Dynamics of 3-body problem and applications to resonances in the Kuiper belt

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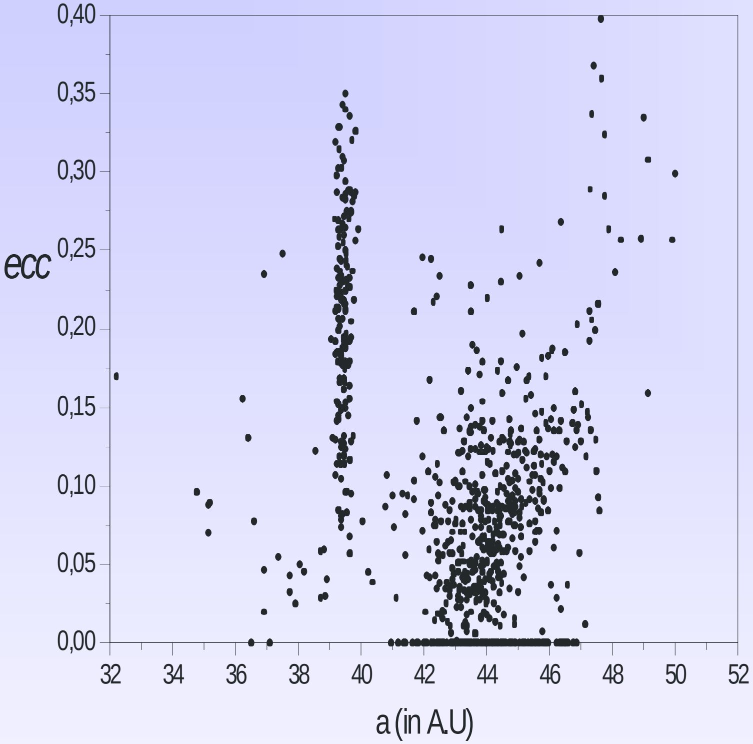
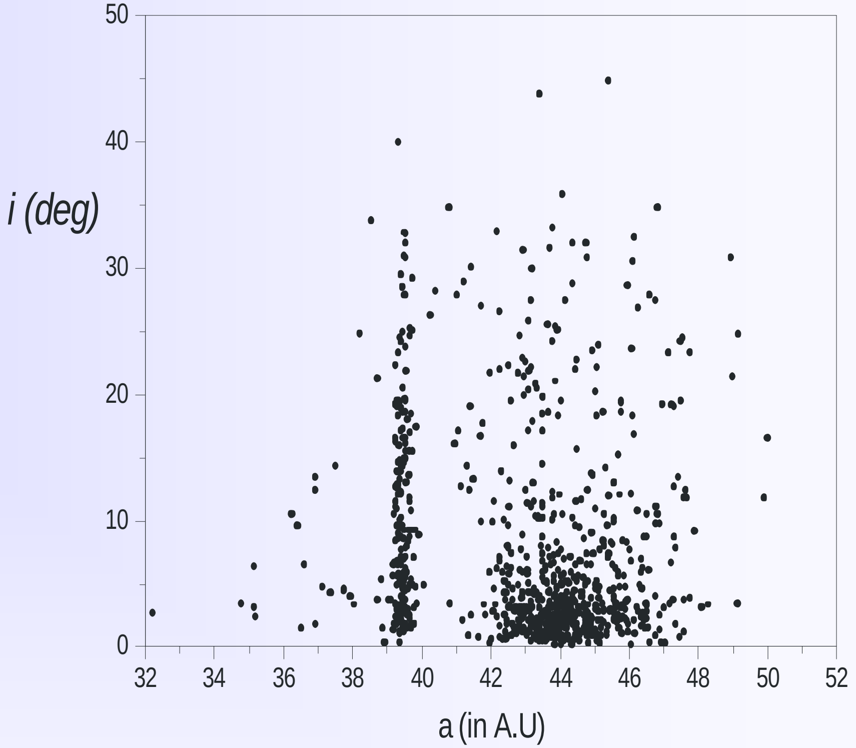
Distribution of TNOs in a-e & a-i planes





The Kuiper belt is divided into two parts:

(a) An internal zone with a<40 A.U. where stable orbits are connected with mean motion resonances of 1st order;

(b) An external zone with a>42 A.U. where the stable orbits are basically non-resonant

Remark: **Many KBOs *<e>≈0.05-0.10* but Plutinos *<e>≈0.21***

Outline

- Framework : Restricted 3-body problem (RTBP)
- Study : 1^{st}, 2^{nd} and 3^{rd} order resonances

  (Part A: **Symmetric** :2/3, 3/4, 4/5, 5/6, 6/7, 3/5, 5/7, 7/9, 4/7, 5/8, 7/10)

  (Part B: **Asymmetric** : 1/2, 1/3, 1/4, 1/5, 1/6)

- Periodic Orbits in the Planar Circular RTBP
- Surfaces of section in Planar Circular model
- Bifurcation Points from 2-D Circular to 2-D Elliptic and 3-D Circular RTBP
- Periodic Orbits in the 2-D Elliptic and 3-D Circular RTBP
- Surfaces of section in 3-D Circular model
- Periodic Orbits in the 3-D Elliptic RTBP
- Conclusions

Circular Planar RTBP

Classical Configuration : Rotating *Oxy* orthogonal system
Sun, Neptune as primaries (μ=0.00005178, T=2π)

Families of periodic orbits

•First kind: $e \approx 0$, n/n' varies along the family (Circular orbits)

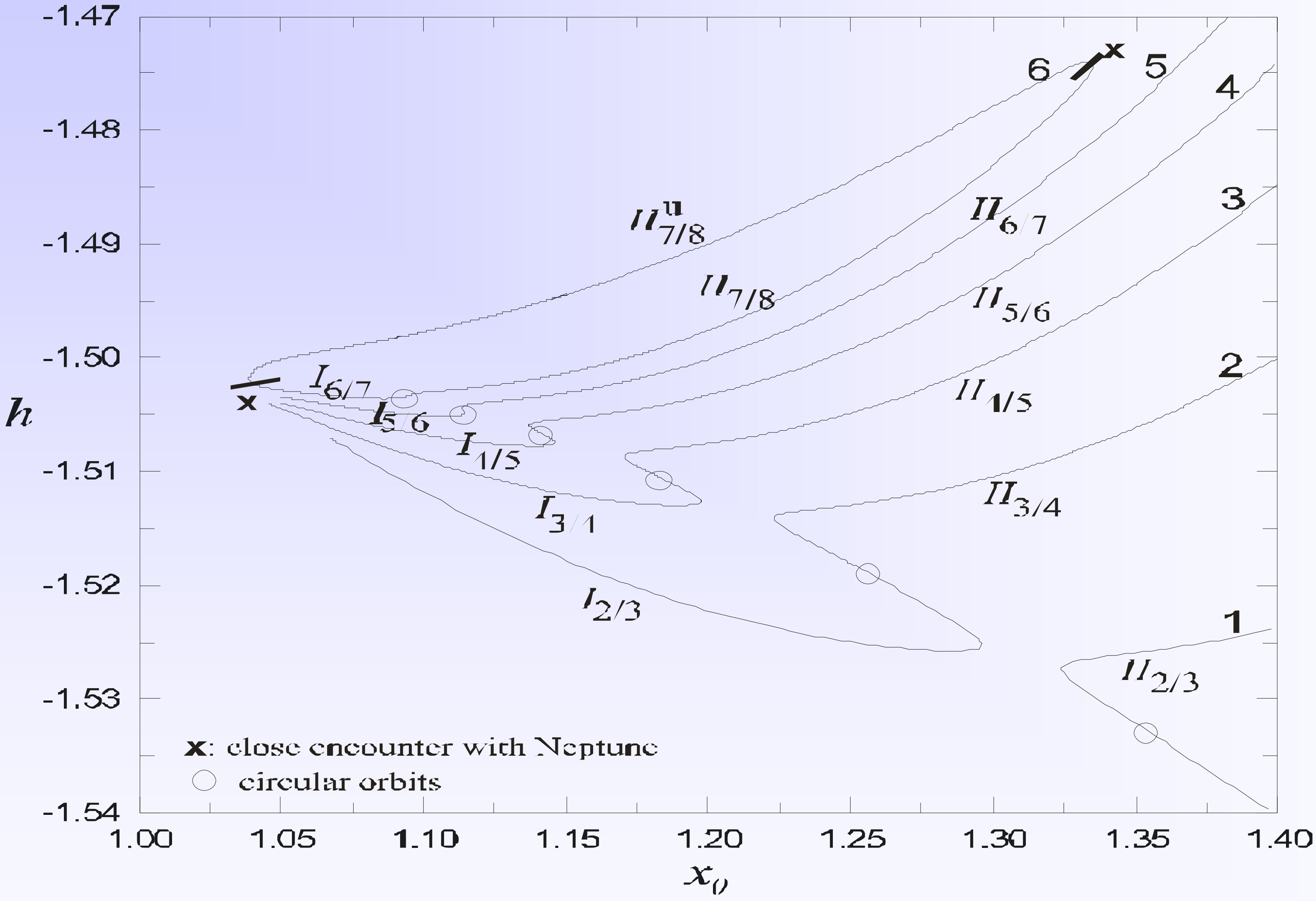
•Second kind: $e \neq 0$, $n/n' \approx$ const. (Elliptic Orbits)

General characteristics:

<u>First kind:</u> Continuation near the second, third –high order resonances for $\mu > 0$

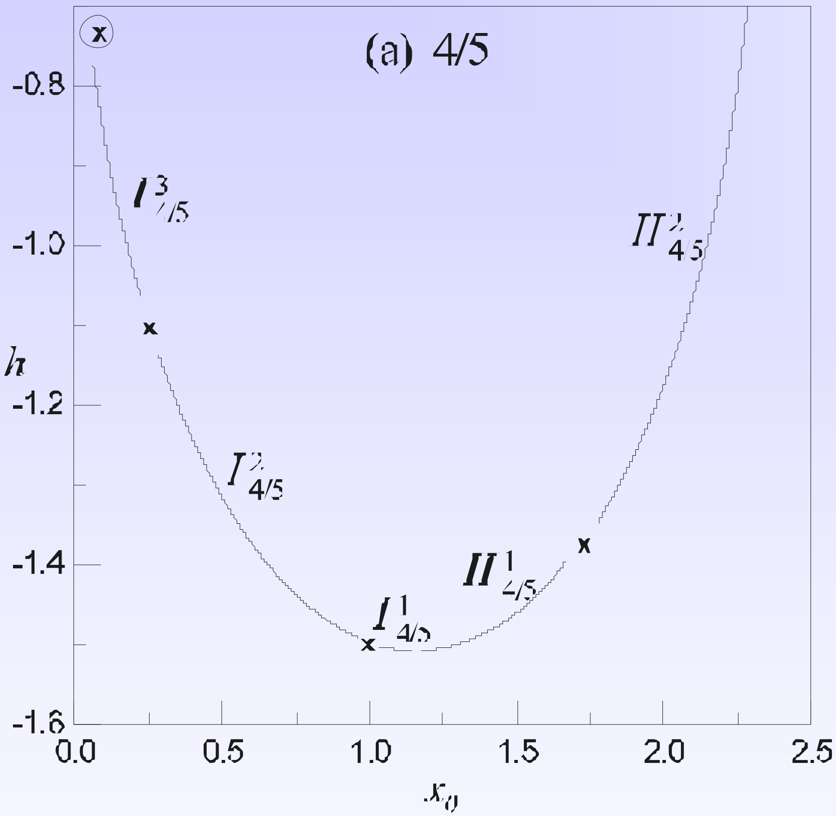
<u>Second kind:</u>
Family I → Unstable; Perpendicular Collision orbit with Neptune when $a(1-e)=1$; Stable and $k \to k+1$, k=multiplicity
Family II→ Stable; Except for a small area: a collision with Neptune occurs

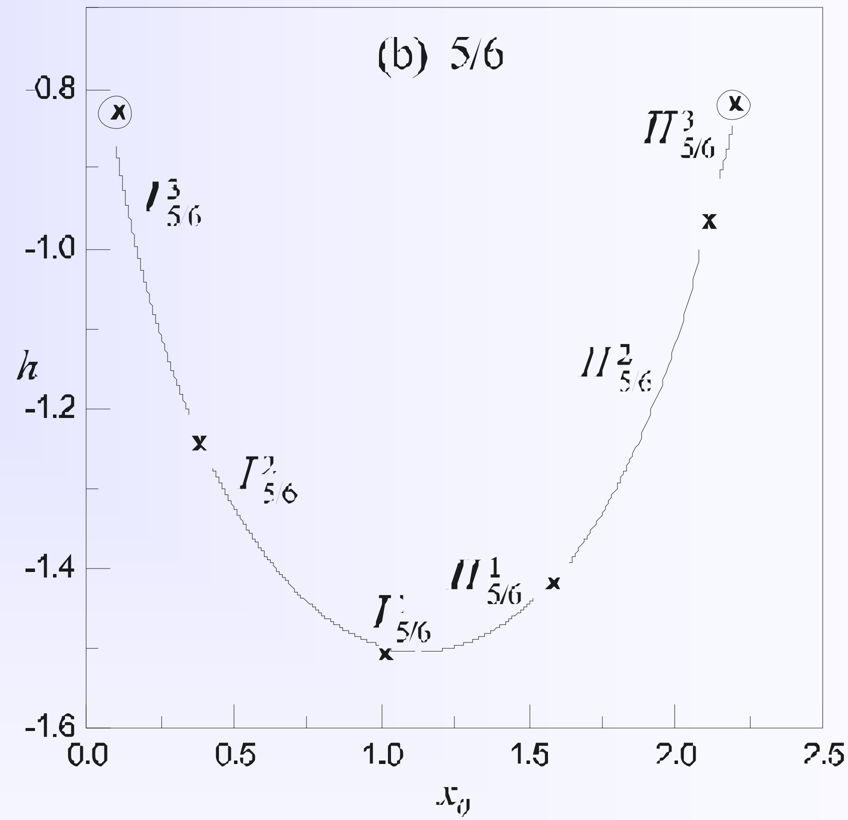
Families of symmetric POs in 1st order NMMRs



