MODERN COSMOLOGY MATHEMATICAL FORMALISM AND PHYSICAL CONSEQUENCES

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- **Introduction - The Cosmological Parameters**

- **Dynamics - Expansion in Cosmological models with dark energy**

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INTRODUCTION

Recent advances in Cosmology have strongly indicated that we are living in a flat, accelerating universe with:

$$\sim \frac{1}{3}\ \text{matter (baryonic and dark)} + \frac{2}{3}\ \text{dark energy} \ .$$

The basic set of experiments that we have:

- SNeIa data [Perlmutter et al. 1999; Tonry et al. 2003]
- CMB anisotropies [Spergel et al. 2003]
- Large Scale Structure [Percival et al. 2002]
- Age of the Globural clusters [Tegmark et al. 2003]
- High Redshift Galaxies [Viel et al. 2003]

Do we know anything regarding the nature of the above exotic dark energy? Possible candidates for quintessence could be:

(I) A time varying Λ-parameter (Caldwell, Dave & Steinhardt 1998a) (III)

An extra "matter" component, which is described by an equation of state $P_Q = w\rho_Q$ where $-1 \leq w < 0$. As a particular case, Λ-models can be obtained from quintessence models with $w = -1$.

- Several results suggest $w < -0.6$ [Efstathiou 1999; Basilakos & Plionis 2003; Tegrmark et al. 2003]

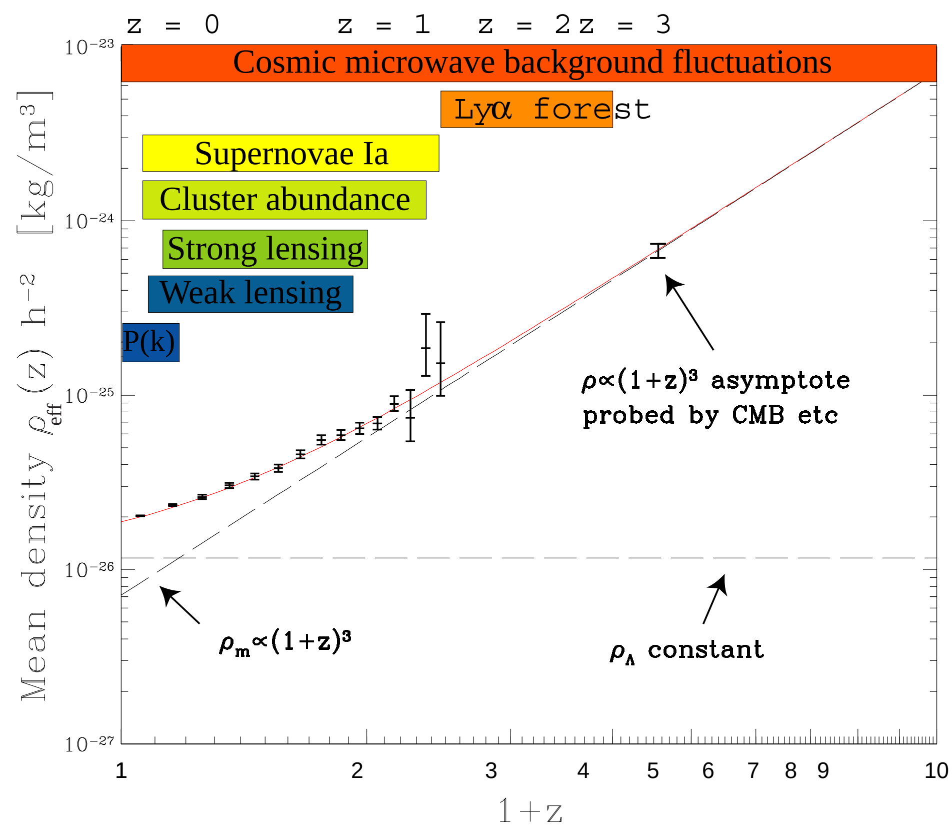

Figure 1: The evolution of the effective cosmic mean density (Tegmark 2002).

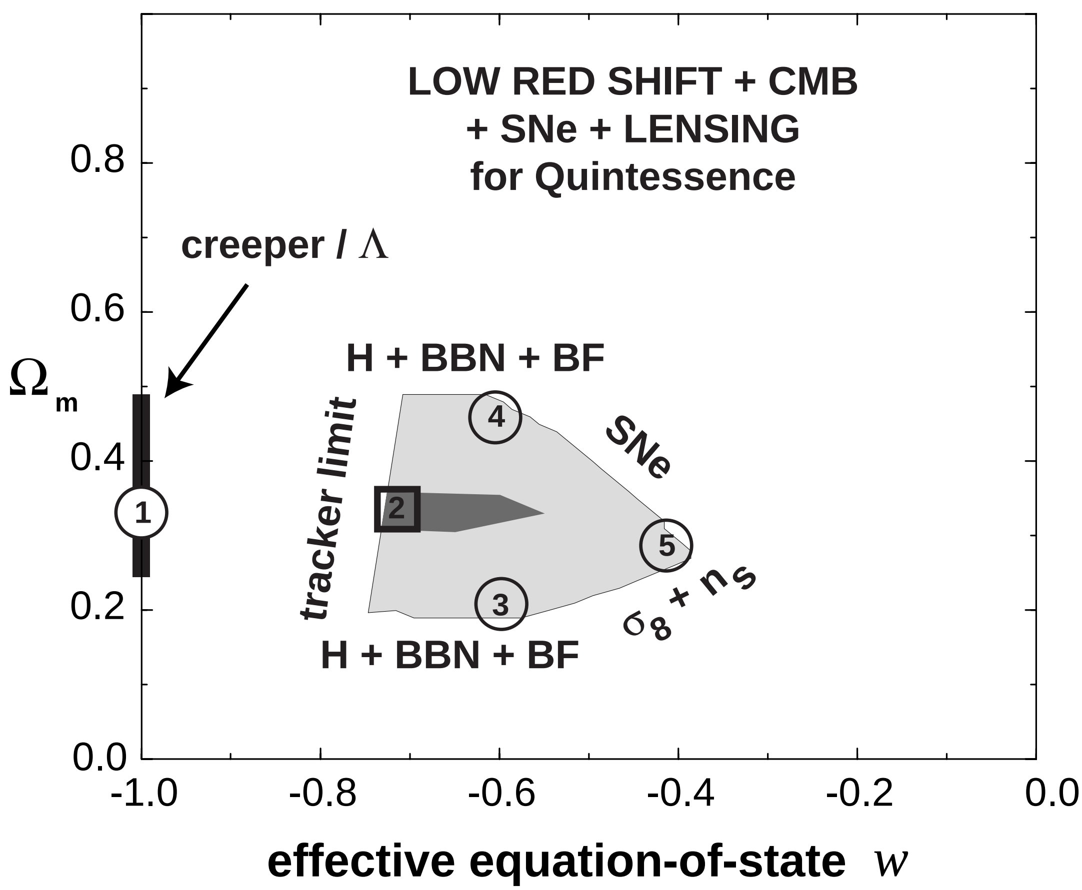

Figure 2: The overall concordance region based low, intermediate, and high redshift tests for quintessence models. The dark shaded region corresponds to the most preferred region (the 2σ maximum likelihood region consistent with the tracker constraint), $\Omega_m \approx 0.33 \pm 0.05$, effective equation-of-state $w \approx -0.65 \pm 0.10$ and $h = 0.65 \pm 0.10$ and are consistent with spectral index $n = 1$. Model 1 is the best fit ΛCDM model and Model 2 is the best fit QCDM model (Wang & Steinhardt 2000).

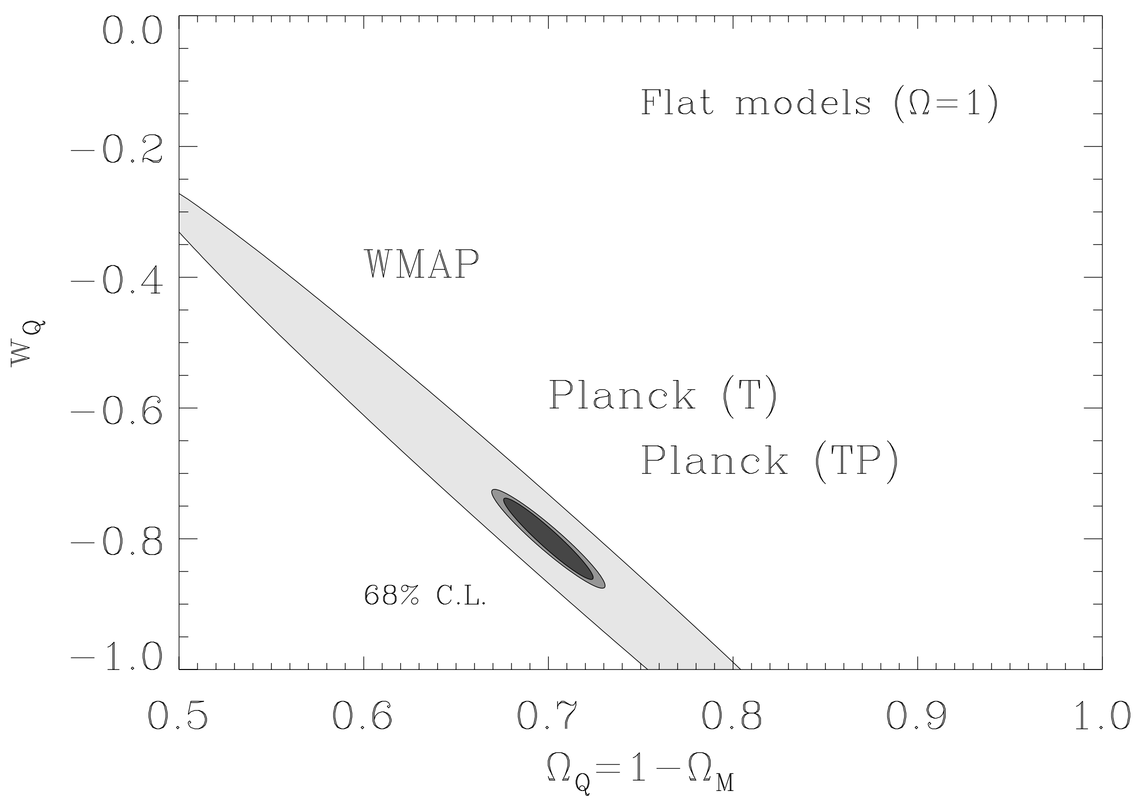

Figure 3: The 68% confidence level constraints (Balbi et al. 2003) in the dark energy parameters space (Ω_Q, w). *Left*: Shaded regions (*lighter to darker*) obtained from *WMAP* and *Planck* (temperature only) and from *Planck* (temperature and polarization).

