accuracy by comparing the results of analytical expansions with those obtained numerically which are 'exact' up to numerical precision.

A more meaningful and perhaps interesting problem was the radiation reaction, since one would be able to gain useful insights into the radiation reaction mechanism in relativistic situations. He mentioned some recent attempts to derive the radiation reaction force term in certain restricted situations. But the status of the problem seemed quite obscure at the moment in his opinion. The main reason was that we don't have a well-justified method to regularize the divergence associated with the point-mass.

Finally, he mentioned the issues of post-Teukolsky expansion and the perturbation approach for background spacetimes other than a black hole. Although there were no good ideas, he emphasized that if we found a way to handle these issues, the perturbation approach would become a much more powerful tool.

Jorge Pullin spoke about the "Close approximation" techniques for dealing with the collision of two black holes. The idea was that if one considered collisions where the two black holes started close to each other one could approximate the situation by a single distorted black hole and apply black hole perturbation theory to the problem. This was shown to work very well in the problem of the head-on collision of two black holes [20]. The challenge next was to apply it to other collision scenarios. For that purpose two things were needed a) initial data and b) some estimate of the degree of trustworthiness of the results.

In order to attack a) they proposed to solve the initial value problem in the close approximation. In this approximation one could find simple yet accurate solutions of the Hamiltonian and momentum constraint for situations like black holes with spins and linear momenta. At the moment they [21] were studying the case of collisions with momentum and the counterrotating collision of black holes with spin [22]. This latter case could be compared with the full numerical simulations of Brandt and Seidel [23]. The case of in-spiralling collision with small total angular momentum could also be treated as a perturbation of Schwarzschild and this was their next step. One could refine this by studying perturbations of Kerr using the Teukolsky equation but this was more involved, although it was feasible. Others are initial data of different symmetries [24].

On the issue of how trustworthy the formalism was, one could construct certain empirical criteria, like how much did the linearized initial data violate the exact Hamiltonian constraint [25]. These criteria were useful as quick tests but may not be accurate enough.

4. Numerical Relativity Challenges

Edward Seidel discussed briefly a number of important challenges and problems for numerical relativity to solve in the coming decade. He felt that although significant progress had been made over the last decade, general, 3D, long term evolutions of systems governed by the Einstein equations were still very difficult.

Numerical relativity has been an active field of research for about 30 years. As analytic techniques were still unable to penetrate the full solution space of Einstein's equations for nonlinear, time dependent systems in 3D, numerical solutions were presently our best hope for unlocking their predictive power for systems without symmetries. Significant progress had been made in the last decade in numerical relativity, leading to evolutions of distorted and colliding black holes in 2D and 3D[26], as well as pure gravitational wave spacetimes, and self-gravitating scalar fields. The entirely new field of critical phenomena came from numerical relativity. It was a very active and exciting area of research, but there was still much work to be done!

In many areas there was a pressing need for both theoretical and numerical study. Particularly important was the interplay between theoretical ideas and numerical implementation. Although much of the current effort in numerical relativity was directed towards 3D, many of the numerical studies could be carried out in simpler spacetimes in 1D and 2D, which were now accessible on powerful workstations. Techniques developed on these more accessible problems could then in many cases be applied to the more complicated 3D cases.

A very basic problem faced by all numerical relativists was the long term, stable evolution of the Einstein equations. At present, all black hole evolutions in 2D and 3D were plagued by numerical instabilities at late times ($t \approx 200M$ or less, where M was the mass of the black hole)[27, 28]. But even in very weakly gravitating systems, such as low amplitude gravitational waves, numerical instabilities could develop after a fairly short time[29]. Some of these instabilities are related to finite differencing across large gradients that develop in metric functions near black holes, some of them are related to the drifting of coordinate lines in 3D, and some of them are unknown in origin.

