THE ASTROPHYSICAL JOURNAL, 233:717–726, 1979 October 15

COLLECTIVE PLASMA EFFECTS ASSOCIATED WITH THE CONTINUOUS INJECTION MODEL OF SOLAR FLARE PARTICLE STREAMS

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Received 1978 August 3; accepted 1979 April 19

ABSTRACT

A modified continuous injection model for impulsive solar flares that includes self-consistently plasma nonlinearities based on the concept of marginal stability is presented. A quasi-stationary state is established, composed of a hot truncated electron Maxwellian distribution confined by acoustic turbulence on the top of the loop and energetic electron beams precipitating in the chromosphere. It is shown that the radiation properties of the model are in accordance with observations.

Subject headings: hydromagnetics — plasmas — Sun: corona — Sun: flares

I. INTRODUCTION

Understanding and modeling nonthermal impulsive phenomena associated with solar flares is among the outstanding problems of solar physics (Svetska 1976). While several models have been proposed, most of them never progressed past qualitative arguments with very little consideration of the self-consistency requirements imposed by plasma physics. It is the purpose of this paper to make a reexamination of the "continuous injection" model suggested first by de Jager and Kundu (1963) and updated by Kane (1974) and to demonstrate that inclusion of proper plasma physics considerations can result in the resolution of several of the mysteries and controversies associated with the modeling of impulsive hard X-ray and microwave bursts. This work should only be considered as a first step toward a comprehensive numerical modeling. The plan of the paper is as follows: In § II we reexamine the continuous injection model including a self-consistent presentation of the relevant plasma physics involved. In § III we discuss the observational consequences and the energetics of the model. The final section summarizes the work and discusses its limitations and possible extensions.

II. SELF-CONSISTENT "CONTINUOUS INJECTION MODEL"

a) *Region I* (*Energy Release*)

A schematic of the model is shown in Figure 1*a*. In a small volume I (Fig. 1*a*) at the top of a loop the plasma electrons, which for the sake of convenience we assume to be Maxwellian, are heated impulsively up to a temperature T_e^{I} by a mechanism that preferentially deposits its energy on the electrons. In a fast time scale (i.e., $t \approx L_{\mathrm{I}}/V_e^{\mathrm{I}} \approx 10^8/8 \times 10^9 \approx 1/80$ s, where L_{I} is the length of region I and V_e^{I} is the electron thermal velocity), electrons will flow out of region I and the requirements for charge neutrality between regions I and II will force a return current. For plasma with $T_e^{\mathrm{I}}/T_i^{\mathrm{I}} \gg 1$ (T_i^{I} is the ion temperature), when the speed V_D of the return current reaches the value $C_s = (T_e^{\mathrm{I}}/M)^{1/2}$, where M is the ion mass, ion acoustic waves can be excited, creating turbulence and reducing particle losses from region I due to local enhancement in the collision frequency (Brown, Melrose, and Spicer 1979; Smith and Lilliequist 1978). Since the time scale for growth and saturation of the ion sound is of the order of $\omega_i^{-1} \approx 10^{-8}$ s, this process can be considered instantaneous with respect to the response time of the ions to the charge imbalance [i.e., $t = (M/m)^{1/2} L_{\mathrm{I}}/V_e^{\mathrm{I}} \approx 0.5$ s, where m is the electron mass]. Therefore, the system will reach a steady state involving a return current in a time faster than 1 s. This allows us to construct a model based on *marginal nonlinear* stability for ion sound waves (Papadopoulos 1977; Manheimer 1977).

The collision frequency ν^* at marginal stability can be computed (Papadopoulos 1977) from the so-called Sagdeev formula

$$\nu^* = 10^{-2}(T_e/T_i)(V_D/V_e)\omega_e\,, \tag{1}$$

which, in conjunction with the marginal stability criterion

$$V_D/V_e \geq T_i/T_e\,, \tag{2}$$

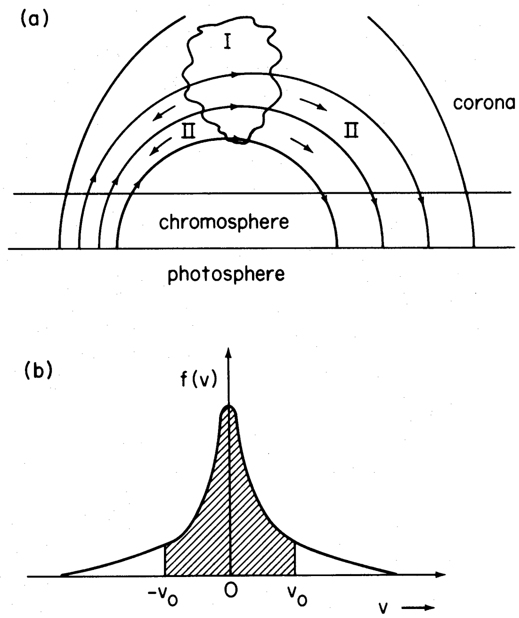

FIG. 1.—(*a*) Magnetic loop structure above the active regions. In region I, electrons are heated and accelerated. In region II, beams with velocity $v > 3V_e$ are formed. (*b*) Possible velocity distribution in region I. Electrons with velocity $v < v_0$ are "trapped" in region I.

