Forced oscillations in accretion disks and QPOs

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Outline

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   - Observations
   - Models
2. Resonances in hydrodynamical disks
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3. Resonances in MHD disks
   - Linear analysis
   - 2D numerical simulations
4. Power spectrum density
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QPOs: Observations (1)

Aim of this work

To find a new model for the high-frequency quasi-periodic oscillations *(kHz QPOs)* observed in accretion disks orbiting around compact objects.

What is a kHz-QPO ?

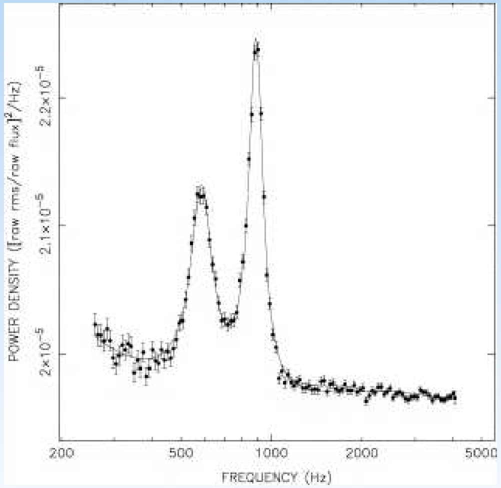
* Discovered in 1996 in 2 low mass X ray binaries (LMXBs) containing a neutron star: Sco X-1 and 4U1608-52.

* A QPO is a modulation in the intensity of the emission observed in the X ray range (1-20 keV).

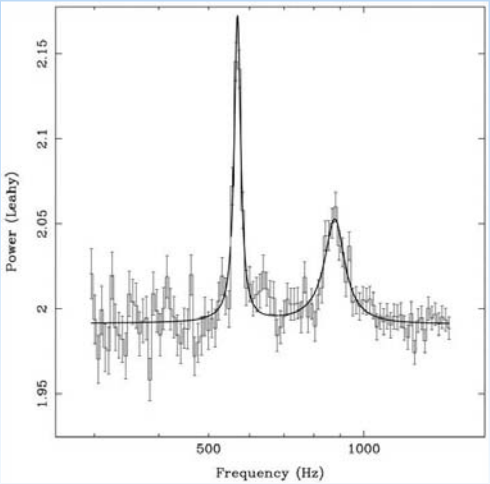
⇒ peak in the Fourier spectrum of the light curve.

Twin peak close to 1 kHz for 2 neutron stars.

Source Sco X-1, van der Klis et al. 1997



Source 4U1608-52, Mendez et al. 1998

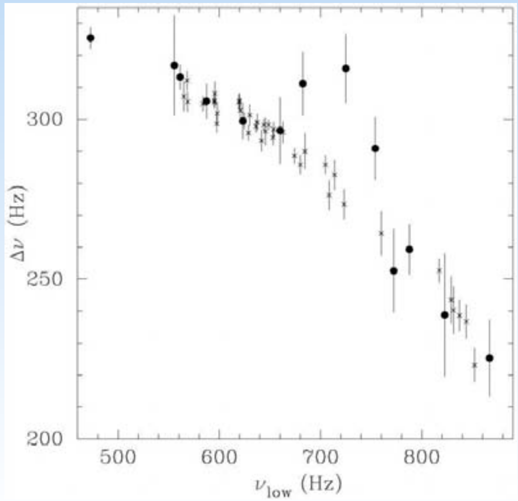


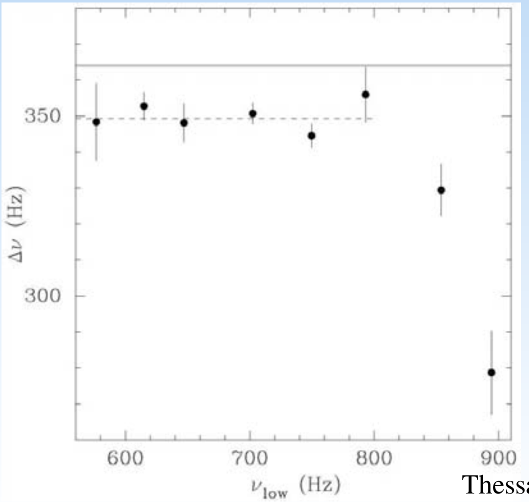
QPOs observations (2)

Some properties

- to date about 20 sources containing a neutron star are known ;
- QPOs appear by pairs (ν_1, ν_2) for 90% of them ;
- frequencies between 300-1300 Hz ;
- peak separation $\Delta\nu$ between 200-400 Hz ;
- QPOs frequencies increase with accretion rate $\dot{M}$ but their separation decreases ;
- sometimes the peak separation $\Delta\nu$ is commensurable with the spin ν_* of the star : $\Delta\nu \approx \nu_*$ or $\Delta\nu \approx \nu_*/2$.

Variation of the twin-peak separation, $\Delta\nu = \nu_2 - \nu_1$ vs. ν_1
(Mendez and van der Klis 1999)

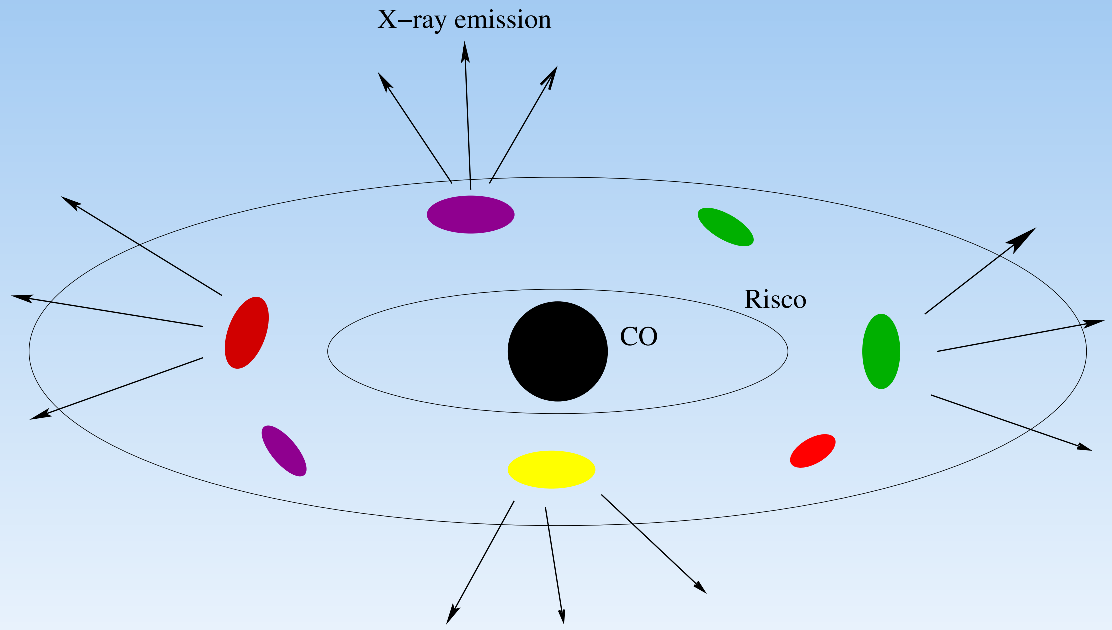




The models

General idea

Inhomogeneities forming in the disk create clumps of matter orbiting around the compact object and generate a modulation in the intensity of the radiation. In the case of interest here, mostly in the X ray range.



The Beat frequency model

Idea

Interaction between the orbital motion at some preferred radius in the accretion disk ν_{orb} and the spin of the neutron star ν_*.

Choice of the preferred radius

- the magnetospheric radius corresponding to the region where the motion of the fluid begins to be dominated by the magnetic field (Alpar & Shaham 1985, Lamb et al. 1985) ;
- the sonic point corresponding to the radius where the radial flow becomes supersonic (Miller et al. 1998).

