Rotating stars and wave extraction: applications of curvature invariants in astrophysical relativity

M Bruni, F White (Portsmouth)

C Beetle (Florida)

E Berti (St. Louis)

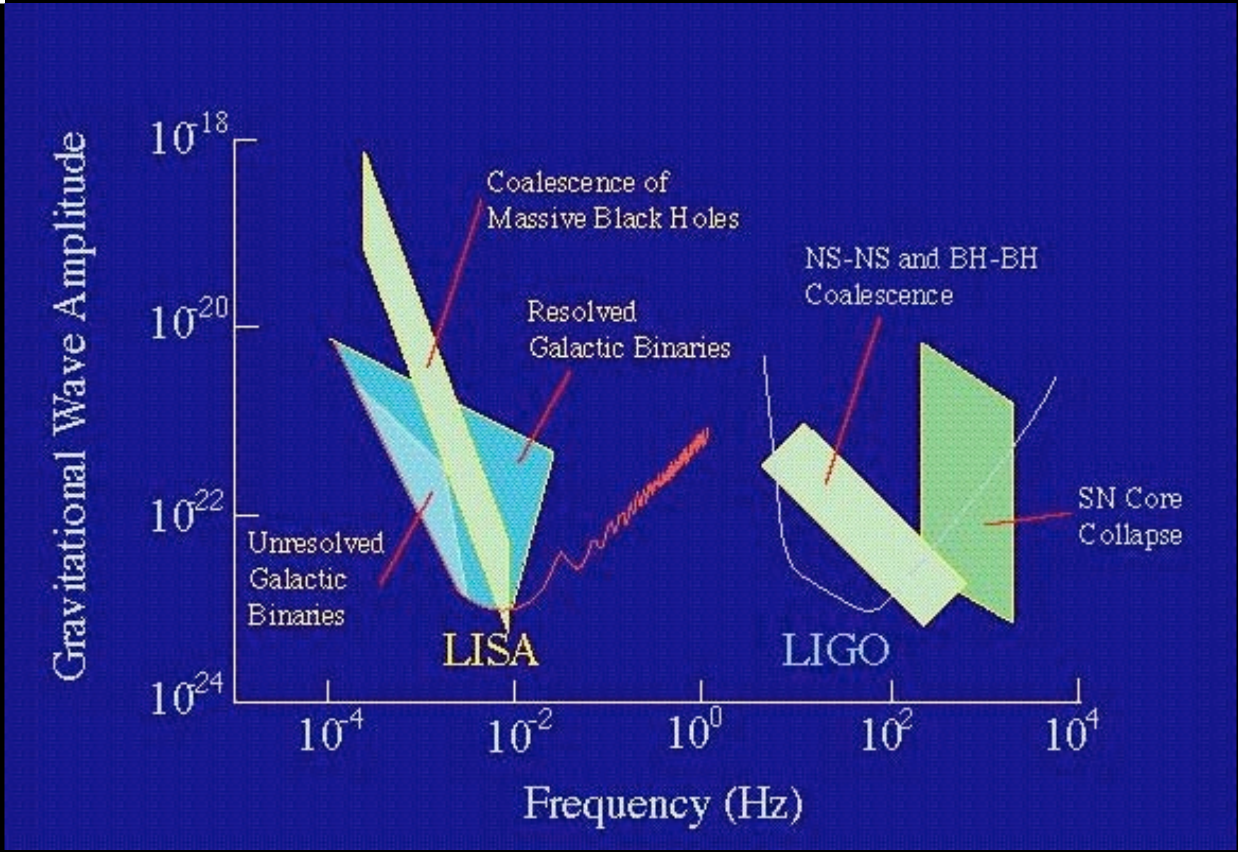
LM Burko (Utah)

M Maniopoulou (Sot'on)

A Nerozzi (Texas)

D Pollney (AEI, Golm)

General motivations



- Study of gravitational wave sources:
  - Isolated neutron stars, proto-NSs, SN remnants
  - BH-BH, BH-NS, NS-NS binaries

Weyl scalars and…

- Non-type-D-ness of rotating neutron stars

  Berti,White,Maniopoulou & MB, MNRAS, submitted (2004)

  gr-qc/0405146

- Wave extraction in numerical relativity

  Nerozzi, Beetle, MB, Burko & Pollney, PRD, submitted (2004)

  gr-qc/0407013

  Beetle, MB, Burko & Nerozzi, PRD, submitted (2004)

  gr-qc/0407012

  A Nerozzi, M Bruni & V Re, in preparation (2004) (A Nerozzi, PhD thesis, Portsmouth, October 2004)

RNS: motivations

- Check reliability of Hartle-Thorne slow-rotation approximation, comparing with numerical RNS.
- Develop perturbation theory for rotating neutron stars (RNS) a-la-Teukolsky (Kerr perturbations).
- Compare Manko metric and numerical RNS.
- Spherical stars and Kerr: Petrov type D, crucial for Teukolsky approach.
- RNS: 1) check that Petrov type is general (I) as expected (Fodor & Perjes 2000, GRG 32, 2319) ;

  2) check that the deviation from type D is small, and quantiyfy this deviation.