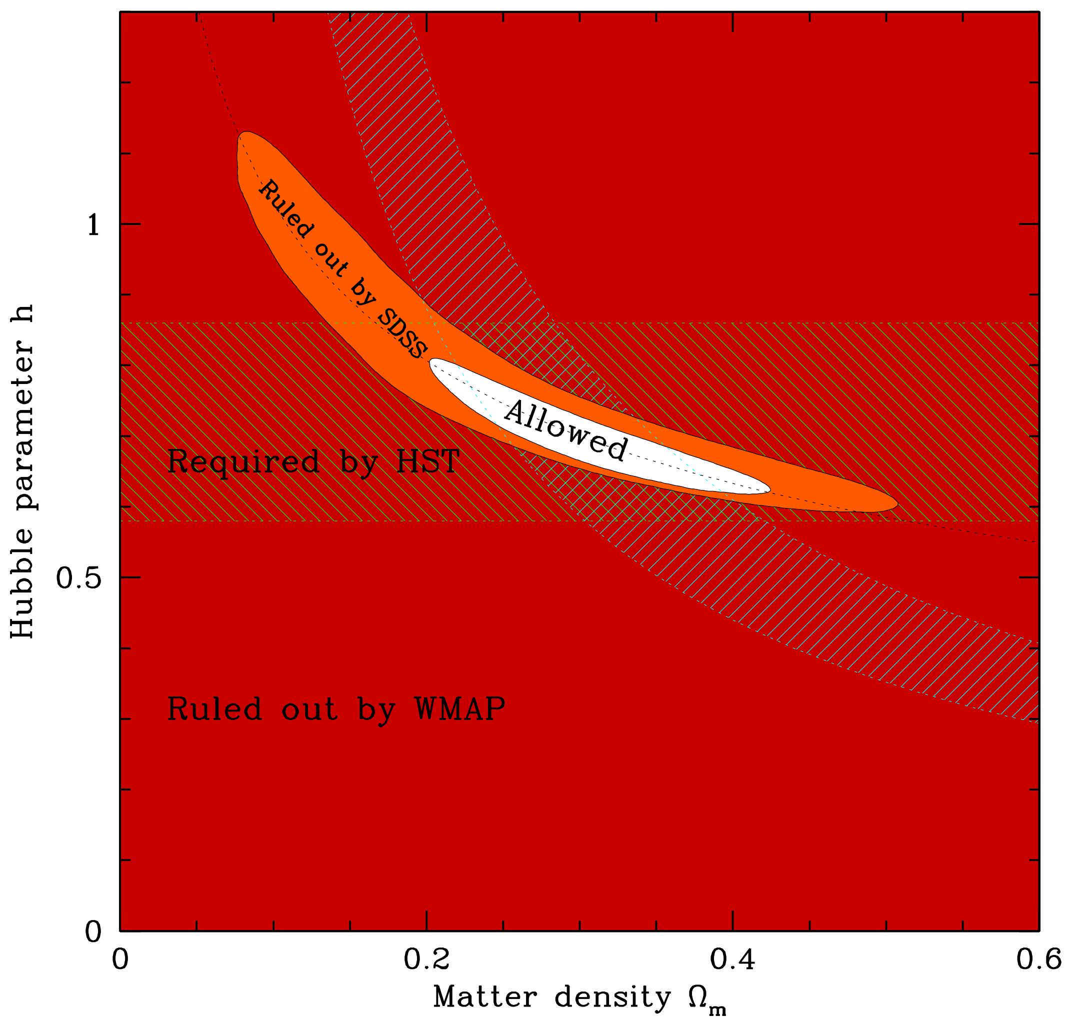

Figure 4: 95% constraints (Tegmark et al. 2003) in the (Ω_{tot}, h).

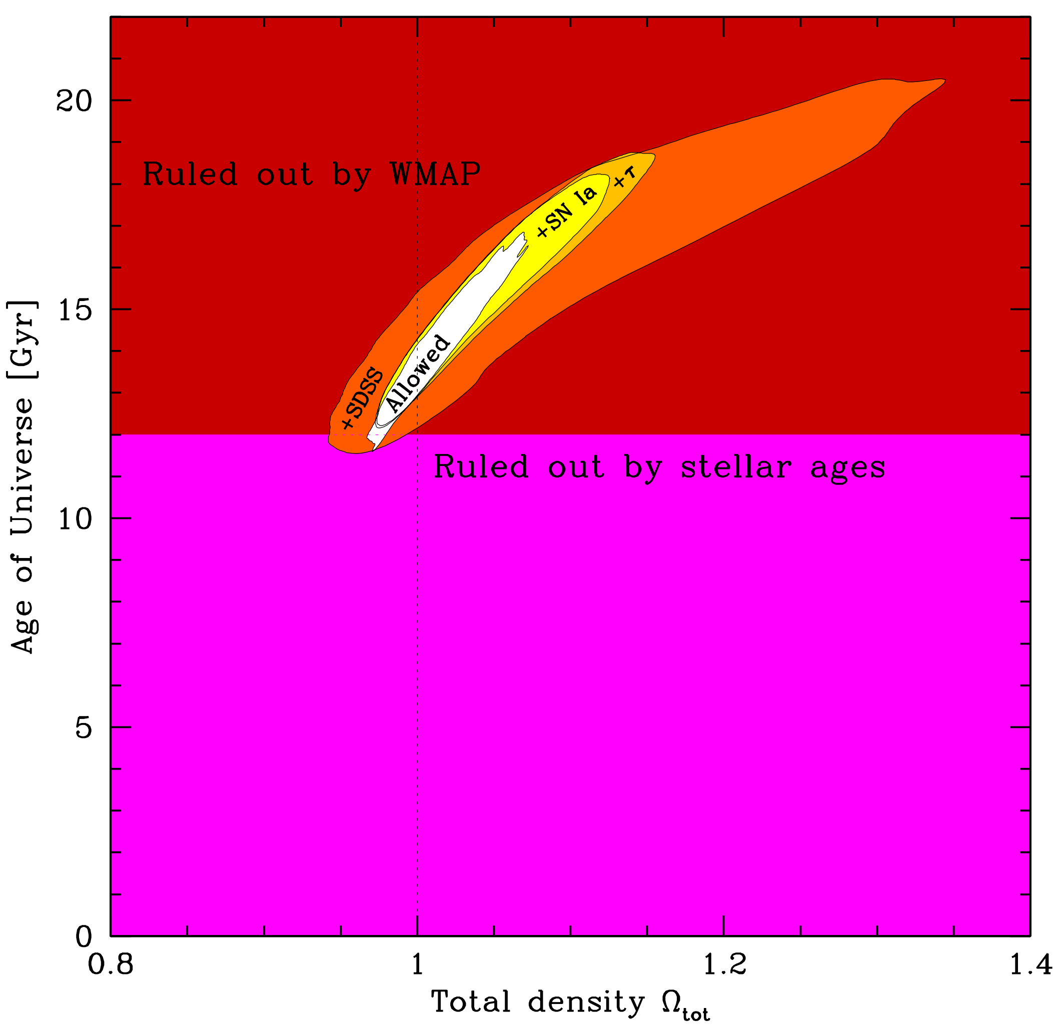

Figure 5: 95% constraints (Tegmark et al. 2003) in the $(\Omega_{tot}, T_\circ)$.

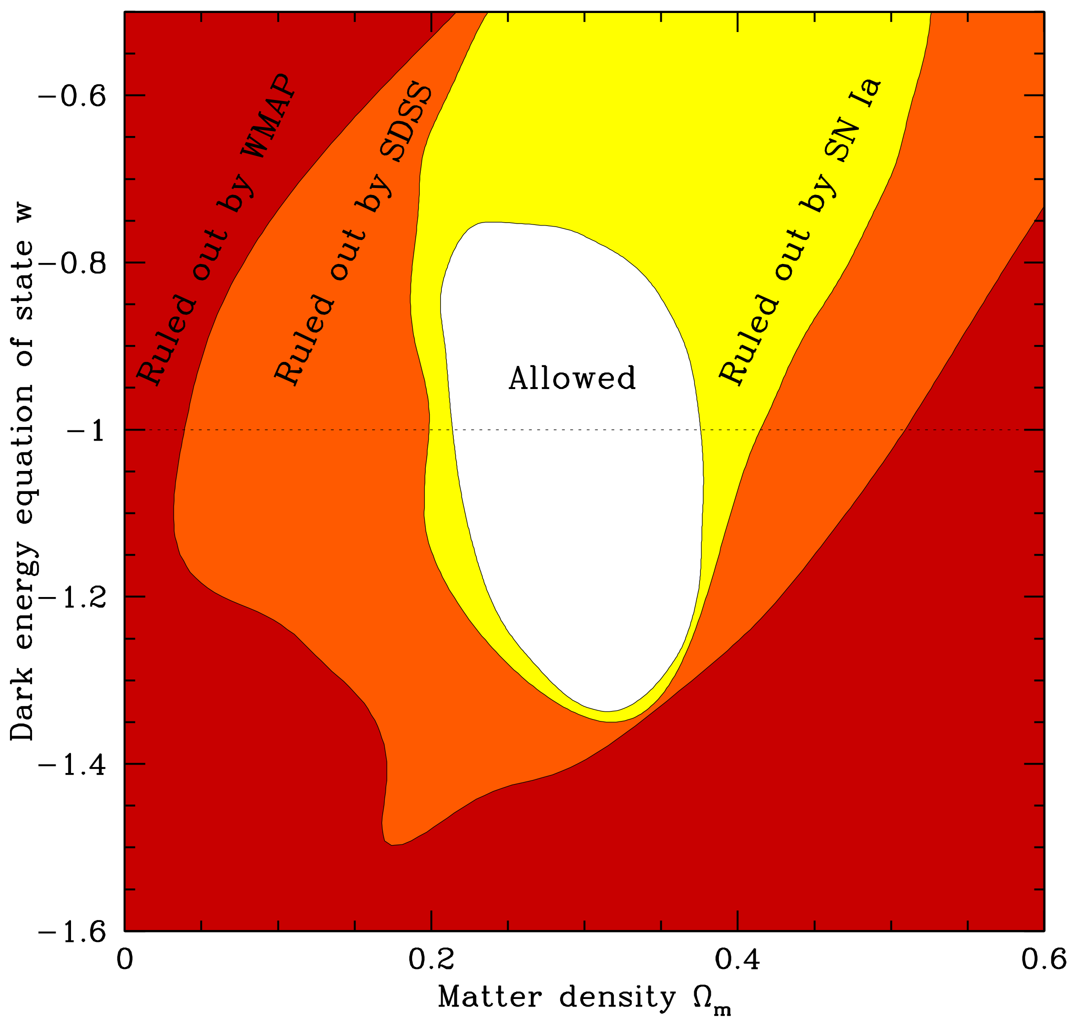

Figure 6: 95% constraints (Tegmark et al. 2003) in the (Ω_m, w).