There were many lines of research that were needed to address these problems. For example, which lapse and shift are needed in 3D was an open question. Good choices would likely be geometric in nature (e.g., minimal distortion shift[30]), would minimize coordinate drift and also the stretching and shearing of metric functions that could lead to problems. The freedom in the lapse has traditionally been used to avoid developing spacetime singularities[31], but it can also be used to control certain properties of the metric. Good choices of slicing and shift conditions in 3D numerical relativity need to be explored. Another important attack against instabilities that develop in numerical evolution was the finite differencing of the equations themselves. In the standard 3+1, ADM approach to numerical relativity[30], the equations were extremely complicated, but in hydrodynamics the equations took on a special form where derivatives appear in the so-called "flux" terms. The physical interpretation of these terms lead to special finite difference operators that preserved important physical characteristics of the solution. In cases where the equations are hyperbolic, the system could be diagonalized and the eigenfields themselves could be evolved. Recently, the Einstein equations had been cast in this first order, flux conservative hyperbolic form (FOFCH)[32, 33]. Dramatic results had been obtained in spherical symmetry, and 3D codes based on this hyperbolic formalism, and others[34], were under construction. But this line of research was just beginning, and much work remained to be done to understand the best way to take advantage of these hyperbolic formulations.
Black holes were among the hardest systems to handle in numerical relativity, even in the stationary case where the solutions were known analytically! Axisymmetric calculations had been carried out for the last decade, but at present, we were still limited to rather crude evolutions in 3D, even in cases with a high degree of symmetry[26, 27]. For the more general case of two spiralling black holes, we were far from the final solution, and much work was needed to be done to achieve that goal.

There was no room to detail all the many areas of research related to black hole evolutions, but he would simply mention the major issues and give references to current work. Recent work on using "apparent horizon boundary conditions"[35, 31, 36] showed that it may be possible to handle singularities in numerical calculations by removing the region inside a horizon from the calculation. This technique allowed one to avoid using pathological singularity avoiding time slicings, and had worked well in spherical symmetry. However, a stable, long term evolution for a true 3D black hole had not yet been tried, and much remained to be done to perfect the technique. In 3D spherical black holes [37], and some axisymmetric systems had been studied, but no spin or angular orbital momentum had been attempted. He expected that this area of research would keep people busy for quite some time to come!

General relativistic hydrodynamics would be a very exciting area, as it encompassed most of astrophysics. Many important problems in astrophysics, such as accretion onto black holes, mergers of neutron stars with other neutron stars or with black holes, stellar core collapse, supernovae, etc., could only be fully treated through a fully self consistent approach to solving the coupled Einstein and hydrodynamic equations. However, very little had been done in this area until now, primarily because of the difficulties of solving the (left hand side of) the Einstein equations[28]. This was a rich area of relativity and astrophysics and would get much more attention in the coming decade.

Pure gravitational waves provided a laboratory for studying the full nonlinear theory without complications associated with matter fields or singularities in the initial data. This area of research was for the most part uncharted territory. The general behavior of 3D strong gravitational waves, including for example gravitational geons and the formation of singularities, was unknown. Many questions about waves in more general 3D spacetimes were waiting to be studied.

The interplay of analytic studies with numerical relativity would become even more important as numerical simulations became more sophisticated. We would need a full set of tools by which to analyze and understand the results of the simulations. An excellent example of this was the recent work using using perturbation theory to explore the dynamics of black hole collisions (See e.g., [38] and references therein.) Also, numerical and theoretical studies of apparent and event horizons had recently led to a better understanding of their dynamics (See, e.g., [27] and references therein.) There was much more to be done along these lines, and others, such as the use of traditionally analytic tools, including Riemann Invariants, principal null directions, etc. This should be an area where more analytically inclined researchers could make an essential contribution to numerical relativity.

In conclusion, numerical relativity had made great strides during the last decade, but many challenges lay ahead. There was room for groups from all areas to contribute, in both theoretical and numerical areas. For those interested in getting started in numerical relativity, 1D, 2D, and even 3D sample codes were available from the NCSA relativity group WWW server at http://jean-luc.ncsa.uiuc.edu.