Wave extraction: motivations

- Aim of simulations of sources of GW is the computation of signal at large distance
- “Wave extraction”, i.e. extraction of GW signal from numerical simulations, is in general non-trivial because of gauges used in the initial value formulation
- Various approaches:
  1) quadrupole formula (Newtonian and spatially conformally flat simulations);
  2) Guess a background spacetime and use perturbation theory;
  3) In characteristic approach, use Ψ_4 and/or news function.
- New approach: do not assume a background, compute Ψ_4 directly in an appropriately defined frame:
  – Quasi-Kinnersley-Frame
- Test the method in a studied case, describing a non-linearly perturbed Black Hole, with a reliable code (“Bondi code” Papadopoulos 2002, PRD 65, 084016) using null coordinates.

Basic notions I

- Weyl scalars Ψ_N (N=1..5): 5 complex scalars, components of the Weyl tensor on a null Newman-Penrose tetrad:

$$\Psi_0 = C_{pqrs}\ell^p m^q \ell^r m^s$$
$$\Psi_1 = C_{pqrs}\ell^p n^q \ell^r m^s$$
$$\Psi_2 = C_{pqrs}\ell^p m^q \bar{m}^r n^s$$
$$\Psi_3 = C_{pqrs}\ell^p n^q \bar{m}^r n^s$$
$$\Psi_4 = C_{pqrs}\bar{m}^p n^q \bar{m}^r n^s$$

- I(Ψ), J(Ψ): scalar curvature invariants (tetrad independent):

$$I = \Psi_4\Psi_0 - 4\Psi_3\Psi_1 + 3\Psi_2^2 \qquad J = det\begin{vmatrix} \Psi_4 & \Psi_3 & \Psi_2 \\ \Psi_3 & \Psi_2 & \Psi_1 \\ \Psi_2 & \Psi_1 & \Psi_0 \end{vmatrix}$$

Basic notions II

- Petrov classification
  - "frame": equivalence (under "type III" null rots.) class of tedrads
  - Type I: in general all Ψs non-zero, but "transverse frames" exists, such that $\Psi_1 = \Psi_3 = 0$
  - Type D: canonical (Kinnersley) frame exists, with Ψ_2 the only non-zero WS.
  - Speciality Index: $S = 27J^2/I^3$
  - S=1 for type D (and other special types).
- Quasi-Kinnersley Frame (QKF)
  - Radiation scalar: $\xi = \Psi_0\Psi_4$
  - QKF: a transverse frame such that

