THE DYNAMICAL PORTRAIT OF THE VERITAS FAMILY REGION

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ABSTRACT

In this paper we present a comprehensive analysis of the dynamics in the region of the Veritas family, aiming to describe the principal mechanisms at work and provide a more reliable estimate of the diffusion rate. Our numerical integrations confirm previous results on the chaotic motion of the family members and on the resonances involved, but also indicate that the members of the family can be classified into four groups exhibiting different dynamical behavior. Using a numerical method, based on the integration of a large number of clones for each member, we calculate the diffusion coefficients in proper eccentricity and inclination. The diffusion rate provides an estimate for the age of the family, confirming that it must be comparatively young (~ 50 Myrs).

Key words: asteroid families; resonances; diffusion.

1. INTRODUCTION

One of the dynamically most interesting and complex regions of the main asteroid belt is the region occupied by the Veritas asteroid family. The Veritas family is a small and compact one, spectroscopically distinct against the background, with no interlopers [7]. It is located in the outer part of the main belt, far enough from the strong mean motion and secular resonances, but in a region cut through by several weak mean motion resonances of different type [1,2], which give rise to a comparatively wide chaotic zone. It has been found [3] that 5 known members of the Veritas family, including the largest body in the family, asteroid (490) Veritas itself, are located inside a chaotic zone and thus exhibit a typical chaotic behavior. The Lyapunov times for these bodies are about 10,000 yrs and all bodies appear to be in the Chirikov regime [4,5]. Chaotic diffusion brings these bodies outside the family boundaries on a 100 Myrs time-scale, which can thus be interpreted as an upper bound to the age of the family [3,6]. Another member of the family is located in a nearby narrower chaotic strip, but its motion is found to be somewhat less chaotic ($T_L \simeq 30,000$ yrs).

The reconstruction of the original velocity field of the

fragments revealed that the Veritas family is highly asymmetric, the initial velocity vectors forming a "jet-like" structure on a well defined plane [7]. Therefore, in terms of the proper semimajor axis, most of the members of the family are found on the same side with respect to (490) Veritas (closer to the Sun), covering a range of some hundredths of an AU. The extent of the whole family in proper semimajor axis is much larger than the aforementioned chaotic zone, so that, apart from the chaotic bodies, there are family members with ostensibly stable motion.

In this paper we present a comprehensive analysis of the dynamics in the region of the Veritas family, with a goal to describe the principal mechanisms at work, to understand the observed behaviors and to provide a more reliable estimate of the chaotic diffusion rates. We first analysed the resonances involved, in particular the harmonics of the (5, -2, -2) three-body mean motion resonance, looking for the possible overlapping of this resonance with other high-order mean motion resonances, or of the harmonics of that resonant multiplet with each other. Next, we tried to explain the episodes of quasistable motion recognised by previous authors [3,6] in the time series of orbital elements and of the standard metrics used to define the families [8]. Finally, using a numerical method [9], based on the integration of clone orbits for each chaotic member, we calculated the diffusion coefficients in proper eccentricity and inclination, in order to better assess the diffusion rates and improve current estimates for the age of the family.

We analysed integrations covering time spans of up to 100 Myrs, performed by means of the Orbit9 integrator. The dynamical model takes into account the direct effects of the four outer major planets and uses a barycentric correction to the initial conditions to account for most of the indirect effect of the inner planets. We considered a purely gravitational model, not taking into account the non-gravitational Yarkovsky effect, because all the considered bodies are too big to be significantly affected on a 100 Myrs time scale.



Figure 1. The logarithm of the maximum LCE $(= 1/T_L)$ as a function of the osculating semi-major axis, a, in the vicinity of the Veritas family. The 12 chaotic family members are super-imposed, as explained in the text.

2. CHAOS IN THE REGION OF (490)-VERITAS

The bodies studied here (Table 1) are 12 members of the Veritas family that follow chaotic orbits ($T_L < 10^6$ yrs, Table I in [10]). The values of the proper semi-major axis, a_P , and of the Lyapunov time, T_L , for numbered objects are taken from the catalogue of synthetic proper elements "numb.syn", available at http://hamilton.dm.unipi.it/astdys. For the multi-opposition objects, a_P is taken from the catalogue of analytically computed proper elements "ufitobs.pro". The corresponding T_L values were calculated with a 3 Myr numerical integration of the variational equations, using again the Orbit9 package.

Asteroid	a_P (AU)	T_L (yrs)	Resonance
490	3.1740	10,175	5 -2 -2
3542	3.1741	9,523	5 -2 -2
22200	3.1742	8,723	5 -2 -2
1981EE ₄	3.1742	8,224	5 -2 -2
2123P-L	3.1741	10,165	5 -2 -2
2147	3.1710	138,504	81-4
2428	3.1700	270,270	33-2
16375	3.1711	111,482	81-4,21/10
1118T-3	3.1698	331,125	33-2
5594	3.1675	34,013	33-2
2934	3.1666	210,084	3 3 -2, 40/19
17626	3.1663	490,196	3 3 -2, 40/19

Table 1. Chaotic members of the Veritas family

From the values of a_P and T_L given in Table 1, it is easy to observe that the family can be split into four groups, shown in descending order of a_P . The members within each group have comparable values of T_L . This implies that the family is crossed by different resonances, which dominate the orbital behavior of bodies in different groups.