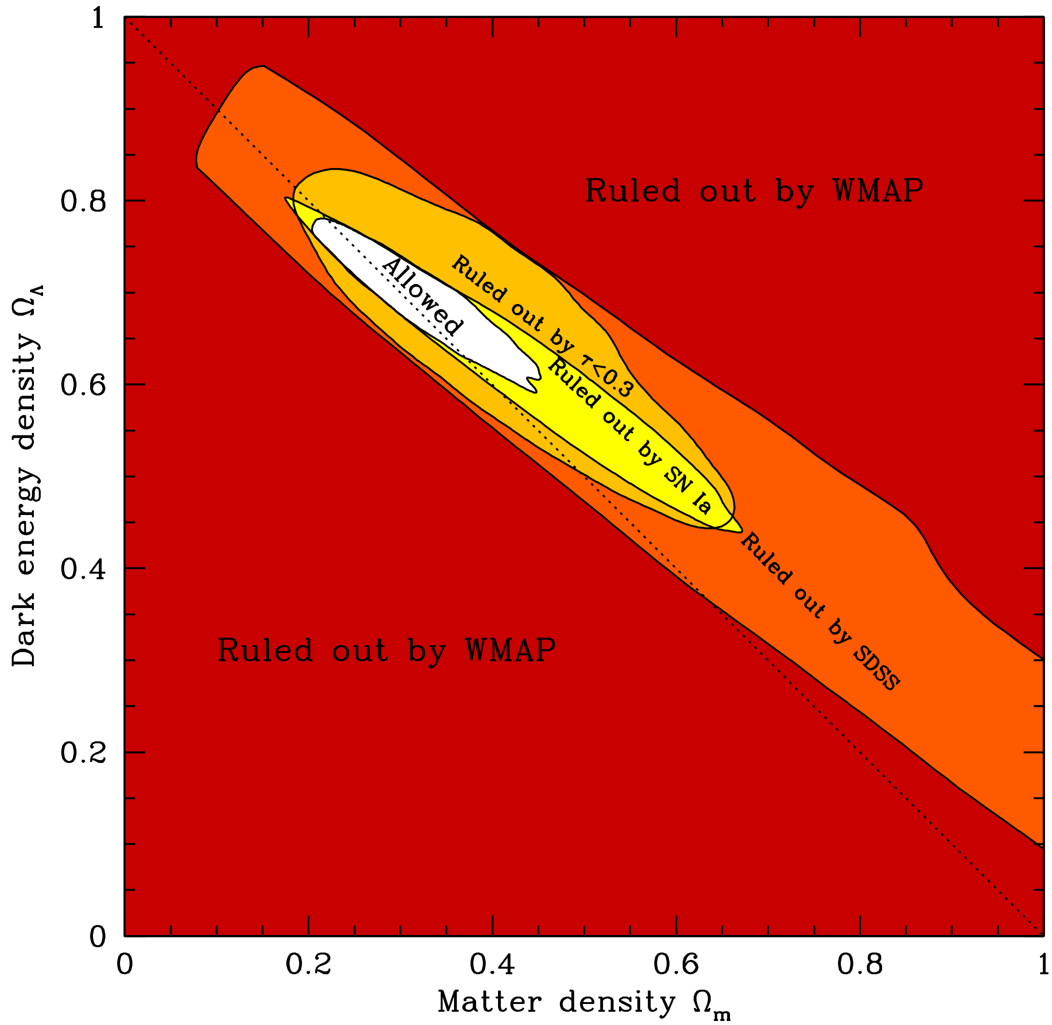

Figure 7: 95% constraints (Tegmark et al. 2003) in the $(\Omega_m, \Omega_\Lambda)$.

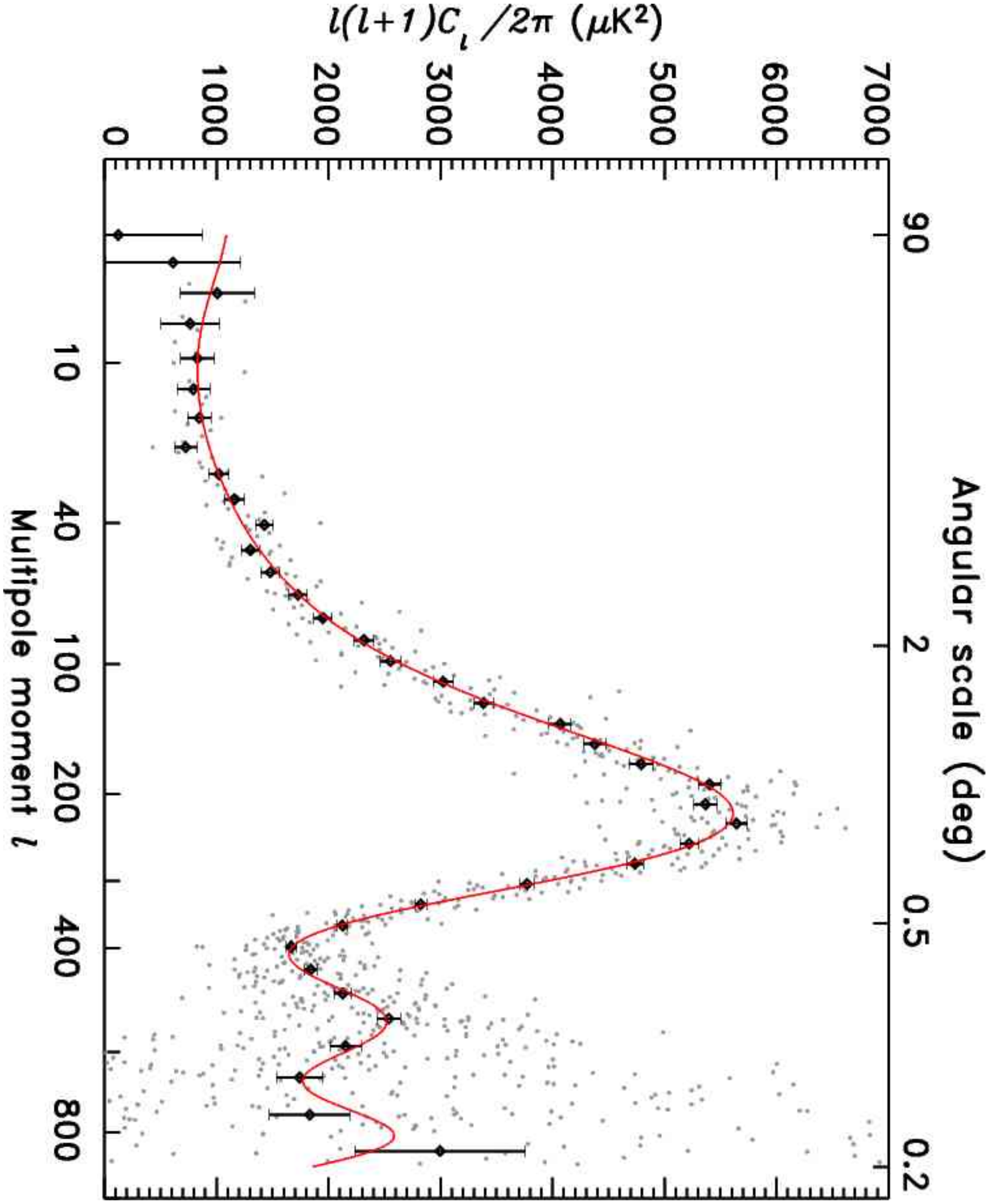

Figure 8: WMAP best fit (Spergel et al. 2003) $(\Omega_m, \Omega_\Lambda) = (0.3, 0.7)$ and $H_\circ = 70/Km/sec/Mpc$.