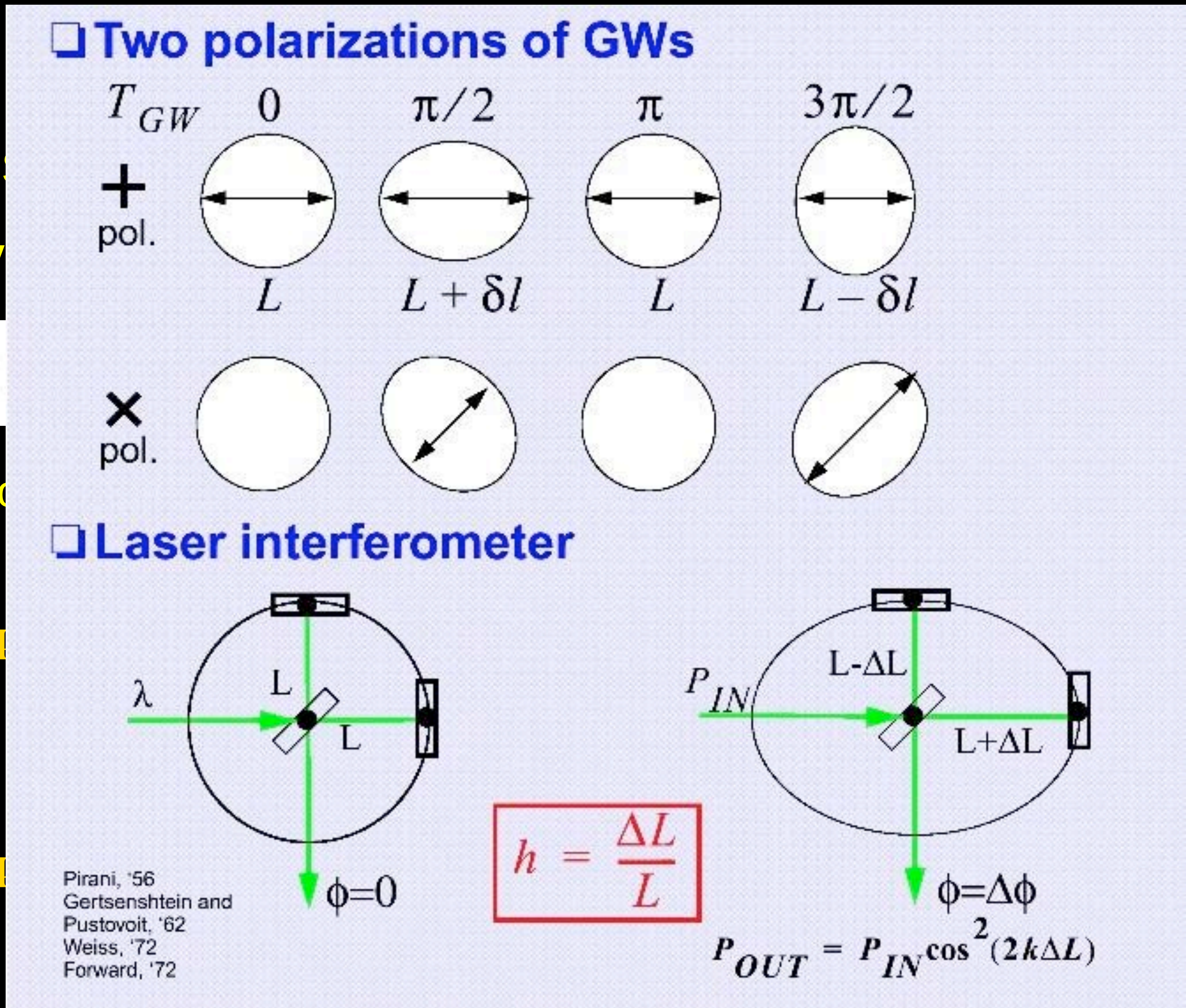
$$\xi \to 0 \quad for \quad S \to 1$$

Basic notions III: Physical significance of WS

- Geode... $u^c u^d$
- Transv...

$E^{pq} = \ldots)e^{pq}_{\mathrm{T}\times}$

  - Ψ_2 "co...
  - Ψ_0 , Ψ...
  - Ψ_1 , Ψ...



The Hartle-Thorne approx.

slow-rotation expansion in $\varepsilon = \Omega / \Omega^*$

- $\Omega^* = \sqrt{M / R^3}$ **Keplerian frequency** of test particle on equator;
- for Newtonian polytropes close to the Keplerian mass shedding frequency;
- so we (roughly) expect the approximation to break down when $\varepsilon \approx 1$

- $\varepsilon = 0$ Standard Tolman-Oppenheimer-Volkoff equations;
- $O(\varepsilon) \Rightarrow J^{(\varepsilon)} = \varepsilon \overline{J}$ Frame-dragging corrections; exterior=Kerr;
- $$O(\varepsilon^2) \Rightarrow \begin{cases} Q^{(\varepsilon)} = \varepsilon^2 \overline{Q} \\ M^{(\varepsilon)} = M + \varepsilon^2 \overline{\delta M} \\ R^{(\varepsilon)} = R + \varepsilon^2 \overline{\delta R} \end{cases}$$

Corrections to sphericity:
The stellar exterior differs from Kerr because of the quadrupole moment Q

Non-type-D-ness of HT

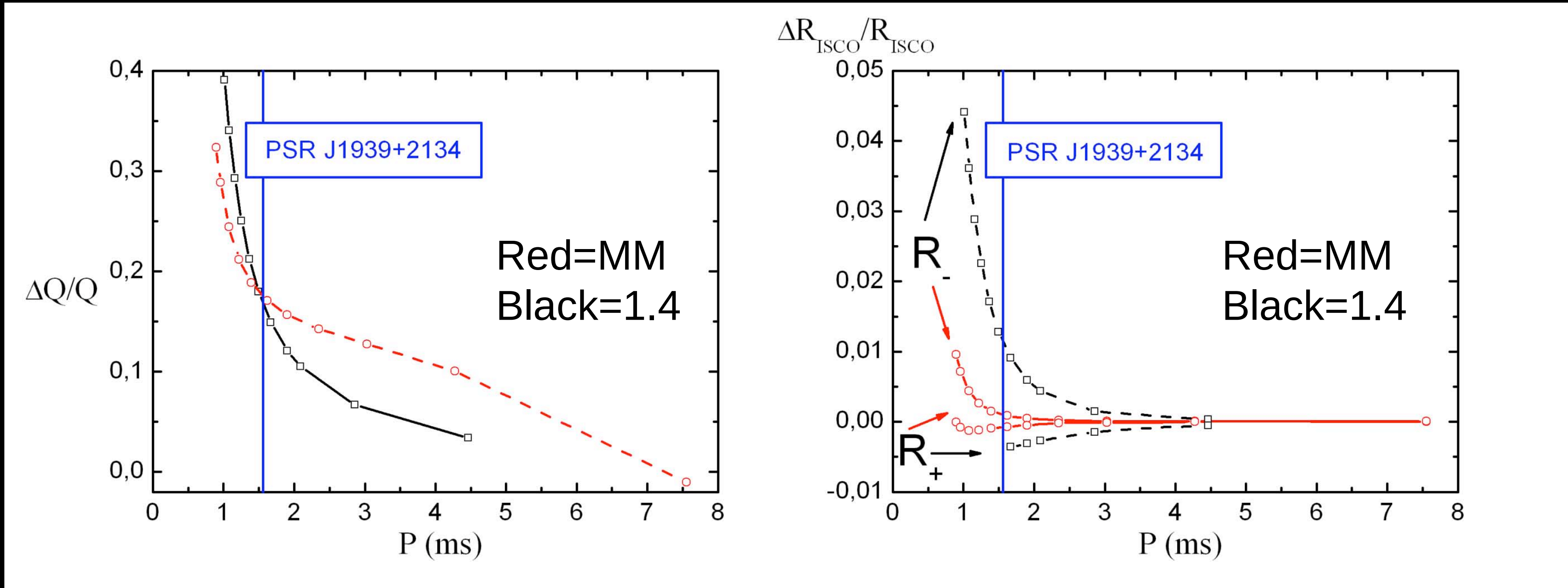
- Def: $\Delta Q = Q - Q_{\mathrm{Kerr}}$ $\quad Q_{\mathrm{Kerr}} = J^2/M$
- WS for HT: QKF with $\Psi_{04} \equiv \Psi_0 = \Psi_4$
  - $(\Psi_{04})_{HT} = F_1(r,\theta;M)\Delta Q$
  - $$(\Psi_2)_{HT} = -\frac{M}{r^3} - \frac{3i\cos\theta J}{r^4} + F_2 J^2 + F_3 \Delta Q$$
- SI: $$1 - S = 3\left(\frac{\psi_{04}^{(2)}}{\psi_2^{(0)}}\right)^2 \epsilon^4 + \mathcal{O}(\epsilon^5)$$