gives

$$\nu^* \gtrsim 10^{-2}\omega_e , \tag{3}$$

where ω_e is the ambient plasma frequency. At this stage it should be noted that the collision frequency given by equation (3) is valid only for electrons with velocity $v \lesssim V_e^{\mathrm{I}}$. When $v > V_e^{\mathrm{I}}$, as shown in Papadopoulos (1977),

$$\nu^*(v) \approx \nu^*(v/V_e^{\mathrm{I}})^{-3} , \tag{4}$$

i.e., similar velocity dependence to the Coulomb collisions. Based on the above steady state, a self-consistent model can be constructed for the loss of energetic particles during the time scale of the impulsive phase of the burst τ_0. The time τ_0 can also be defined as the duration of the energy input in region I. The energetic particle current $n_b e V_b$ of our region I is balanced by the return current J_{cold} of cold electrons drawn from region II. The velocity v_0, below which particles are confined for time τ_0 inside the region I, can be found by equating the diffusion length $L_D(v)$ as a function of velocity, to L_{I}. We find that

$$L_D(v_0) = v_0[\tau_0(\nu^*)^{-1}(v_0/V_e^{\mathrm{I}})^3]^{1/2} = L_{\mathrm{I}} . \tag{5}$$

Equation (5) gives

$$v_0/V_e^{\mathrm{I}} = (L_{\mathrm{I}}/\lambda_D)^{2/5}(\omega_e\tau_0)^{-1/5}(\nu^*/\omega_e)^{1/5} . \tag{6a}$$

For $L_{\mathrm{I}} \approx 10^8$ cm, $\lambda_D \approx 1$ cm, $\omega_e \approx 6 \times 10^9\ \mathrm{s}^{-1}$, $\tau_0 \approx 100$ s, and ν^*/ω_e given by equation (3), we find

$$v_0 \approx 2.8V_e^{\mathrm{I}} . \tag{6b}$$

The number of particles escaping from region I can be found by integrating over the Maxwellian, i.e.,

$$n_b \approx \frac{1}{2}\frac{n}{(2\pi)^{1/2}V_e}\int_{v_0}^{\infty} dv \exp\left[-\frac{1}{2}\left(\frac{v}{V_e^{\mathrm{I}}}\right)^2\right] \approx \frac{1}{2}n \exp\left[-\frac{1}{2}\left(\frac{v_0}{V_e^{\mathrm{I}}}\right)^2\right] \approx 10^{-2}n \tag{7}$$

for $v_0 \approx 2.8V_e^{\mathrm{I}}$, where n is the electron density taken as $10^{10}\ \mathrm{cm}^{-3}$. The energy flux per unit volume will be

$$n_b V_b E_b \frac{\pi R^2}{\pi R^2 L_{\mathrm{I}}} \approx n_b V_b E_b / L_{\mathrm{I}}$$

($V_b \approx 3V_e^{\mathrm{I}}$, $E_b \approx \frac{1}{2}mV_b^2$, and R is the radius of the loop). The important conclusions derived from the above discussion are that while the main body of the Maxwellian remains trapped in region I (Fig. 1*b*), the tails ($v > v_0$) can flow out of the region toward the chromosphere in the form of an electron beam propagating in region II.

b) *Energy Balance in Region I*

The energy balance equation in region I can be written as

$$\frac{\partial T^{\mathrm{I}}}{\partial t} = G_{\mathrm{fl}} + G_{\mathrm{ret}} - L_{\mathrm{exp}} - L_{\mathrm{ep}} - L_R - L_{\mathrm{wave}} - L_{\mathrm{cond}} \tag{8a}$$

where G_{fl} is the energy input from the energy release mechanism (e.g., magnetic reconnection), G_{ret} is the energy input from the return current heating, L_{exp} are the expansion losses, L_{ep} are the energetic particle losses, L_R are the radiation losses, L_{wave} are the ion acoustic wave losses, and L_{cond} are the heat conduction losses in the boundary of region I. For steady state,

$$G_{\mathrm{fl}} + G_{\mathrm{ret}} = L_{\mathrm{exp}}\left(1 + \frac{L_{\mathrm{ep}}}{L_{\mathrm{exp}}} + \frac{L_R}{L_{\mathrm{exp}}} + \frac{L_{\mathrm{W}}}{L_{\mathrm{exp}}} + \frac{L_{\mathrm{cond}}}{L_{\mathrm{exp}}}\right). \tag{8b}$$

It can easily be shown that

$$L_R = 2 \times 10^{24} n_{10}{}^2 L_8 R_8{}^2 [T_e{}^{\mathrm{I}}(\mathrm{eV})]^{-1}, \tag{8c}$$

$$L_{\mathrm{W}} = 5 \times 10^{20} n_{10} R_8{}^2 \left(\frac{W_s}{nT_e}\right)_{-2} [T_e{}^{\mathrm{I}}(\mathrm{eV})]^{3/2}, \tag{8d}$$

and

$$L_{\mathrm{cond}} = 3.2 \times 10^{18} \left(\frac{W}{nT_e}\right)_{-2}^{-1} n_{10} R_8{}^2 [T_e{}^{\mathrm{I}}(\mathrm{eV})]^{3/2}. \tag{8e}$$