Model

- the high frequency peak ν_2 associated with orbital motion ν_{orb} of the disk at the preferred radius ;
- the low frequency peak ν_1 related to the beat frequency given by $\nu_{beat} = \nu_{orb} - \nu_*$.

Consequence

The separation between the twin peaks remains constant and equal to $\Delta\nu = \nu_2 - \nu_1 = \nu_*$.
however $\Delta\nu$ was observed not to be perfectly constant
$\Rightarrow$ leads to some refinement of the beat frequency model (Lamb & Miller Lamb2001) and also to other models like the relativistic precession model.

Characteristic frequencies around rotating BHs

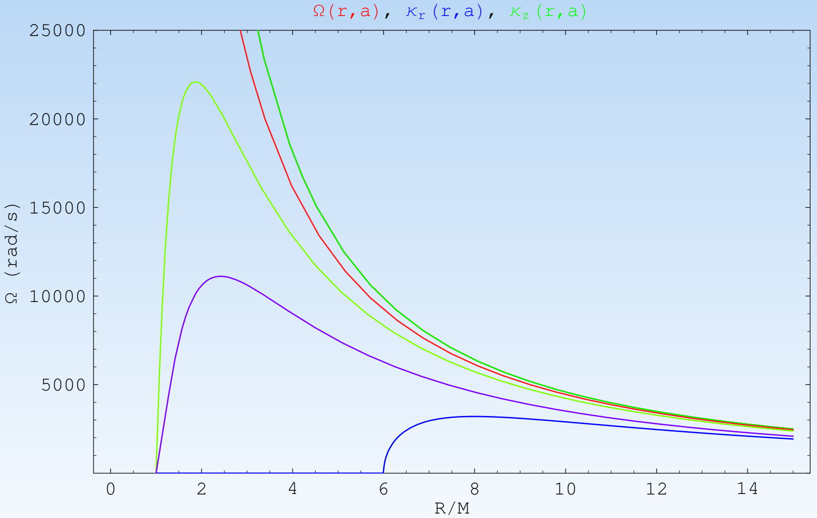
We distinguish 3 characteristic frequencies in accretion disks :

- the circular orbital frequency Ω ;
- the radial epicyclic frequency κ_r ;
- the vertical epicyclic frequency κ_z.

In Newtonian theory, orbital, radial and vertical epicyclic frequencies are all equal : $\Omega = \kappa_r = \kappa_z$.

In General Relativity, they are all different and independent. Moreover they depend not only on the mass M_* of the star but also on its angular momentum a_*.

Orbital Ω, radial epicyclic κ_r and vertical epicyclic κ_z frequencies around a rotating black hole



Innermost stable circular orbit (ISCO)

The circular orbit of a single particle becomes unstable when $\kappa_r \leq 0$
$\Rightarrow$ disk truncated at the inner boundary R_{ISCO} such that $\kappa_r(R_{ISCO}, a) = 0$.
For the Schwarzschild spacetime :

$$R_{ISCO} = 6\,GM_*/c^2 = 3\,R_s$$

The maximal orbital frequency is then :

$$\nu_{ISCO} = 2198\,\frac{M_\odot}{M_*}\,\mathrm{Hz}$$

For a 1.4 $M_\odot$ neutron star :

$$\nu_{ISCO} = 1571\,\mathrm{Hz}$$

Therefore for such systems :

$$\nu(kHz - QPO) \leq \nu_{ISCO} = 1571\,\mathrm{Hz}$$

Knowing the mass of the black hole, we can constrain its angular momentum.

The relativistic precession model

Idea

To take into account the characteristic frequencies of circular orbits around rotating black holes (Stella & Vietri 1998,1999).

Model

- the highest kHz-QPO ν_2 associated with the orbital motion ν_{orb} at some preferred radius ;
- the lowest kHz-QPO ν_1 associated with the precession of the periastron $\nu_{prec} = \nu_{orb} - \nu_r$;
- the low frequency QPO ($\leq$ 10 Hz) related to the Lense-Thirring precession (ν_{LT}).

Consequences

Predicts two quantitative relations :

- a first relation between ν_r and $\Delta\nu$ such that :

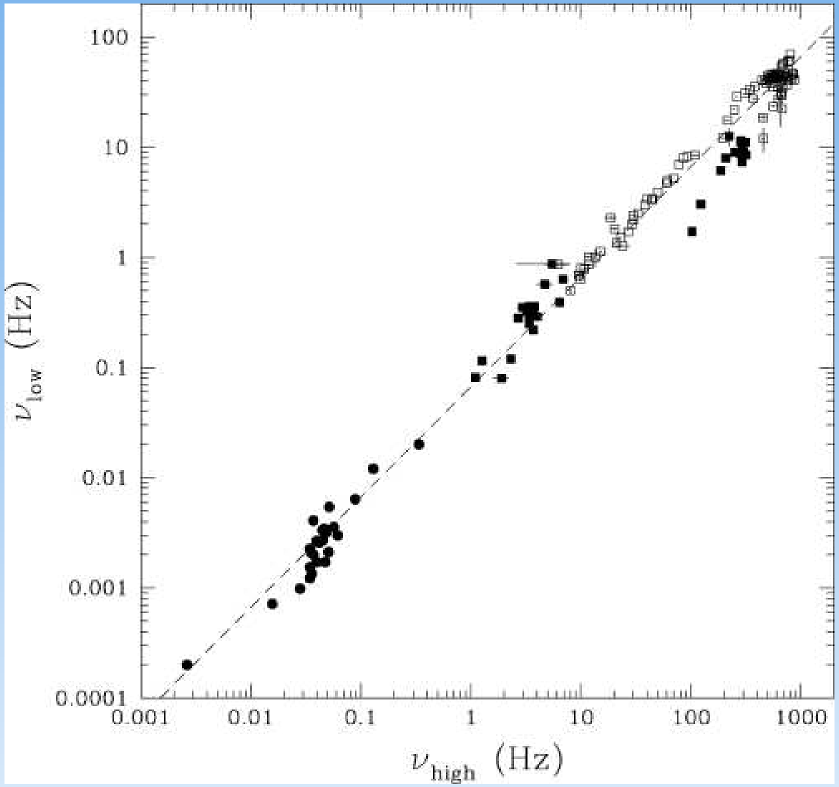
$$\Delta\nu = \nu_2 - \nu_1 = \nu_r$$

- a second relation between low frequency QPOs and kHz-QPOs such that :

$$\nu_{LT} = \frac{8\,\pi^2\,I_*}{c^2\,M_*}\,\nu_*\,\nu_{orb}^2$$

QPOs Observations (3)

Correlation between low (≲10 Hz) and high (≈1 kHz) frequency QPOs in WD, NS and BH.
(Psaltis et al. 1999, Mauche 2002, Warner et al. 2003)



$$\nu_{high} \approx 15\,\nu_{low}$$

There must be one same physical mechanism producing these QPOs irrespective of the nature of the compact object.

Idea

The physical mechanism

To show that an accretion disk evolving in :

- either a gravitational potential ;
- or a magnetic field ;

which possesses the two following essential properties :

- an asymmetry with respect to the rotation axis of the disk ;
- a rotating motion compared to the disk ;

will be subject to some instabilities.

The method

This study is done in two steps :

1. hydrodynamical disk evolving in a quadrupolar gravitational perturbation ;
2. magnetized disk evolving in a dipolar magnetic perturbation.

The tools

The main tools at hand are :

1. a linear analysis of the stability ;
2. 2D numerical (M)HD simulations.

The predictions

Power spectrum density of the accretion disk.