issues:
- PC & perturbations
- PC: global vs local

- Leading terms at large distance:

$$(\psi_{04})_{HT} \sim \frac{3\sin^2\theta(Q - Q_{\mathrm{Kerr}})}{2r^5} \quad (\psi_2)_{HT} \sim -\frac{M}{r^3} \quad \left|\frac{\psi_{04}}{\psi_2}\right| \sim \frac{3\sin^2(\theta)}{2Mr^2}\Delta Q$$

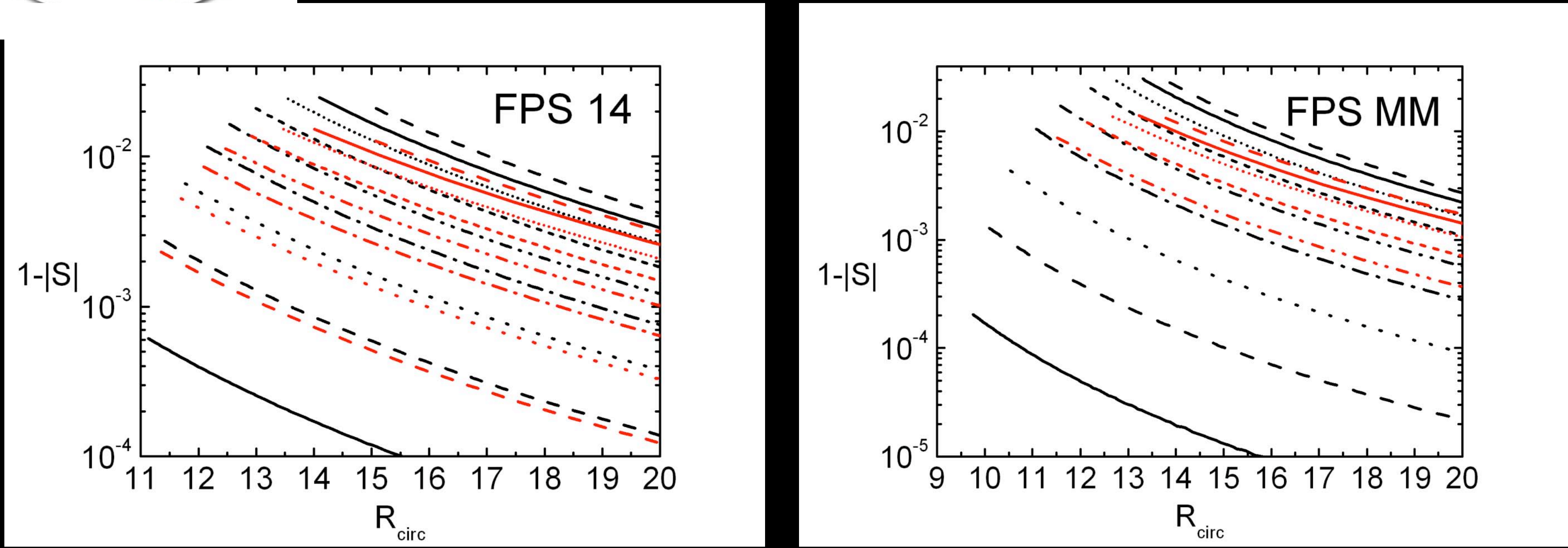
Accuracy of Hartle-Thorne models: astrophysical relevance