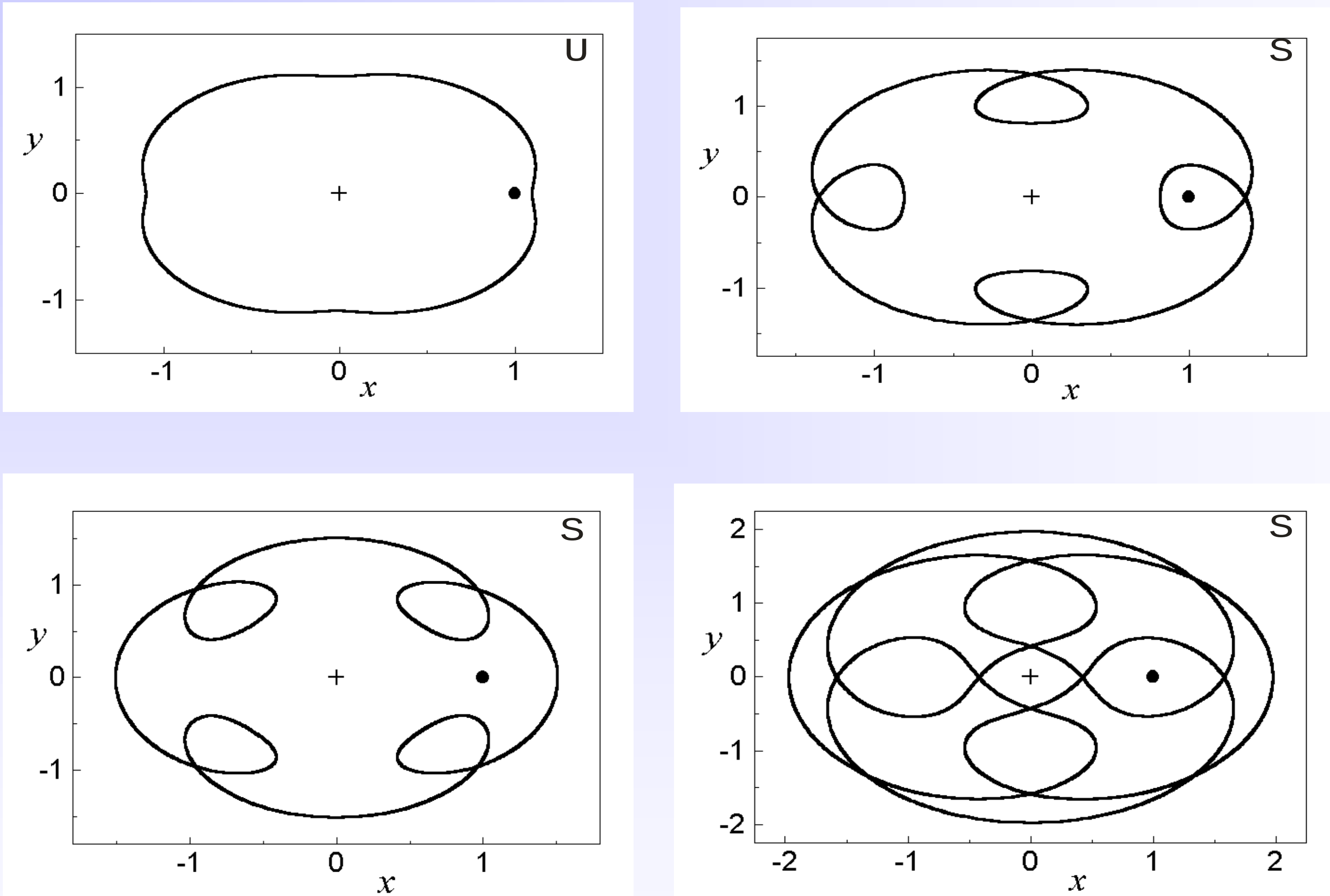
(**Voyatzis and Kotoulas, 2005: Planetary and Space Science 53, 1189-1199**)

An example: the 4/5 and 5/6 cases



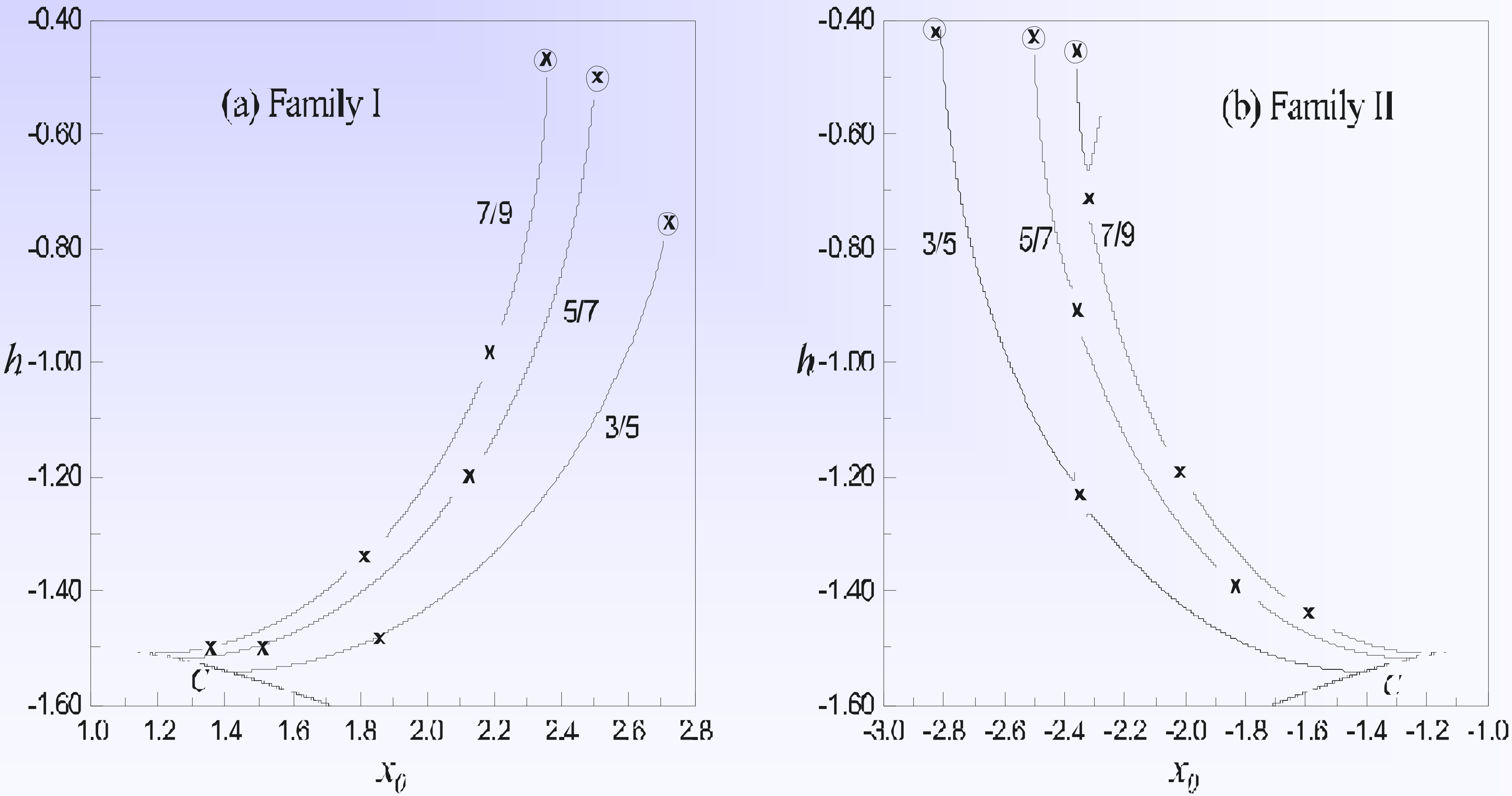


Samples of periodic orbits (res:4/5)