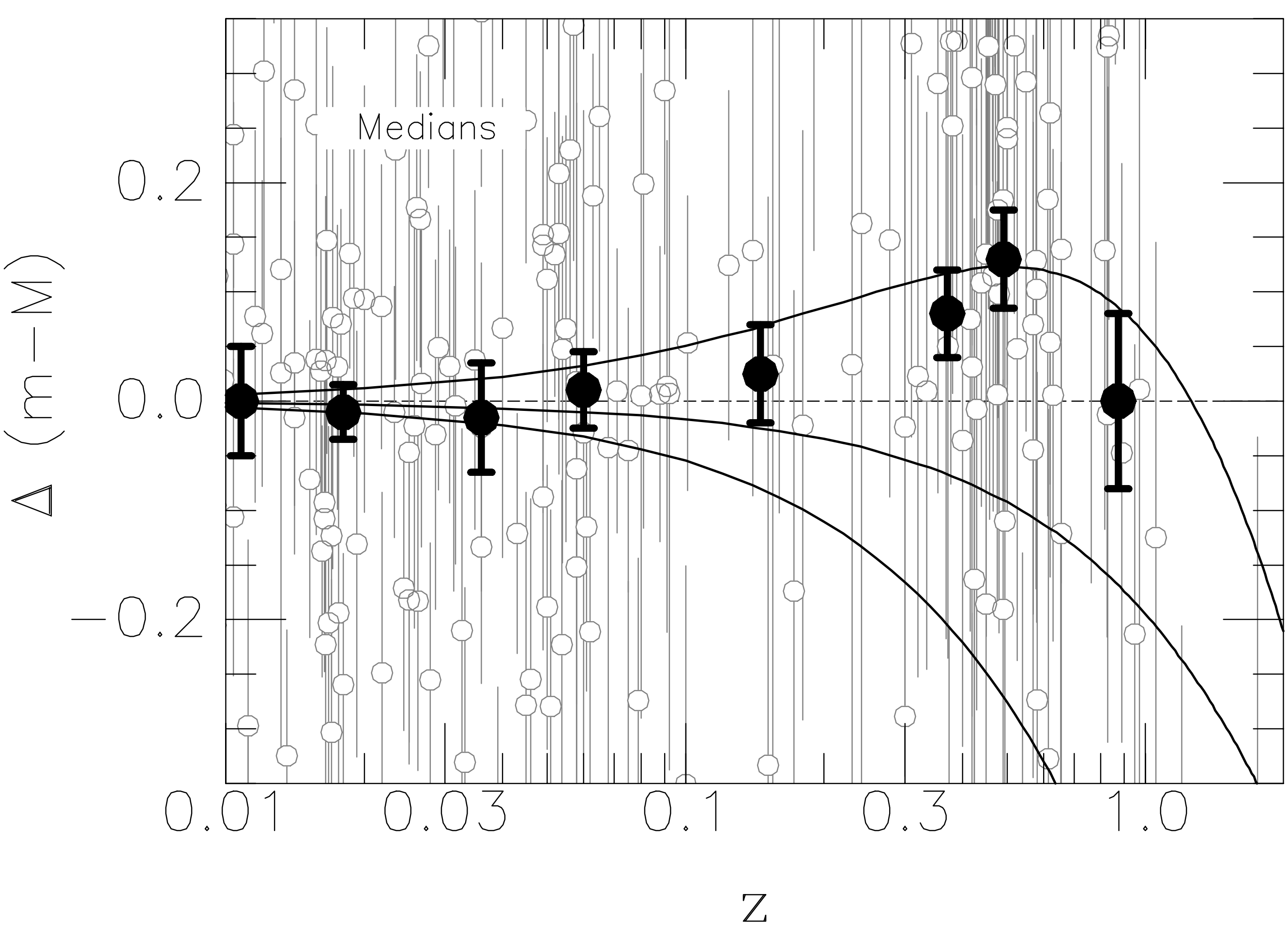

Figure 9: The residual Hubble diagram (Tonry et al. 2003) with respect to an empty universe. In this plot the highlighted points correspond to median values in eight redshift bins. From top to bottom the curves show $(\Omega_m, \Omega_\Lambda) = (0.3, 0.7)$, (0.3,0.0), and (1.0,0.0), respectively.

DYNAMICS IN MODELS WITH DARK ENERGY

For homogeneous and isotropic cosmologies, the time evolution equation of the models is determined by the **Friedmann-Lemaitre** equations:

$$\left(\frac{\dot{\alpha}}{\alpha}\right)^2 = \frac{8\pi G\rho}{3} - \frac{k}{\alpha^2} \tag{1}$$

and

$$\frac{\ddot{\alpha}}{\alpha} = -4\pi G(\rho + 3P) \ . \tag{2}$$

THE TOTAL MASS-ENERGY DENSITY AS WELL AS THE TOTAL PRESSURE:

$$\rho = \rho_m + \rho_r + \rho_Q \quad \text{and} \quad P = P_m + P_r + P_Q$$

- For Matter: $\rho_m = \rho_{mo}(1+z)^3$ and $P_m = 0$.
- For Radiation: $\rho_r = \rho_{ro}(1+z)^4$ and $P_r = \frac{1}{3}\rho_r$.
- For Dark energy: $\rho_Q = \rho_{Qo}(1+z)^{3(w+1)}$ and $P_Q = w\rho_Q$.

Considering the matter epoch ρ_r and P_r are negligible.

$$\left(\frac{\dot{\alpha}}{\alpha}\right)^2 = \frac{8\pi G}{3}(\rho_{\rm m} + \rho_Q) - \frac{k}{\alpha^2} \tag{3}$$

and

$$\frac{\ddot{\alpha}}{\alpha} = -4\pi G[(w + \frac{1}{3})\rho_Q + \frac{1}{3}\rho_{\rm m}] \ , \tag{4}$$

Note that for $w = -1$ we have the ΛCDM case ($Q = \Lambda$).

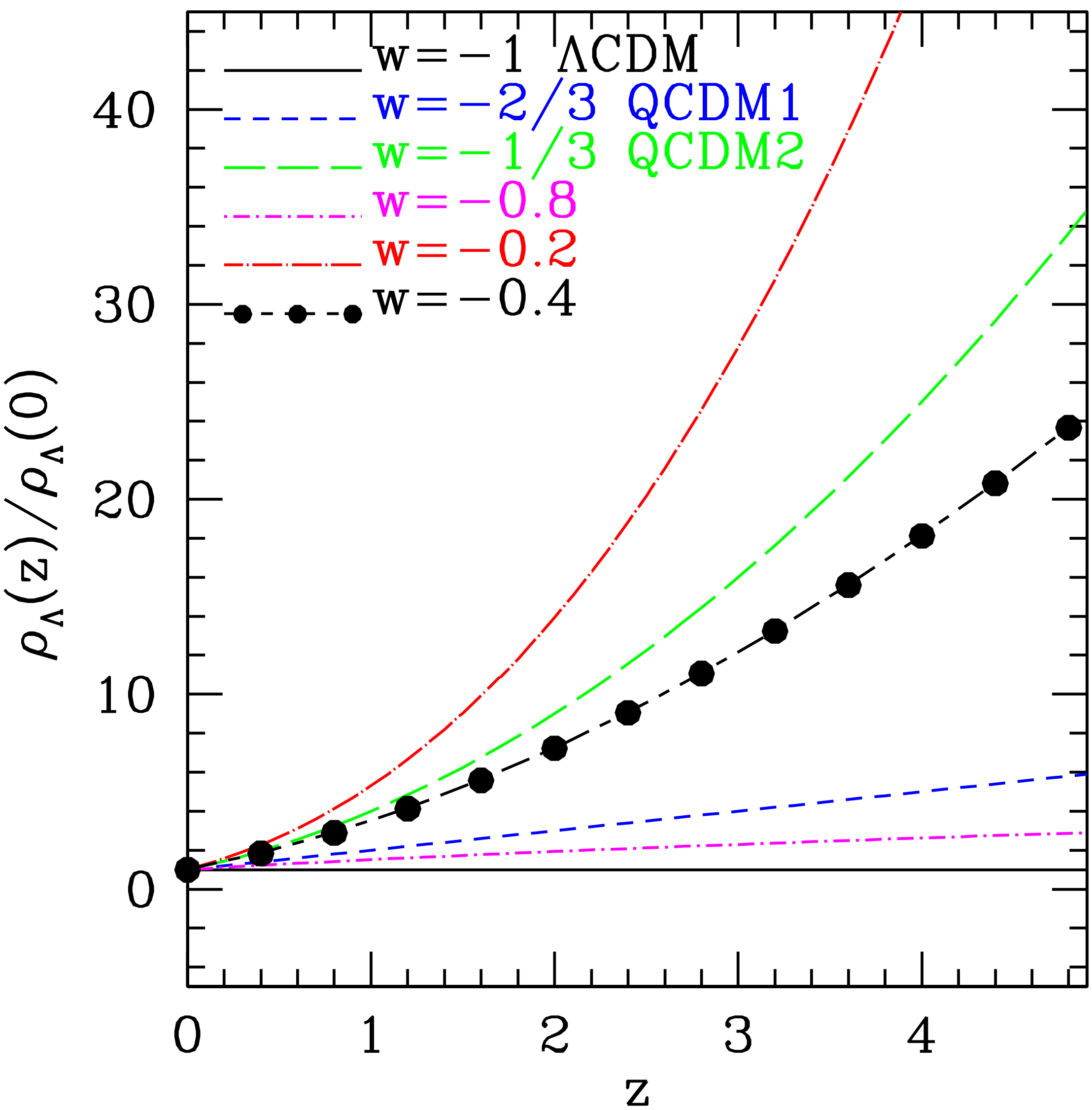

Figure 10: The evolution of the dark energy density for different cosmological models.

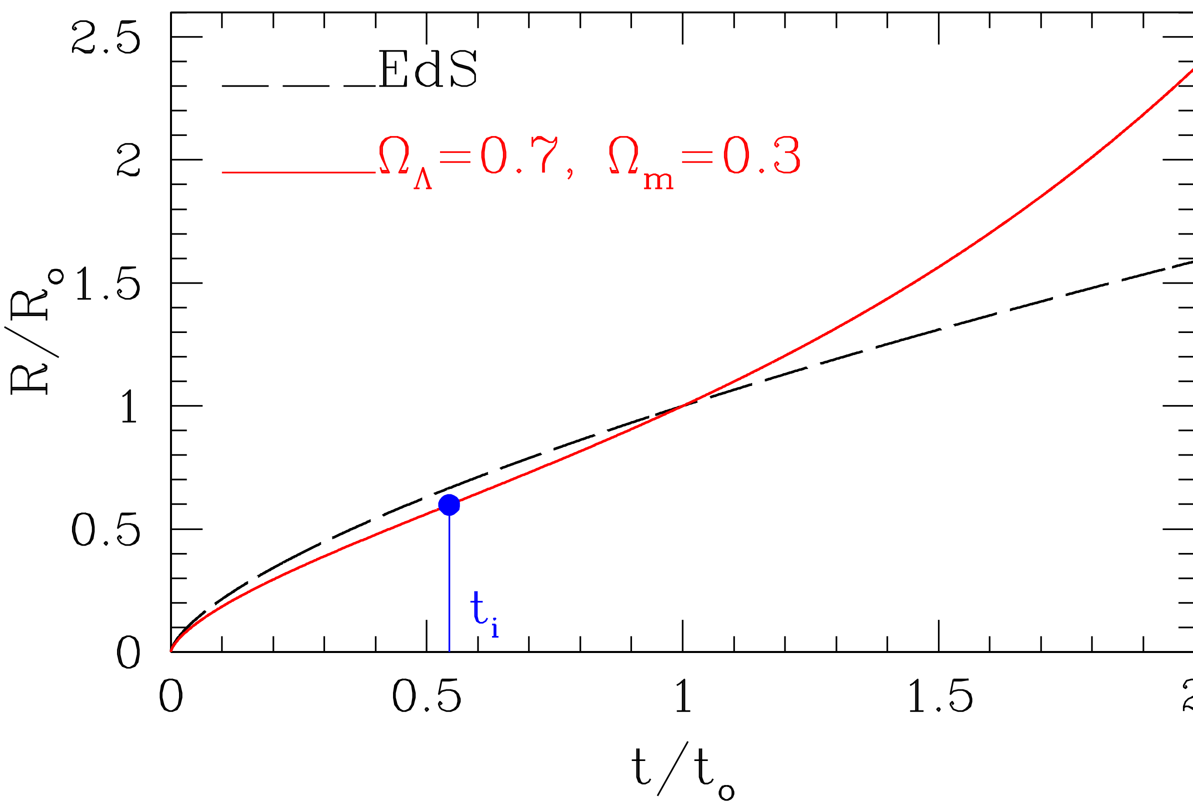

Figure 11: The expansion of the Universe in an Einstein de-Sitter (EdS) and in the *preferred* Λ model. We indicate the inflection point beyond which the expansion accelerates. It is evident that in this model we live in the accelerated regime and thus the age of the Universe is larger than the Hubble time ($H_{\circ}^{-1}$). Note that $R(t) = \alpha(t)$.

