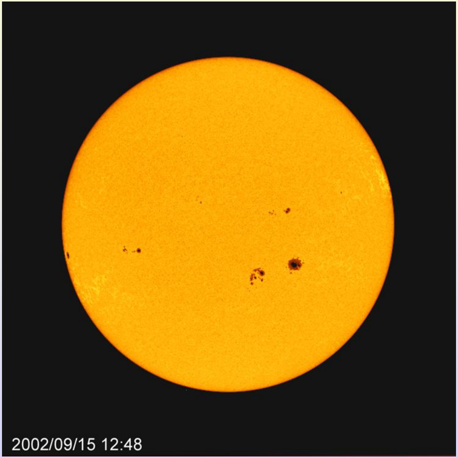
Simplified simulations of MHD turbulence in a coronal loop

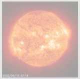
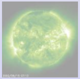
Éric Buchlin

Dipartimento di Astronomia e Scienza dello Spazio
Università di Firenze, Arcetri, Florence, Italy

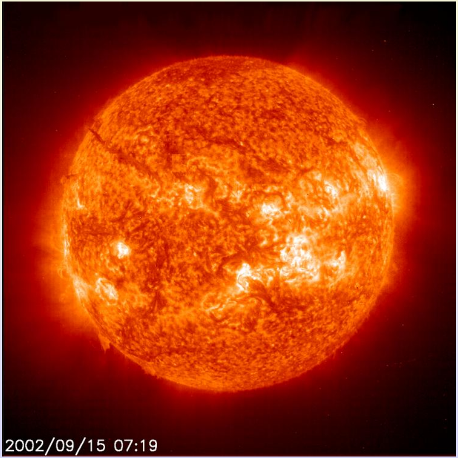
Α. Π. Θ.
2 Feb 2005

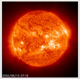
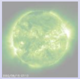
The Solar atmosphere



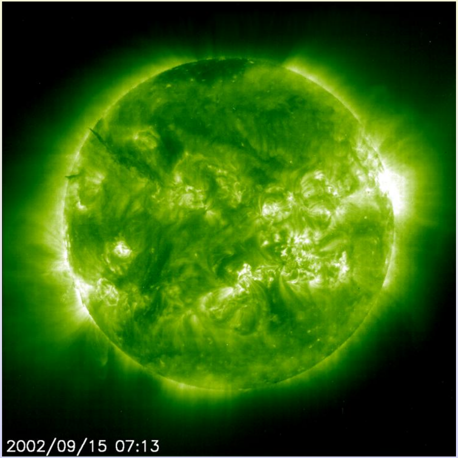
-  Visible surface (photosphere), 6000 K
-  High chromosphere / transition region, 50 000 K, altitude: 2000 km
-  Corona, $> 10^6$ K, altitude: > 5000 km.

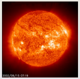
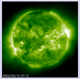
The Solar atmosphere



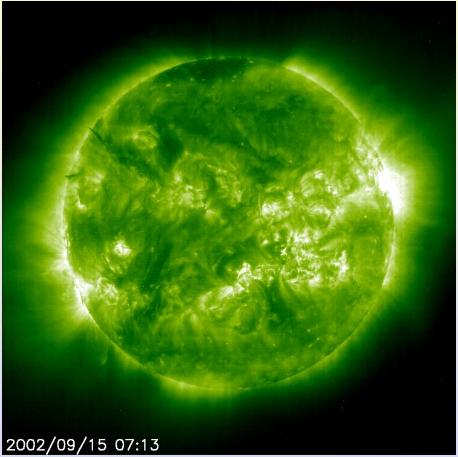
-  Visible surface (photosphere), 6000 K
-  High chromosphere / transition region, 50 000 K, altitude: 2000 km
-  Corona, $> 10^6$ K, altitude: > 5000 km.

The Solar atmosphere



-  Visible surface (photosphere), 6000 K
-  High chromosphere / transition region, 50 000 K, altitude: 2000 km
-  Corona, $> 10^6$ K, altitude: > 5000 km.

Coronal structures

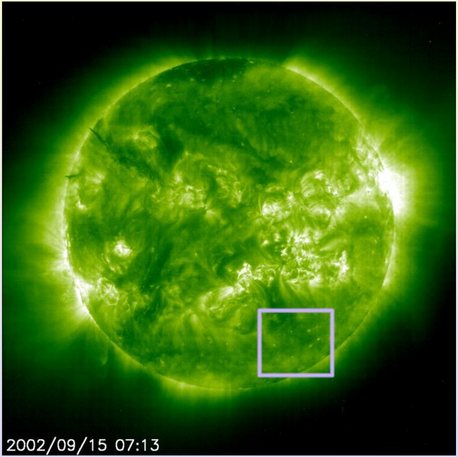


EIT 195, 2002

- Active regions
- Quiet Sun, bright points

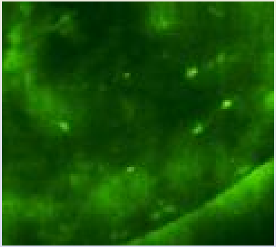
High energy dissipation $\longrightarrow$ bright structure in UV