5. Data Analysis: What do we agree upon?

In his presentation B.S. Sathyaprakash presented a critical resume of the data analysis endeavours. He pointed out aspects of gravitational wave data analysis that had in his view, gained consensus amongst people working in those areas and indicated problems that needed further exploration. The sources discussed include coalescing compact binaries, non-spherical neutron stars, stochastic background and burst sources.

The first in our list of candidate sources was the coalescing compact binary. It was now reasonably well agreed upon that matched filtering would be employed to extract chirp signals buried in noisy data. The number of templates required to span the parameter space of the inspiral wave form sensitively depended on the lower limit in masses of binaries [39] that we intended to search as well as the post-Newtonian order of the template of wave forms[40, 41, 42]. By now it was well established that the restricted post-Newtonian wave forms, only incorporating the phase corrections to the wave form, were good enough for detection[2].

Amplitude corrections at worst induced biases in the estimation of parameters and these could be taken care in off-line analysis. It was however not known how important were the eccentricity induced modulations of the wave form. For a search covering the range $[0.5, 20]M_\odot$ of the total mass of the binary one would need as many as 2×10^4 post-Newtonian templates and a computing power of 2 GFLOPS for an on-line search while the corresponding numbers for $[1.0, 20]M_\odot$ were 3500 post-Newtonian templates and 250 MFLOPS computer power[40, 42]. A parameter called the chirp time, which was a certain combination of the masses of the two stars had been found to be a good parameter for the purpose of making a choice of templates [43]. It remained to be seen as to what was a good second parameter.

There had been several attempts to find a solution to the inverse problem of reconstructing the full wave form (the two independent polarisation amplitudes, direction of the incoming wave form and the parameters of the wave form) with a knowledge of the response function from a network of three or more detectors[44, 45, 46, 47], but an optimal solution was still lacking. In the area of parameter estimation Cramer-Rao error bounds were now available at various post-Newtonian orders[48, 49, 50, 51, 52, 53]. However, we needed estimates of biases induced in estimation when inaccurate templates were employed in detection [42]. Moreover, Cramer-Rao bounds were not useful since they did not tightly constrain the errors. Recent work employing Monte Carlo simulations had shown that errors in the measurement of parameters were typically larger by a factor of 2–3 than

Cramer-Rao bounds [52, 53, 54]. There was considerable work on testing theories and models which had shown that gravitation theories [55] and cosmological models [56] could be constrained at high confidence levels with observation of few to many events. Further work was needed in finding how well astrophysical models could be tested.

Second in our list of sources were the sources that emitted periodic gravitational waves continuosly, namely spinning, non-spherical neutron stars that could or could not be observable electromagnetically. The extent of amplitude and frequency modulation caused due to Earth's motion relative to the solar system barycentre had been worked out[57, 58] and attempts had been made to compute the Fourier transform of the modulated wave form [57]. The signal was envisaged to be detected by Doppler de-modulating and Fourier transforming data that was typically several months long [59]. The problem was that demodulation depended on the direction to the source. Present estimates of the number of patches in the sky that needed to be corrected for modulation effects (so that there was no appreciable loss in signal-to-noise ratio for all sources located in a given patch) was in excess of 10^{13} while searching a year's worth of data[58]. Computing power needed to carry out an on-line, all-sky, all-frequency search was certainly out of reach even by the standard of computers that were expected to become available towards the turn of the century. It would however be possible to do on-line search for known pulsars and possibly all-sky, all-frequency search for pulsars in a week's worth of data. However, large proper motions observed in the case of some pulsars caused additional drop in signal-to-noise ratio and it needed to be explored as to what was the magnitude of this motion and to what extent this could be corrected for. There had not been much work on the estimation of parameters such as the extent of nonsphericity, neutron star magnetic fields, direction to the source, etc.