As we will show below, L_{exp} is much larger than L_R, L_{W}, and L_{cond} for all values of $T_e{}^{\mathrm{I}} \gtrsim 100$ eV. Therefore, keeping only the dominant terms, equation (8b) becomes

$$G_{\mathrm{fl}} + G_{\mathrm{ret}} \approx L_{\mathrm{exp}} + L_{\mathrm{ep}}. \tag{8f}$$

i) The value of G_{ret} is given by

$$G_{\mathrm{ret}} \approx \eta_{\mathrm{an}} j_{\mathrm{cold}}{}^2 2\pi R^2 \Delta L, \tag{9}$$

where η_{an} is the anomalous resistivity $\eta_{\mathrm{an}} \approx 4\pi\nu/\omega_e{}^2$ or $\eta_{\mathrm{an}} \approx (W_s/nT_e)\omega_e{}^{-1}$, W_s/nT_e is the nonlinear saturation level of the ion acoustic waves, and ΔL is the thickness of the conduction front. Estimating ΔL by equating the conduction time with the heating time of the thermal electrons (Brown, Melrose, and Spicer 1979), we find

$$\Delta L \simeq 5 \times 10^{-1} n_{10}{}^{-1/2} (W_s/nT_e)^{-1} [T_e{}^{\mathrm{I}}(\mathrm{eV})]^{1/2} \ \mathrm{cm}, \tag{10}$$

where $x_n = x/10^n$. Therefore, from equations (9) and (10) we find that

$$G_{\mathrm{ret}} \approx 1.6 \times 10^{21} n_{10}{}^{3/2} R_8{}^2 [T_e{}^{\mathrm{I}}(\mathrm{eV})]^{3/2} \ \mathrm{ergs\ s^{-1}}. \tag{11}$$

ii) L_{ep} is given by

$$L_{\mathrm{ep}} \approx 2[n_b E_b (V_b/L_{\mathrm{I}})]\pi R^2 L_{\mathrm{I}} = 3 \times 10^{21} d_2{}^{-1} R_8{}^2 n_{10} [T_e{}^{\mathrm{I}}(\mathrm{eV})]^{3/2} \ \mathrm{ergs\ s^{-1}}, \tag{12}$$

where $d = n/n_b$.

iii) The expansion losses are given by

$$L_{\mathrm{exp}} \approx 3nT_e{}^{\mathrm{I}} V_F \pi R^2 \approx 7 \times 10^{22} \alpha n_{10} R_8{}^2 [T_e{}^{\mathrm{I}}(\mathrm{eV})]^{3/2} \ \mathrm{ergs\ s^{-1}}, \tag{13}$$

where V_F is the expansion velocity $\sim \alpha C_s$. From equations (8), (11), (12), and (13) we find that the steady-state temperature

$$T_{\mathrm{max}}{}^{\mathrm{I}} \approx (7\alpha \times 10^{22} + 3d_2{}^{-1} 10^{21} - 1.6 \times 10^{21})^{-2/3} G_{\mathrm{fl}}{}^{2/3} n_{10}{}^{-2/3} R_8{}^{-1/3} \ \mathrm{eV}, \tag{14}$$

where α and d are defined as follows: d is given from equation (7) and depends on the velocity distribution function in region I, and α is of the order of or larger than one (Brown, Melrose, and Spicer 1979). Exact values of α can be found only from a numerical simulation similar to the one already performed by Smith and Lilliequist (1978).

c) *Region II* (*Beam Propagation*)

The propagation of the beam through region II and the preservation of its positive slope down to the lower corona has been a major cause for concern in the astrophysical literature (Lifshitz and Tomozov 1974; Smith

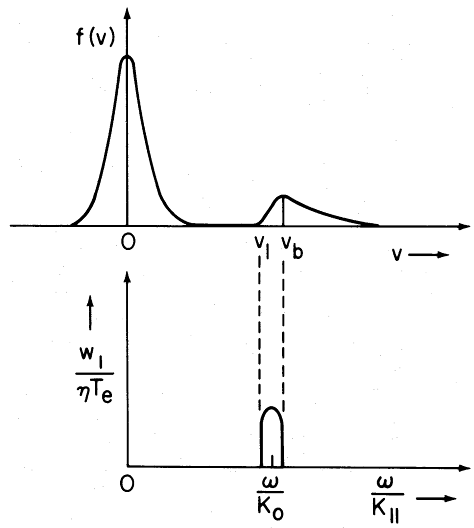

FIG. 2.—Velocity distribution in region II and linearly excited Langmuir wave spectrum

1975). The reason is that the presence of a beam with a positive slope (Fig. 2) excites Langmuir waves propagating parallel to the magnetic field with phase velocity between $V_1 < \omega_e/k_\parallel < V_b$ and growth rate

$$\gamma_L \approx (n_b/n)(V_b/\Delta V_b)^2 \omega_e \,, \tag{15}$$

where $V_b \approx \frac{1}{3}V_b$, and $k_\parallel$ is the wave vector parallel to the magnetic field. We review briefly below the mechanisms involved in the nonlinear evolution of the instability and their consequences. The evolution of the beam entering in region II from region I is given by Vedenov and Ryutov (1976)

$$V_{\text{gr}}(\partial W_1/\partial S) = 2\gamma_L W_1 + \text{nonlinear terms} \,, \tag{16a}$$

$$V_b \frac{\partial f}{\partial S} = \frac{\partial}{\partial v} D_1 \frac{\partial f}{\partial v} \,, \tag{16b}$$

where S is the distance from region I along the magnetic field, V_{gr} is the group velocity, D_1 is the velocity diffusion coefficient, and W_1 is the wave energy in resonance with the beam.