Splitting the family into four groups is further justified

from the numerical results shown in Fig. 1. A 3 Myrs numerical integration of 250 fictitious objects was performed, in order to calculate the value of T_L for a range of values of a covering the family. These particles had the same e, i, Ω, ω and M as (490) Veritas, while a was selected in the range 3.155-3.180 AU, with a step of 0.0001 AU. Fig. 1 is an artificial composition of images, since the 250 values of a are referred to the epoch of osculation, while in Table 1 we give the proper values. We can find an approximate location of the family members, with respect to the resonances, by subtracting the difference $\delta a = a_P - a(0) = 0.0061$ AU of (490) Veritas from the a_P values of each of the 12 members. In this way one can see that the first group of Table 1 (noted as "490 et al." in Fig. 1) is located inside the 5 -2 -2 three-body mean motion resonance. This is the strongest and widest resonance for $a \sim 3.17 \,\mathrm{AU}$. The two other most prominent peaks shown in Fig. 1 correspond to the 3 3 -2 (left) and the 7 -7 -2 (right) resonance, respectively. Asteroid (5594) seems to be locked in the 3 3 -2 resonance. The rest of the family members are located either to the left of the 3 3 -2 or in between the 3 3 -2 and 5 -2 -2 resonances, where a number of peaks are seen in Fig. 1 as "side-bands" of the main resonances.

3. ANALYSIS OF RESONANCES

In order to check which bodies are influenced by each of the aforementioned resonances, we performed a 300,000 yrs integration of the 12 bodies. Short periodic fluctuations were filtered out from the elements' time series, the dark band of the filter covering periods in the interval 0.2-20 yrs. For a better representation of the mean semi-major axis we filtered out all periods up to 100 yrs.

The most significant resonances that are located in the region of the Veritas family are (i) the 5 -2 -2, 3 3 -2 and 8 1 -4 three-body mean motion resonances of the Jupiter-Saturn-asteroid type, and (ii) the 21/10 and 40/19 mean motion resonances between Jupiter and the asteroid. The former ones are expected to be more significant, since their strength and width are much larger than those of simple mean motion resonances of order q > 10. The corresponding critical arguments (for the planar problem) are

$$\sigma_{p,q,r} = p\lambda_J + q\lambda_S + r\lambda + k\varpi_J + m\varpi_S + n\varpi \quad (1)$$

and

$$\sigma_{(p+q):p,k} = (p+q)\lambda_J - p\lambda - (q-k)\varpi_J - k\varpi \quad (2)$$

for three-body and simple mean motion resonances respectively. Having eliminated short periods from the elements, the fast angles used in Eqs. (1) and (2) are the *mean* mean longitudes, while the secular angles of the planets are represented by linear functions of time, given by the synthetic secular theory of [11]. For the threebody resonances we checked 15-17 harmonics of each multiplet, while for the 21/10 and 40/19 resonances we checked q + 1 = 12 and 22 harmonics, respectively. The most important resonances for each of the 12 bodies are given in Table 1.



Figure 2. The filitered semi-major axis of asteroid (22200) is shown in the top frame. The critical argument of the strongest term of the 5 -2 -2 multiplet is shown in the lower frame. Note the correlation between the librations / circulations of the argument and the respective variations of a.

All 5 bodies of the "(490) et al." group (see Fig. 1) are dominated by the 5 -2 -2 harmonics. In fact, all of the 5 -2 -2 critical arguments show the same behavior: the angle changes from libration to circulation in an erratic manner. This means that all harmonics are in fact overlapping. This results to the strong chaos ($T_L \sim 10^4$ yrs) observed in this resonance. Fig. 2 shows the evolution of the filtered semi-major axis and of the strongest 5 -2 -2 harmonic for the asteroid (22200). The correlation between the librations / circulations of the argument and the oscillations of a is obvious. Note also the existence of brief time intervals, during which a is "locked" in smaller (the same holds for larger) values of a. The behavior of the critical argument indicates that the orbit falls out of this resonance harmonic. If $\dot{\sigma} < 0$ (or > 0) the locking occurs at $a < a_P$ (or $a > a_P$). Note, however, that one should simultaneously inspect many critical arguments (which correspond to overlapped harmonics of comparable strength), in order to be able to fully understand the intermittent behavior of a.

For the asteroid (5594) all 5 -2 -2 arguments are circulating fast, as it is located far away from this resonance. On the contrary, several (but not all) terms of the 3 3 -2 multiplet show the characteristic pattern of chaos. Thus many, but not all, of the 3 3 -2 terms are overlapping, creating the wide peak of $T_L \sim 3 \cdot 10^4$ yrs shown in the left part of Fig. 1.