Coronal structures



EIT 195, 2002

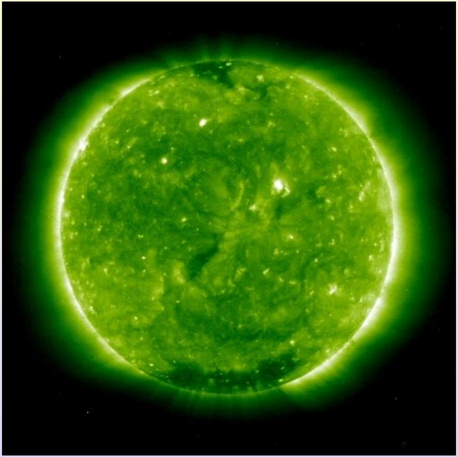
- Active regions
- Quiet Sun, bright points



Structures even smaller may exist

High energy dissipation $\longrightarrow$ bright structure in UV

Coronal structures



EIT 195, 1997

- Active regions
- Quiet Sun, bright points

Structures even smaller may exist

High energy dissipation $\longrightarrow$ bright structure in UV

The coronal heating problem

- The corona is very hot (known since 1943)
- The Sun produces enough energy (in its core) to heat it

But:

- This energy needs to be transported to the corona, in a non-thermal way. Solutions:
  - Sound waves and slow magnetosonic waves excluded (do not reach the corona)
  - Fast magnetosonic and Alfvén waves
- It needs then to be dissipated, but:
  - The observed events of energy dissipation are not enough
  - The physical mechanisms of dissipation:
    - wave-particle interactions
    - reconnexion, resistivity (Joule)

    are too slow (not enough efficient)

The coronal heating problem

- The corona is very hot (known since 1943)
- The Sun produces enough energy (in its core) to heat it

But:

- This energy needs to be transported to the corona, in a non-thermal way. Solutions:
  - Sound waves and slow magnetosonic waves excluded (do not reach the corona)
  - Fast magnetosonic and Alfvén waves
- It needs then to be dissipated, but:
  - The observed events of energy dissipation are not enough
  - The physical mechanisms of dissipation:
    - wave-particle interactions
    - reconnexion, resistivity (Joule)

    are too slow (not enough efficient)

The coronal heating problem

- The corona is very hot (known since 1943)
- The Sun produces enough energy (in its core) to heat it

But:

- This energy needs to be transported to the corona, in a non-thermal way. Solutions:
  - Sound waves and slow magnetosonic waves excluded (do not reach the corona)
  - Fast magnetosonic and Alfvén waves
- It needs then to be dissipated, but:
  - The observed events of energy dissipation are not enough
  - The physical mechanisms of dissipation:
    - wave-particle interactions
    - reconnexion, resistivity (Joule)

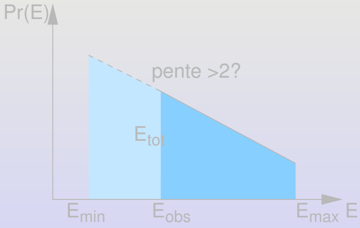
    are too slow (not enough efficient)

How to (finally) solve the problem?

A possible way: small scales (spatial and temporal)

- Dissipation mechanisms are more efficient at small scales (nanoflares, Parker 1988)
- The smallest events (non-observables) contribute perhaps the most to the heating of the corona

- Diffusion time: L^2/η, with $\eta \approx 1\,\mathrm{m^2/s}$
- Hudson (1991):

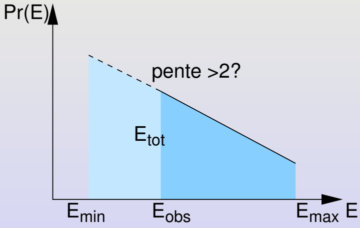


How to (finally) solve the problem?

A possible way: small scales (spatial and temporal)

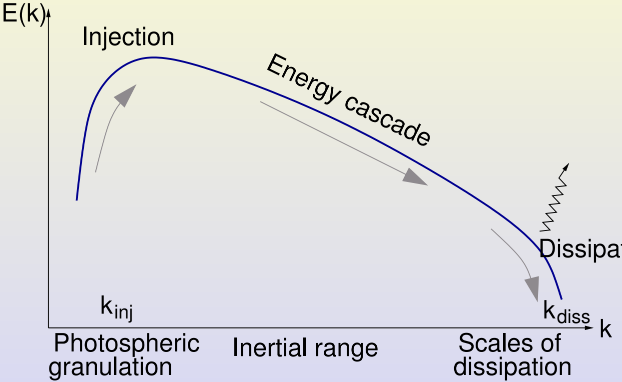
- Dissipation mechanisms are more efficient at small scales (nanoflares, Parker 1988)
- The smallest events (non-observables) contribute perhaps the most to the heating of the corona

- Diffusion time: L^2/η, with $\eta \approx 1\,\mathrm{m}^2/\mathrm{s}$
- Hudson (1991):



The origin of small scales

May be created by turbulence:



In the corona: $R_e = UL/\eta \approx 10^{13} \gg 1$
(for $U = 1\,\mathrm{Mm/s}$, $L = 10\,\mathrm{Mm}$, $\eta = 1\,\mathrm{m^2/s}$)

Smallest (dissipative) scales: $10\,\mathrm{m}$!

