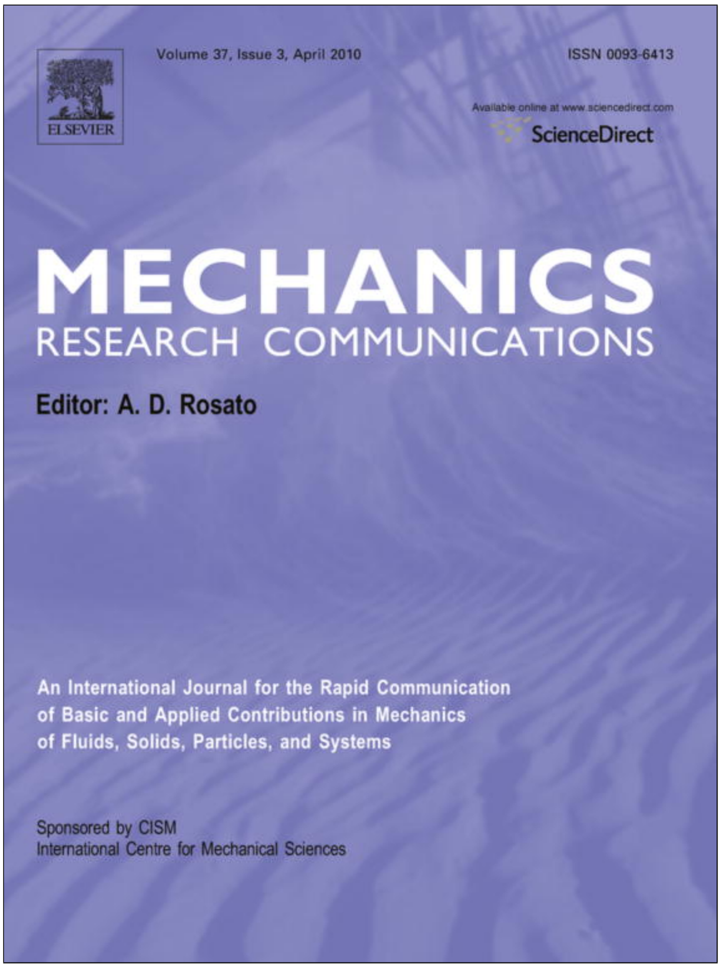
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Comment on the paper "Chaotic orbits in a 3D galactic dynamical model with a double nucleus" by N.D. Caranicolas and E.E. Zotos

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ABSTRACT

A questionable approximation in the calculation of the period of revolution of a pair of extended bodies, appearing in the paper mentioned in the title, is pointed out. An exact method to perform this task is suggested.

In issue 8/volume 36 (December 2009) of Mechanics Research Communications, Caranicolas & Zotos (2009) published a paper discussing the motion of stars in a galaxy, which consists of two nuclei and a disk. The galaxy is modeled, by the authors, as two bodies revolving about their center of mass in circular orbits at a constant angular velocity. Body 1 consists of the disk and one nucleus and body 2 consists of the other nucleus. The gravitational potential of each body is given in Eqs. (1) and (2) of the paper and the angular frequencies of each body are given in Eqs. (8) and (9). For reasons of completeness of the present comment, we reproduce here the above equations, keeping the numbering of the paper. The potential of the first body is

$$V_1(x,y,z) = -\frac{M_d}{\left[x^2+y^2+(a+\sqrt{h^2+z^2})^2\right]^{1/2}} - \frac{M_{nl}}{(x^2+y^2+z^2+c_{nl}^2)^{1/2}} \tag{1}$$

and the potential of the second

$$V_2(x,y,z) = -\frac{M_{n2}}{(x^2+y^2+z^2+c_{n2}^2)^{1/2}} \tag{2}$$

The frequencies of revolution of the two bodies are

$$\Omega_{1p} = \sqrt{\frac{1}{x_1}\left(\frac{-dV_2(r)}{dr}\right)_{r=R}}, \quad \Omega_{2p} = \sqrt{\frac{1}{x_2}\left(\frac{-dV_1(r)}{dr}\right)_{r=R}} \tag{9}$$

where

$$x_1 = \frac{M_{n2}}{M_t}R, \quad x_2 = R - \frac{M_{n1}}{M_t}R \tag{8}$$

and where $M_t = M_d + M_{n1} + M_{n2}$, while R is the distance between the centers of the two bodies.

Discussing Eq. (9), the authors conclude that "as the two bodies are not mass points, the two angular frequencies are not equal" and they make them equal "by choosing properly the parameters a, h, c_{n1}, c_{n2} of the system". But if the angular frequencies of the two bodies are, in general, different, the method used by Caranicolas & Zotos cannot be used. If we define the center of mass of the two bodies in the usual way, then it does not, in general, lie at the origin of the coordinate system, which is the center of revolution. If we assume that the center of mass is at the origin of the coordinate system, then the corresponding frame of reference is not inertial. Another way to understand that the method, followed by the authors, to calculate the frequency of revolution of the system leads to a questionable result is by realizing that $\Omega_{1p} = \Omega_{2p}$ is an equation in four unknowns (a, h, c_{n1}, c_{n2}), which has, in general, a triple infinity of solutions, from which a physically meaningful choice would have to be made. Which is the "real" one, that will be "obeyed" by the system, if it was possible to make the experiment? The question is really important, since a small difference from the accurate frequency may turn a chaotic orbit to an ordered one and vice-versa, thus affecting the results of the Caranicolas & Zotos paper.

The exact approach would be to write down the equation of motion of each body in an inertial frame of reference, and then add and subtract the two equations. In this way the authors would get a "generalization" of the Kepler's third law, containing the sum of the masses of the two bodies and a function depending on the mass distribution of the bodies (e.g. see Subrahmanyam, 1983,

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Eq. (7)). To do the above calculations in detail is not an easy task, mainly due to the fact that Plummer-type potentials, as those of Eqs. (1) and (2), correspond to mass densities that vanish only at infinity. Such problems of interpenetrating bodies have been discussed in the literature (e.g. see Alladin, 1965; Subrahmanyam, 1980, 1983; Subrahmanyam and Narasimhan, 1989; Soares, 1990) for spherically symmetric galaxies (in which case a Plummer-type potential corresponds to a polytrope of index 5). The difficulty lies in the calculation of the forces entering the equations of motion, through the differentiation of the potential energy of the system (e.g. see Alladin, 1965). However here the full solution is probably a problem of academic only interest, since the model used by the authors is a highly idealized one. Because the centers of the two bodies are close to one another, one would expect pronounced tidal phenomena (e.g. excitation of spiral waves), which would destroy the symmetries of the model. Therefore, one probably would not get much useful information for the real system, even if able to find the correct orbital frequency of the two bodies.

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