For the objects lying in between the 5 -2 -2 and the 3 3 -2 resonances, all terms of the 5 -2 -2 resonance circulate relatively fast, while some terms of the 3 3 -2 reso-

nance seem to be important as well. Moreover, the 8 1 -4 and 21/10 resonances prove also to be important. In particular, one of the 8 1 -4 harmonics is constantly in libration for the asteroid (2147). Several other terms of this resonance present slow and quite disturbed circulation. Also some terms of the 3 3 -2 and 21/10 resonances are slowly circulating. A similar picture holds for the asteroid (16375). One of the 8 1 -4 terms is changing from libration to circulation, while several other terms of the 3 3 -2, 8 1 -4 and 21/10 resonances present a slow and disturbed circulation. We note that (2147) and (16375) are the ones having $T_L \sim 10^5$ yrs and are most closely spaced in terms of a_P (in Fig. 1 they are the ones closer to the left side of the 5 -2 -2 resonance). For (2428) the most important term belongs to the 3 3 -2 multiplet. We note that this term is circulating fast for (5594), i.e. it does not belong to the ones comprising the strong peak seen in Fig. 1. The asteroid is most of the time out of the libration zone and only brief crossings of the separatrix occur. During most of these periods the asteroid undergoes a single libration. The same behavior is also found for 1118T-3.

The two remaining objects shown in Table I are lying to the left of the 3 3 -2 resonance. None of the 3 3 -2 arguments shows a chaotic behavior, but some of them are slowly circulating with $\dot{\sigma} < 0$. The small T_L values probably come from the action of the 40/19 mean motion resonance, something which is more evident for the asteroid (17626) than for (2934).

4. CALCULATION OF THE DIFFUSION RATE

In the region of the 5 -2 -2 resonance all harmonics seem to overlap, creating a connected stochastic zone. In this regime one can assume that asteroids, initially placed in this region, exhibit a random walk in the space of proper elements and the variations of e_P and i_P are governed by a diffusion equation of the Fokker-Planck type [12]. Thus, the mean squared displacement of an action-like element grows linearly with time, at a rate given by the value of the *diffusion coefficient*

$$D(I) = \frac{\langle (\Delta I)^2 \rangle}{2\,\delta t} \tag{3}$$

where I stands for any action-like element. In order to calculate the coefficients in e_P and i_P , we integrated the orbits of 100 clones for each object in the 5 -2 -2 region. For each orbit we obtained the time series for the proper elements, with the aid of the analytical theory of [13]. The quantity $\langle (\Delta I)^2 \rangle$ is calculated as a function of time, where $\langle \cdot \rangle$ denotes the ensemble average over all objects. The value of D(I) is then taken as the slope of a least-squares linear fit.

Although the variance of a_P initially grows with time, it saturates relatively fast, reaching a value dictated by the width of the respective resonance. Thus, the semi-major axis presents no macroscopic diffusion within our model. On the other hand, the mean squared displacement in e_P and $\sin(i_P)$ grows almost linearly with time, as shown in Fig. 3.



Figure 3. The mean squared diplacement in e_P (top) and $\sin i_P$ (bottom) as functions of time. Two curves are shown in each panel, corresponding to the 5 -2 -2 and the 3 3 -2 groups, as indicated on the plot. The slope of the least squares linear fit yields the value of the diffusion coefficient.

The calculations for the 5 -2 -2 group were performed (i) for each body seperately, using the ensemble of its 100 clones and (ii) for all bodies together (the graph of Fig. 3). In all cases the graphs as well as the values of the coefficients were almost the same. This result simply confirms the fact that the bodies in the 5 -2 -2 resonance evolve as one group, despite their small differences in e_P and i_P . The values of the coefficients are $D(e_P) =$ $0.38 \cdot 10^{-11} \text{ yr}^{-1}$ and $D(\sin i_P) = 0.76 \cdot 10^{-12} \text{ yr}^{-1}$. Note that the results are clearly different for the group of (5594) clones (3 3 -2 resonance). There is almost no diffusion, although the degree of stochasticity (i.e. the value of T_L) of the trajectories is of the same order of magnitude as for the 5 -2 -2 group.

Assuming that the diffusion coefficient is almost constant, for a range in proper eccentricity equal to the observed spreading of the 5 -2 -2 members ($\delta e_P = 7.2 \cdot 10^{-3}$), the value of $D(e_P)$ implies that the age of the family is of the order of $\delta t \sim 2 (\delta e_P)^2 / D \sim 27$ Myrs. A similar value is obtained using our results for the proper inclination ($\delta \sin i_P = 2.5 \cdot 10^{-3}$, $\delta t \sim 16$ Myrs). Although these values for δt do not constitute accurate estimates for the age of the family, they agree with previous results and confirm the fact that the Veritas family should be quite young.

5. CONCLUSIONS

The dynamical behavior of the chaotic members of the Veritas family can be explained, primarily accounting for the action of three-body resonances. The splitting of the family into four sub-groups is justified by our results. No indication of overlapping between the main three-body resonances is found. Thus, the chaotic phase-space region that the family members span does not seem to be connected. This result is also certified by the different vaues of $D(e_p)$ and $D(i_P)$ between the 5 -2 -2 group and the ensemble of (5594) clones, which practically do not diffuse. The values of the diffusion coefficients of the 5 -2 -2 group indicate a very short age for this family, in agreement with previous results.

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