Need for statistics of coronal heating

- Small flares, small scales:
  - cannot be observed directly
  - Hudson... $\longrightarrow$ small-scale statistics extrapolated from observations
- Theoretical description of turbulence: statistics are a privileged means of tackling its complexity

When used at the same time to analyze observations and simulations: a comparison is possible

Outline

Numerical models of a coronal loop

MagnetoHydroDynamics (MHD)

Starting-point: incompressible MHD

Equations, with $\rho_0 = 1$, $\boldsymbol{\nabla} \cdot \mathbf{v} = 0$ and $\boldsymbol{\nabla} \cdot \mathbf{B} = 0$:

$$\partial_t \mathbf{v} + (\mathbf{v} \cdot \boldsymbol{\nabla})\mathbf{v} = \frac{1}{\mu_0}(\boldsymbol{\nabla} \times \mathbf{B}) \times \mathbf{B} + \nu \boldsymbol{\nabla}^2 \mathbf{v} \tag{1}$$

$$\partial_t \mathbf{B} = \boldsymbol{\nabla} \times (\mathbf{v} \times \mathbf{B}) + \eta \boldsymbol{\nabla}^2 \mathbf{B} \tag{2}$$

Includes velocity $\mathbf{v}$, magnetic field $\mathbf{B}$, and:

- Non-linear terms, which allow the creation of small scales by turbulence
- Diffusion terms, which allow energy dissipation (mainly at small scales)
- Alfvén waves (as fluctuations)

Need for simplifications

Direct numerical simulations (DNS) of MHD:

- Small grid sizes ($\approx 1024^3$ max)
- Reynolds numbers too small (1000)
- Too slow (long computations for just a few events)

$\Longrightarrow$ Need for simplified simulations of MHD:

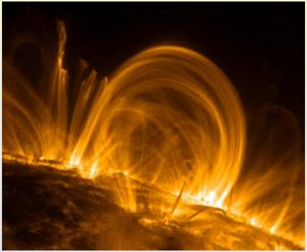
- Reduce number of spatial dimensions
  $\longrightarrow$ Einaudi, Velli, Georgoulis; Galtier...
- Reduce the number of active modes in turbulence, and the complexity of their interactions
  $\longrightarrow$ shell-models: Carbone, Giuliani...
  $\longrightarrow$ cellular automata: Lu & Hamilton; Isliker & Vlahos, Krasnoselskikh...

Coronal loops

Our models represent a coronal loop: loop of magnetic field, containing plasma.

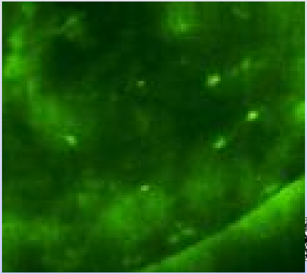
Large loops in active regions:

150 000 km

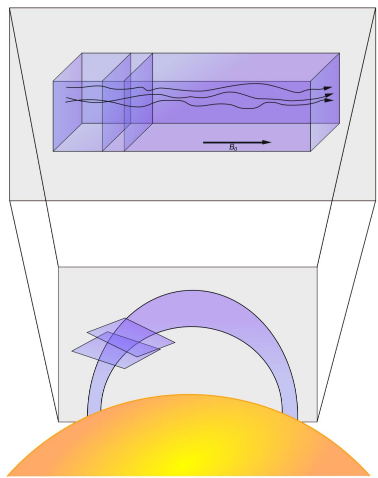


We are mainly interested in small loops of the quiet Sun (bright points and smaller):

120 000 km

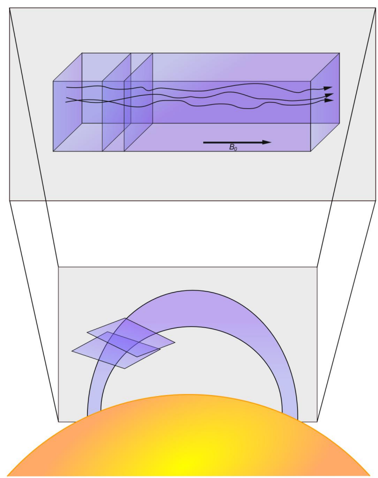


Common framework to our loop models



- The box represents a loop, MHD
- Weak forcing at the "photosphere", $\mathbf{\Pi} = -(\mathbf{v}_{\perp,\mathrm{ph}} \cdot \mathbf{B}_{\perp})\mathbf{B}_0/\mu_0$
- Propagation of Alfvén waves along $\mathbf{B}_0$ (along the loop)
- Non-linear interactions between these waves
- Energy dissipation