Stochastic background of gravitational waves was the next in the list. Here detection was envisaged by cross correlating data from two near by interferometric detectors or an interferometric detector in a narrow band operation and a cryogenic bar with a compatible operating frequency and sensitivity [59]. In either case the signal-to-noise ratio was enhanced in proportion to $T^{1/4}$ where T was the observing time. There had been estimates of the amplitude and power spectrum of background gravitational waves produced in models of inflation but how such models may be constrained with the aid of data from gravitational wave detectors remained to be estimated. We did not know how the noise intrinsic to the detector contaminated the cosmological background. Monte Carlo simulations of detection of the background would greatly help in learning more about these sources.

The last in the list of sources was the burst of radiation emitted in a supernova event and other short lived burst sources. It could be possible to dig out bursts buried in noise by looking for generic features such as a characteristic frequency and a ring down time. Unfortunately there was a great variety of wave forms predicted in a supernova depending on the nature and extent of asymmetry in the collapse, equation of state of gravitating matter, details of nuclear astrophysical processes, etc. and thus supernova events fell into the category of unknown sources as far as their detection was considered. It could be that they could only be detected by cross correlating data from different nearby detectors or looking at high-sigma events in the time series. Much work was needed to be done with regard to data analysis of bursts.

6. On the Detection of Gravitational Waves from Pulsars

Sanjeev Dhurandhar summarised the status of the studies related to gravitational waves from pulsars. The problem of detecting gravitational waves from known and unknown pulsars or rotating neutron stars was discussed. In the case of known pulsars a general data analysis scheme for detecting gravitational waves from millisecond pulsars with resonant bar antennas was proposed [60]. As a specific example, the case of the nearest known millisecond pulsar PSR 0437-4715 and the resonant bar antenna at the University of Western Australia was considered, since the gravitational wave frequency of the pulsar coincides with the resonant frequency of the bar within a fraction of a Hertz. The key idea was to rotate the phase plane with appropriate angular velocity and thus correct for the Doppler shift in the apparent pulsar frequency. For the University of Western Australia niobium antenna tuned to PSR 437 - 4715, astrophysically relevant sensitivity could be achieved with an improved transducer technology. It was concluded that the best candidate for detection in this way is PSR 437 - 4715. Also a list of pulsars was given, which in principle could be detected by resonant bar antennas operating between frequencies 500 Hz to 1300 Hz.

The other problem which is far more difficult is of detecting unknown pulsars or rotating neutron stars since in this case we do not know from electromagnetic observations the frequency or the location of the pulsar in the sky. Schutz [59] has analysed the problem of the *all sky all frequency search* and shows that one may have to search for 10^{13} patches or directions in the sky for reasonable parameters. This could involve to the tune of 10^{23} floating point operations. Different schemes therefore need to be tried. One such scheme using differential geometric techniques of scanning the pulsar signal manifold with appropriate basis functions on the celestial sphere was proposed. A promising set of basis functions are the Gelfand functions. These functions have proved useful in the past in elegantly describing the antenna patterns of laser interferometric gravitational wave detectors and resonant bars [44].

7. Limit on rotation of relativistic stars

John Friedman summarised the state of our understanding of the rotation limits of neutron stars. The angular velocity of a uniformly rotating star could not exceed that of a satellite in orbit at the star's equator. This limiting *Kepler frequency*, Ω_K, was about half the frequency of a satellite at the equator of the corresponding spherical star, because the star's radius increased sharply as Ω approached Ω_K. For fixed mass, its value was sensitive to the equation of state (EOS): a soft equation of state implied a centrally condensed star with larger binding energy and a correspondingly larger value of Ω_K than that of a stiff EOS. The uncertainty in the equation of state above nuclear density was large, and the stiffest proposed EOSs gave stars with moments of inertia at $\Omega = \Omega_K$ about 8 times greater for the stiffest equations of state than for the softest consistent with our knowledge.

The limiting angular velocity could be reached only if the magnetic field was small. For neutron stars that were born with a sufficiently weak magnetic field, a nonaxisymmetric instability probably set a limit that was slightly more stringent limit than the Kepler frequency. The instability, however, was damped by bulk viscosity for temperatures above 2×10^{10} K and by an effective shear viscosity below the superfluid transition temperature. As a result, it was not likely not to play a role in neutron stars spun up by accretion.