HD disk: linear analysis (1)

Hydrodynamical equations of an accretion disk with adiabatic motions :

$$
\begin{aligned}
\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho\, \vec{v}) &= 0 \\
\rho \left[\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \vec{\nabla}\,)\, \vec{v} \right] &= \rho\, \vec{g} - \vec{\nabla}\, p \\
\frac{D}{Dt} \left(\frac{p}{\rho^\gamma} \right) &= 0
\end{aligned}
$$

Perturbing the equilibrium state with respect to the Lagrangian displacement $\vec{\xi}$ and making allowance for a perturbation in the gravitational field, the Lagrangian displacement satisfies a second order linear partial differential equation :

$$
\rho\, \frac{D^2 \vec{\xi}}{Dt^2} - \vec{\nabla}(\gamma\, p\, \vec{\nabla} \cdot \vec{\xi} + \vec{\xi} \cdot \vec{\nabla}\, p) - \vec{\nabla} \cdot (\rho\, \vec{\xi}\, \vec{v} \cdot \vec{\nabla}\, \vec{v}) + \vec{g}\, \vec{\nabla} \cdot (\rho\, \vec{\xi}) + (\vec{\nabla} \cdot (\rho\, \vec{\xi}) - \rho)\, \delta \vec{g} = 0
$$

We introduced the convictive derivative by $D/Dt = \partial_t + \Omega\, \partial_\varphi$.

HD disk: linear analysis (2)

Simplification :

Study of the Lagrangian displacement in each direction independently.

- In the radial direction $\vec{\xi} = (\xi_r, 0, 0)$:

$$\frac{D^2\xi_r}{Dt^2} - \frac{1}{\rho\, r}\,\frac{\partial}{\partial r}\left(\rho\, c_s^2\, r\,\frac{\partial \xi_r}{\partial r}\right) + \kappa_r^2\,\xi_r + \frac{1}{\rho\, r}\,\frac{\partial}{\partial r}\,(r\,\rho\,\xi_r)\;\delta g_r = \delta g_r$$

- In the vertical direction $\vec{\xi} = (0, 0, \xi_z)$:

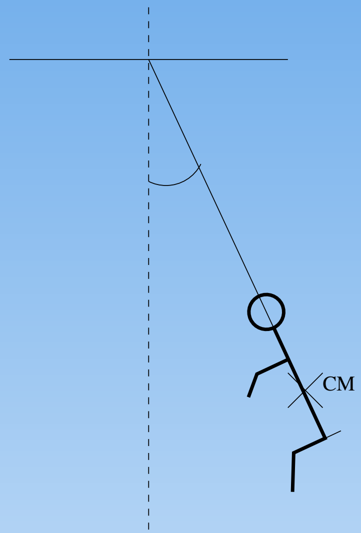
$$\frac{D^2\xi_z}{Dt^2} - \frac{1}{\rho}\,\frac{\partial}{\partial z}\left(\rho\, c_s^2\,\frac{\partial \xi_z}{\partial z}\right) + \kappa_z^2\,\xi_z + \frac{1}{\rho}\,\frac{\partial}{\partial z}\,(\rho\,\xi_z)\;\delta g_z = \delta g_z$$

with c_s the sound speed, κ_r and κ_z the radial and vertical epicyclic frequencies.
Both equations look very similar with the following parts :

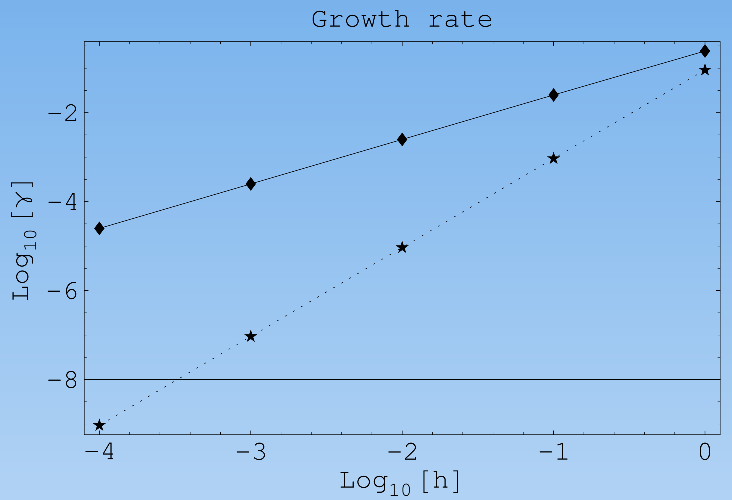
- a sound wave propagation in a tube of varying cross section : no instability ;
- an harmonic oscillator at the epicyclic frequency ;
- a Mathieu equation giving rise to a parametric resonance ;
- a resonance due to a driving force $\delta g_{r/z}$.

Parametric resonance

A simple physical example : the swing



Growth rate for different excitation amplitudes h.



Prototype example, Mathieu equation :

$$\theta''(t) + \omega_0^2 \left[1 + h \, \cos\left(\gamma \, t\right)\right] \theta(t) = 0$$

Resonance conditions : $\gamma = \frac{2\,\omega_0}{n}$ with n integer.

We deduce the relation *growth rate-amplitude* : $\gamma_n \propto h^n$.

HD disk: linear analysis (3)

Three kind of resonances are expected :

- a corotation resonance at the radius where the angular velocity of the disk equals the rotation speed of the star, only possible for prograde motion : $\Omega = \Omega_*$
- an inner and outer Lindblad resonance at the radius where the radial/vertical epicyclic frequency equals the rotation rate of the the gravitational potential perturbation as measured in the frame locally corotating with the disk :

$$2\,|\Omega - \Omega_*| = \kappa_{r/z}$$

- a parametric resonance related to the periodically time-varying radial/vertical epicyclic frequency, (Mathieu equation) :

$$|\Omega - \Omega_*| = \frac{\kappa_{r/z}}{n}$$

with

- n: integer ;
- Ω_*: spin of the star ;
- Ω: rotation rate of the disk ;
- $\kappa_{r/z}$: radial/vertical epicyclic frequency.

Newtonian disk: results

For a thin disk, the rotation is roughly Keplerian and $\kappa_r \approx \kappa_z \approx \Omega_k = \frac{G\,M_*}{r^{3/2}}$.
Resonance conditions are expressed as :

$$\frac{\Omega_k}{\Omega_*} = \frac{n}{n-1} = -, 2, 3/2, 4/3, ...$$
$$\frac{\Omega_k}{\Omega_*} = \frac{n}{n+1} = 1/2, 2/3, 3/4, ...$$

$$\frac{\Omega_*}{2} \le \Omega_k \le 2\,\Omega_*$$

Therefore the QPOs associated with the orbital motion are :

$$\frac{\nu_*}{2} \le \nu_{QPO} \le 2\,\nu_*$$

General-relativistic disk: results

Application to a neutron star with spin ν_*.

$$\Omega(r, a_*) \pm \frac{\kappa_{r/z}(r, a_*)}{n} = \Omega_*$$

For a given angular momentum a_*, we have to solve these equations for the radius r. For a neutron star, we adopt the typical parameters :

- mass $M_* = 1.4\, M_\odot$;
- angular velocity $\nu_* = \Omega_*/2\pi = 300 - 600$ Hz ;
- moment of inertia $I_* = 10^{38}\ kg\, m^2$;
- angular momentum $a_* = \frac{c\, I_*}{G\, M_*^2}\, \Omega_* = 5.79 * 10^{-5}\, \Omega_*$.

rank n	Orbital frequency $\nu(r, a_*)$ (Hz)			
	Vertical		Radial	
	$\nu_* = 600$ Hz	$\nu_* = 300$ Hz	$\nu_* = 600$ Hz	$\nu_* = 300$ Hz
1	—- / 301	— / 150	1196 / 330	800 / 159
2	1175 / 401	597 / 200	870 / 432	480 / 209
3	893 / 451	449 / 225	769 / 478	405 / 234

HD disk: detailed 2D linear analysis

Expansion of the solution in plane wave form

$$\xi_r(r,\varphi,t) = \xi_r(r)\, e^{i(m\varphi - \sigma\, t)}$$

We look for the eigenvalues σ satisfiyng the prescribed boundary conditions, namely the Lagrangian pressure perturbation $\Delta p = 0$ at the inner edge.
The radial Lagrangian displacement is solution of the Schrödinger type equation, with $\psi(r) = \xi_r \sqrt{r\,p}$:

$$\psi''(r) + V(r)\,\psi(r) = F(r)$$

with the potential and the Doppler shifted frequency given by :

$$V(r) = \frac{\omega^2 - \kappa^2}{c_s^2}$$
$$\omega = \sigma - m\,\Omega$$

HD disk: detailed 2D linear analysis

This Schrödinger equation possesses two different kind of solutions :

- a free wave solution travelling in the unperturbed gravitational potential, corresponding to the homogeneous part with $F(r) = 0$;
- a non-wavelike disturbance due to the asymmetric potential represented by the non homogeneous part of the equation.