Even for the fastest known millisecond pulsar, PSR J1939+2134:

1) **DQ/Q is smaller than ~20 %**
2) **DR/R for (corotating/counterrotating) ISCOs is smaller than ~1%**

Non-Type-D-ness



Black lines: "speciality index" S for the numerical models

* Far away from the star the spacetime becomes special (D, S->1)

* Fast rotating stars "deviate more from speciality" (effect of the quadrupole moment)

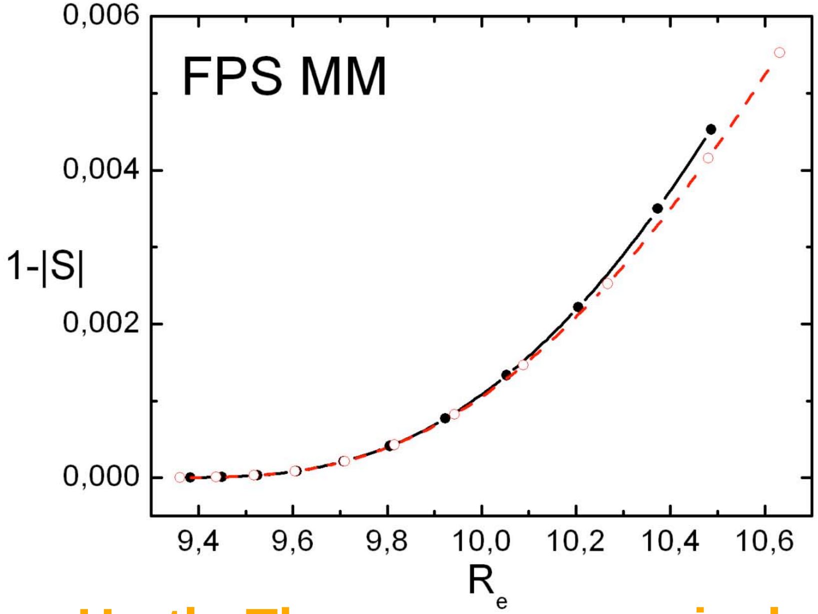
Red lines: "speciality index" S for the Manko matching models

* They are qualitatively similar to the full numerical models

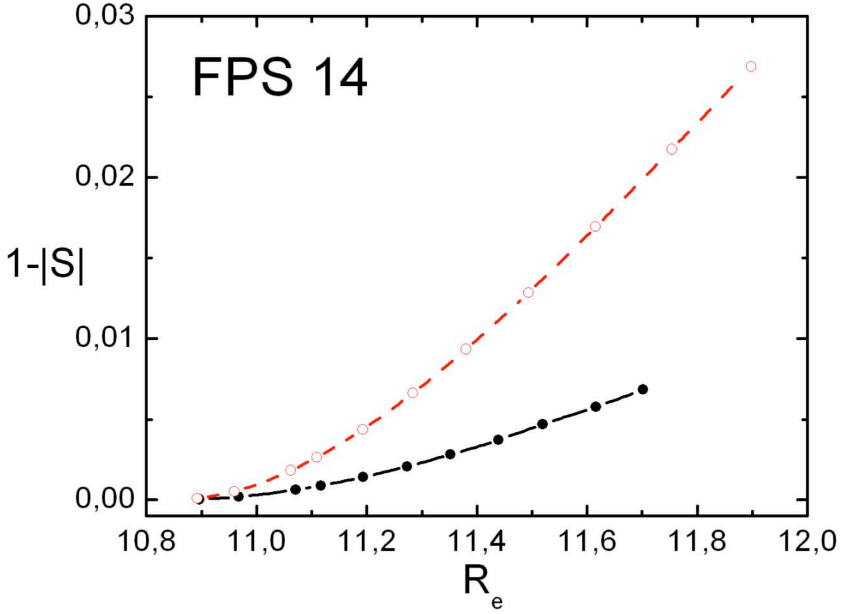
* They are typically **closer to Type D**