If the wave energy level at saturation allows neglect of the nonlinear terms, the beam will evolve into a quasi-linear plateau at a distance of a few Debye lengths (λ_D) from the injection point [i.e., $(n/n_b)(V_e/V_b)\lambda_D$, where V_e is the thermal velocity in region II]. The associated level of turbulence is given by Vedenov and Ryutov (1976)

$$\frac{W_m}{nT_e} \approx \frac{1}{15}\left(\frac{n_b}{n}\right)\left(\frac{V_b}{V_e}\right)^4 , \tag{17}$$

where T_e is the electron temperature in region II taken as 1 keV. This situation has caused concern in the flare modeling community, since it implies that the beam reaches the chromosphere with a plateau as well as losing most of its energy—a fact seemingly inconsistent with observations. In order to avoid plateau formation, the instability should saturate at a level much lower than the one given by equation (17). This led to an examination of nonlinear process that can influence the beam relaxation length.

Any nonlinear wave-wave stabilization mechanism operates on the principle that at some level W_1 of wave energy in resonance with the beam the energy is transferred outside the resonance region ($V_1 < \omega_k/k_\parallel < V_b$, Fig. 2) at a rate $\gamma_{\text{NL}}(W_1)$ faster than γ_L. Therefore, if we denote by W_2 the wave energy out of the resonance region, the set of equations describing the system evolution will be given by (Papadopoulos and Coffey 1974; Papadopoulos, Goldstein, and Smith 1974; Papadopoulos 1975)

$$\partial W_1/\partial t = 2\gamma_L W_1 - 2\gamma_{\text{NL}}(W_1)W_2 \,, \tag{18a}$$

$$\partial W_2/\partial t = 2\gamma_{\text{NL}}(W_1)W_2 - \nu_L W_2 \,, \tag{19a}$$

where $\gamma_{\text{NL}}(W_1)$ is the appropriate nonlinear rate and ν_L the damping rate of the waves W_2.

Lifshitz and Tomozov (1974) considered the possibility that $\gamma_{\rm NL}(W_1)$ is given by induced scattering of thermal ions. In this case (Tsytovich and Shapiro 1965)

$$\gamma_{\rm NL}(W_1) = \frac{1}{30}\left(\frac{V_b}{V_e}\right)\frac{W_1}{nT_e}\left(\frac{m}{M}\right)\omega_e\,. \tag{20}$$

This mechanism is valid only within the context of weak turbulence theory $[W_1/nT_e < (k_0\lambda_{\rm D})^2 \approx (V_e/V_b)^2]$, and the direction of energy transfer is toward longer wavelengths ($k \to 0$). The approximate level at which saturation occurs will be given by $\gamma_L \approx \gamma_{\rm NL}$. From equations (15) and (20), we find

$$\frac{W_1}{nT_e} \approx 30\left(\frac{n_b}{n}\right)\left(\frac{M}{m}\right)\left(\frac{V_e}{V_b}\right)\left(\frac{V_b}{\Delta V_b}\right)^2, \tag{21a}$$

subject to the conditions of weak turbulence theory, i.e., $W_1/nT_e \ll (V_e/V_b)^2$. From equation (21a) we find that, for induced ion scattering stability,

$$\frac{n_b}{n} \ll \frac{1}{30}\left(\frac{V_e}{V_b}\right)\left(\frac{m}{M}\right)\left(\frac{\Delta V_b}{V_b}\right)^2, \tag{21b}$$

which renders this mechanism inapplicable to all but trivial situations.

The resolution of the puzzle came when it was noted (Papadopoulos 1973) that for turbulence levels such that $W_1/nT_e > (k_0\lambda_{\rm D})^2$, the dispersive structure of the Langmuir waves is destroyed. This can be seen by considering the dispersion relation $\omega_{ek} = \omega_e[1 + (3/2)(k\lambda_{\rm D})^2]$ in the presence of the ponderomotive force of the beam excited waves. Using the pressure equation we find

$$\delta nT + \frac{1}{2}\frac{E^2}{8\pi} = 0 \qquad \text{or} \qquad \frac{\delta n}{n} \approx -\frac{1}{2}\frac{W_1}{nT_e}\,.$$

Therefore, the nonlinear frequency of the Langmuir waves is given by

$$\omega_{ek} = \omega_{e0}\left(1 + \frac{3}{2}(k\lambda_{\rm D})^2 + \frac{1}{2}\frac{\delta n}{n}\right) = \omega_{e0}\left(1 + \frac{3}{2}(k\lambda_{\rm D})^2 - \frac{1}{4}\frac{W_1}{nT_e}\right)\,. \tag{22}$$