Analytical approximate solutions for the free wave solutions given by a linear combination of the Airy functions Ai and Bi by :

$$
\begin{aligned}
\omega_1(r) &= -\left[-\frac{3}{2}\int_{r_L}^{r}\sqrt{V(s)}\,ds\right]^{2/3} \text{ for } r \leq r_L \\
\omega_1(r) &= \left[\frac{3}{2}\int_{r_L}^{r}\sqrt{-V(s)}\,ds\right]^{2/3} \text{ for } r \geq r_L \\
\psi_{1/2}(r) &= \frac{Ai/Bi(\omega_1(r))}{\sqrt{|\omega_1'(r)|}} \\
\psi(r) &= C_1\,\psi_1(r) + C_2\,\psi_2(r)
\end{aligned}
$$

HD disk: Free wave solutions

In the WKB approximation

$$\psi(r) = \Phi(r)\, e^{i \int^r k(s)\, ds}$$

The dispersion relation is then given by :

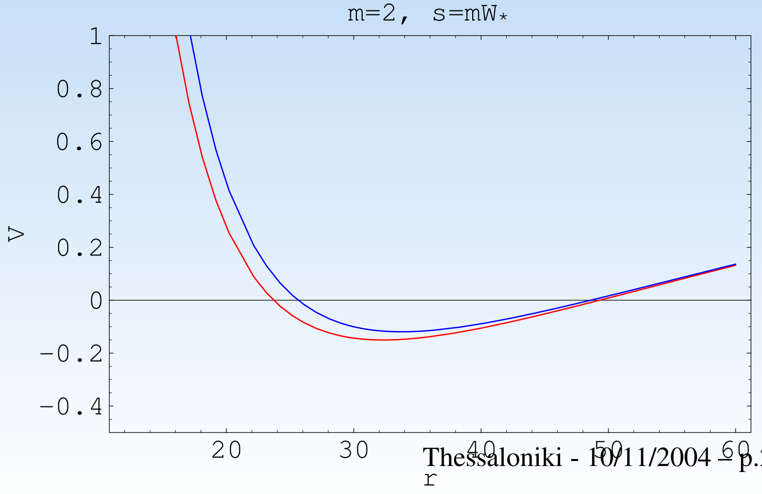
$$\omega^2 = \kappa^2 + c_s^2\, k^2$$

The wave can only propagate in regions where

$$\omega^2 - \kappa^2 \geq 0$$

Example of potential V for $m = 2$ and $\sigma = m\,\Omega_*$

The frontier between propagating and damping zone is defined by the Lindblad radius r_L defined by $V(r_L) = 0$.



HD disk: solutions of the eigenvalue problem

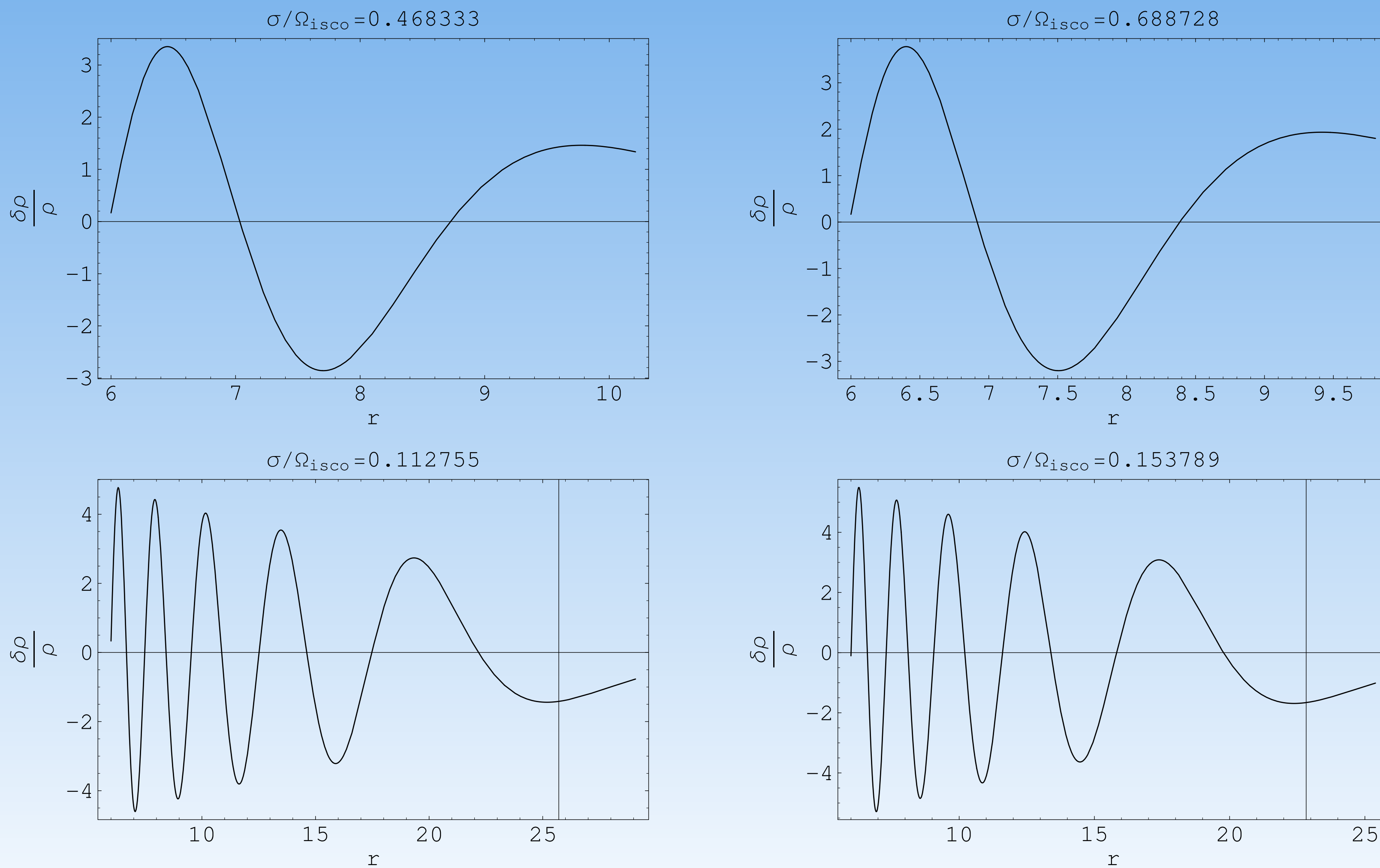
The highest eigenvalues

Eigenvalues σ/Ω_{ISCO}			
Newtonian		Schwarzschild	
$m = 2$	$m = 5$	$m = 2$	$m = 5$
0.838519	3.55997	1.34337	4.01528
0.60303	2.9742	0.916075	3.2938
0.468333	2.6023	0.688728	2.84633
0.373567	2.32075	0.53807	2.51832
0.302154	2.09279	0.42896	2.25799