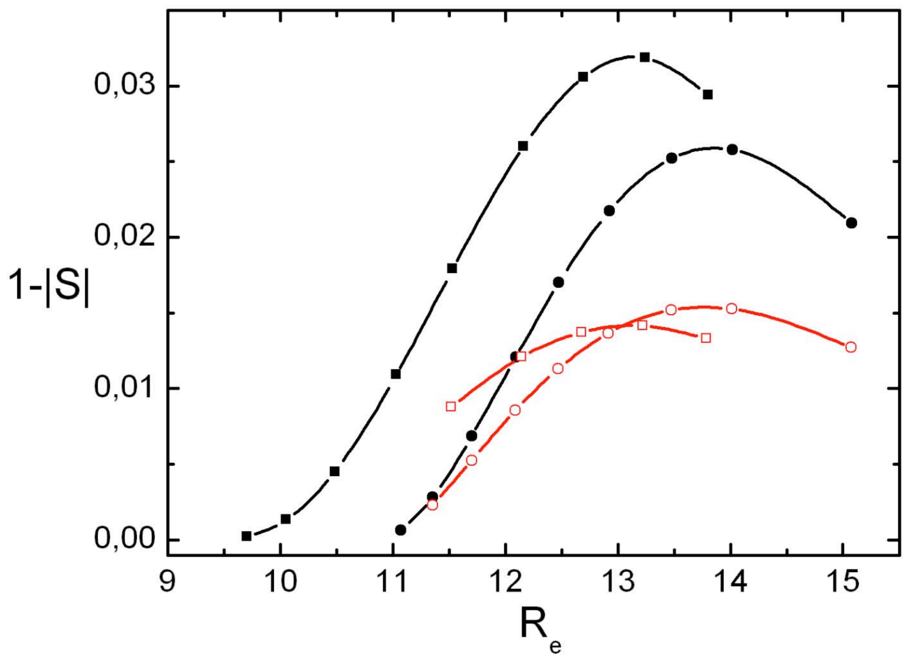
A single number for the Non-Type-D-ness of a star?

Manko vs. numerical: the idea doesn't work!



Hartle-Thorne vs. numerical: ok!





News function: Ψ_4 vs $\gamma_{,vv}$

- γ: unconstrained metric function on Bondi metric; Initial data on a light cone;
- $\gamma_{,v}$ known as "news function", carries info on the radiated energy:

$$\frac{dE_{tot}}{dv} = -\frac{1}{4\pi}\int (\gamma_{,v})^2 sin\theta d\theta d\phi$$

- We use a Bondi axisymmetric code, with

$$\Psi_4 = -\frac{1}{2}\left(\frac{\partial^2 h_{\theta\theta}^{TT}}{\partial v^2}\right) , \qquad h_{\theta\theta}^{TT} = 2\gamma \ ;$$