Common framework to our loop models

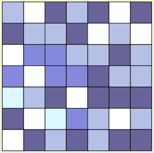


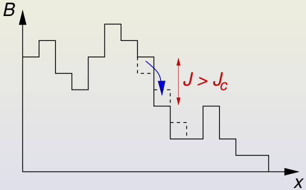
- The box represents a loop, MHD
- Weak forcing at the “photosphere”, $\mathbf{\Pi} = -(\mathbf{v}_{\perp,\mathrm{ph}} \cdot \mathbf{B}_\perp)\mathbf{B}_0/\mu_0$
- Propagation of Alfvén waves along $\mathbf{B}_0$ (along the loop)
- Non-linear interactions between these waves
- Energy dissipation

Simplification: simplification of the non-linear interactions in each cross-section of the loop:

1. Cellular automata
2. Shell-models

Coupled cellular automata

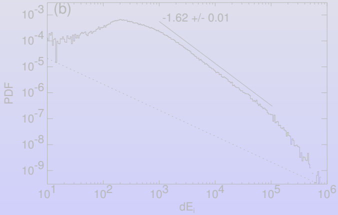


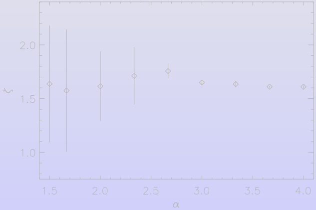


Each cross-section is a cellular automaton.

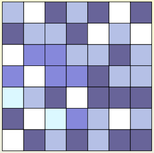
Non-linear interaction between Alfvén waves: avalanches, with threshold J_c on current density J.

$\longrightarrow$ Buchlin et al. A&A 2003





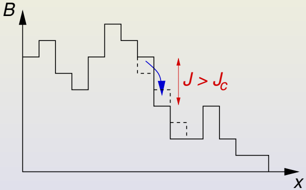
Coupled cellular automata

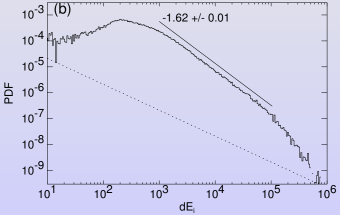


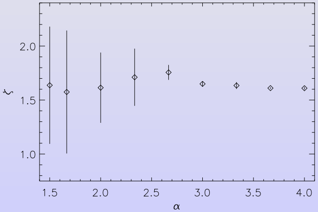
Each cross-section is a cellular automaton.

Non-linear interaction between Alfvén waves: avalanches, with threshold J_c on current density J.

$\longrightarrow$ Buchlin et al. A&A 2003

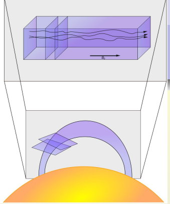




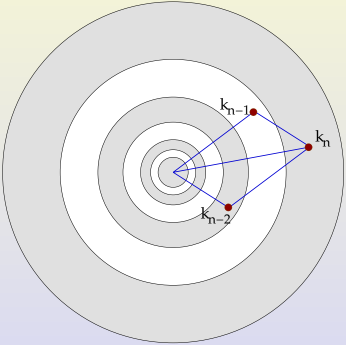


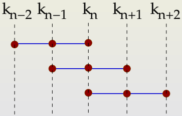
Coupled shell-models

Coupled shell-models



Each cross-section $\perp \mathbf{B}_0$ is a shell-model (Giuliani and Carbone, 1998):





- Logarithmic spacing of shells (modes) (2D Fourier space)
- Non-linear interactions between neighboring modes (triads)

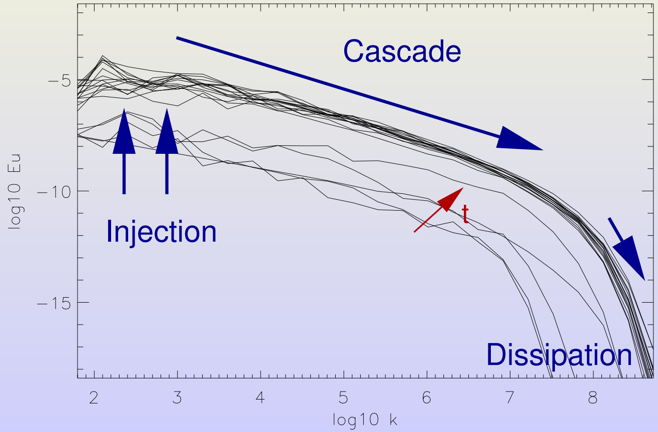
Advantages of shell-models

- Large spread of wavenumbers (of scales) with just a few modes (24)
  $\longrightarrow$ large Reynolds numbers (10^6 instead of ≈ 1000 for DNS)
  $\longrightarrow$ intermittency is possible
- Good model of local non-linear interactions between modes of MHD
- No free parameters (coefficients determined by conservation of 2D MHD invariants