The first term in equation (22) is due to the electron inertia, the second represents the kinetic energy of the oscillations, while the third is equivalent to the potential energy. When $W_1/nT_e \gg k_0{}^2\lambda_{\rm D}{}^2$, the potential energy term dominates and leads to collapse (i.e., similar to the gravitational one) and creation of shorter wavelengths. This results in a transfer of the oscillations to lower phase velocities. This process is equivalent to the oscillating two-stream instability with finite wavelength pump (Manheimer and Papadopoulos 1975). The details of these can be found in Papadopoulos (1975) and confirmation by particle computer simulations in Rowland and Papadopoulos (1977). Within the context of equations (18a) and (19a) the above processes can be described by using for $\gamma_{\rm NL}(W_1)$ the value

$$\gamma_{\rm NL} = (m/M)^{1/2}(W_1/nT_e)^{1/2}\omega_e\,. \tag{23a}$$

As mentioned above, the beam will be stabilized only if $\gamma_{\rm NL} > \gamma_L$, or, requiring $W_1/nT_e \lesssim W_m/nT_e$, we find

$$\frac{n_b}{n} < \frac{1}{15}\left(\frac{m}{M}\right)\left(\frac{\Delta V_b}{V_b}\right)^4\left(\frac{V_b}{V_e}\right)^4 \tag{23b}$$

for which $\Delta V_b/V_b \sim \frac{1}{3}$ and $V_b \gtrsim 10V_e$ will give $n_b/n \leq 10^{-2}$.

It should be mentioned that the initial criticism of the theory as applied to type III bursts (Kaplan, Pikel'ner, and Tsytovich 1974; Smith 1974) was due to the fact that the above authors did not appreciate the extensive region of validity of the theory and it was later rescinded (Tsytovich, Stenflo, and Wilhelmsson 1975; Smith 1977).

In order to calculate the asymptotic turbulence level as well as the nonlinear beam relaxation, one should consider in addition to the above equations the fact that the above processes emit sound waves even in an isothermal plasma, which can nonlinearly stabilize the instability due to effects similar to Dawson-Oberman (1962, 1963) high-frequency resistivity, as well as the fact that the waves W_2 are forming high-energy tails, which can damp them. Defining

$$\frac{W_s}{nT_e} = \frac{E^2}{8\pi nT_e}\frac{\partial}{\partial\omega}(\epsilon\omega) \approx (\delta n/n)^2\,,$$

we have, in addition to equations (18a) and (19a),

$$\partial W_s/\partial t = 2\gamma_{\rm NL}W_s - \nu_1 W_s\,, \tag{24}$$

where ν_1 is the damping of the ion density fluctuations.

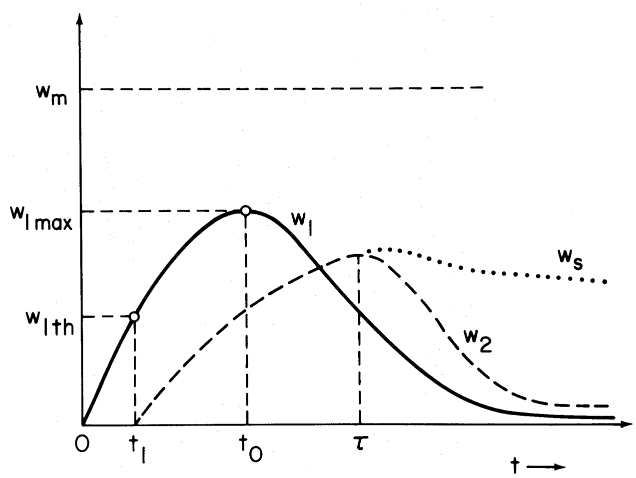

FIG. 3.—Schematic evolution of the wave spectrum; W_1 is the wave energy of the Langmuir waves in resonance with the beam, W_2 is the wave energy of the waves scattered toward lower phase velocities, W_s is the wave energy of the ion density fluctuations, and W_m is the quasi-linear saturation level given by eq. (17).

The evolution of the wave spectrum can be described qualitatively as follows: waves in resonance with the beam (W_1) are initially growing exponentially with growth rate γ_L. As soon as W_1 becomes greater than the collapse threshold $[W_{1_{\rm th}} > (k_0\gamma_{\rm D})^2]$, the waves with energy W_2 and W_s and wave vector k start growing with a rate $\gamma_{\rm NL}$ (equations [19a] and [24]) at the expense of W_1. If $\gamma_{\rm NL} > \gamma_L$, the wave energy in resonance with the beam will reach a maximum value $W_{1\max}$ at time t_0 as shown in Figure 3 (Papadopoulos 1975). Ion density fluctuations play an important role in the subsequent stabilization of the beam: They will soon reach a nonthermal level and will scatter (Dawson and Oberman 1962, 1963) the growing Langmuir waves to shorter wavelengths with a rate

$$\alpha_{\rm NL} \approx \frac{(\delta n/n)^2}{(k\lambda_{\rm D})^2}\,\omega_e = \frac{W_s}{k^2}\,\omega_e\,.$$