HD disk: solutions of the eigenvalue problem

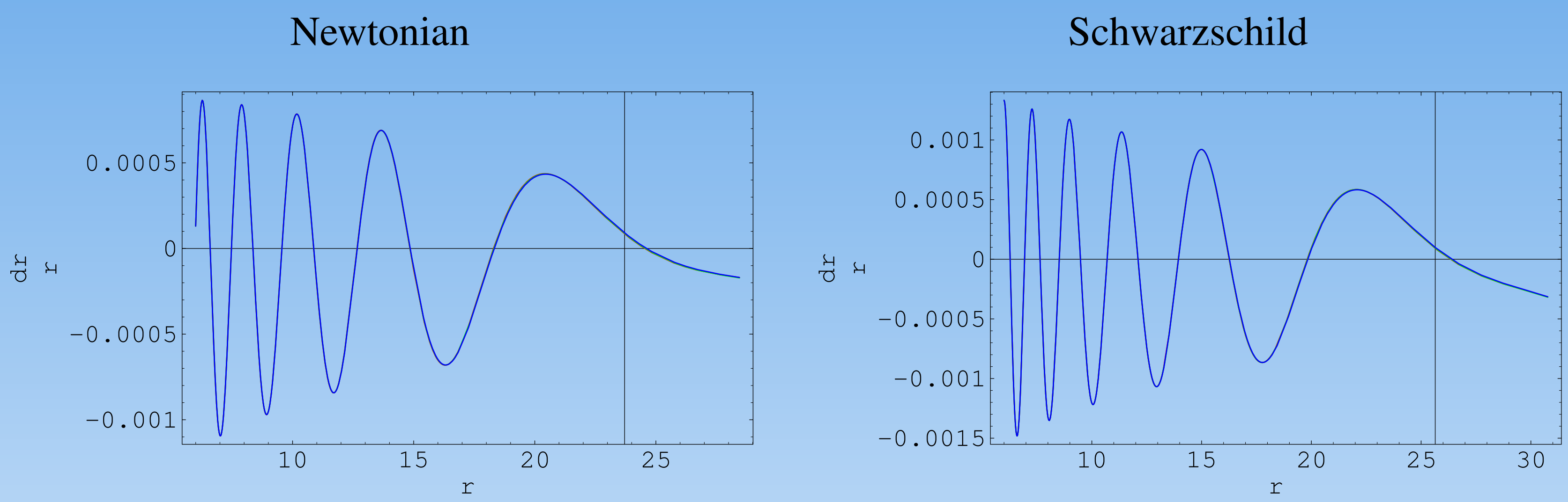
The corresponding eigenfunctions for the density waves

Newtonian   Schwarzschild



HD disk: Non-wavelike disturbance

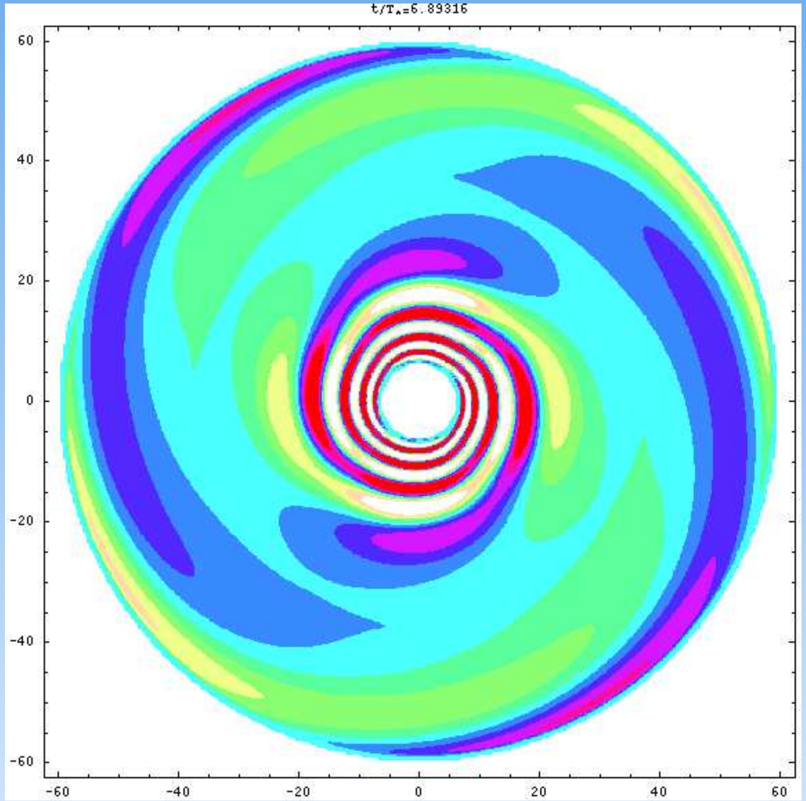
Non-wavelike density perturbation for the mode $m = 2$ and the speed pattern $\sigma = 2\,\Omega_*$.



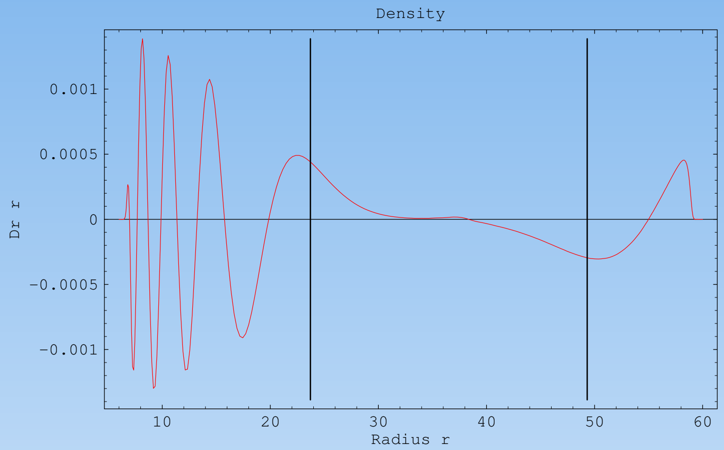
The amplitude is no more arbitrary but fixed by the strength of the gravitational petrurbation.

Newtonian disk: High resolution 2D Simulation

Final snapshot

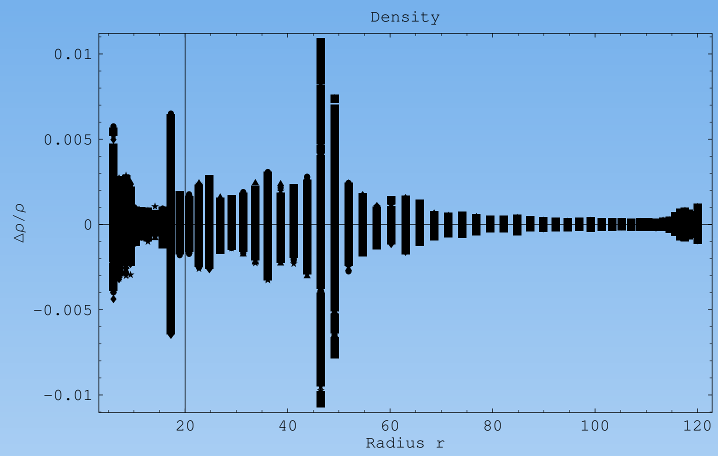


Cross section

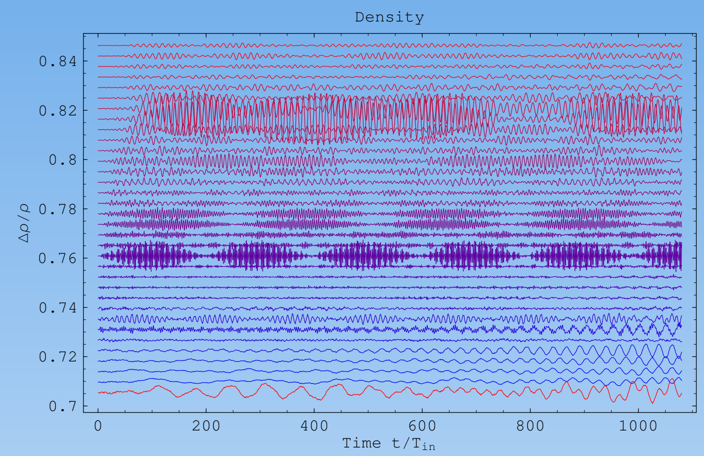


Newtonian disk: 2D Simulation (2)