An observational upper limit on neutron star rotation could constrain the equation of state of matter above nuclear density. In particular, the two fastest of the known pulsars have angular velocities within 3% of each other (about 640 Hz); if they turned out to be rotating at nearly the upper limit on rotation for a 1.4 $M_\odot$ neutron star, then the equation of state above nuclear density was unexpectedly stiff.

One could set an upper limit on spin that was independent of the (unknown) equation of state above nuclear density. Causality and the observation of neutron stars (or, not to bias the case, against, say, quark stars or stars with quark interiors) with mass at least as large as 1.44 $M_\odot$ constrained the equation of state. With these minimal assumptions, the minimum period of a relativistic star was 0.30 ms [61, 62, 63, 64].

8. Contributed Papers

8.1. Theoretical

A. Gopakumar [65] presented his work on the 2PN evolution of inspiralling compact binaries in general orbits using the post-Minkowskian approach [13]. They have computed the 2PN accurate mass quadrupole moment for a system of two point masses, and calculated the 2PN contributions to energy and angular momentum fluxes. Both the quadrupole moment and the energy loss agree with those obtained by Will and Wiseman using the Epstein-Wagoner approach. They have also calculated the 2PN waveforms for general orbits. The results are compared with those obtained using the perturbation methods .

8.2. Detectors and Technologies

In this section the status of the prospects and the present and future sensitivity of both bar and laser interferometric detectors were discussed.

David Blair from Australia reported on the work of his bar detector group. He pointed out that in the high frequency regime $\approx 1000Hz$ where the the resonant detectors operate they are now as sensitive as the LIGO-I and in two to three years they would achieve sensitivities a bit lower than the expected ones for LIGO-II (which would be at least a decade in the future).

Massimo Bassan from Rome also pointed out the high sensitivity of the present generation of the bar detectors, which are the only operational detectors at present. An important point in his talk was the proposal for an array of 20-30 small detectors which would be sensitive in the bandwidth of a few kHz, since each one separately was not sensitive enough one needed a group of them for accumulating sensitivity [66]. Nevertheless, such an array would use existing technology without the need of advanced cryogenics and the cost of such arrays would not be high. The arrays of detectors would be useful for the detection of burst signals from collapsing objects to neutron stars or small black holes.

Ju Li reported a promising result i.e. the use of sapphire for mirrors in the laser interferometers. Sapphire with its excellent mechanical and thermal properties is an attractive material to use for beam splitter and test masses in laser interferometer gravitational wave detectors. The internal thermal noise amplitude of a sapphire test mass would be 16 times better than that of a fused silica test mass with same dimensions. If large sapphire samples maintained similar performance the total local noise sources in laser interferometers could be reduced by factors of 16. Consequently laser interferometric detectors with a few hundred meters arm (AIGO400) would be comparable to that of the LIGO with arm length of 4 Kms or alternatively the kilometre armlength interferometers would have enhanced sensitivity. The uncertainity principle sets the fundamental limit which is almost close to the limits envisaged [67, 68, 69].

Vijay Chickarmane discussed the possibility of enhancing the sensitivity of a dual recycled interferometer (with Fabry-Perot cavities in the arms), using the squeezed state technique [70]. They calculated the sensitivity for both the broad and narrow band modes of operation of the interferometer. The result was that in the broad band case the squeezing was not so useful, however, in the narrow band case squeezed light could be used to enhance the sensitivity. For that case of 60% squeezing, the gain in sensitivity was by a factor 2.5.

David McClelland also discussed the dual recycling and reported on 'bench top' recycling experiments underway at the Australian National University. He pointed out the importance of the dual recycling for mid-baseline gravitational wave detectors like GEO600, TAMA300 and AIGO400 since with the combination of innovative suspension systems and optical materials they would be able to achieve sensitivities in excess of first stage LIGO and VIRGO over a bandwidth of a few hundred Hertz.