AS A FUNCTION OF REDSHIFT:

The Friedmann-Lemaitre equation (3) divided by $H = \dot{\alpha}/\alpha$ we have:

$$1 = \frac{8\pi G\rho_m}{3H^2} + \frac{8\pi G\rho_Q}{3H^2} - \frac{k}{\alpha^2 H^2} \longrightarrow 1 = \Omega_m + \Omega_Q + \Omega_k \tag{5}$$

$$\Omega_m(z) = \frac{\Omega_\circ(1+z)^3}{E^2(z)} \tag{6}$$

$$\Omega_Q(z) = \frac{\Omega_{Qo}(1+z)^{3(1+w)}}{E^2(z)} \tag{7}$$

and

$$\Omega_k(z) = \frac{\Omega_{ko}(1+z)^2}{E^2(z)} \tag{8}$$

where

$$E(z) = \left[\Omega_\circ(1+z)^3 + \Omega_k(1+z)^2 + \Omega_{Qo}(1+z)^{3(1+w)}\right]^{1/2}$$

$\Omega_\circ$ (density parameter), Ω_{ko} (curvature parameter), Ω_{Qo} (dark energy parameter) at the present time.

Different values of w could yield cosmological models where:

- Λ-models can be described by quintessence models with w strictly equal to -1.
- if $w = 0$ the equation of state behaves like that of pressureless matter. In other words, $P_Q = 0$ plays a similar role to the cold dark matter (CDM).
- Having a flat low-$\Omega_\circ$ with $w = -1/3$ model, the dark energy density can be given by $\rho_Q \propto (1+z)^2$. Therefore, the equation of state $P_Q = w\rho_Q$ leads to the same expansion as in an open universe. In other words, P_Q plays a similar role to the curvature, despite the fact that this quintessence model has a spatially flat geometry!
- Having a flat low-$\Omega_\circ$ with $-1 < w < -1/3$ model, for the dark energy density we have $\rho_Q \propto \alpha^{-(3(1+w)} < \alpha^{-3} \propto \rho_{\rm m}$, which means that the dark energy density falls off at a slower rate than cold dark matter. This is very important because the dark energy component starts to dominate the mass density in the universe, especially at the late times, thus creating an accelerating expansion.

NEOCLASSICAL TESTS

(A) TIME EVOLUTION:

$$H_\circ t(z) = \int_z^\infty \frac{\mathrm{d}x}{(1+x)E(x)} \tag{9}$$

(B) ANGULAR SIZE DISTANCE:

$$y(z) = H_\circ \alpha_\circ r(z) = \frac{H_\circ \alpha_\circ}{\mid \Omega_{ko} \mid^{1/2}} S_k \left[\mid \Omega_{ko} \mid^{1/2} \int_0^z \frac{\mathrm{d}x}{E(x)} \right] \tag{10}$$

where

$S_k(\chi) = \mathrm{sinh}\chi$ for open , $S_k(\chi) = \mathrm{sin}\chi$ for close , $S_k(\chi) = \chi$ for flat Geometry

(C) THE DISTANCE MODULUS:

$$m - M = 25 + 5\log[3000(1+z)y(z)] - 5\log h \tag{11}$$

$$H_\circ = 100h\mathrm{Km\ sec^{-1}\ Mpc^{-1}} \quad 0.5 \leq h \leq 1$$

(D) LINEAR PERTURBATION THEORY $\delta < 1$:

Considering the matter as an ideal fluid and utilizing the Newtonian mechanics (mass conservation, Euler and Poisson equations) we can obtain the time evolution equation for the mass density contrast: $\delta = (\delta\rho_{\rm m}/\rho_{\rm m})$, with general solution of the growing mode (cf. Peebles 1993) being $\delta \propto D(t)$, is:

$$\frac{\partial^2 \delta}{\partial t^2} + 2H(t)\frac{\partial \delta}{\partial t} = 4\pi G \rho_{\rm m} \delta \ \ , \tag{12}$$

while the growth factor of the linear density contrast can be given by (Silveira & Waga 1994; Basilakos & PLionis 2003)

$$D(z) = (1+z)^{-1} F[-\frac{1}{3w}, \frac{w-1}{2w}, 1 - \frac{5}{6w}, \frac{(\Omega_\circ - 1)(1+z)^{-3w}}{\Omega_\circ}] \tag{13}$$

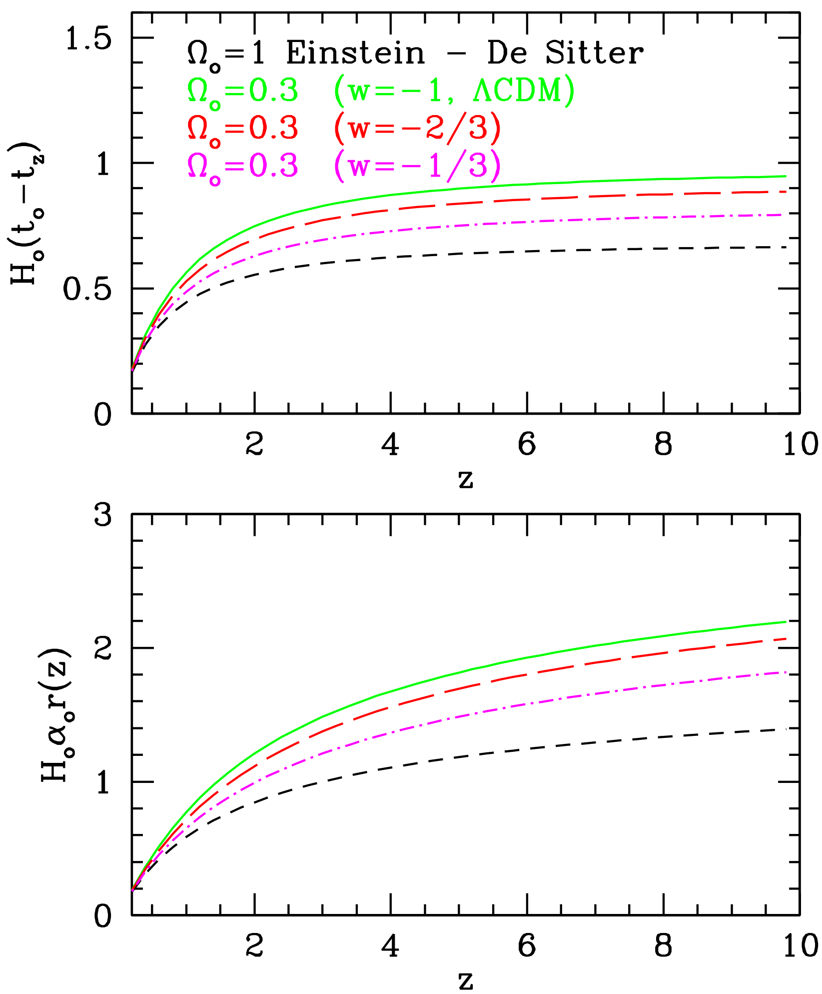

Figure 12: The angular size distance and the time evolution for different cosmological models.

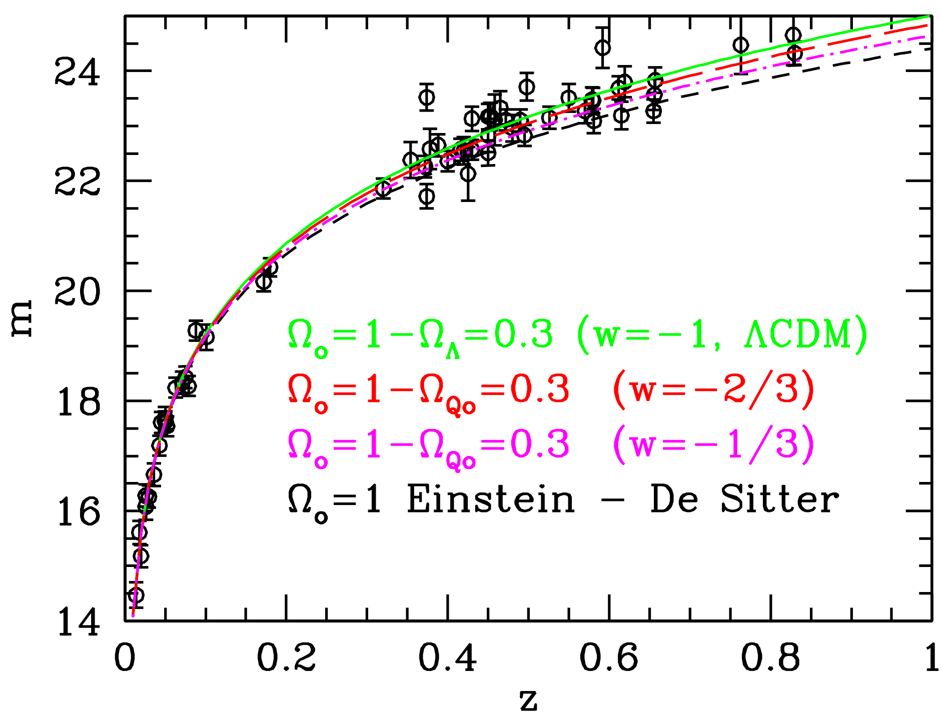

Figure 13: The apparent magnitude for SNeI.

COSMOLOGICAL PARAMETERS

- **Matter Density:** $\Omega_\circ = 0.27 \pm 0.04$

- **Baryon Density:** $\Omega_b = 0.044 \pm 0.004$

- **Dark Energy Density:** $\Omega_{Q\circ} = 0.73 \pm 0.04$

- **Equation of State:** $w < -0.6$**, most probable** $w = -1$

- **Hubble Constant:** $H_\circ = 71^{+4}_{-3}$**Km/sec/Mpc**

- **Age of the Universe:** $T_\circ = 13.7 \pm 0.2$**Gyr**

- **Matter Power Spectrum Normalization:** $\sigma_8 = 0.84 \pm 0.04$

- **Decoupling Redshift:** $z_{dec} = 1089 \pm 1$

- **Redshift of Matter/Radiation Equality:** $z_{eq} = 3233^{+194}_{-210}$

- **Spectral Index at** $k = 0.05$**Mpc**$^{-1}$**:** $n_s = 0.99 \pm 0.03$

- **Baryon/Photon Ratio:** $n = (6.1^{+0.3}_{-0.2}) \times 10^{-10}$

CDM COSMOLOGIES

Now we present the cosmological models that we use in this work. For the power spectrum of our CDM models, we consider $P(k) \approx k^{n_s} T^2(k)$ with scale-invariant ($n_s = 1$) primeval inflationary fluctuations. We utilized the transfer function parameterization as in Bardeen et al. (1986):

$$T(k) = \frac{\ln(1+2.34q)}{2.34q}[1 + 3.89q + (16.1q)^2 + (5.46q)^3 + (6.71q)^4]^{-1/4} \ .$$

Here $q = k/h\Gamma$, $k = 2\pi/\lambda$ is the wavenumber in units of h Mpc^{-1} and Γ, is the CDM shape parameter, in units of $(h^{-1}\mathrm{Mpc})^{-1}$, given approximately by Sugiyama's (1995) formula:

$$\Gamma \approx \Omega_\circ h \exp[-\Omega_b - (2h)^{1/2}\Omega_b/\Omega_\circ] \quad ,$$

where Ω_b is the baryon density.