For continuous injection equations (18a) and (19a) are replaced by

$$\partial W_1/\partial t = 2\gamma_L W_1 - \alpha_{\rm NL} W_1\,, \tag{18b}$$

$$\partial W_2/\partial t = \gamma_{\rm NL} W_1 - \nu_L W_2\,. \tag{19b}$$

The set of equations for (24), (18b), and (19b) can be solved for the steady state if ν_1 and ν_L are known. The value of ν_1 is nothing else but the sound absorption of electrons,

$$\nu_1 = \omega_e (m/M)^{1/2} k\lambda_{\rm D}\,. \tag{25}$$

The value of ν_L has to be found self-consistently, since it is due to the tails created by the waves W_2 (Fig. 4). Following Papadopoulos (1975) or Hammer and Papadopoulos (1976), but with the dominant loss mechanism being convection with a time $t_c = L_{\rm II}/v$, where $L_{\rm II}$ is the length of region II, we find that for steady state the slope of the tails is

$$\frac{\partial F_T}{\partial v} = \frac{1}{1 + D_2 t_c/v^2}\,\frac{\partial f_M}{\partial v}\,,$$

where $\partial f_M/\partial v$ is the slope of a Maxwellian, and $D_2 = \frac{1}{4}(W_2/nT_e)V_e^2\omega_e$. Therefore, if the number of particles in the tail is $n_T \approx 10^{-3}n$,

$$\nu_L = 4\,\frac{n_T}{n}\,\frac{v^3}{V_e^2\omega_e L_{\rm II}}\,\frac{nT_e}{W_2}\,\omega_e \approx 4\times 10^{-3}\left(\frac{v}{V_e}\right)^3\frac{\lambda_{\rm D}}{L_{\rm II}}\,\frac{nT_e}{W_2}\,\omega_e\,.$$

For tails of $v \approx 3.4V_e$, $L_{\rm II}/\lambda_{\rm D} \approx 10^{10}$ we find

$$\nu_L/\omega_e \approx 10^{-11} nT_e/W_2\,. \tag{26}$$

From (18b) and (19b)

$$W_1 = \nu_L W_2/2\gamma_L \approx 10^{-11}(n/n_b)(\Delta V/V_b)^2\,. \tag{27}$$

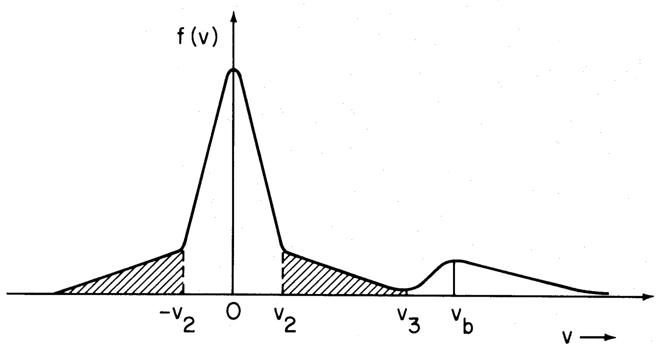

FIG. 4.—Steady-state velocity distribution in region II

The various relaxation lengths can be computed from (27). We find that the relaxation length for momentum loss is given by

$$l_1 \approx 10^{11} \frac{n_b}{n} \left(\frac{V_b}{V_e}\right)^3 \lambda_D ,$$

while the relaxation length for thermalization (i.e., plateau formation) is

$$l_2 \approx 4 \times 10^{11} \frac{n_b}{n} \left(\frac{V_b}{V_e}\right)^3 \gamma_D .$$

The above relations are valid if relationship (21b) is not satisfied. We see that l_1, l_2 are comparable, and for typical parameters (i.e., $V_b/V_e \gtrsim 10$, $n_b/n \lesssim 10^{-2}$) they exceed the distance L_{II}.

From equations (18b) and (19b) we find that a marginally stable state exists only if the ion density fluctuations are maintained undamped. Ion density fluctuations can be supported from W_1 or W_2, depending on which one is damped faster. We know that the threshold for $W_1{}^T$ is $\sim k^2$ and for $W_2{}^T$ is $\sim 4\nu_L$ (Papadopoulos 1975). We then conclude that if $k^2 > \nu_L$, which is the case in our calculations, the marginally stable values for the waves excited from the beam are $W_2{}^0 \approx 10^{-7}$, $W_1{}^0 \approx 10^{-9}$, and $W_S{}^0 \approx 10^{-5}$. The emitted microwave power at $2\omega_e$ from the plasma waves with energy $W_2{}^0 \approx 10^{-7}$ is given by (Sturrock, Ball, and Baldwin 1965)

$$S(f \approx 2f_p) = \frac{2\pi V_{\rm rad}}{D^2}\left[\frac{1}{2}\frac{3\pi^3}{a}\left(\frac{V_e}{c}\right)^5 (W_2{}^0)^2 n T_e{}^{\rm II}\right], \tag{28}$$

where D is the distance of the radiating source from the Earth, $\sim 2 \times 10^{+13}$ cm, f_p is the plasma frequency, $\sim 10^4 n^{1/2}$, $V_{\rm rad}$ is the radiating volume, $\sim 2\pi R^2 L_{II} \approx 6 \times 10^{25}$, $a \approx 1$, $T_e{}^{\rm II} \approx 1$ keV, $n \approx 10^{10}$, and $W_2{}^0 \approx 10^{-7}$, we find that $S(f \approx 2\text{ GHz}) \approx 10^{-19}$–$10^{-20}$ W m^{-2} Hz^{-1}.