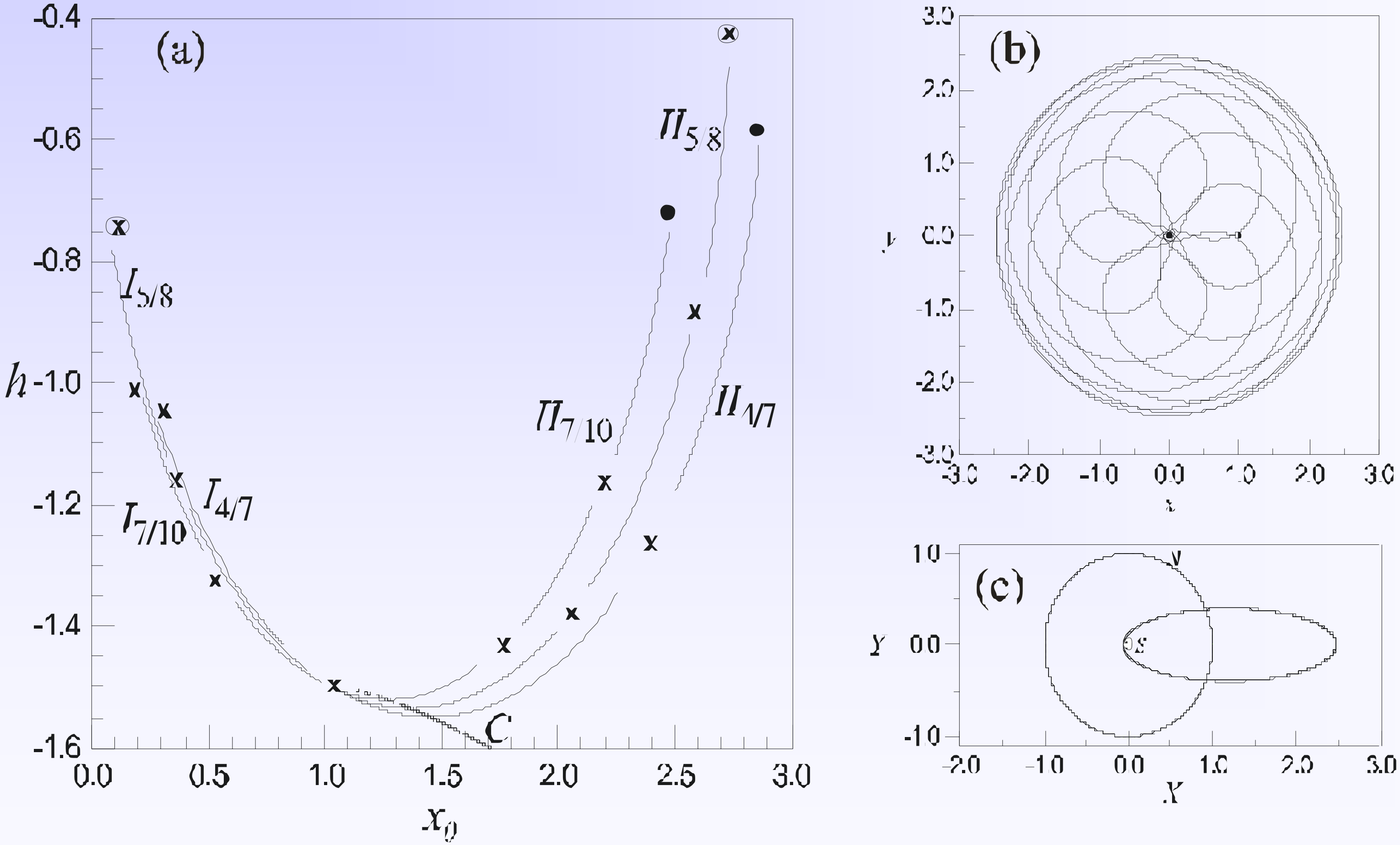
+ Position of Sun
Position of Neptune

Families of POs in 2nd order NMMRs

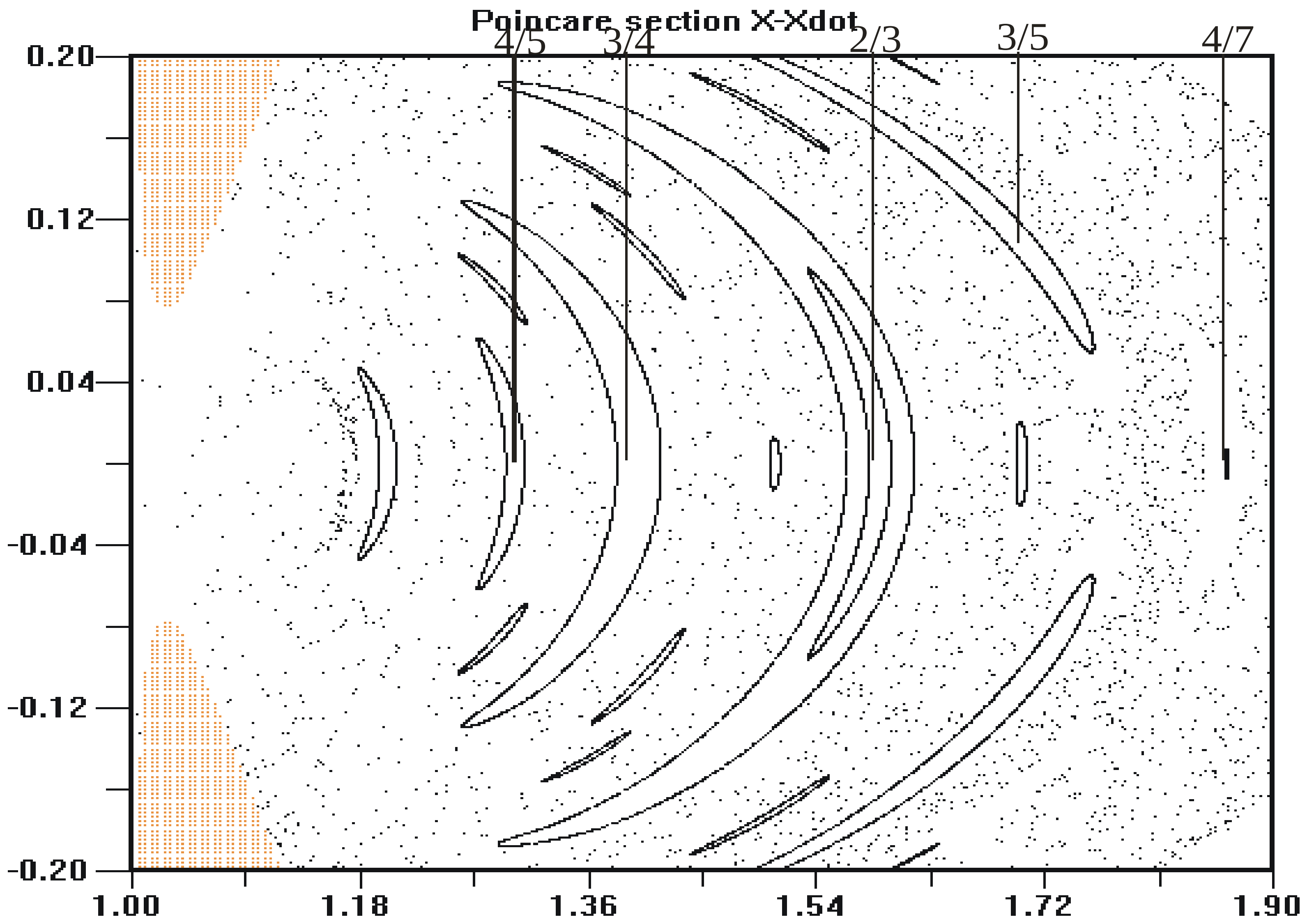


(Voyatzis and Kotoulas, 2005: Planetary and Space Science 53, 1189-1199)

Families of POs in 3^{rd} order NMMRs



Surface of section (y=0, dy/dt<0)



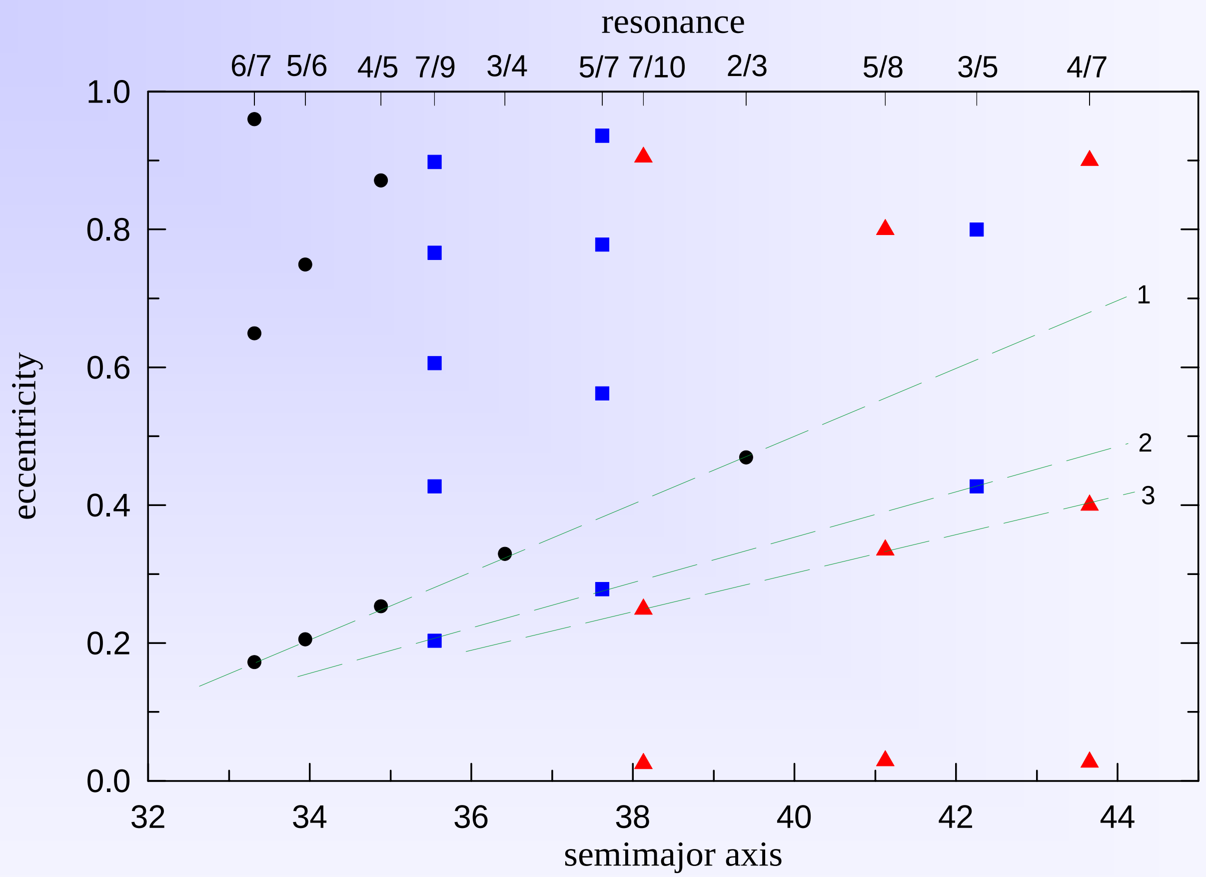
From Planar Circular to Planar Elliptic Problem

Bifurcation points : Periodic Orbits of the Circular problem with period $T=2m\pi;\ m \in Z.$

Table 1: Bf points (eccentricity values)

Resonance	a (A.U)	Period	Bf1	Bf2	Bf3	Bf4
2/3	39.40	6π	0.469			
3/4	36.41	8π	0.329			
4/5	34.88	10π	0.253	0.871		
5/6	33.95	12π	0.205	0.749		
6/7	33.31	14π	0.172	0.649	0.960	
3/5	42.26	10π	0.427	0.800		
5/7	37.62	14π	0.278	0.562	0.778	0.936
7/9	35.54	18π	0.203	0.427	0.606	0.898
4/7	43.65	14π	0.027	0.400	0.900	
5/8	41.12	16π	0.029	0.335	0.800	
7/10	38.13	20π	0.025	0.249	0.905	

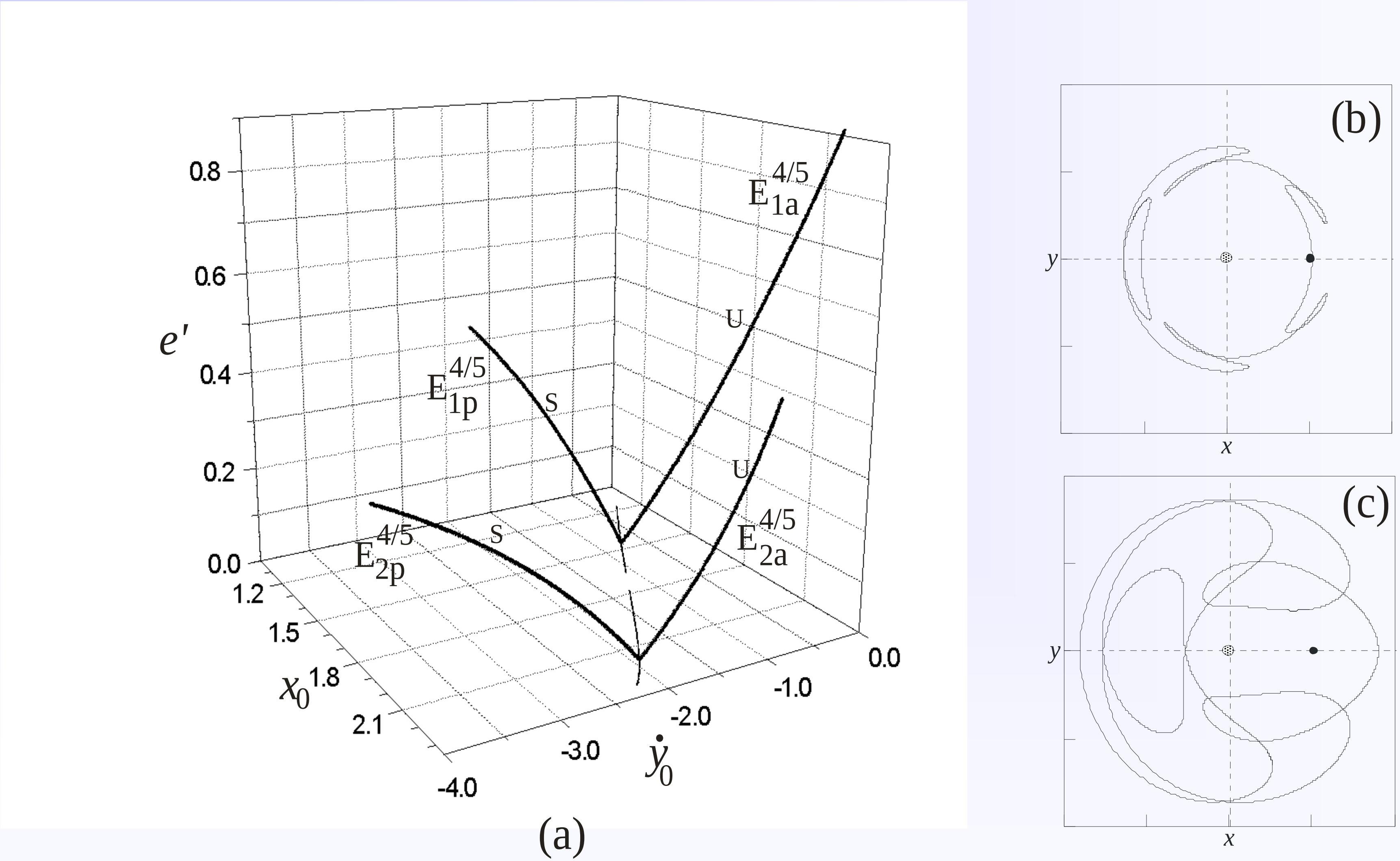
BFPs from Planar Circular to Planar Elliptic