Cold Dark Matter Models (CDM):

- **SCDM model:** $\Omega_\circ = 1$, $H_\circ = 50$ **Kms^{-1}Mpc^{-1}, and** $\Gamma \sim 0.5$.
- **OCDM model:** $\Omega_\circ = 0.3$, $H_\circ = 65$ **Kms^{-1}Mpc^{-1}, and** $\Gamma \sim 0.2$.
- **ΛCDM model:** $\Omega_\circ = 1 - \Omega_\Lambda = 0.3$, $w = -1$, $H_\circ = 70$ **Kms^{-1}Mpc^{-1} and** $\Gamma \sim 0.2$.
- **QCDM1 model:** $\Omega_\circ = 1 - \Omega_{Qo} = 0.3$, $H_\circ = 70$ **Kms^{-1}Mpc^{-1}**, $w = -2/3$, **and** $\Gamma \sim 0.2$.
- **QCDM2 model:** $\Omega_\circ = 1 - \Omega_{Qo} = 0.3$, $H_\circ = 70$ **Kms^{-1}Mpc^{-1}**, $w = -1/3$, **and** $\Gamma \sim 0.2$.

Finally, the latter cosmological models are normalized to have fluctuation amplitude in 8 h^{-1}Mpc scale of

$$\sigma_8 = 0.50 \pm 0.1\Omega_\circ^{-\gamma} \quad \text{(Wang \& Steinhardt 1998)}$$

with $\gamma = 0.21 - 0.22w + 0.33\Omega_\circ$.

CLUSTER FORMATION IN MODELS WITH DARK ENERGY

Why Clusters of Galaxies? because being the largest physical laboratories in the universe, they appear to be ideal tools for studying large-scale structure, testing theories of structure formation and extracting invaluable cosmological informations ($\Omega_\circ$, $H_\circ$ etc).

Non-Linear Evolution of Perturbations $\delta >> 1$:

The simplest approach to non-linear evolution is to follow the spherical "TOP-HAT" collapse model.

- Suppose that at the epoch defined by $1 + z_1 = 10^3$ the universe is characterized by an unperturbed Hubble flow.
- The matter fluctuation field has a Gaussian distribution and the density of mass scales of clusters is normally distributed about the mean density with variance σ_8.
- We take into account also the dark-energy (Λ for $w = -1$)
- Therefore we can write for the perturbations:

$$\Omega_{Q*} = \Omega_Q(z_v) \quad \text{and} \quad \Omega_{m*} = \Omega_m(z_v)(1 + \Delta)$$

CLUSTER FORMATION:

- For the bound perturbations that we consider, the matter fluctuation field has a Gaussian distribution

$$dF(\Delta) = \frac{1}{\sqrt{2\pi}\sigma_8} \exp\left(-\frac{\Delta^2}{2\sigma_8^2}\right) d\Delta \tag{14}$$

- The solution of the integral, describes the fraction of the universe (characterized by $\Omega_\circ$, Ω_{Qo} and σ_8) on some specific mass scale that has already collapsed at time t and is given by (see also Richstone et al. 1992):

$$F\left(\frac{t}{T_\circ}\right) = \frac{1}{2}\left[1 - \mathrm{erf}\left(\frac{\Delta(t/T_\circ)}{\sqrt{2}\sigma_8}\right)\right] \tag{15}$$

- Then, we can obtain the ratio (cf. Richstone et al. 1992) of the collapse time to the current age of the (unperturbed) universe:

$$\frac{t}{T_\circ} = 2\int_z^\infty \frac{\mathrm{d}x}{(1+x)M(\Delta, x)} \left[\int_0^\infty \frac{\mathrm{d}x}{(1+x)E(x)}\right]^{-1} \tag{16}$$

where

$$M(\Delta, z) = [\Omega_\circ(1+\Delta)(1+z)^3 + (1 - \Omega_\circ(1+\Delta) - \Omega_{Qo})(1+z)^2 + \Omega_{Qo}(1+z)^\beta]^{1/2} \ .$$

- The next step is to normalize the cluster formation to give the ratio of the number of clusters which have already collapsed by the epoch t (cumulative distribution), divided by the number of clusters which have collapsed at the present epoch:

$$\mathcal{F} = \frac{F(t/T_\circ)}{f} \tag{17}$$

with

$$f = \frac{\langle n \rangle M}{\rho_c \Omega_\circ} \ . \tag{18}$$

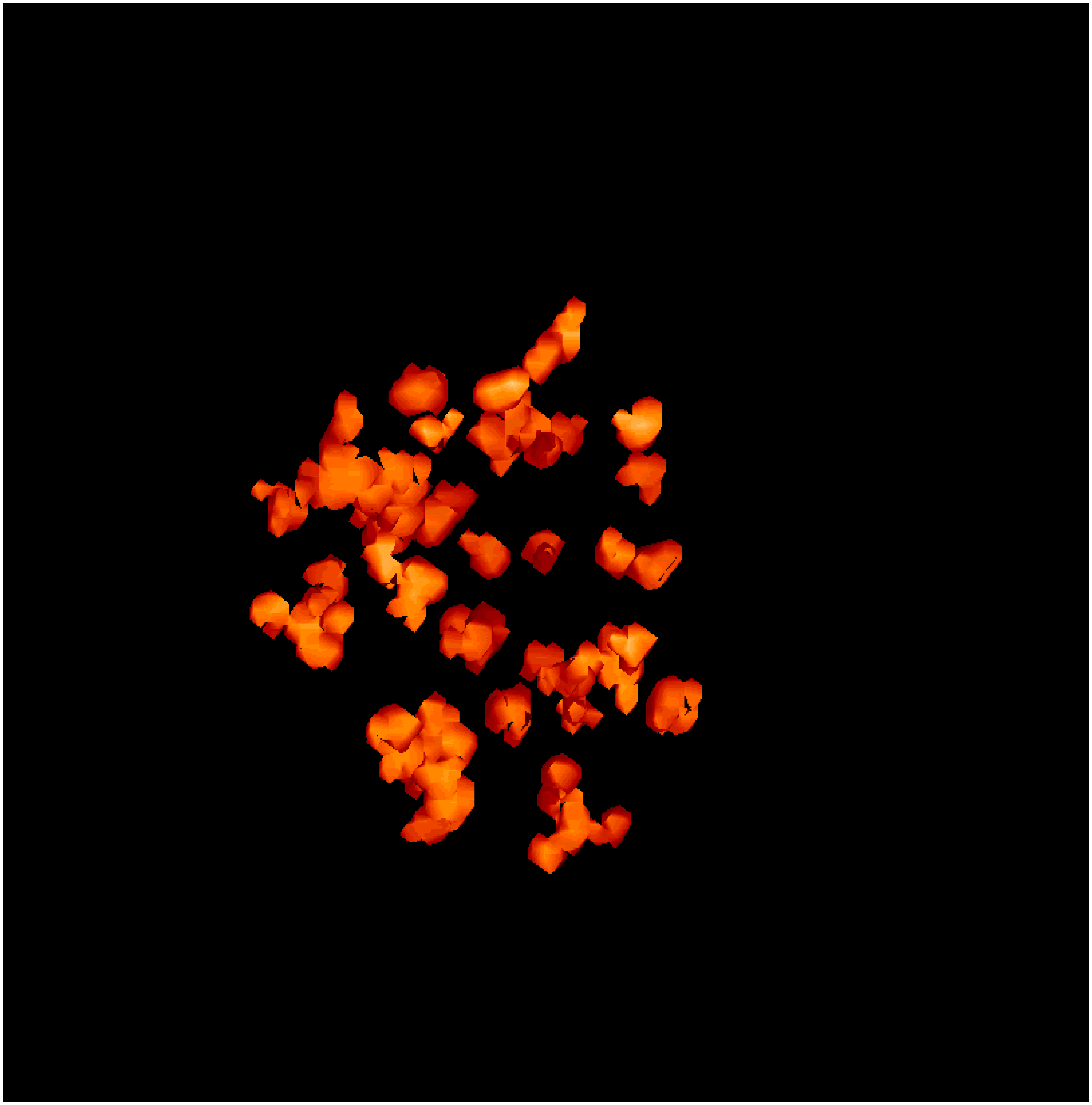

Figure 14: The 3D map of the local superclusters (Basilakos, Plionis, Robinson 2001) within $200h^{-1}$Mpc.