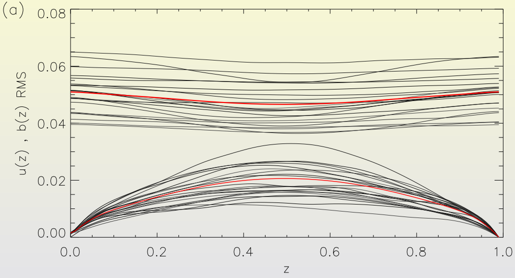
Spectrum development

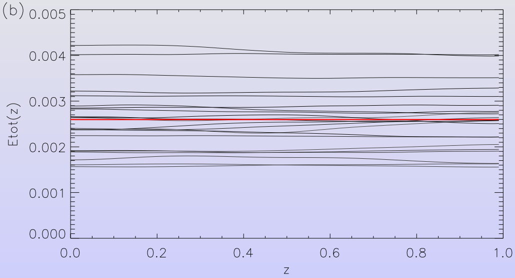
Small scales, created by turbulent cascade
(due to non-linear terms of MHD)

Spectra of $\perp$ kinetic energy in a loop cross-section as a function of time:

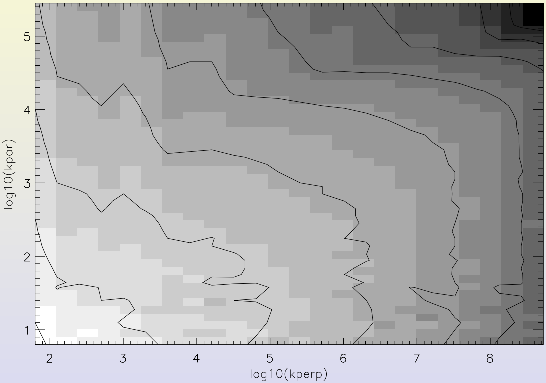


Profile of kinetic and magnetic energy along the loop

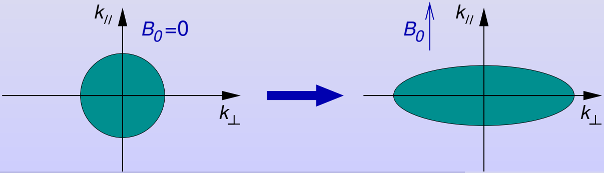




2D spectra



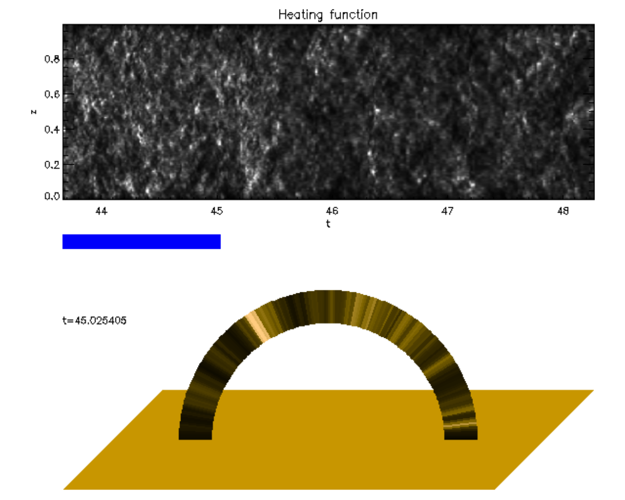
Obtained by Fourier transform along the loop.



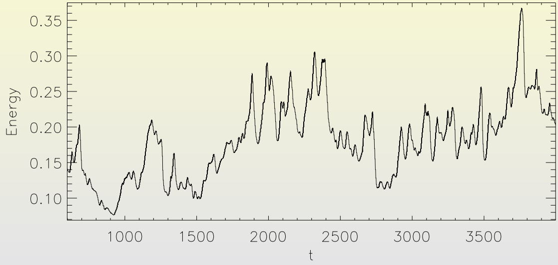
Anisotropic?