Cross section



Time evolution

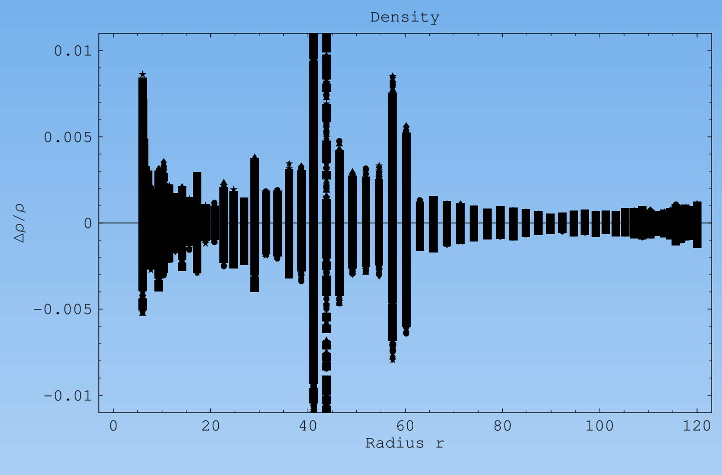


Disk properties

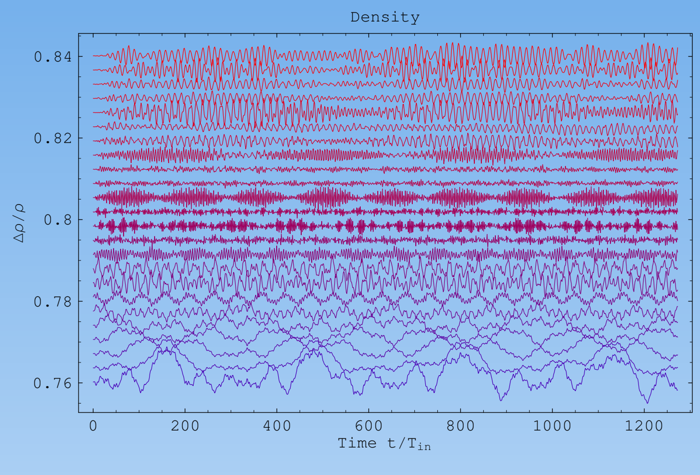
- growing and persistence of the corotation and Lindblab resonances ;
- a periodic variation of the density produced :
  1. the Keplerian rotation around the accreting source ;
  2. a beat phenomenon at much lower frequencies.
- the density evolution corresponds to :
  1. a non-wavelike perturbation due to the external periodic force ;
  2. a free wave propagation between the Lindblad resonances and the disk edges.
- results are qualitatively the same for a pseudo-Schwarzschild or a pseudo-Kerr geometry.

Schwarzschild disk: 2D Simulation (2)

Cross section



Time evolution



Disk properties are the same as before, no qualitative change.

MHD disk: linear analysis (1)

Ideal MHD equations for the accretion disk with adiabatic motions :

$$\begin{aligned}
\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \, \vec{v}) &= 0 \\
\rho \left[\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \vec{\nabla}\,) \, \vec{v} \right] &= \rho \, \vec{g} - \vec{\nabla} \, p + \vec{j} \wedge \vec{B} \\
\frac{D}{Dt} \left(\frac{p}{\rho^\gamma} \right) &= 0 \quad ; \quad \frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \wedge \ (\vec{v} \wedge \vec{B}) \quad ; \quad \vec{\nabla} \wedge \ \vec{B} = \mu_0 \, \vec{j}
\end{aligned}$$

The magnetic field $\vec{B}$ has two distinct components :

1. the stellar magnetic field which is dipolar and rotating ;
2. the magnetic field induced by the flow in the disk.

Introducing again the Lagrangian displacement $\vec{\xi}$, it satisfies a second order linear PDE :

$$\begin{aligned}
\rho \, \frac{D^2 \vec{\xi}}{Dt^2} &= \vec{\nabla} \, \Pi + \frac{1}{\mu_0} \left(\vec{B} \cdot \vec{\nabla} \, \vec{Q} + \vec{Q} \cdot \vec{\nabla} \, \vec{B} \right) + \vec{\nabla} \cdot (\rho \, \vec{\xi} \, \vec{v} \cdot \vec{\nabla} \, \vec{v} - \vec{g} \, \vec{\nabla} \cdot (\rho \, \vec{\xi}) \\
&\quad + \frac{1}{\mu_0} \, \vec{\nabla} \wedge (\vec{B} + \delta \vec{B}) \wedge \delta \vec{B}_*
\end{aligned}$$

with $\vec{Q} = \vec{\nabla} \wedge (\vec{\xi} \wedge \vec{B})$ and $\Pi = \gamma \, p \, \vec{\nabla} \cdot \vec{\xi} + \vec{\xi} \cdot \vec{\nabla} p - \frac{1}{\mu_0} \, \vec{B} \cdot \vec{Q}$.

MHD disk: linear analysis (2)

Simplification :

Study of the Lagrangian displacement in each direction independently.

- In the radial direction $\vec{\xi} = (\xi_r, 0, 0)$:

$$\frac{D^2\xi_r}{Dt^2} - \frac{1}{r\,\rho}\,\frac{\partial}{\partial r}\left[r\,\rho\,(c_s^2 + c_{az}^2)\,\frac{\partial \xi_r}{\partial r}\right] + \kappa_r^2\,\xi_r + \frac{\delta B_*^z}{\mu_0\,\rho}\,\left[\frac{\partial^2}{\partial r^2}\,(\xi_r\,B_z) + \frac{\partial B_z}{\partial r}\right] = 0$$

- In thevertical direction $\vec{\xi} = (0, 0, \xi_z)$:

$$\frac{D^2\xi_z}{Dt^2} - \frac{1}{\rho}\,\frac{\partial}{\partial z}\left[\rho\,(c_s^2 + c_{ar}^2)\,\frac{\partial \xi_z}{\partial z}\right] + \kappa_z^2\,\xi_z - \frac{\delta B_*^r}{\mu_0\,\rho}\,\left[\frac{\partial^2}{\partial z^2}\,(\xi_z\,B_r) - \frac{\partial B_r}{\partial z}\right] = 0$$

$c_{a,z/r}$ Alfven speed, κ_r and κ_z the radial and vertical epicyclic frequencies.

- a sound wave propagation ;
- an harmonic oscillator ;
- a Mathieu's equation ;
- a driving force.

MHD disk: linear analysis (3)

3 kind of resonances

- a corotation resonance : $\Omega = \Omega_*$
- an inner and outer Lindblad resonances :

$$|\Omega - \Omega_*| = \kappa_{r/z}$$

- a parametric instability :

$$|\Omega - \Omega_*| = 2\,\frac{\kappa_{r/z}}{n}$$

with

- n: integer ;
- Ω_*: spin of the star ;
- Ω: rotation rate of the disk ;
- $\kappa_{r/z}$: radial/vertical epicyclic frequency.