d) *Energy Balance in Region II*

As it was shown in § II*c*, the beam is losing an insignificant amount of energy in high and low frequency waves. The energy balance equation for region II can be written as

$$\partial T_e{}^{\rm II}/\partial t = G_{\rm ret}' + L_{\rm cond} - L_R' - L_{\rm cond}' , \tag{29}$$

where $G_{\rm ret}'$ is anomalous ohmic heating due to the return current driven by the beam, L_R' is the radiation loss from region II, and $L_{\rm cond}'$ is the conduction loss in the transition region. Using equation (8f), we find

$$G_{\rm ret}' \approx 10^{23}[W_S{}^0]_{-5} n_{10}{}^{3/2} d_{-2}{}^2 R_8{}^2 [L_{\rm II}]_{10} T_e{}^{\rm II}(\text{eV}) , \tag{30}$$

where $W_S{}^0$ is the steady-state level of the ion density fluctuations excited by the beam as calculated in § II*c*. The other losses are given by

$$L_R' \approx 2.2 \times 10^{26} n_{10}{}^2 R_8{}^2 [L_{\rm II}]_{10} [T_e{}^{\rm II}(\text{eV})]^{-1} , \tag{31}$$

$$L_{\rm cond} \approx 3.2 \times 10^{21} [W_S{}^0]_{-5}{}^{-1} n_{10} R_8{}^2 [T_e{}^{\rm II}(\text{eV})]^{3/2} . \tag{32}$$

For $T_e{}^{\rm II} \gtrsim 100$ eV we find that $L_R/G_{\rm ret}' \lesssim 10^{-2}$. Keeping only the dominant terms in equation (20) and solving for $T_e{}^{\rm II}$, we find

$$T_{\rm max}{}^{\rm II} \approx 10^3 [W_S{}^0]_{-5}{}^2 n_{10} d_{-2}{}^2 [L_{\rm II}]_{10}{}^2 \text{ eV} . \tag{33}$$

The values for $T_{max}{}^{I}$ and $T_{max}{}^{II}$ given by equations (33) and (14) are only rough estimates of the temperatures at regions I and II at the impulsive phase of the flare.

The emerging general model then is that energetic electrons leave region I, forming an energetic beam which propagates with little loss of energy through region II to the lower corona. The main body of the Maxwellian in region I remains trapped due to the excitation of ion sound by the return current. In addition, enhanced turbulence levels will appear in region II near ω_e. In the next section we will examine the radiation properties of the system and compare them with the observations.

III. OBSERVATIONAL AND ENERGETIC REQUIREMENTS OF THE MODEL

We proceed next to examine the observational consequences of the self-consistent injection model discussed above, as well as the energy and electron density requirements. It is important to note that the only required input for the development of the model is the heating rate G_{fl} in region I while the subsequent distribution functions and turbulence levels follow in a self-consistent fashion according to the analysis given in § II. An outline of the physical processes and radiation signatures of the various regions of the flaring solar arch is shown in Figure 5.

Following the chain of events as shown in Figure 5, we assume a heating rate G_{fl} in region I, which for the sake of numerical definiteness we take as 10^{29} ergs s^{-1} (see Table 1 for more values of G_{fl}), and proceed to examine the emerging state of the plasma. Notice that we only need the heating rate and we do not specify the mechanism which produced it. Therefore, the model has a rather general validity and is independent of whether tearing, wave heating, or any other process provides the energy for the initiation of the impulsive burst. On the basis of equation (14) the emerging steady state is a truncated Maxwellian with $T_{max}{}^{I} = 10$ keV confined in region I. At the same time a substantial level of ion turbulence is induced by the return current which confines the particles. In the boundary between region I and II an electron beam with energy of about 80 keV and $(n_b/n) \approx 10^{-3}$ is streaming down toward the lower corona. This beam is nonlinearly stabilized by plasma effects as explained in § II*b*. The propagation of this beam through region II produces enhanced electron plasma waves, ion sound waves, as well as superthermal tails in the local Maxwellian. The distribution shown in Figure 4 ends up in the dense chromosphere, where it produces a substantial number of radiative effects.

The observed spectrum will therefore be composed from the following radiation sources (Fig. 5). Region I will radiate predominantly soft and hard X-ray bursts with energies up to $9T_{max}{}^{I}$. Region II radiates microwave

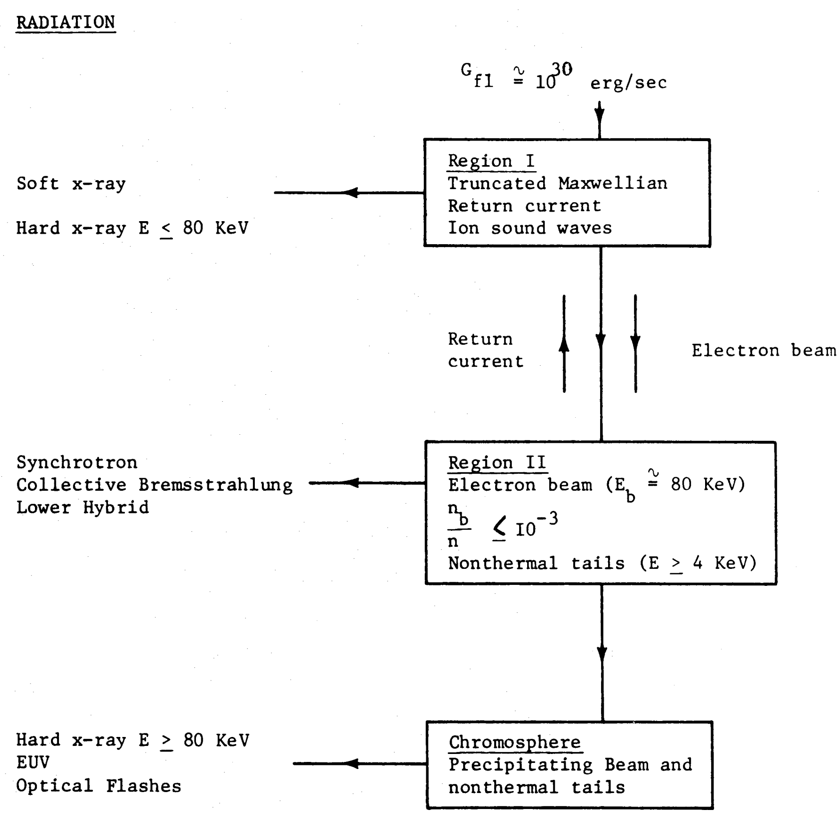

FIG. 5.—Energy flow and radiation diagram for the modified continuous injection model