Comment : One stable and one unstable family of POs arise from them

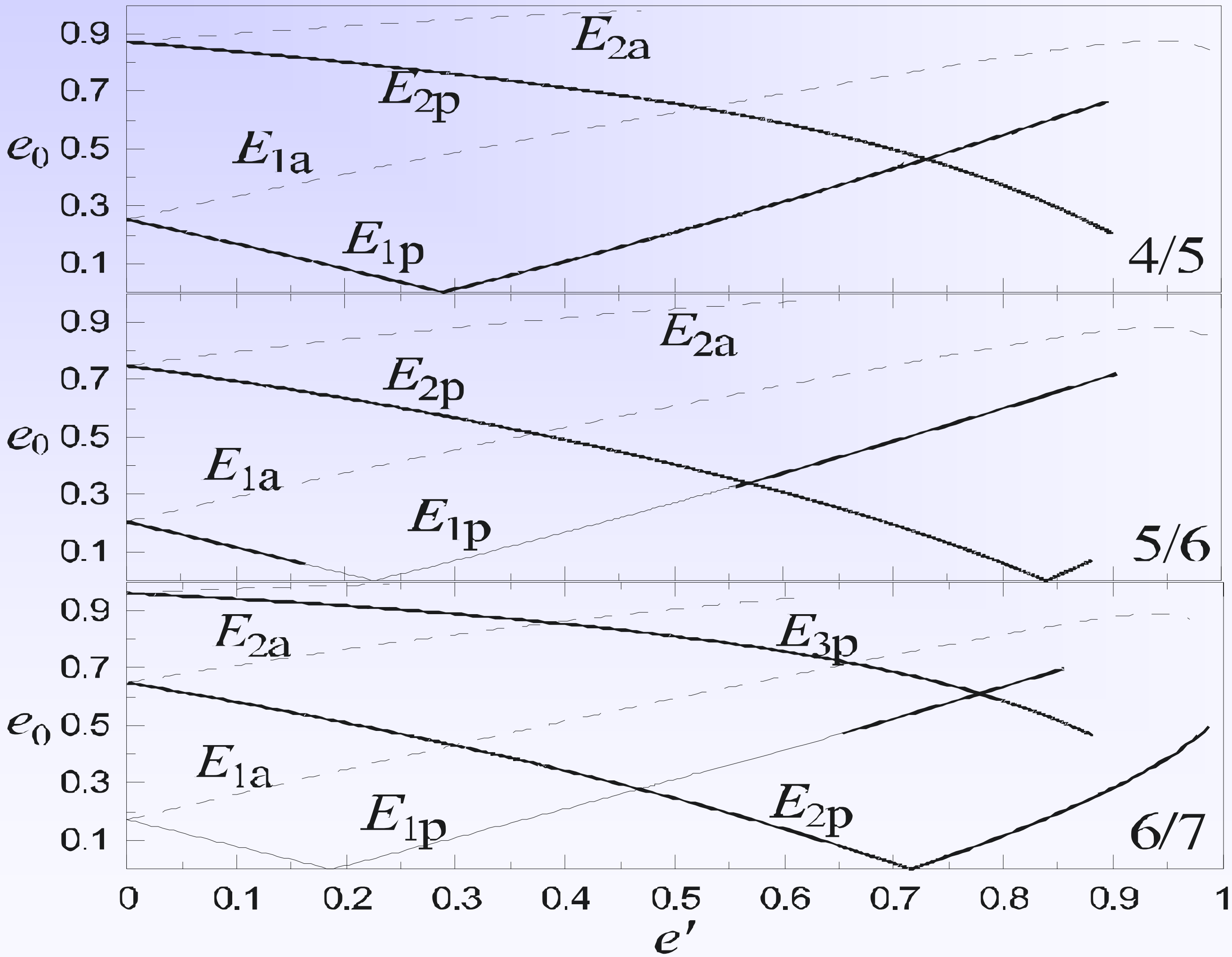
(Kotoulas and Voyatzis, 2004: Proceedings of the 197 IAU)

An example: Families of symmetric POs at N4/5

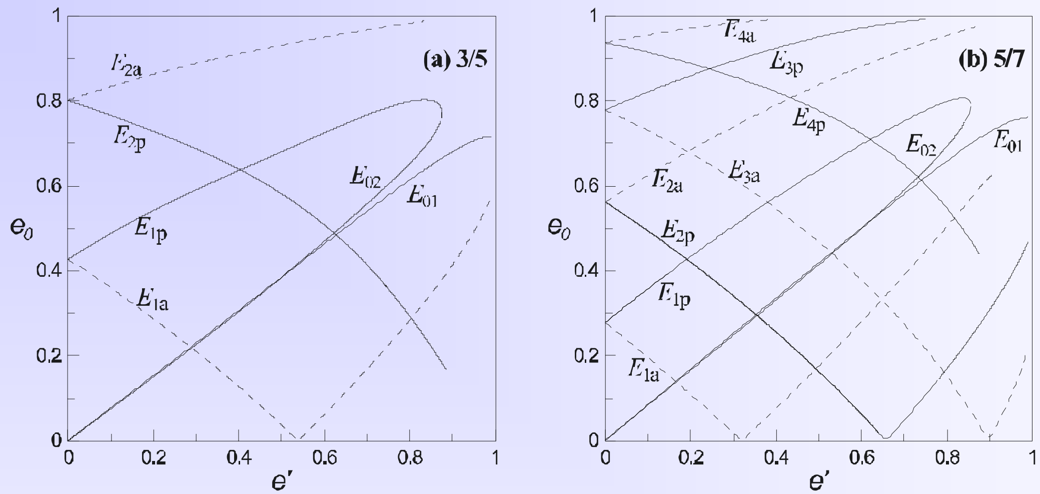


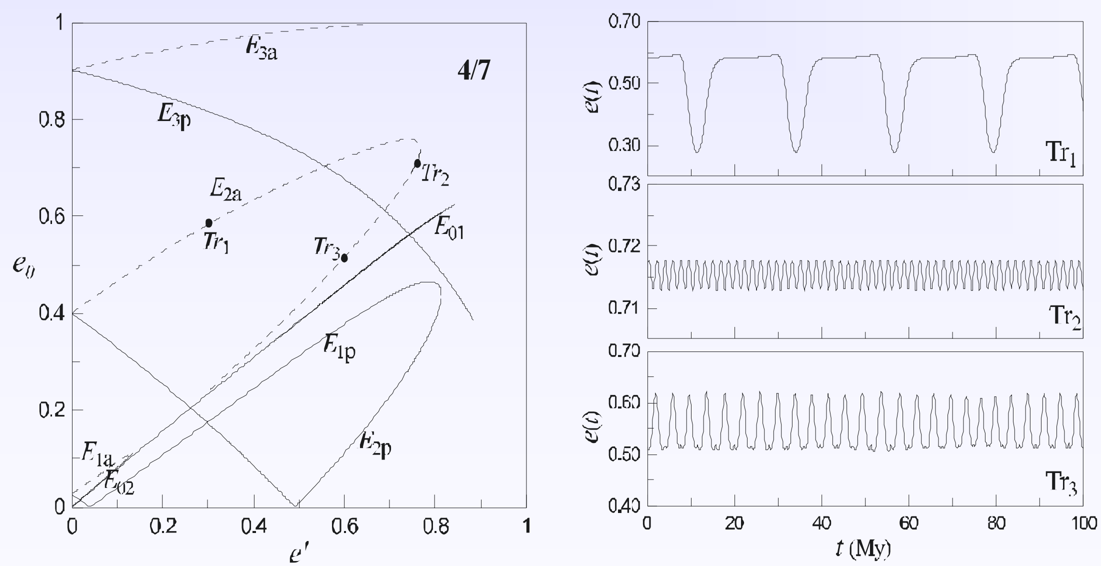
(Voyatzis and Kotoulas, 2005: Planetary and Space Science 53, 1189-1199)

Projection of families of POs in the plane e_0-e'



2^{nd} and 3^{rd} order mean motion resonances (2-D elliptic problem)





(a) (b)

BFPs from Planar Circular to 3D Circular

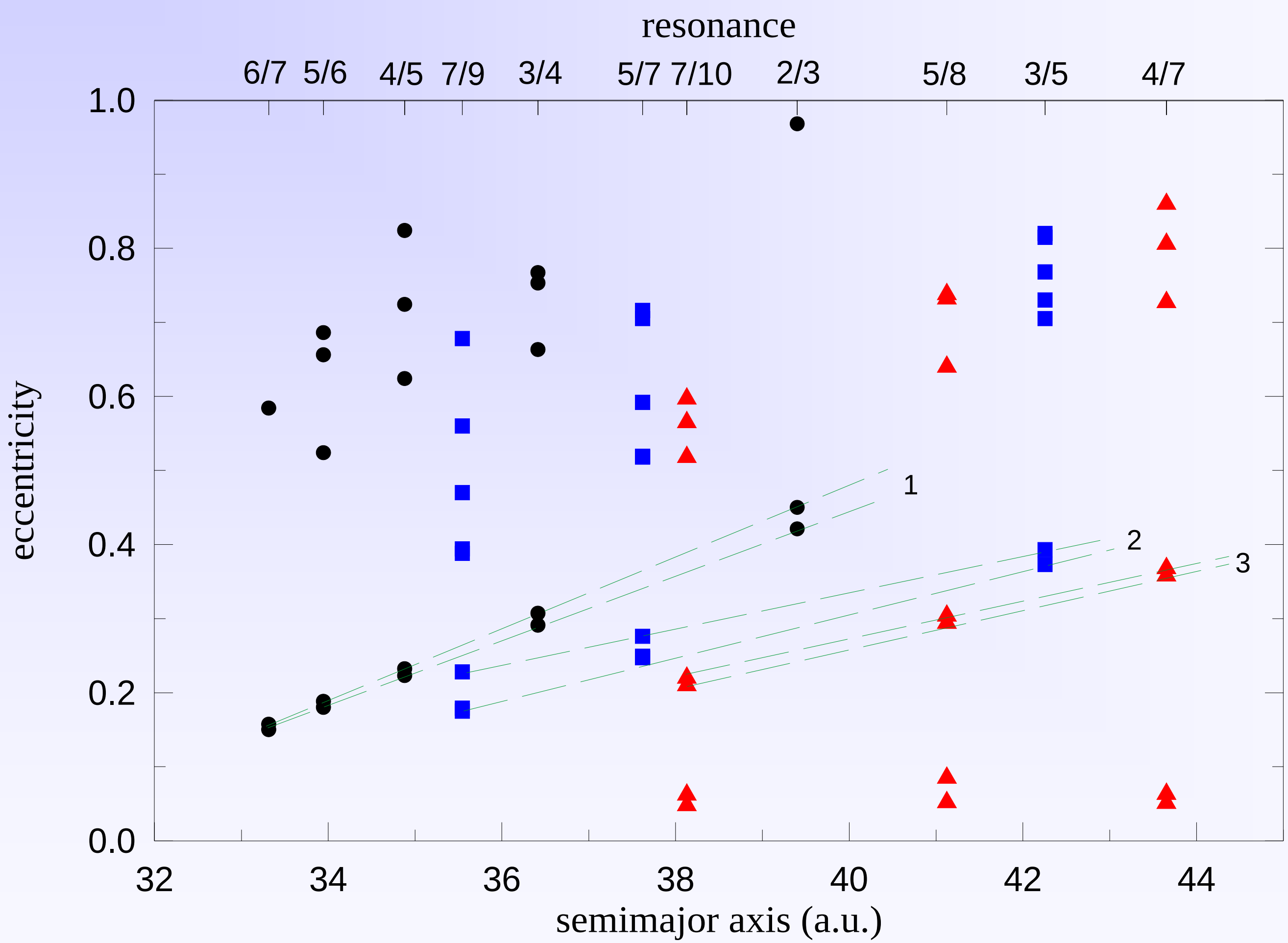
Bifurcation points : Periodic Orbits of the Circular problem with vertical critical stability.

Periodic orbits : F-Symmetry with respect to *xz*-plane, **G-**symmetry with respect to *x*-axis

Table 2: Bf points, eccentricity values (**FS**, **FU**, **GS**, **GU**)

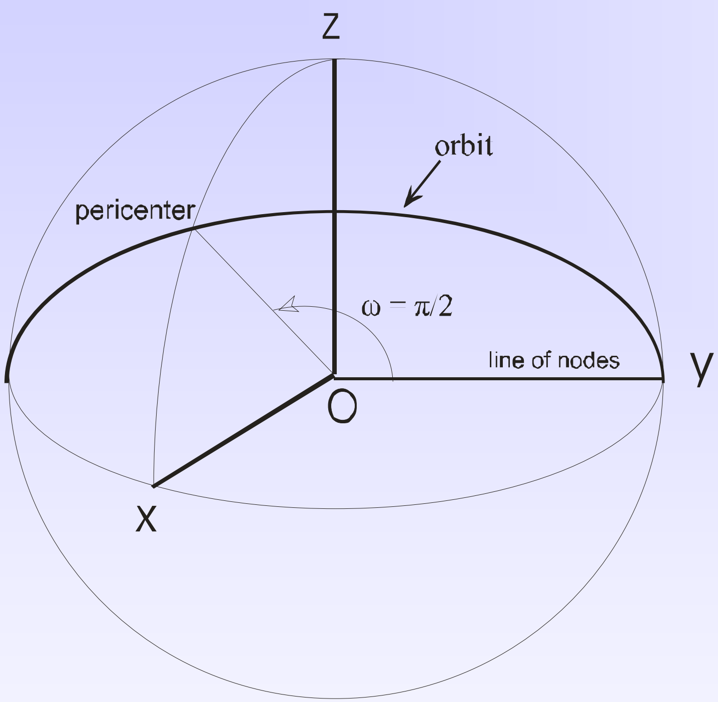
Res.	a (A.U)	Bf1	Bf2	Bf3	Bf4	Bf5	Bf6	Bf7	Bf8
2/3	39.40	0.421	0.450	0.968					
3/4	36.41	0.291	0.307	0.663	0.753	0.767			
4/5	34.88	0.222	0.233	0.624	0.729	0.825			
5/6	33.95	0.179	0.188	0.525	0.652	0.686			
6/7	33.31	0.150	0.157	0.578					
3/5	42.26	0.373	0.393	0.705	0.730	0.768	0.815	0.820	
5/7	37.62	0.248	0.251	0.281	0.518	0.519	0.592	0.716	
7/9	35.54	0.175	0.179	0.228	0.388	0.394	0.470	0.560	0.678
4/7	43.65	0.051	0.064	0.359	0.369	0.727	0.808	0.860	
5/8	41.12	0.050	0.063	0.293	0.303	0.640	0.732	0.738	
7/10	38.13	0.049	0.062	0.210	0.222	0.517	0.565		