- In the linearized regime (one polarization only); we must have

$$\Psi_4 = -\frac{\partial^2 \gamma}{\partial v^2}$$

- This is the statement that we want to verify numerically

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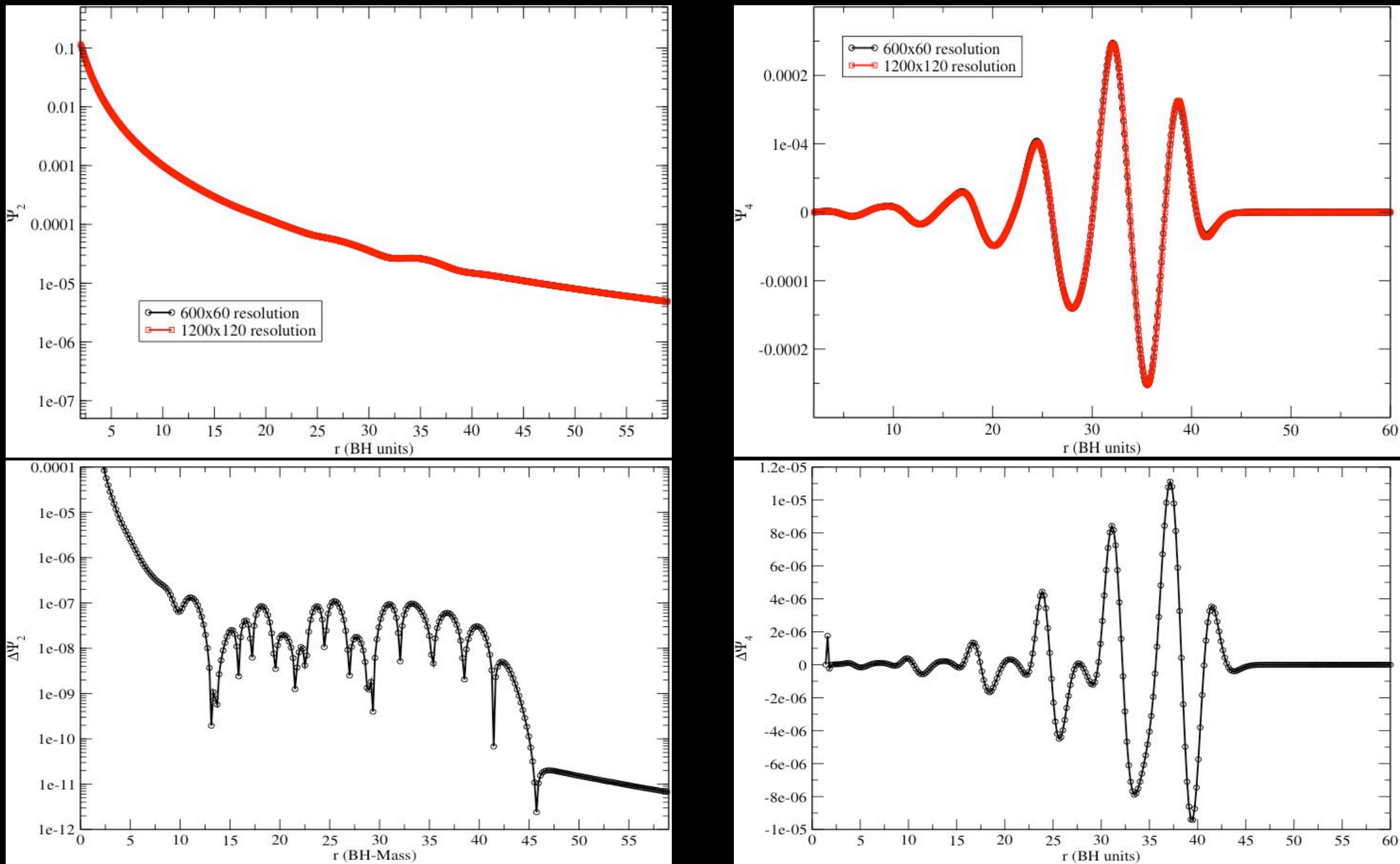
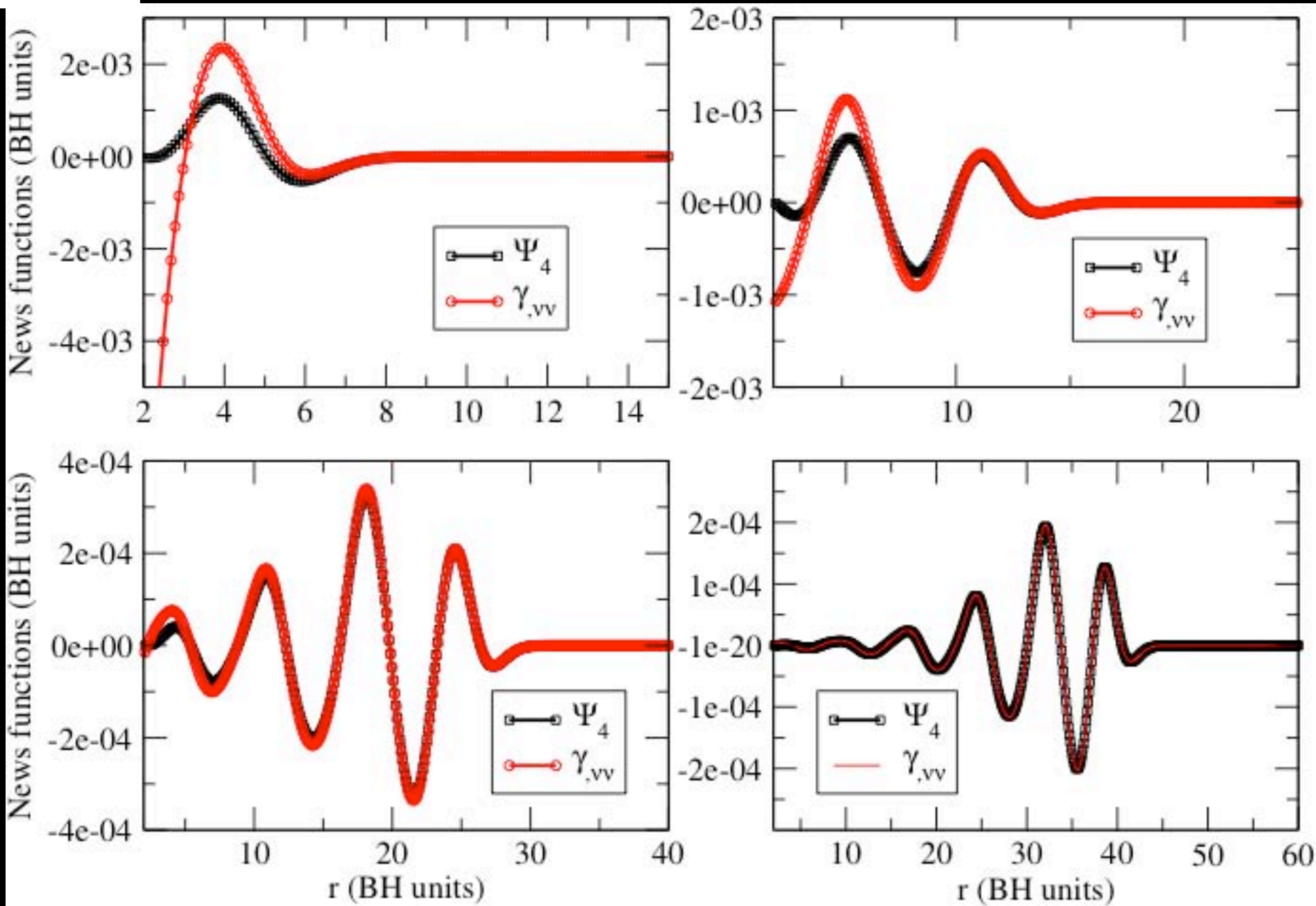
News function: Ψ_4 vs $\gamma_{,vv}$