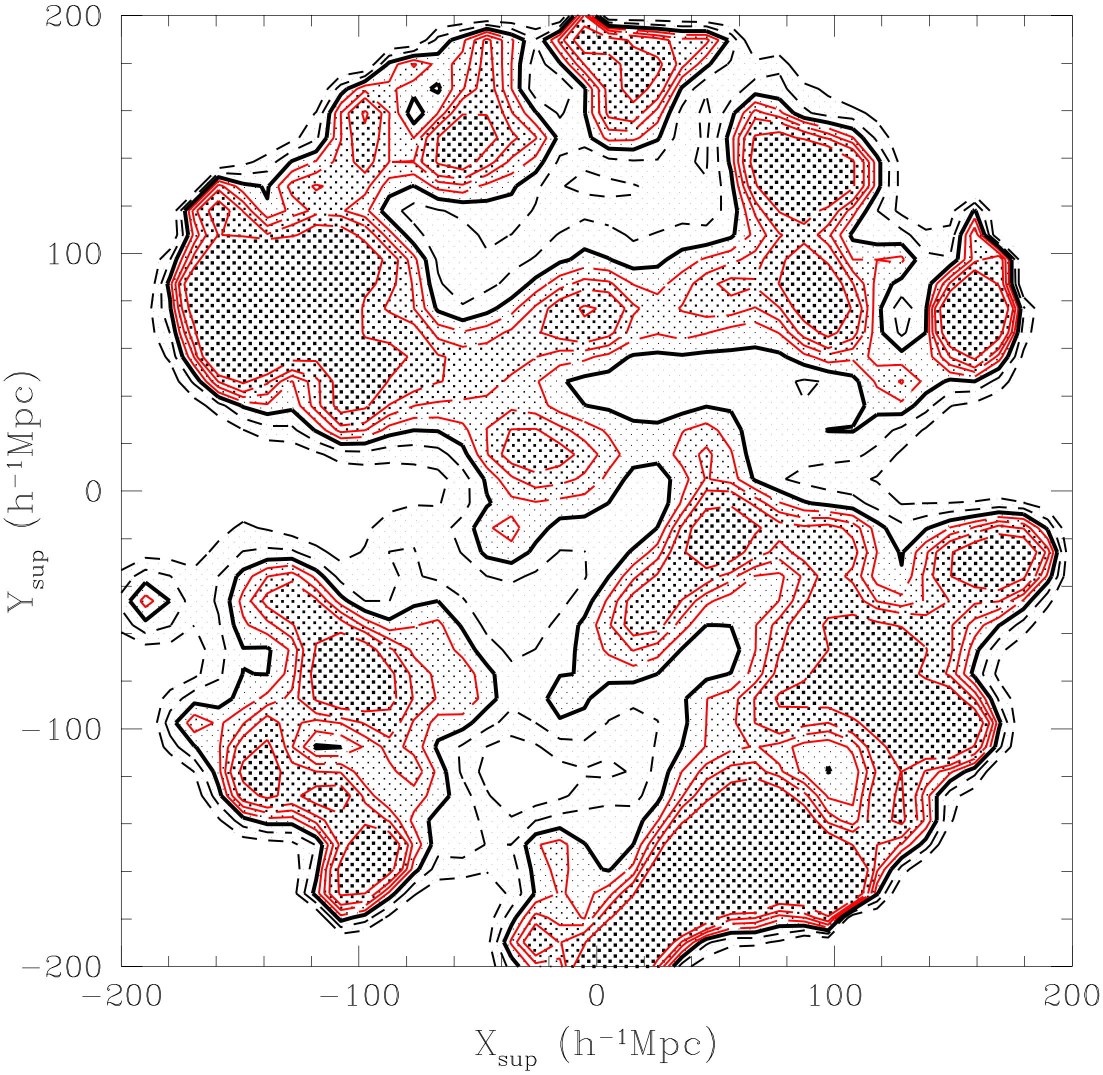

Figure 15: Contour plot of the smoothed galaxy distribution on the supergalactic plane. Well known structures appear in this plot; the largest and most evident is the Shapley concentration located at $(X_{sup}, Y_{sup}) \approx (-120, 60)$, the Perseus-Pisces supercluster at $(X_{sup}, Y_{sup}) \approx (60, -40)$, the Coma supercluster at $(X_{sup}, Y_{sup}) \approx (-20, 70)$, the Ursa-Major supercluster at $(X_{sup}, Y_{sup}) \approx (100, 100)$, the Pisces-Cetus supercluster at $(X_{sup}, Y_{sup}) \approx (50, -140)$ while the Great Attractor (Hydra-Centaurus complex ?), at $(X_{sup}, Y_{sup}) \approx (-30, 30)$ appears in the foreground of the Shapley concentration.

- $\rho_c \simeq 2.78 \times 10^{11} h^2 M_\odot$ Mpc^{-3} is the critical density, $\langle n \rangle$ is the number density of clusters which have collapsed prior to the present epoch. The parameter $\langle n \rangle$ can be defined utilizing the Abell/ACO cluster catalogue, which is a volume-limited sample within $\sim 180 - 200h^{-1}$ Mpc and which a general agreed value is: $\langle n \rangle \simeq 1.8 \times 10^{-5} h^3$ Mpc^{-3}. Therefore, considering virialized clusters of the mass scale of rich Abell clusters, $M \simeq 10^{15} h^{-1} M_\odot$, it is a routine to obtain the ratio of collapsed matter at the present time
$$f(10^{15} M_\odot) = 0.065 \Omega_\circ^{-1}$$
.

- It is obvious that the above generic form (eq.15) depends on the choice of the background cosmology. Indeed the relationship between Δ, z, $t/T_\circ$ and σ_8 is different in different cosmologies (cf. Mo & White 1996; Magliocchetti et al. 2001)
$$\frac{\Delta(z)}{\sigma_8} = \frac{\delta_c}{D(z)\sigma_8} = \frac{\delta_c}{\sigma(z)} \quad . \tag{19}$$
The value $\delta_c = 1.686$ corresponds to the spherical top-hat model in $\Omega_\circ = 1$, but it has been shown that δ_c depends only weakly on the cosmology (Eke, Cole & Frenk 1996).

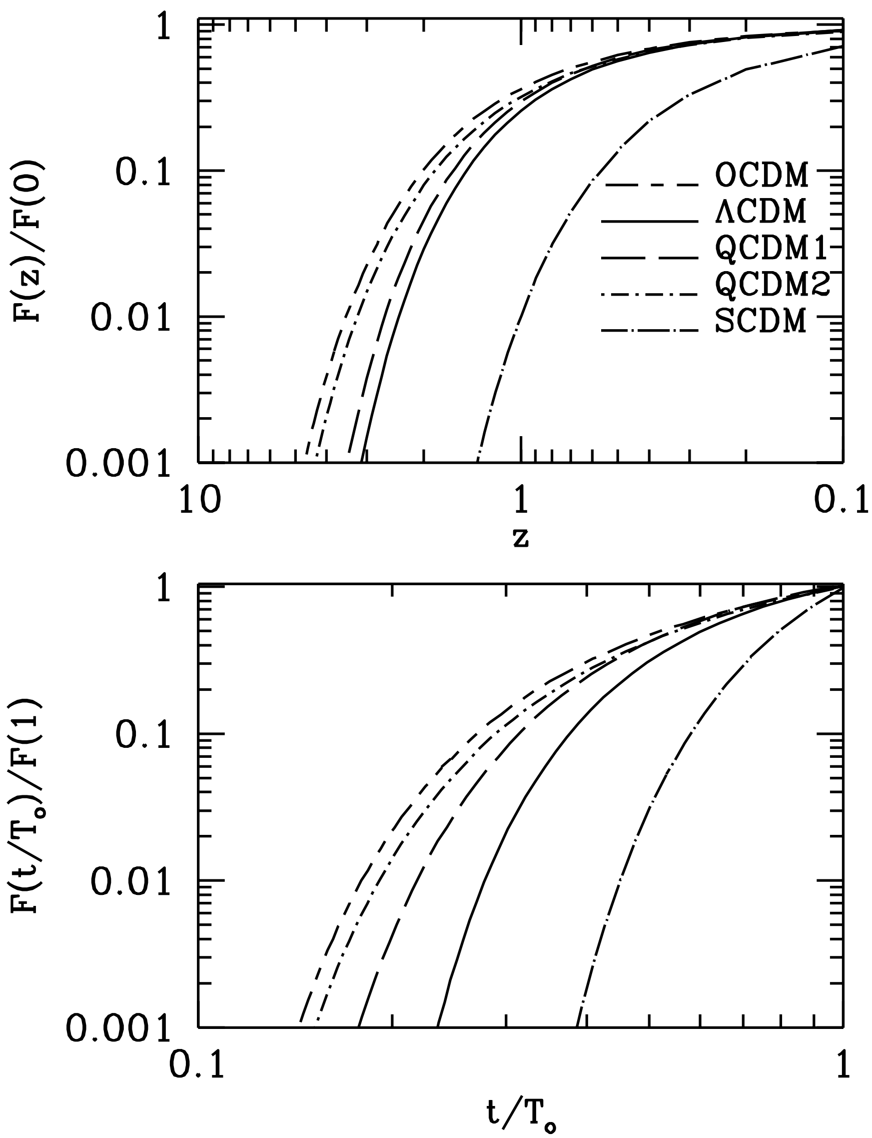

Figure 16: Theoretical predictions of the fractional rate of cluster formation as a function of redshift and fractional time $t/T_{\circ}$ for different cosmological models (Basilakos 2003).

VIRIALIZATION IN QCDM

Using the virial theorem we can write for the final state

$$T_v = -\frac{1}{2}U_{G,v} + U_{Q,v} \quad . \tag{20}$$

Since the energy is conserved, assuming that the sphere remained uniform we can write:

$$E = T_v + U_{G,v} + U_{Q,v} = U_{G,ta} + U_{Q,ta} \longrightarrow \frac{1}{2}U_{G,v} + 2U_{Q,v} = U_{G,ta} + U_{Q,ta} \ . \tag{21}$$

with

- The matter potential energy:

$$U_{G,i} = -\frac{3}{5}\frac{GM^2}{R_i}$$

- The dark energy potential:

$$U_{Q,i} = -\frac{4}{5}\pi G \rho_{Q,i} M R_i^2$$

- Total mass:

$$M = \frac{4\pi}{3}\rho_{ta} R_{ta}^3$$

where $i = v$ **and** $i = ta$.

The above analysis leads to the following cubic equation for the ratio $\frac{R_v}{R_{ta}}$:

$$2n_v(\frac{R_v}{R_{ta}})^3 - (2 + n_{ta})(\frac{R_v}{R_{ta}}) + 1 = 0 \tag{22}$$

where

$$n_v = \frac{2\Omega_{Qo}(1+z_v)^{3(1+w)}}{j\Omega_\circ(1+z_{ta})^3} \quad n_{ta} = \frac{2\Omega_{Qo}(1+z_{ta})^{3w}}{j\Omega_\circ} \quad j = (\frac{3\pi}{4})^2\Omega_\circ^{-0.79+0.26\Omega_\circ-0.06w}$$

ELEMENTS FROM ALGEBRA: Given a cubic equation:

$$x^3 + a_1 x^2 + a_2 x + a_3 = 0 \quad .$$

Let D be the discriminant:

$$\mathcal{D} = a_1^2 a_2^2 - 4a_2^3 - 4a_1^3 a_3 - 27a_3^2 + 18a_1 a_2 a_3$$

and

$$x_1 = -a_1^3 + \frac{9}{2}a_1 a_2 - \frac{27}{2}a_3 \, , \qquad x_2 = -\frac{3\sqrt{3\mathcal{D}}}{2}$$

then, the roots of the above equation are:

$$r_1 = -\frac{a_1}{3} + \frac{1}{3}[q_1 + q_2]$$

$$r_2 = -\frac{a_1}{3} + \frac{1}{3}[\epsilon^2 q_1 + \epsilon q_2]$$

$$r_3 = -\frac{a_1}{3} + \frac{1}{3}[\epsilon q_1 + \epsilon^2 q_2]$$

where $q_n = (x_1 \pm i x_2)^{1/3}$ and $\epsilon = \frac{-1+\sqrt{-3}}{2}$. If $\mathcal{D} < 0$, we have one real root (r_1) and a pair of complex conjugate roots. If $\mathcal{D} = 0$, all roots are real and at least two of them are equal. If $\mathcal{D} > 0$, all roots are real (irreducible case). In that case r_1, r_2 and r_3 can be written:

$$r_1 = -\frac{a_1}{3} + \frac{2}{3}R$$

$$r_2 = -\frac{a_1}{3} - \frac{1}{3}[R + \sqrt{3}M]$$

$$r_3 = -\frac{a_1}{3} - \frac{1}{3}[R - \sqrt{3}M]$$

where R and M are given by

$$R = r^{1/3}\cos(\frac{\theta}{3}) \quad \text{and} \quad M = r^{1/3}\sin(\frac{\theta}{3}) \tag{23}$$

with $r = \sqrt{x_1^2 + x_2^2}$ and $\theta = \mathrm{Arctan}(x_2/x_1)$.