BFPs from Planar Circular to 3D Circular



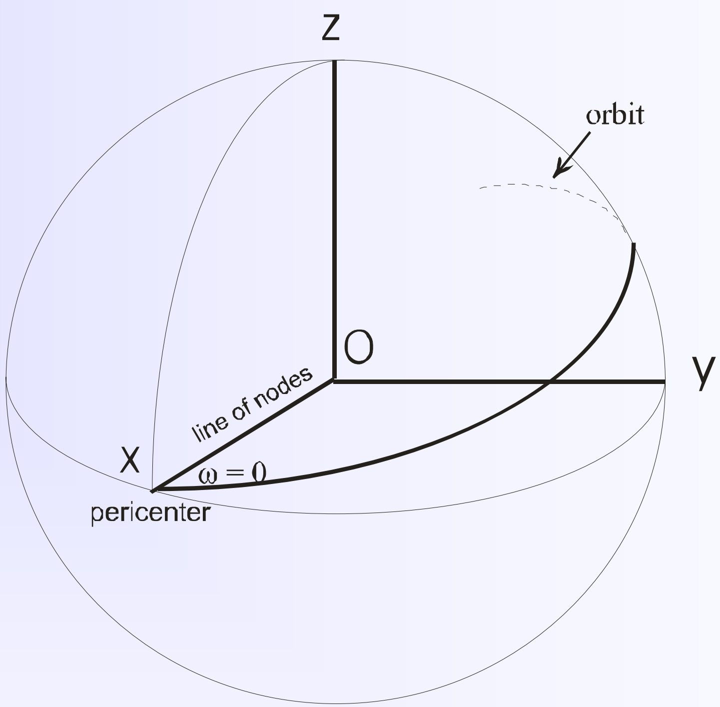
(Kotoulas and Voyatzis, 2004: Proceedings of the 197 IAU)

Types of 3-D periodic orbits



3-D periodic orbit of type 1

(a)

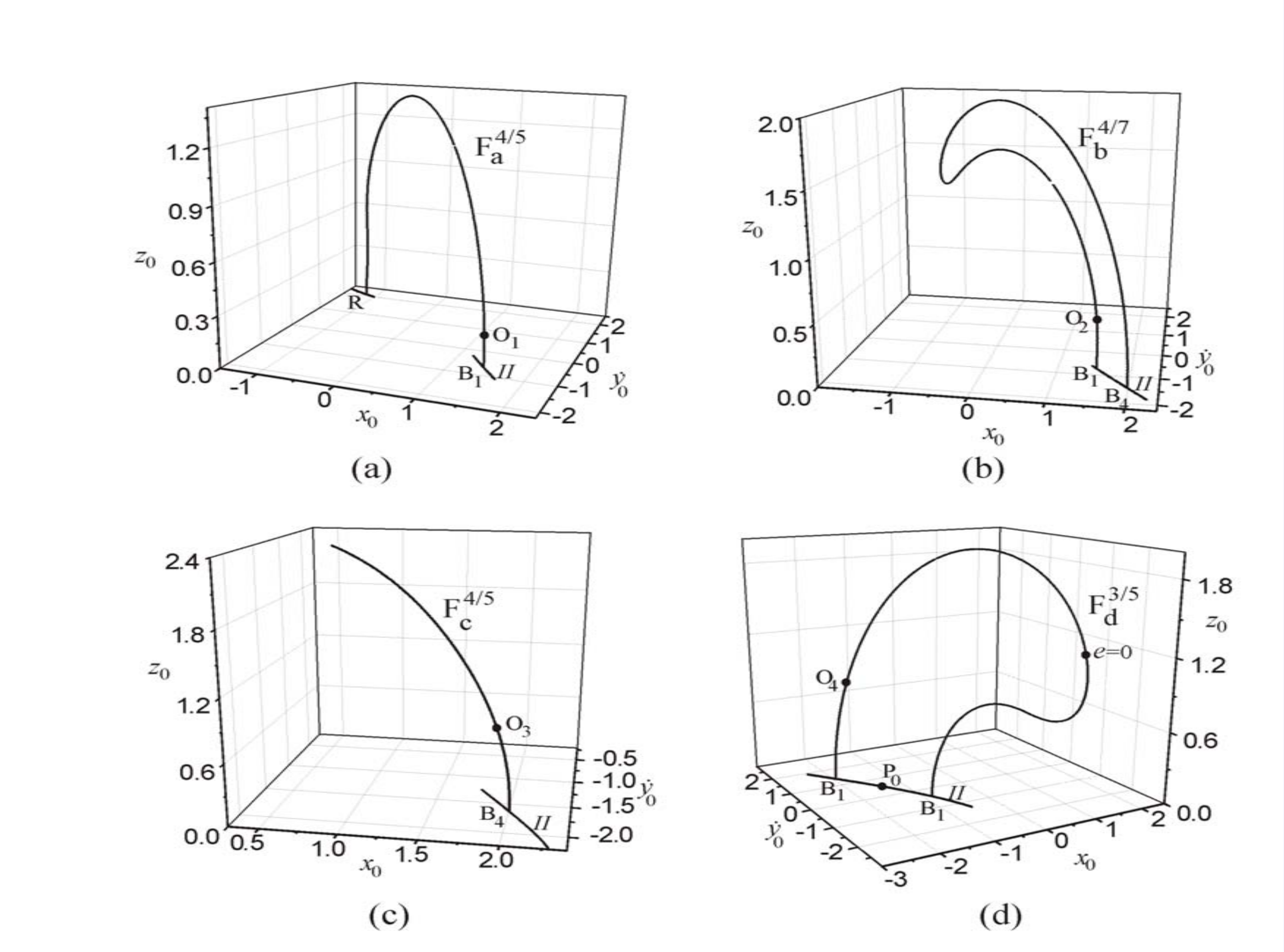


3-D periodic orbit of type 2

(b)

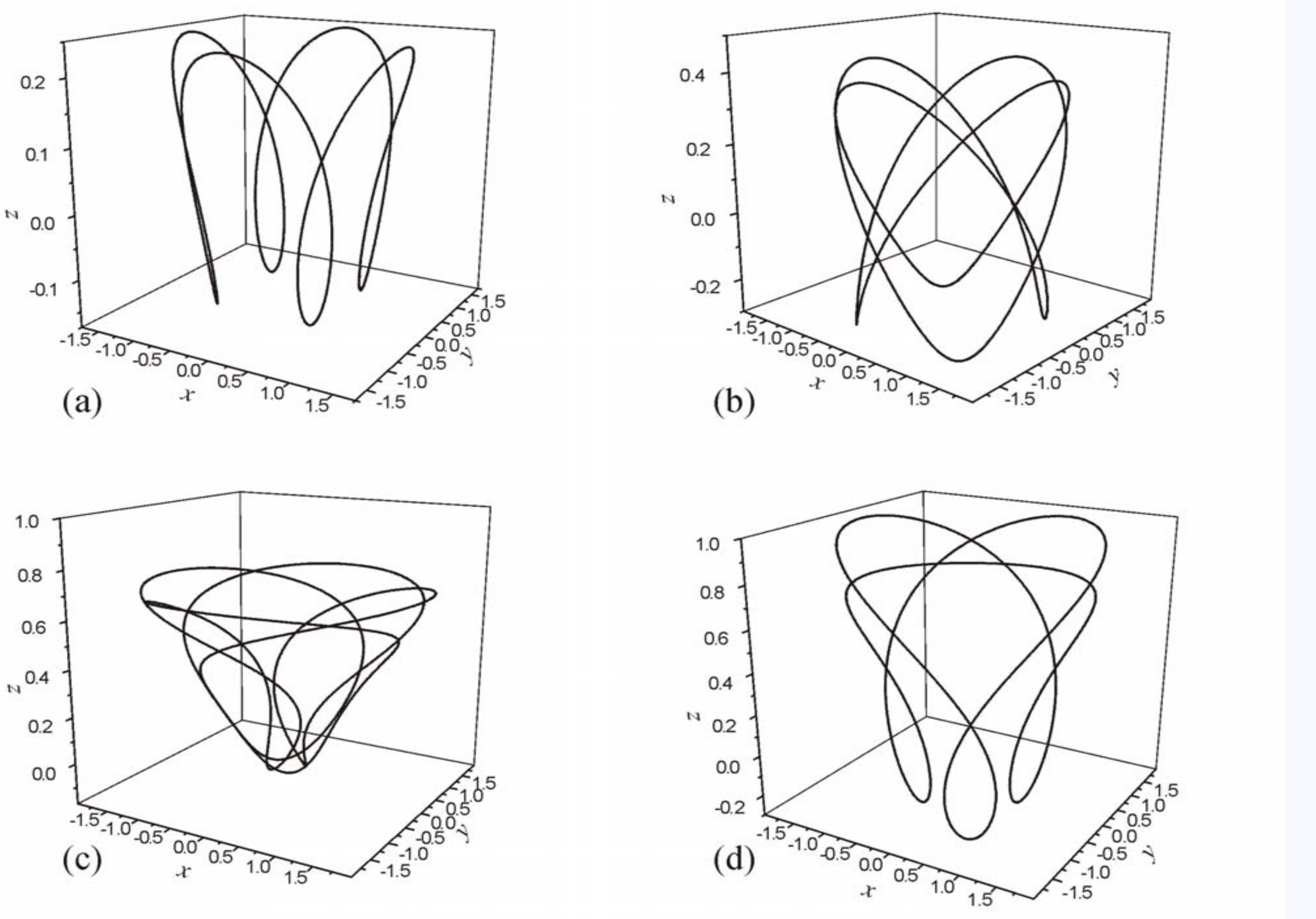
(Kotoulas and Hadjidemetriou, 2002: Earth, Moon and Planets 91: 63-93)

Families of POs in 3-D circular RTBP

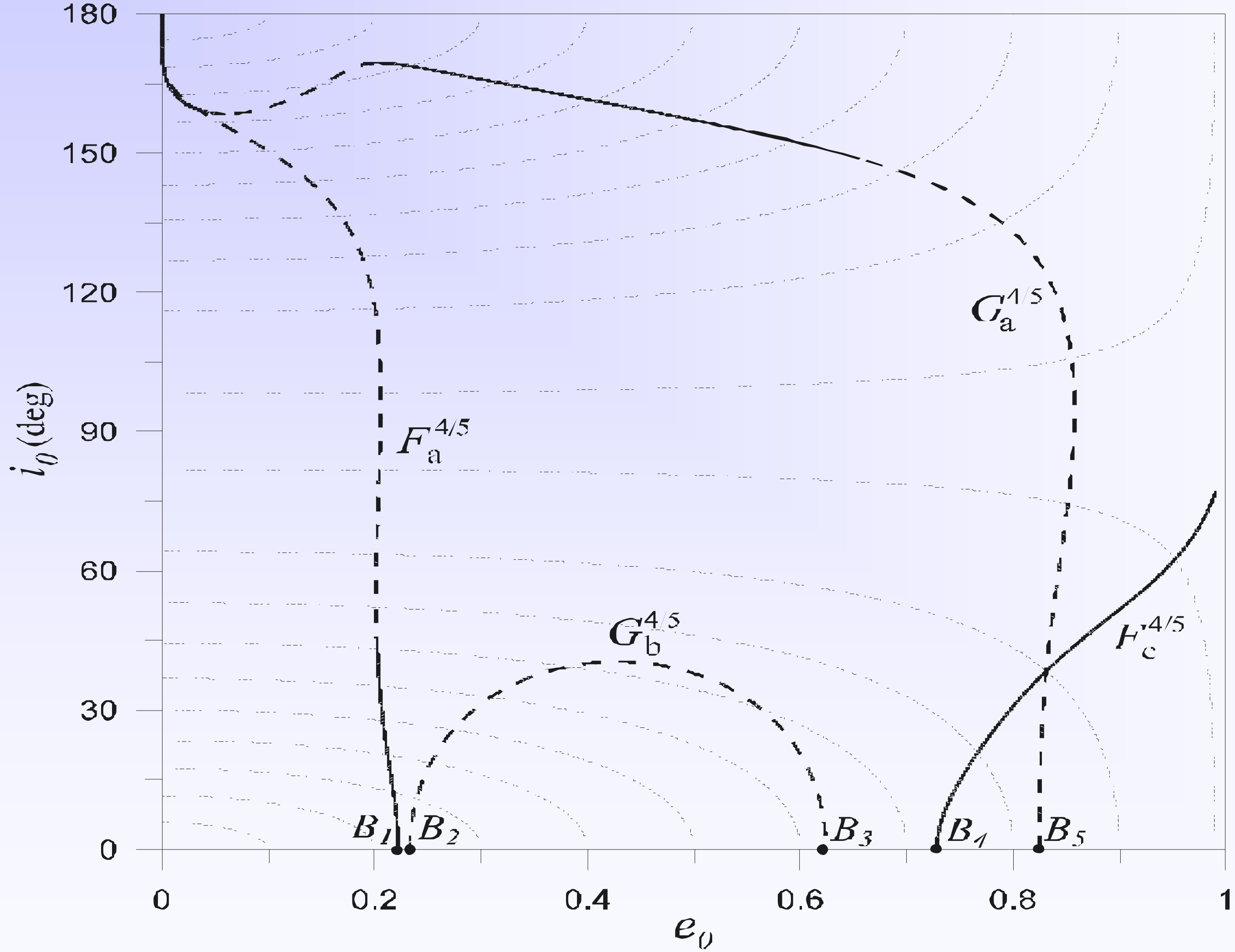


(Kotoulas and Voyatzis, 2005: A&A 441, 807-814)