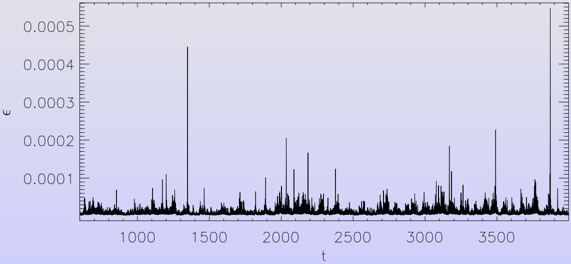
Heating function

Energy dissipation as a function of time and position along the loop:



Time series: energy and dissipation





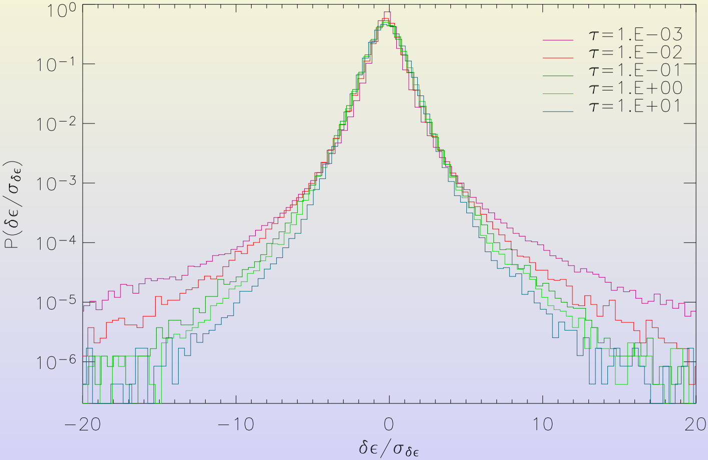
Physical values of model units:

- Time: $10\,\mathrm{s}$
- Energy: $10^{17}\,\mathrm{J}$
- Power: $10^{16}\,\mathrm{W}$

4 days of computation
$5 \cdot 10^7$ time steps
100 planes
(cross-sections)

Intermittency of the dissipation power time series

Distributions of $\delta_\tau \epsilon \equiv \epsilon(t+\tau) - \epsilon(t)$ for different τ's:



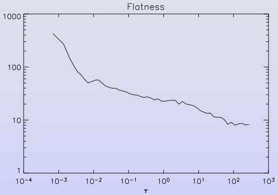
(normalized by their standard deviations)

Intermittency of the dissipation power time series (2)

$\delta_\tau \epsilon \equiv \epsilon(t+\tau) - \epsilon(t)$

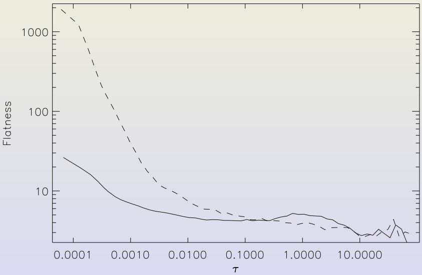
Intermittency:

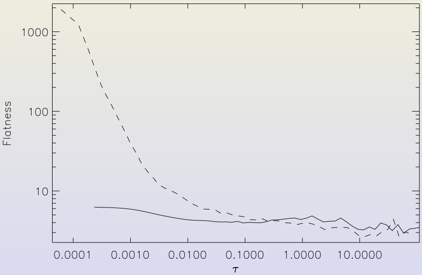
- Distributions of $\delta_\tau \epsilon$ have a shape which depends on the scale τ, Deviation from the Kolmogorov 1941 turbulence theory: Turbulence is not self-similar (fractal), but multi-fractal
- Exponents ζ_q of the structure functions $S^q(\tau) \equiv \langle |\delta_\tau \epsilon|^q \rangle \propto \tau^{\zeta_q}$ are a non-linear function of q.
- In particular, the flatness $F(\tau) \equiv S^4(\tau)/(S^2(\tau))^2$ grows at small scales:



Intermittency as a function of the parameters

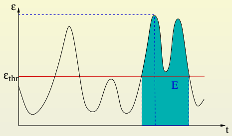
Reference run (- - -): $\nu = 10^{-13}$, $a = L/\ell = 10$



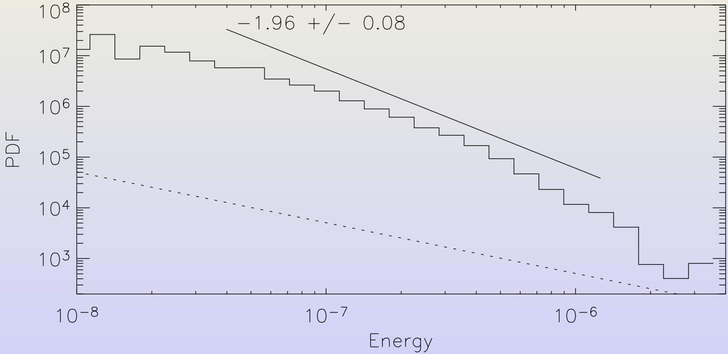


$\nu = 10^{-11}$ $a = 80$

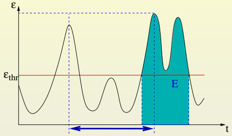
Distributions of events: energies



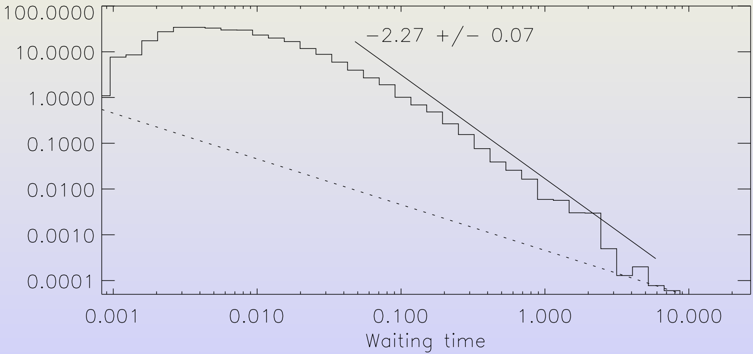
Distribution of energies of events
(defined by a threshold):