Note the difference with the hydrodynamical case corresponding to $m = 2$.
More generally, for a perturbation of azimuthal mode m, the resonance conditions are :

- for the inner and outer Lindblad resonances : $m\,|\Omega - \Omega_*| = \kappa_{r/z}$
- for the parametric instability : $m\,|\Omega - \Omega_*| = 2\,\frac{\kappa_{r/z}}{n}$

Newtonian MHD disk: linear analysis

For a thin disk, the rotation is roughly Keplerian and $\kappa_r \approx \kappa_z \approx \Omega_k = \frac{G\,M_*}{r^{3/2}}$.
Resonance conditions are expressed as :

$$\frac{\Omega_k}{\Omega_*} = \frac{n}{n-2} = -1, -, 3, 2, ...$$
$$\frac{\Omega_k}{\Omega_*} = \frac{n}{n+2} = 1/3, 1/2, 3/5, 2/3, ...$$

$$\frac{\Omega_*}{3} \leq \Omega_k \leq 3\,\Omega_*$$

Therefore

$$\frac{\nu_*}{3} \leq \nu_{QPO} \leq 3\,\nu_*$$

General-relativistic MHD disk: linear analysis

Application to a neutron star with standard parameters.

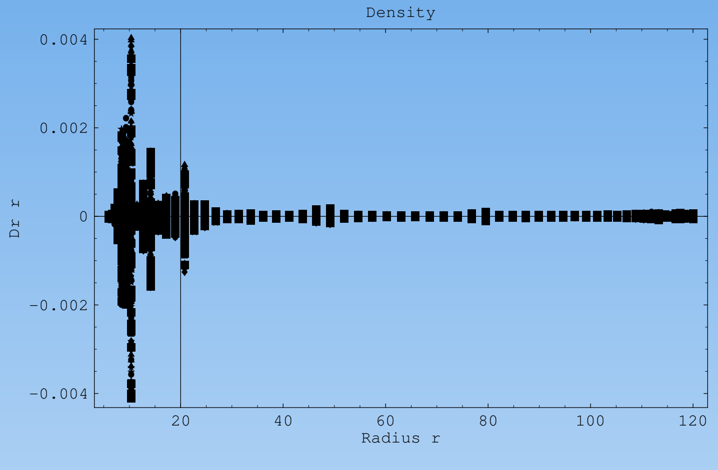
- mass $M_* = 1.4 \, M_\odot$;
- angular velocity $\nu_* = \Omega_*/2\pi = 300 - 600$ Hz ;
- moment of inertia $I_* = 10^{38} \; kg \, m^2$;
- angular momentum $a_* = \frac{c \, I_*}{G \, M_*^2} \, \Omega_*$.

$$\Omega(r, a_*) \pm 2 \, \frac{\kappa_{r/z}(r, a_*)}{n} = \Omega_*$$

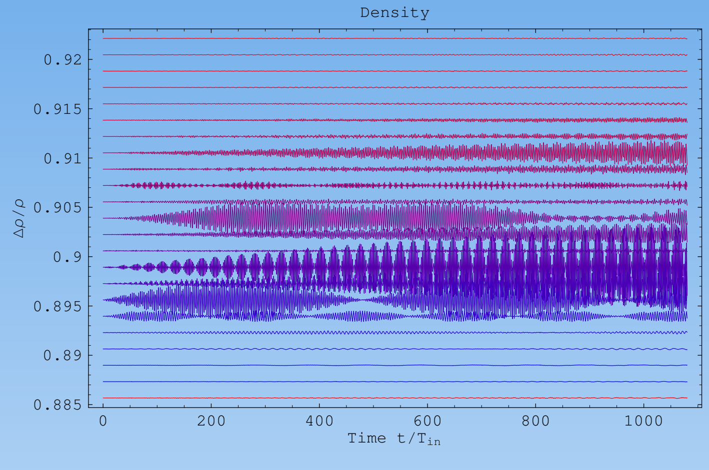
rank n	frequency $\nu_k(r, a_*)$ (Hz)			
	Vertical		Radial	
	$\nu_* = 600$ Hz	$\nu_* = 300$ Hz	$\nu_* = 600$ Hz	$\nu_* = 300$ Hz
1	—- / 200	— / 100	1596 / 220	1332 / 106
2	—- / 301	— / 150	1196 / 330	800 / 159
3	1695 / 361	885 / 180	980 / 392	573 / 190

Newtonian MHD disk: 2D Simulation (2)

Cross section



Time evolution

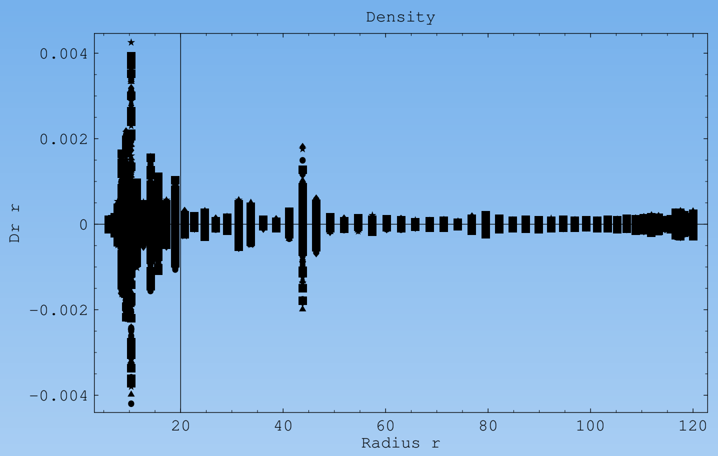


Properties of the disk

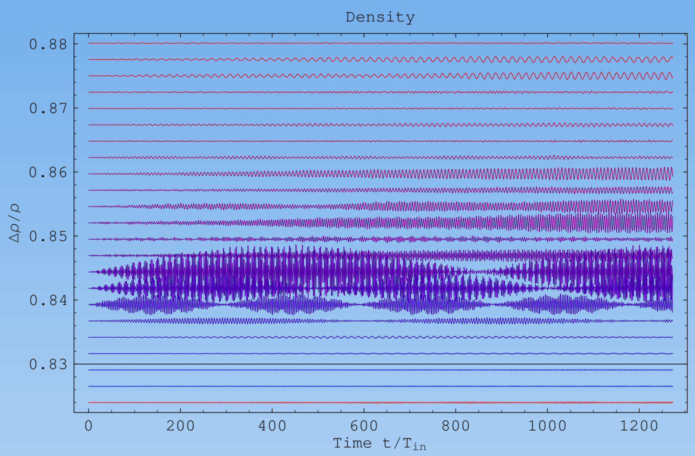
- growing and maintain during time evolution of the corotation and driven resonances ;
- a periodic variation of the density produced :
  1. a Keplerian rotation around the accreting source ;
  2. a beat phenomenon at much lower frequencies.
- the density evolution corresponds to :
  1. a non-wavelike perturbation due to the external periodic force ;
  2. a free wave propagation between the Lindblad resonances and the disk edges.
- results are qualitatively the same for a pseudo-Schwarzschild or a pseudo-Kerr geometry.

Schwarzschild MHD disk: 2D Simulation (2)

Cross section



Time evolution



Properties of the disk :

same as before

What are the observational consequences of these instabilities?

Spectrum of the accretion disk

To estimate the power spectrum density of the accretion disk related to the 2D simulations shown previously ⇒ light curves in a curved spacetime.

Main characteristics :

- the Doppler redshift due to the motions in the disk ;
- the gravitational redshift induced by spacetime curvature ;
- the light ray deflection.

Hypothesis :

- a sample of punctual source in the disk emitting isotropically ;
- take only the primary image (above emitting part of the disk) into account.