Samples of 3-D symmetric POs

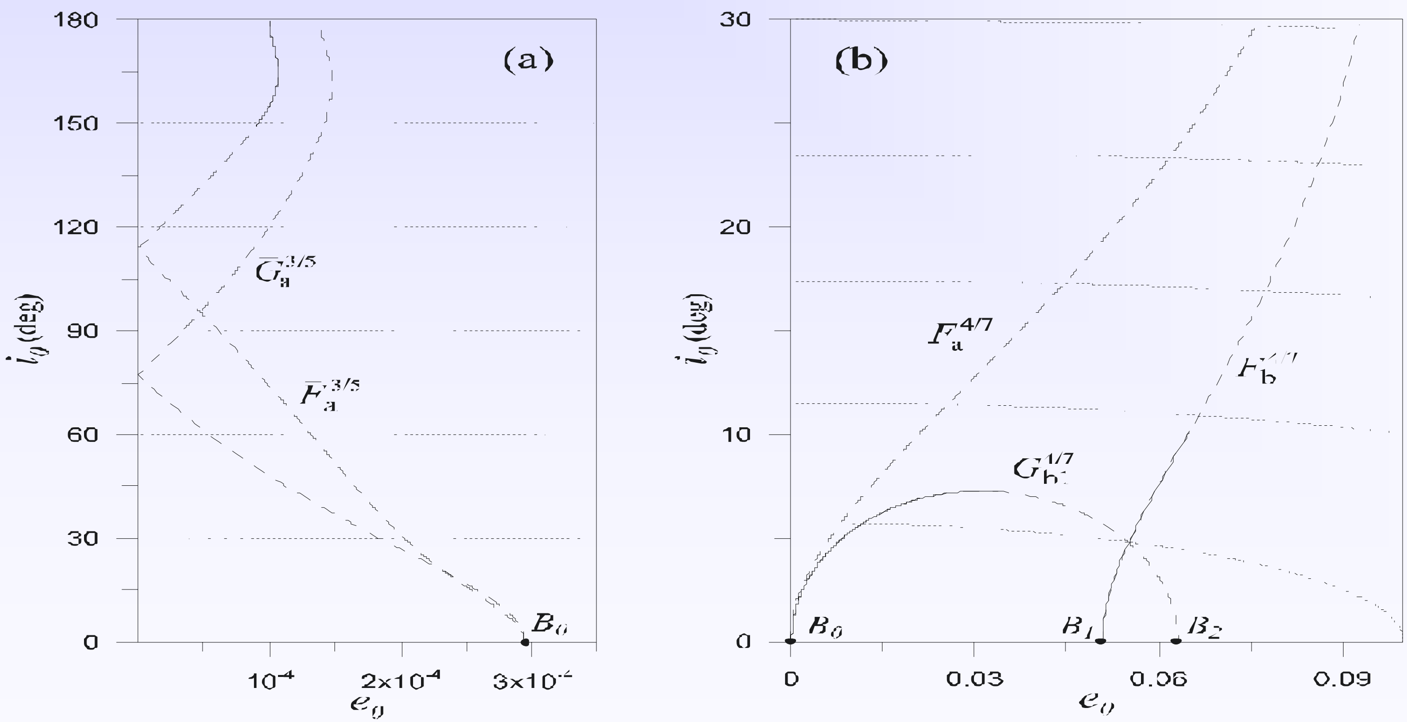


Projection of the families of POs in the plane e_0-i_0: The 4/5 case

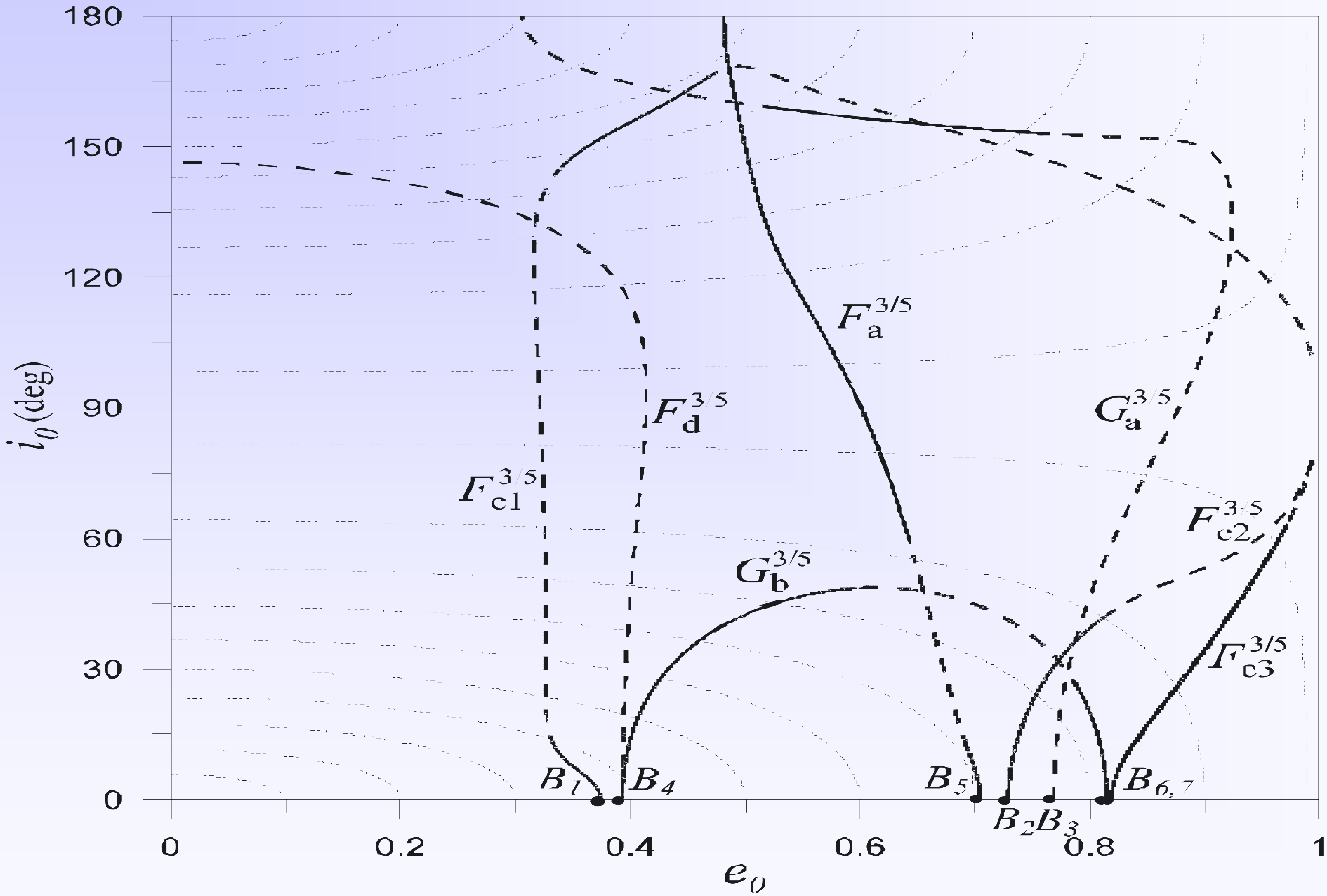


Families of 3-D POs generating from orbits with e≈0

Res	h_0	BP_0	Res	h_0	BP_0
3/5	-1.541	FU , GU	4/7	-1.549	FS , GU
5/7	-1.518	FU , GU	5/8	-1.535	FS , GU
7/9	-1.510	FU , GU	7/10	-1.520	FS , GU