Distributions of events: waiting-times

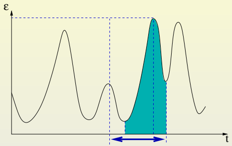


Distribution of waiting times between events (defined by a threshold):



Non-Poissonian, long-duration correlations

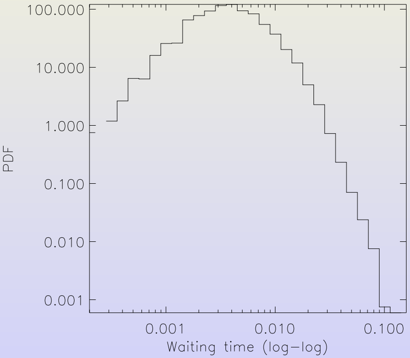
About the definition of events



Other definition:

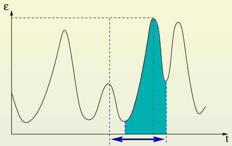
Each peak can be considered as an event.

Distribution of waiting-times:

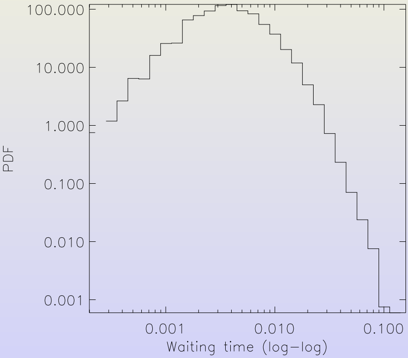


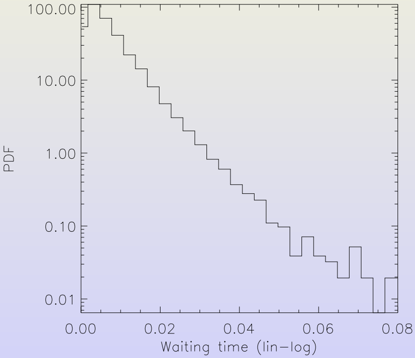
About the definition of events

Other definition:
Each peak can be considered as an event.



Distribution of waiting-times:





Still Poissonian?

About the definition of an event (2)

In the general case:

- Events statistics do depend on the definition of events (of course...)
  Thus conclusions about the Poisson nature of the flaring process, or about the nano-flares hypothesis, depend on the definition
- Higher sensibility of statistics to the definition
  when intermittency is low
- Are there really clear events in a time series / structures in a MHD field?
  If not, it is better to use statistics which do not need events to be defined (structure functions, spectra...)

⟶ Buchlin et al. 2005 (submitted to A&A)

Conclusions

Conclusions

- Models designed to explore coronal heating at small scales created by MHD turbulence.
- Simplified so as to be able to produce statistics
- First models of this type with geometry of a loop and energy loading at footpoints.

Some results of the shell-model:

- Spectra of turbulence, large Reynolds numbers
- Intermittency (also as a function of parameters of the model)
- Heating function: dissipation power as a function of time and position
- Events statistics distributed as power-laws: -2 for energy, -2.3 for waiting-times.
  But these distributions depend on the definition of an event

Conclusions

- Models designed to explore coronal heating at small scales created by MHD turbulence.
- Simplified so as to be able to produce statistics
- First models of this type with geometry of a loop and energy loading at footpoints.

Some results of the shell-model:

- Spectra of turbulence, large Reynolds numbers
- Intermittency (also as a function of parameters of the model)
- Heating function: dissipation power as a function of time and position
- Events statistics distributed as power-laws: -2 for energy, -2.3 for waiting-times.
  But these distributions depend on the definition of an event