Gabriella Gonzalez discussed the motivation and current status of the MIT phase noise experiment. Above 200 Hz the sensitivity is determined by how well the phase difference between the two beams can be determined at the detector. Given the limit of photon shot noise LIGO would require 70 Watts of laser light incident on the beam splitter to reach the required sensitivity. To study noise at high power levels a team at MIT was putting together a recycled Michelson interferometer. The system was a asymmetric single bounce Michelson interferometer with recycling gain 100 incorporating on a small scale the LIGO suspension design and active and passive LIGO seismic isolation.

8.3. Data Analysis

R. Balasubramanian discussed strategies for analyzing data from coalescing binaries and presented results of Monte Carlo estimation of the parameters of the binary system [52, 53]. He presented a formalism using differential geometric techniques which generalises the problem of choosing an optimal set of filters to detect the chirp waveform. He pointed out the need for finding sets of convenient parameters and he showed that even after the inclusion of 2PN corrections, the waveform could essentially be detected by using a one-dimensional lattice of templates. He also presented the results of a Monte Carlo simulation using the above formalism. The results of the simulations have shown that the covariance matrix underestimates the actual errors in the estimation of parameters, even when the signal to noise ratio is as high as 20. He concluded that since the detection of events with high signal to noise ratio would be very rare the covariance matrix was inadequate to describe the errors in the measurement of the parameters of the waveform.

Piotr Jaranowski presented his work [46, 47] on the estimation of parameters of the gravitational-wave signal from a coalescing binary by a network of laser interferometers. The solution of the inverse problem (using maximum likelihood and least squares methods) for the network of 3 detectors was generalized to the network of N detectors. This enabled, from measurements at individual detectors of the network, optimal estimation of the astrophysically interesting parameters of the binary system: its distance from Earth, its position in the sky, and the chirp mass of the system. The accuracy of the estimation of the parameters was assessed from the inverse of the Fisher information matrix. Extensive Monte Carlo simulations were performed to assess the accuracy of the estimation of the astrophysical parameters by networks of 3 and 4 detectors. He reported that the addition of the fourth node to LIGO/VIRGO network in Australia increases the number of detectable events roughly by two times and accuracy of position determination by 3-4 times.

S.D. Mohanty presented an alternative method for analysing data from coalescing binaries, which is a *modified periodogram* [71] and discussed the scaling laws for chirps. In this approach all the features of the standard matched filtering appear nevertheless it was a less time consuming method which had the advantage of being statistically independent for the same sample of noisy data.

Kanti Jotania talked about the analysis of gravitational waves from pulsars [72]. He mentioned various obstacles related to this detection since the signals are expected to be very weak and thus the observation times should be long (a few months). This created additional problems since a monochromatic signal became both frequency and amplitude modulated due to the rotation and the orbital motion of the earth. The effect of these two modulations was to smear out the monochromatic signal into a small bandwidth about the signal frequency of the wave. He showed the results of his study on the Fourier transform of the pulsar signal taking into account the rotation of the Earth for one day observational period. Finally, he showed an analytic form of the Fourier transform considering the rotation of the Earth including the orbital corrections.

Biplab Bhawal discussed the coincidence detection of broadband signals by the planned interferometric gravitational wave detectors [73]. He had taken into account the six planned detectors (2 LIGOs, VIRGO, GEO600, AIGO400, TAMA300) and performed coincidence experiments for the detection of broadband signals coming either from coalescing compact binaries or burst sources. He showed results on the comparisons of the achievable sensitivities of these detectors under different optimal configurations and found that a meaningful coincidence experiment could only be performed by a network where the LIGOs and VIRGO are operated in power recycling mode and other medium scale detectors (GEO, AIGO, TAMA) are operated in dual recycling mode with a narrower bandwidth. Finally, for effectively optimizing the values for different possible networks he calculated the time-delay window sizes. The effect of filtering on calculation of thresholds and the volume of sky covered by the networks were also obtained.

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