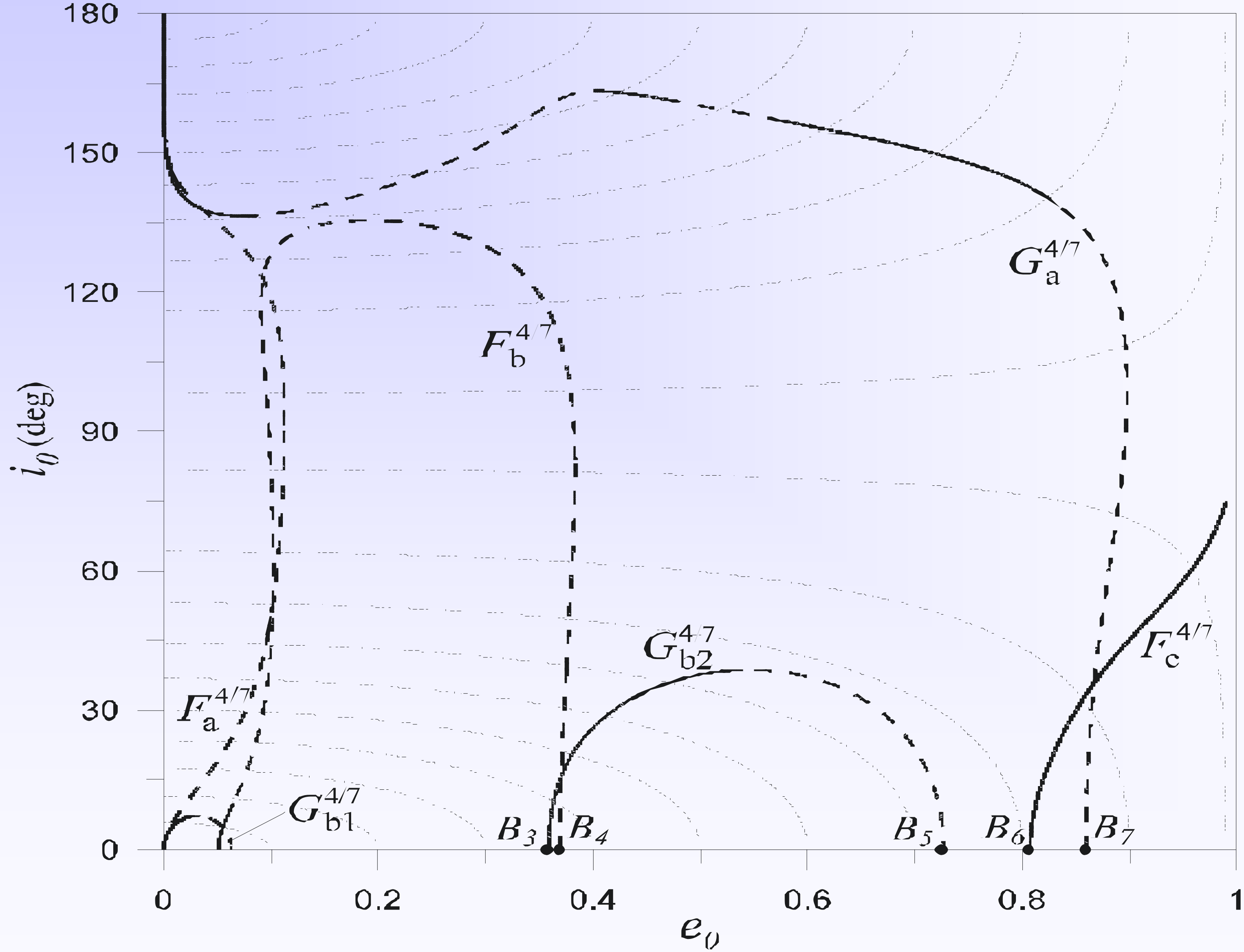


2^{nd} order resonances: The 3/5 case

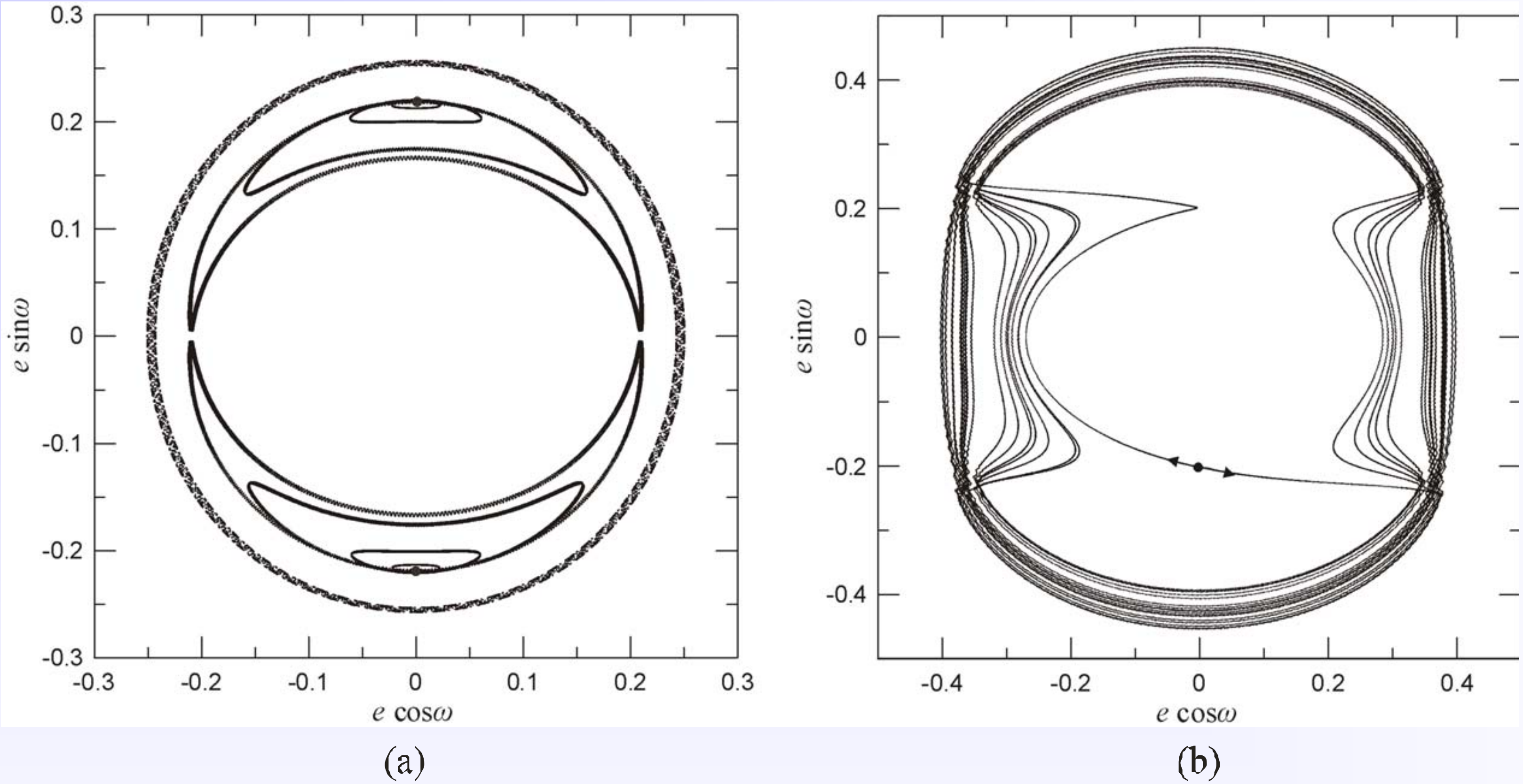


(Kotoulas and Voyatzis, 2005: A&A 441, 807-814)

3^{rd} order resonances: The 4/7 case



Projections of 4-D surfaces of section (y=0, dy/dt<0)



Comments: (a) A section in the region of a stable PO

(b) A section of 2 trajectories near an unstable PO

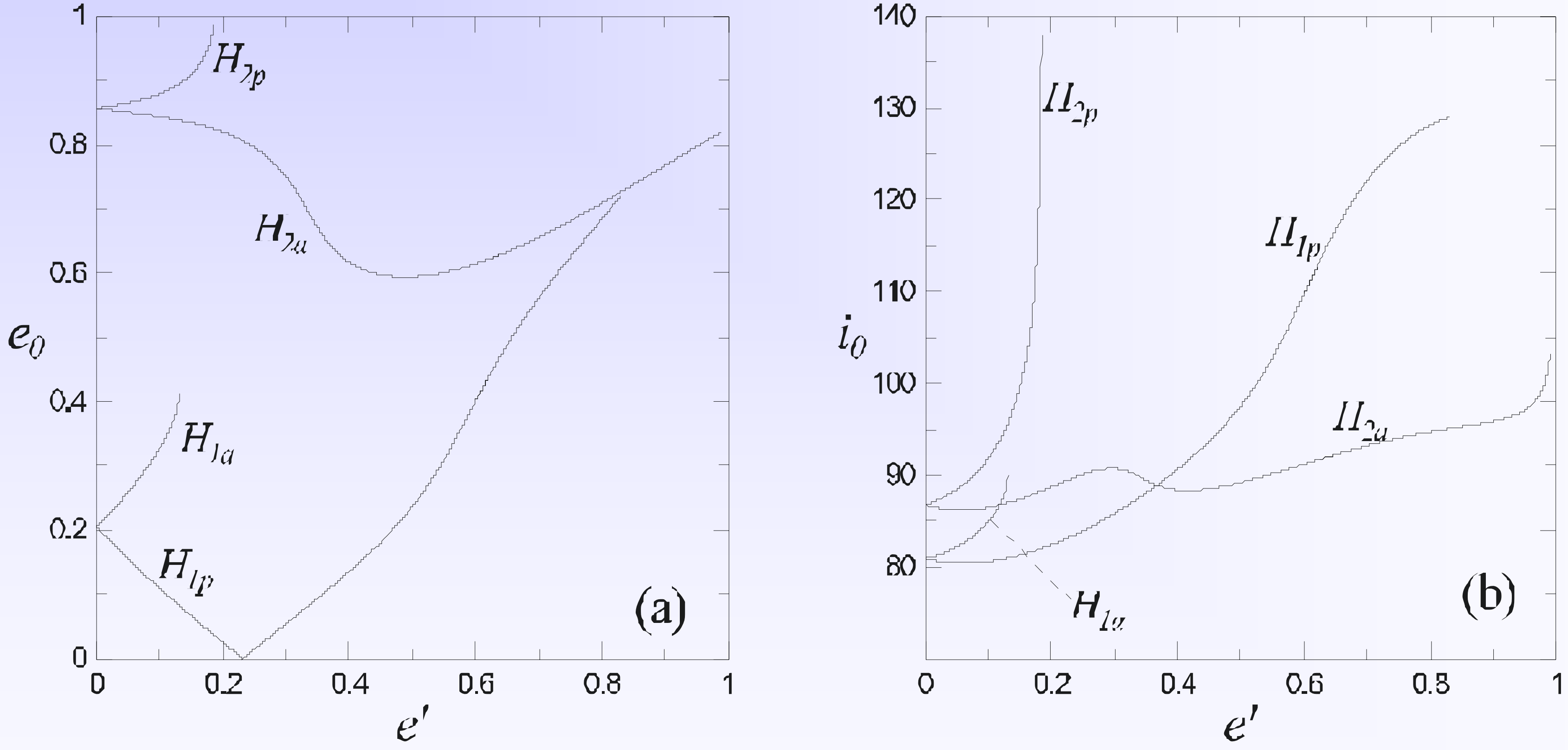
(Kotoulas and Voyatzis, 2005: A&A 441, 807-814)

3-D Elliptic RTBP

Bifurcation points : **1**. Periodic Orbits of the 3-D Circular RTBP with T=2mπ, m Є ***Z***

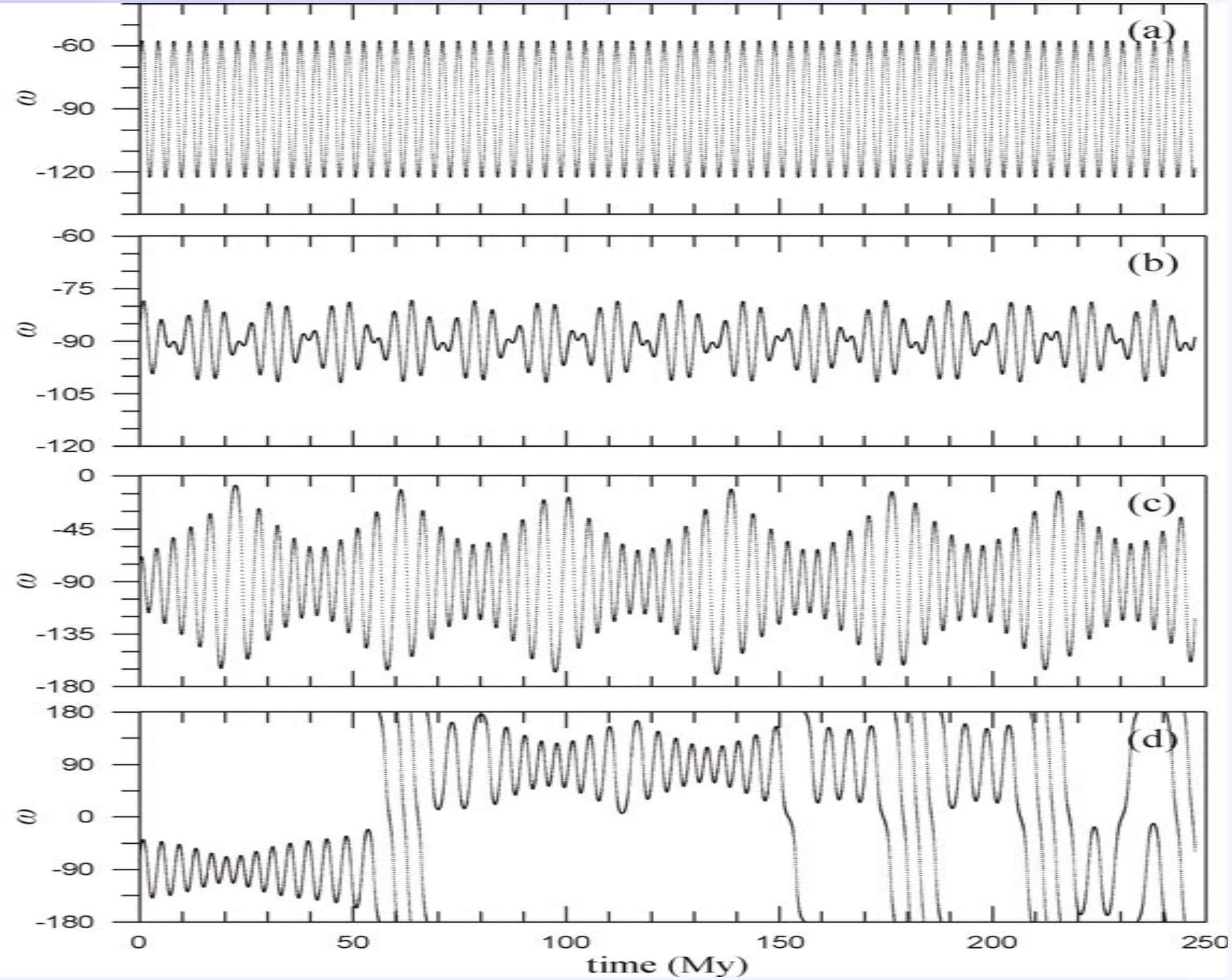
2. Periodic Orbits of the 2-D Elliptic RTBP with vertical stability

Types of symmetry : F, **G** as in the 3-D Circular model



Comment : All of them are linearly **unstable**

Evolution of an orbit at the 4/5 NMMR



Initial conditions: α=34.88 A.U., e=0.219, i=10 deg, M=180 deg, Ω=-90 deg
a)ω=-76 deg, e´=0 , b) ω=0 deg, e´=0.01, M'=0 deg, ω'+Ω'=0
c) ω=-76 deg, e´=0.01, M'=0, ω'+Ω'=180 deg
d) ω=-76 deg, e´=0.01, M'=0, ω'+Ω'=0 deg

Asymmetric Resonances

Circular Planar RTBP

Family I : the same characteristics as in the previous cases.

Family II : Stable→(Bf1)Unstable (Bf2)→Stable.