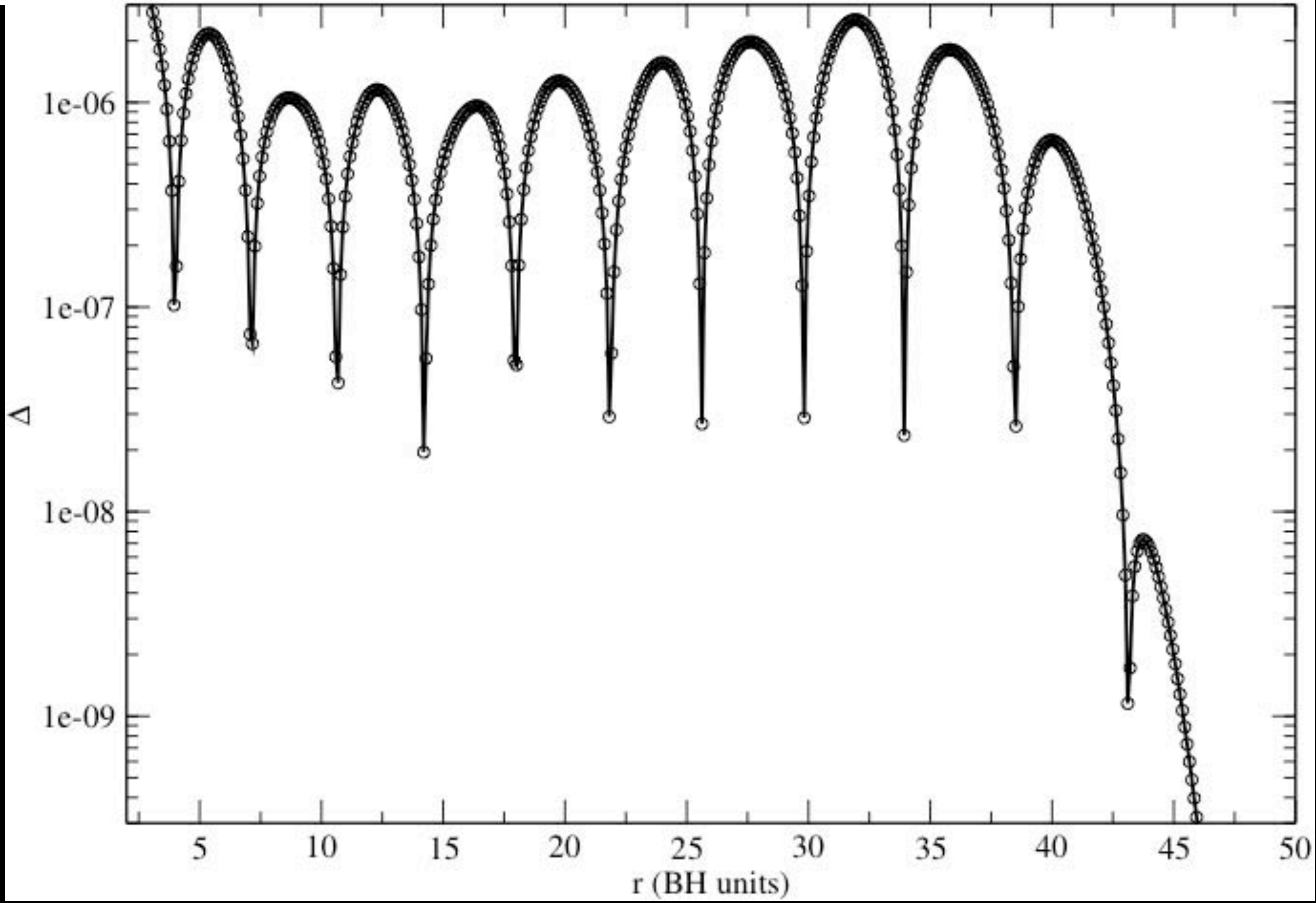
V=80: $\Psi2$ and $\Psi4$ at two different resolutions, and $\Delta\Psi2$, $\Delta\Psi4$

News function: Ψ_4 vs $\gamma_{,vv}$



Ψ_4 vs $\gamma_{,vv}$ for v_0=1, v_1=20, v_2=50, v_3=80

News function: Ψ_4 vs $\gamma_{,vv}$



The difference $\Delta = |\Psi_4 + \gamma_{,vv}|$ of the two news at $v_3=80$

Conclusions

- QKF defined, general way of computing it found using NP formalism
- Scheme to implement the QKF in ADM formulation worked out
- Work in progress on how to chose in a general context (simulation) a specific tetrad from the QKF frame
- Direct use of QKF in Bondi coordinates gives wave extraction from Ψ_4, in agreement with previous “metric based” work using the “news function”
- HT approximation: in general very good up to rotational speeds of the fastest observed ms pulsar
- Deviation from type D of spacetime of RNS very small
- A perturbative analysis of RNS a-la-Teukolsky is viable