COMMENTS:

- For $\Omega_{Qo} = 0$ (**Einstein-de Sitter case**), $n_v = n_t = 0$ the solution is:

$$\frac{R_{\rm f}}{R_{ta}} = \frac{1}{2}$$

- The discriminant is:

$$\mathcal{D}(n_u, n_t) = \frac{2(2+n_t)^3 - 27n_u}{4n_u^3} \quad .$$

Owing to the fact that we are living in an accelerating universe $\Omega_{Qo} > 0$ with low matter (baryonic and dark matter) density, the condition for an overdensity shell to turn around is $0 < n_t \leq n_u < 1$ which gives $\mathcal{D} > 0$ and therefore all roots o f the cubic equation are real (irreducible case) but one of them

$$r_2 = \frac{R_{\rm f}}{R_{\rm ta}} = -\frac{1}{3}[R + \sqrt{3}M]$$

corresponds to expanding shells. Thus, according to the solution described above, figure 3 shows the surface behavior of the exact virial solution in parametric form with:

$$r^{1/3} = \left(\frac{27^2}{16n_u^2} + \frac{27\mathcal{D}}{4}\right)^{1/6} \quad \text{and} \quad \frac{\theta}{3} = \frac{1}{3}\text{Arctan}\left(\frac{2n_u\sqrt{3D}}{9}\right)$$

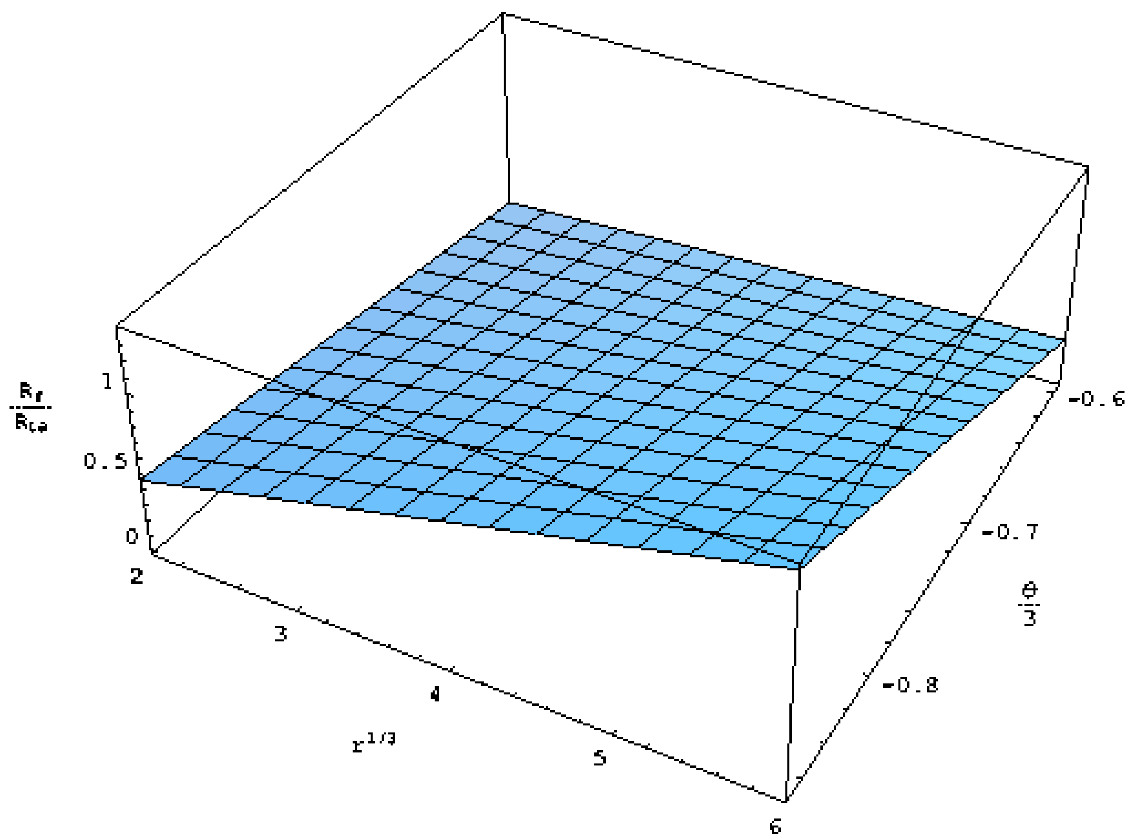

Figure 17: Ratio of the final (virial) to turnaround radius of a virialized cluster utilizing a QCDM cosmology (Basilakos 2003). We give the surface in parametric form.

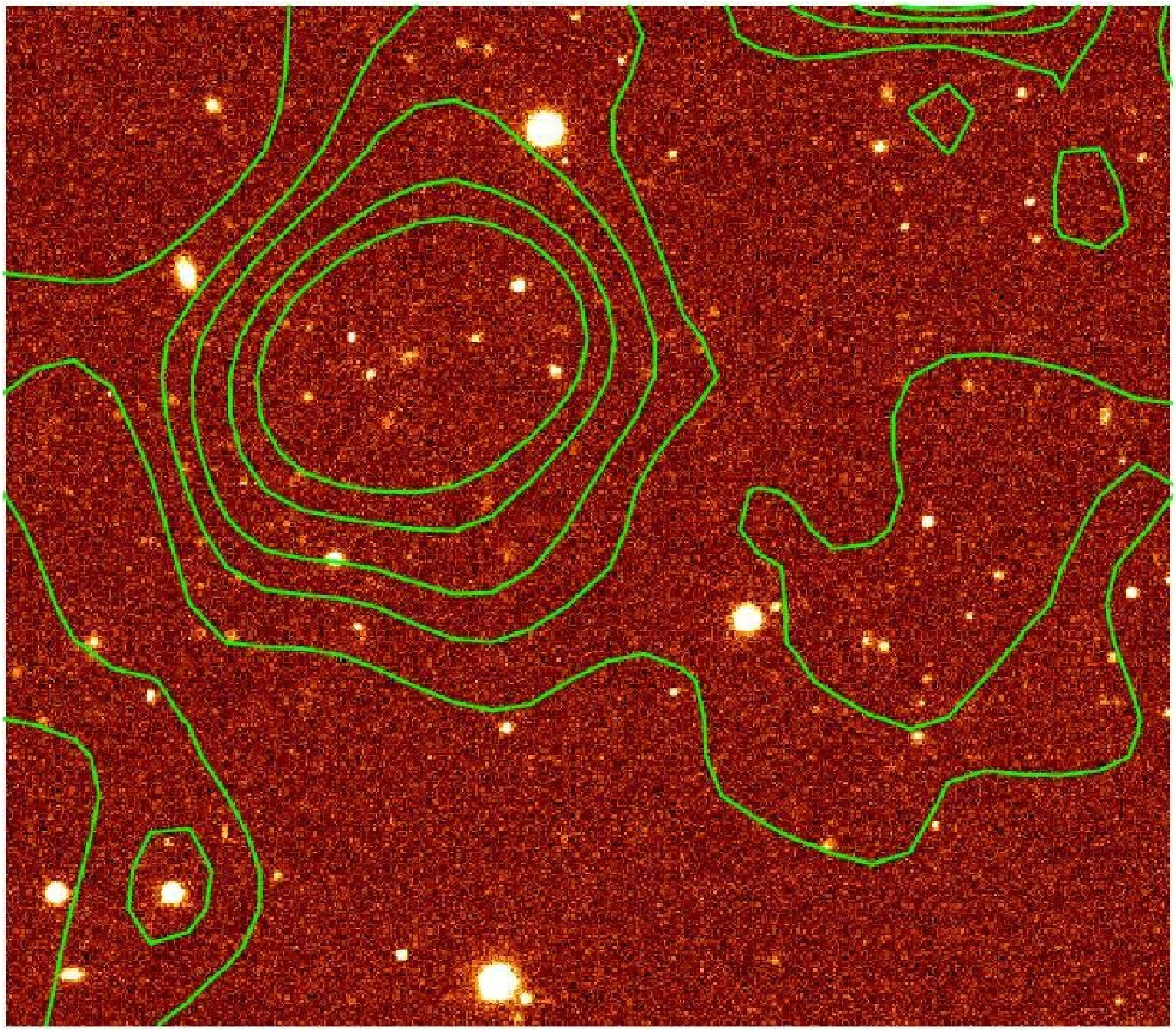

Figure 18: In this figure we plot as an example the optical SDSS image (cluster loccated at $z = 0.67$) is overlaid with the X-ray contours from the XMM data.

CONCLUSIONS

- **We are living in a flat, accelerating Universe with** 30% **matter (baryonic+dark),** 70% **dark energy,** $H_\circ \simeq 70$**Km/sec/Mpc and** $T_\circ \simeq 13.7$**Gyr.**

- **Rich clusters of galaxies formed at** $z \sim 2$ **in Cold Dark Matter models with dark energy (negative pressure).**

- **The cluster formation rate in quintessence models** $(-1 < w < 0)$ **is an intermediate case between the open and** Λ**CDM models respectively.**

- **XMM cluster project (Basilakos, Plionis, Georgantopoulos, Georgakakis, Kolokotronis, Gaga, G. C. Stewart-Un. of Leicester, T. Shanks-Un. of Durham)**

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