*1/5 (reciprocal process)

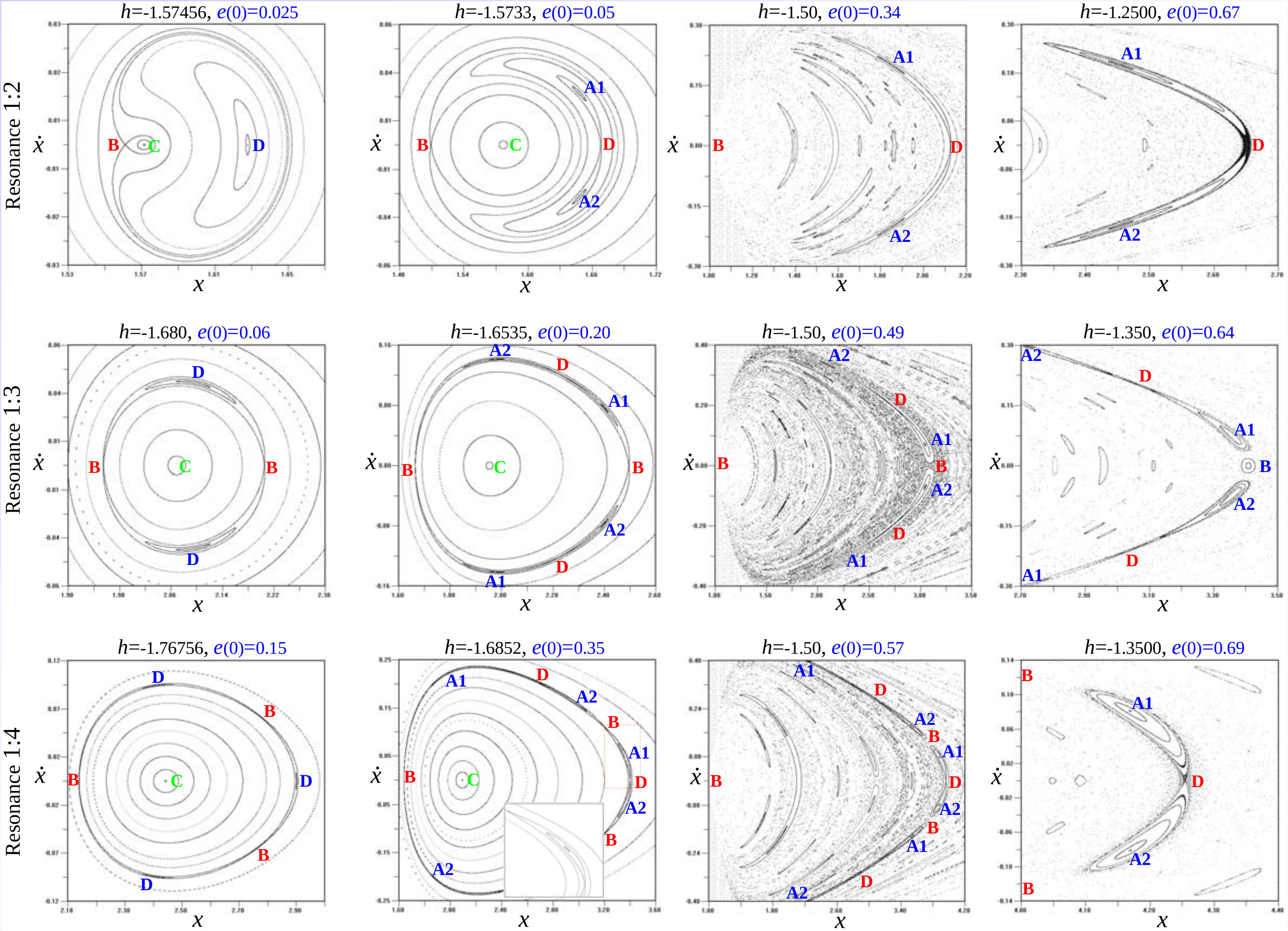
Res	a (A.U)	Bf1	Bf2
1/2	47.78	0.035	0.960
1/3	62.53	0.123	0.972
1/4	75.75	0.201	0.978
1/5	87.90	0.262	0.981
1/6	99.26	0.322	0.984

Table 3 : Bfps for asymmetric POs

Table 4 : Bfps from Planar Circular to Planar Elliptic and 3D circular RTBP

Res	a (A.U)	Bf(el) 1	Bf(el) 2	Bf(el) 3	Bf(3D) 1	Bf(3D) 2
1/2	47.78	0.070	0.637		0.059	0.066
1/3	62.53	0.135	0.759	0.950	0.112	0.595
1/4	75.75	0.815				
1/5	87.90					
1/6	99.26	0.870				

Poincare maps for 1/2, 1/3, 1/4 NMMRs



Asymmetric periodic orbits

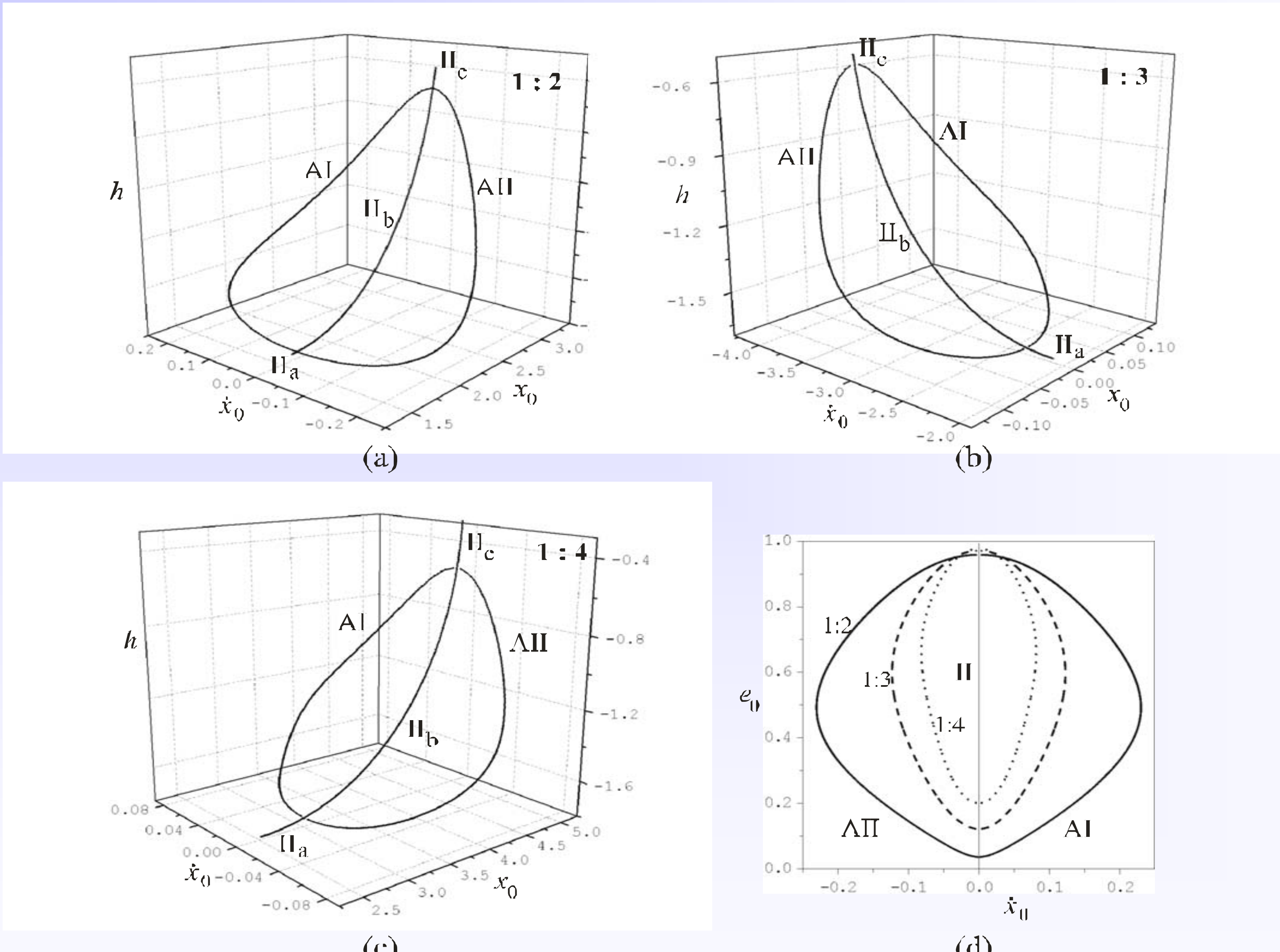
- An asymmetric periodic orbit is a periodic orbit which is not invariant under the fundamental symmetry

- Σ: (x,y,v_x,v_y,t): $(x,-y,-v_x,v_y,t)$, (v_x=dx/dt, v_y=dy/dt).

- Consequently, if an asymmetric periodic orbit T1 exists for particular initial conditions, then another asymmetric periodic orbit T2 exists which is "symmetrical" of T1 because of the symmetry Σ [Henon, 1997]. Therefore, asymmetric orbits appear in pairs.

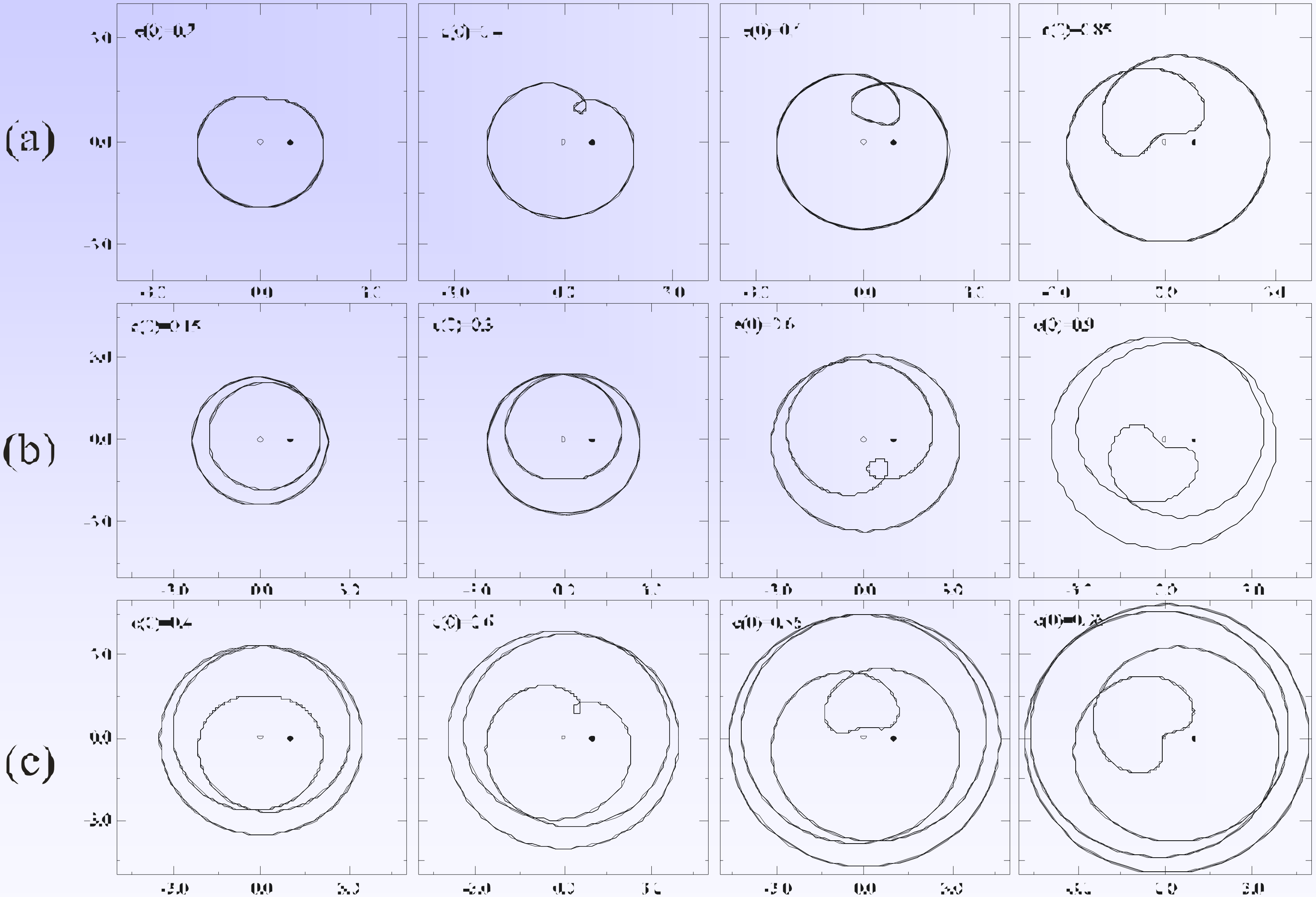
- Their stability can be examined in a similar way as in the case of symmetric periodic orbits. Symmetrical asymmetric periodic orbits (i.e. orbits T1 and T2) have the same stability type.

Families of asymmetric Pos in 1/2, 1/3, 1/4 NMMRs



(Voyatzis et al. 2005, Cel. Mech. And Dyn. Astr. 91: 191-202)

Samples of asymmetric POs



(Voyatzis et al. 2005, Cel. Mech. and Dyn. Astr. 91: 191-202)

Conclusions

•Planar circular model: Families of periodic orbits in 1st, 2nd, 3rd order resonances

•Bifurcation points from the planar circular to planar elliptic and to the three-dimensional circular one for the 1st, 2nd and 3rd order mean motion resonances.

•Planar elliptic problem: one family is stable (Neptune at perihelion) and the other one is unstable (Neptune is at aphelion)

•Stable POs→ surrounded by regular librations
•Unstable POs→ formation of phase space regions with chaotic motion.

•3D circular problem: both stable and unstable periodic orbits→ These families extend up to high inclination values.

•3D elliptic problem: all the basic periodic orbits are unstable → These families start from high inclination values and continue till the rectilinear problem

•The asymmetric resonances
Bf points: planar circular → planar elliptic. Exception: **1/5**
Bf points: planar circular → 3D circular only for cases **1/2**, **1/3**
Bf points: Asymmetric periodic orbits extend to high-eccentricity values.

The END

Thank you!