Definition of the redshift :

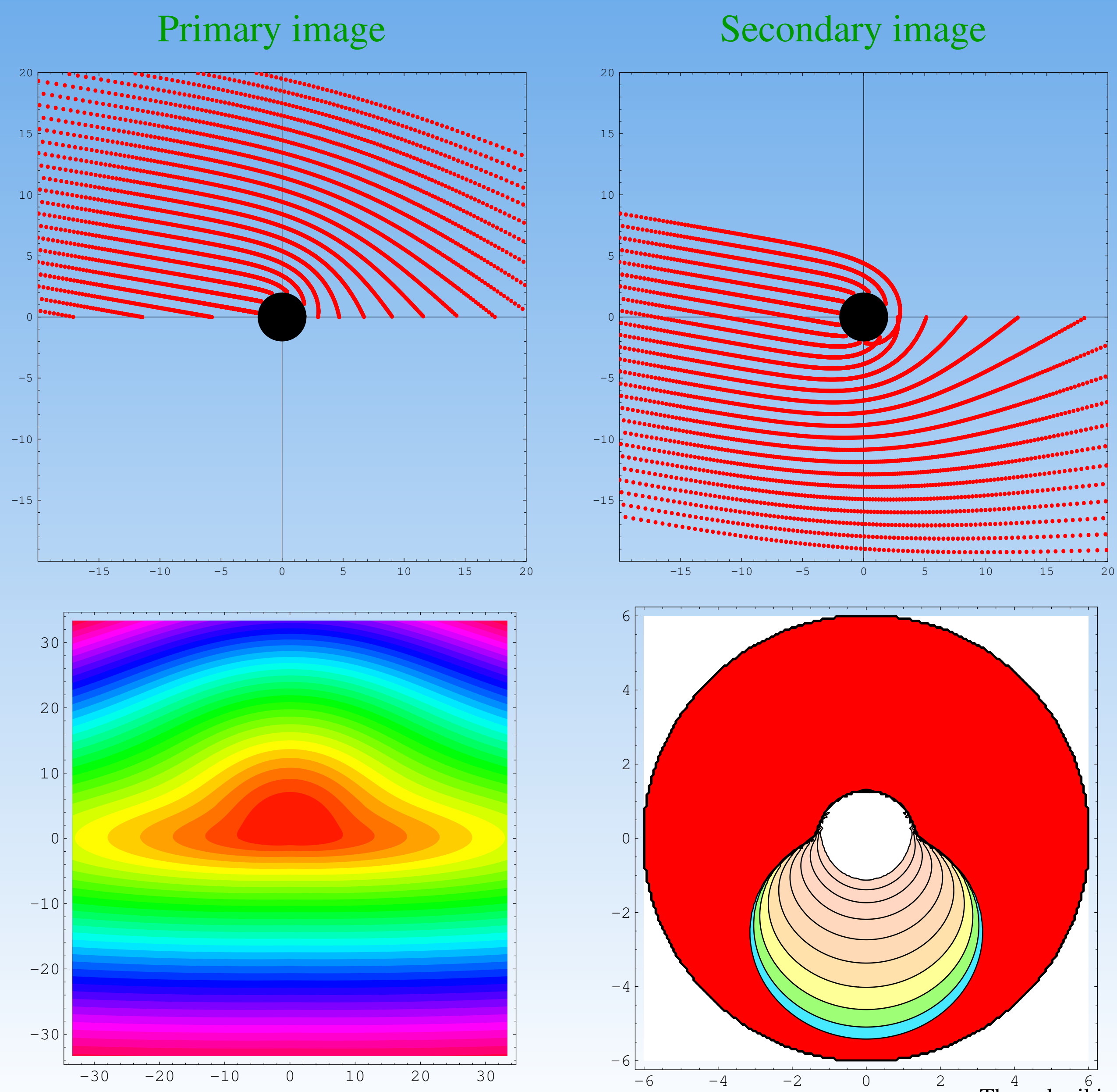
$$g = \frac{E_{obs}}{E_{em}} = \frac{(k^i\, u_i)_{obs}}{(k^i\, u_i)_{em}}$$

- k^i : 4-wave number of the photon ;
- u^i : 4-velocity of the particle in the disk (em) and observer (obs) frame.

Then the intensity measured by a distant observer is :

$$I_{obs} = g^4\, I_{em}$$

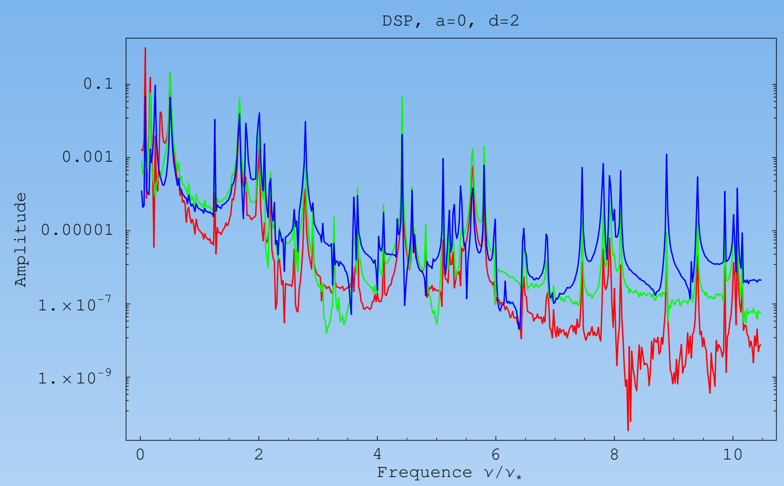
Primary image vs secondary image



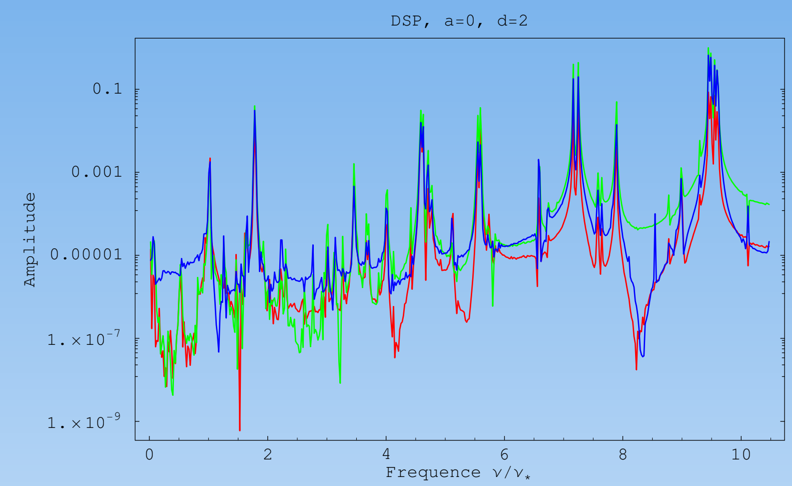
PSD: Results

Power spectrum density for different line of sight inclination, $i = 10^o, 45^o, 80^o$

HD disk



MHD disk



- intensity depends on the dissipation law in the disk, put in another way on the efficiency of converting kinetic energy into radiation ;
- frequencies around $0.5 - 2\,\nu_*$ are dominant for HD disk ;
- high frequencies are dominant, close to the frequency of the ISCO for the MHD disk ;
- shape of the PSD depends only slightly on inclination of the line of sight ;

Conclusions

- the resonances appear in accretion disks due to a rotating non axisymmetric gravitational or magnetic field ;
- these instabilities possess a small radial extension ;
- the physical origin of these instabilities is the same in the HD and MHD case ;
- the high quality factor $Q \geq 20$ explained by :
  1. instabilities localized in narrow radial extension ;
  2. a long enough life and coherence time of the inhomogeneities ;
- the line of sight inclination has only a small influence on the Fourier spectrum ;

Perspectives

- include a viscous term and therefore a stationary inwards flux of matter ;
- replace polytropic equation of state by conservation of energy and radiative transfer mechanism ;
- take into account the finite size of the emitting regions ;
- study the fully general-relativistic (HD or MHD?) case :
  1. linear analysis of the resonance conditions ;
  2. 2D simulations ;
- effect of the warping and precession of the disk orbital plane $\Rightarrow$ generalization to 3D simulations.