TABLE 1

(τ = 100 s)

$G_{\rm fl}$ (ergs s^{-1})	$T_{\rm max}{}^{\rm I}$ (K)	N	$L_{\rm ep}$ (ergs)
10^{27}	10^7	3×10^{33}	10^{28}
10^{28}	4×10^7	6×10^{33}	7×10^{28}
10^{29}	2×10^8	10^{34}	3×10^{29}
10^{30}	10^9	3×10^{34}	10^{31}

bursts which can be due to collective bremsstrahlung at $2\omega_e$ and synchrotron radiation. Slottje (1978) has recently observed these two components of radiation. His observed frequency and intensity are close to the numbers predicted by the above model. Finally, hard X-ray bursts with energies larger than $9T_{\rm max}{}^{\rm I}$, EUV, and optical flashes will be the product of the precipitating electrons in the chromosphere. The detailed accounting of the radiation spectrum and intensities based on the above model is presently in preparation and will be published elsewhere.

In examining the energy and particle requirements of the above model and using the canonical numbers of steady-state temperature of $T_{\rm max}{}^{\rm I} \approx 10$ keV and heating rate (impulsive phase) of 10^{29} ergs s^{-1} we have the following: the total number of particles drawn from the corona to the chromosphere is given by

$$N(E > 9T_{\rm max}{}^{\rm I}) \approx 10^{34} d_2{}^{-1}(T_{\rm max}{}^{\rm I})_4{}^{1/2} R_8{}^2 n_{10} \tau_{02}\,, \tag{34}$$

and the total energy flux in the chromosphere is

$$L_{\rm ep} \approx 3 \times 10^{29} d_{-2} n_{10} R_8{}^2 [T_{\rm max}{}^{\rm I}]_4{}^{3/2} \tau_{02}\ \text{ergs}\,, \tag{35}$$

where $\tau_0 \approx 100$ s is the duration of the impulsive phase of the burst (see Table 1).

It was also noted in the nonlinear stabilization mechanism of the beam that nonthermal tails will be produced in region II with energies larger than 4 keV. These electrons can also radiate low-energy impulsive X-ray bursts (Kahler and Kreplin 1971). It can be seen from the above that both the particle and energy requirements are rather modest.

From the above and from Figure 5 it is obvious how the continuous injection model has become free from previous deficiencies.

1. By attributing (Brown, Melrose, and Spicer 1979) the total X-ray spectrum to one region I as thermal X-rays and to the chromosphere as nonthermal X-rays, we avoid the necessity of unacceptably large numbers of electrons which have to be drawn in to the lower corona (Hoyng 1977).
2. By considering the strong turbulence theory, we have demonstrated that the electron beam can stream from region I to the lower corona without losing its energy or developing a plateau (Brown 1971; Lin 1974; Smith 1975).
3. The self-consistent formation of 3–5 keV electron tails due to the nonlinear stabilization of the beam provides a new interpretation of the impulsive 3–5 keV bursts (Kahler and Kreplin 1971).
4. The temporal confinement of the bulk of the Maxwellian in region I has led us to the conclusion that only a fraction ($n_b/n \lesssim 10^{-2}$) of the electrons producing hard X-rays is available to produce microwave, EUV, and optical flashes, in accordance with the existing theoretical models (Takakura 1973; Brown, Canfield, and Paterson 1977; Emslie, Brown, and Donnelly 1978).
5. The collective bremsstrahlung radiation at $2\omega_e$ in region II can explain the recently observed emission in microwave bursts by Slottje (1978).

IV. SUMMARY AND CONCLUSION

The above work should only be considered as a step toward developing a rather complete numerical model of the impulsive solar flare phenomena. Extreme care should be exercised so that the various conclusions and ideas are not taken as the ultimate word but only as a guideline to further work and exploration of the subject. Various effects, such as the magnetic field, the existence of lower hybrid waves, micropulsation, drift waves, have not been taken into account. In addition, the heated region I will start expanding with velocity $V_F \leq 30$ km s^{-1} as suggested by Brown, Melrose, and Spicer (1979) and Smith and Lilliequist (1978), thereby limiting the validity of our model to time shorter than $t \approx L_{\rm II}/V_F$. However, a major conclusion of the present paper is that collective plasma effects which mostly have been excluded in previous studies can improve our understanding of the evolution of the solar structure dynamics.

Several discussions with Professors D. Wentzel, J. C. Brown, M. R. Kundu, and D. S. Spicer are gratefully appreciated. This work was partially supported by NASA grant NGR 21-002-199, and by ONR.

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