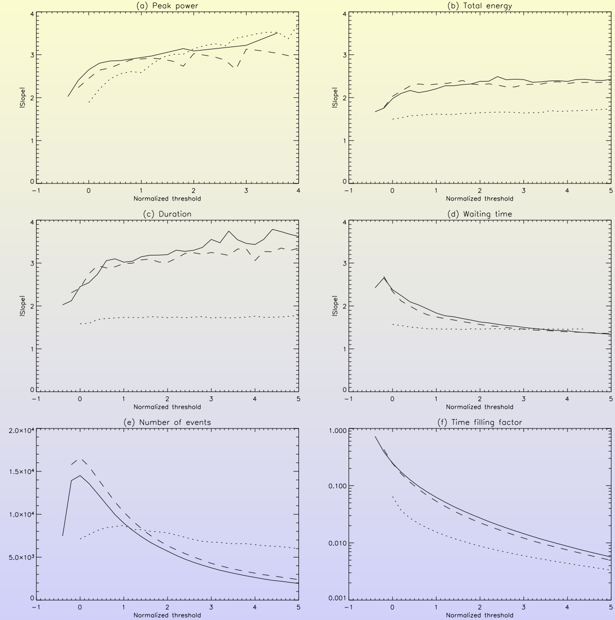
Current and future developments

To allow a better comparison between model output and observations:

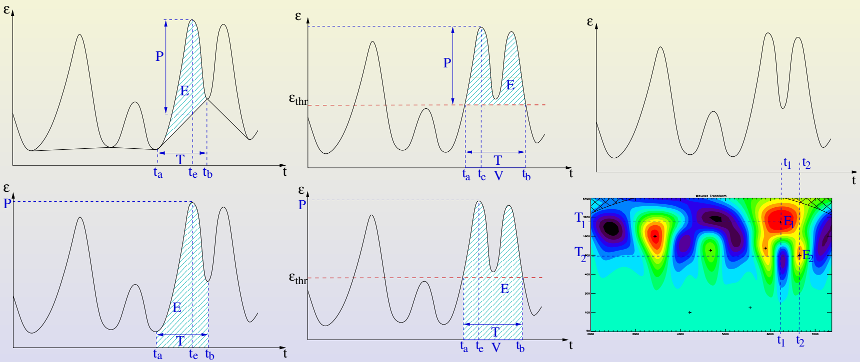
- Computation of radiative output
  (via thermodynamics and atomic models)
- More realistic:
  - Stratification: $B_0(z)$...
  - Parameters and energy input deduced from observations
- Produce images of the luminosity of the loop
  (geometry from extrapolations)

Appendix

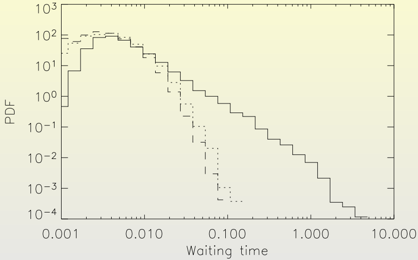
- Variation of the slope of the distributions as a function of the threshold
- Definitions of events
- Waiting-time statistics, definitions, and intermittency



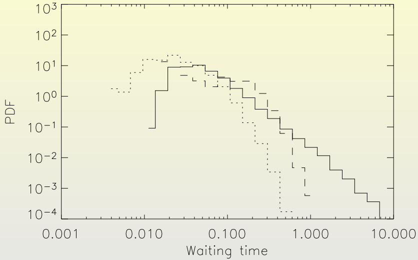
(a) Peak power
(b) Total energy
(c) Duration
(d) Waiting time
(e) Number of events
(f) Time filling factor
|Slope|
Normalized threshold


ε
P
E
T
t_a t_e t_b
t
ε
P
ε_{thr}
E
T
t_a t_e V t_b
t
ε
t_1 t_2
t
ε
P
E
T
t_a t_e t_b
t
ε
P
ε_{thr}
E
T
t_a t_e V t_b
t
Wavelet Transform
T_1
T_2
E_1
E_2
t_1 t_2

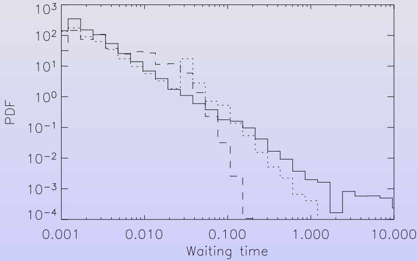
Time series $< 1 >$:



Time series $< 2 >$:



Time series $< 3 >$:



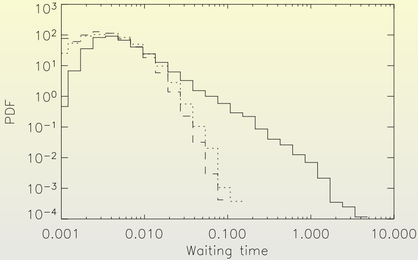
——: threshold

- - -: peaks

. . .: wavelets

More intermittency $\longrightarrow$ less sensitivity

Time series $< 1 >$:

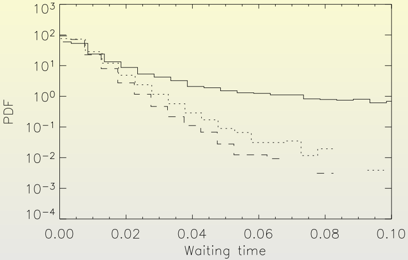


——: threshold

- - -: peaks

. . .: wavelets

Time series $< 1 >$:



——: threshold

- - -: peaks

. . .: wavelets